

International workshop on baryon and lepton number violation (BLNV 2024)

Recent theory development in baryon number violating processes in the context of GUTs

Michal Malinský
IPNP Charles University in Prague

KIT Karlsruhe, October 8 2024

Perturbative BNV from the SM perspective

see the talk by J. Gargalionis

d=6 : 2-body B & L violating nucleon decay (B-L conserving)

$$p \rightarrow \pi^0 e^+, \dots$$

d=7 : nucleon decay “exotics”, high-scale baryogenesis (B-L violating)

$$n \rightarrow e^- K^+, \quad n \rightarrow e^- \pi^+, \quad p \rightarrow \pi^+ \nu$$

d=8 : multibody BLNV nucleon decay (B-L conserving)

$$p \rightarrow \pi^+ \pi^- e^+, \dots$$

d=9 : (Majorana) neutron-antineutron oscillations,...

$$n \leftrightarrow \bar{n}, \dots$$

...

see next talk by D. Milstead

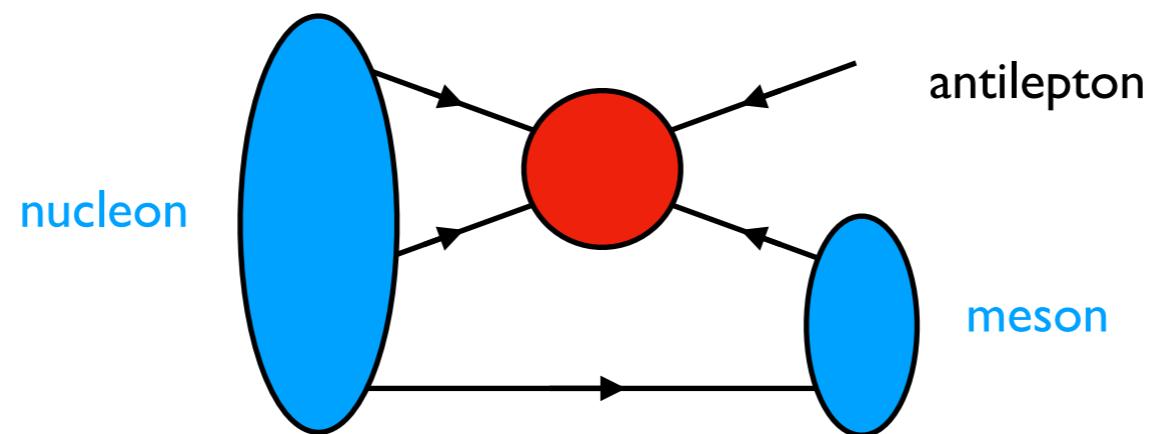
d=6 BNV anatomy

$$p \rightarrow \pi^0 e^+$$

$$p \rightarrow K^+ \bar{\nu}$$

$$n \rightarrow \pi^- e^+$$

...



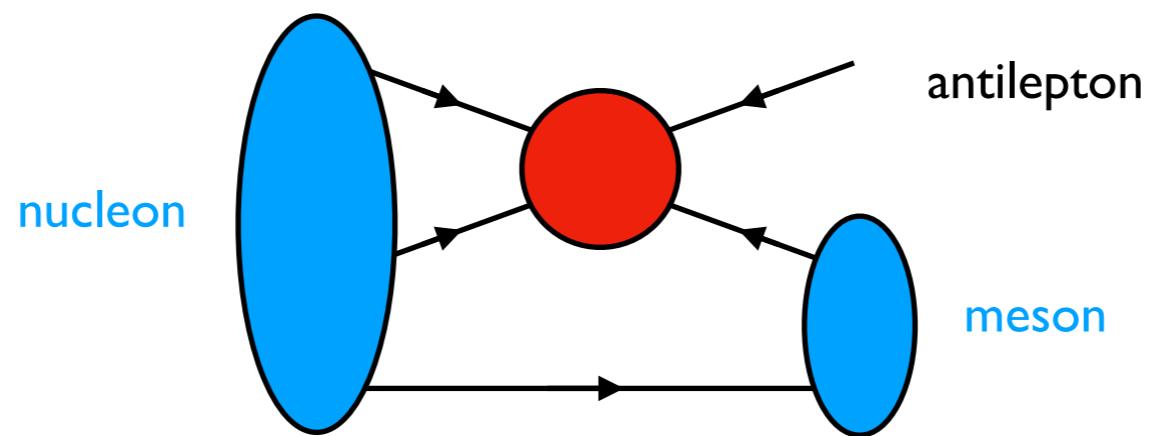
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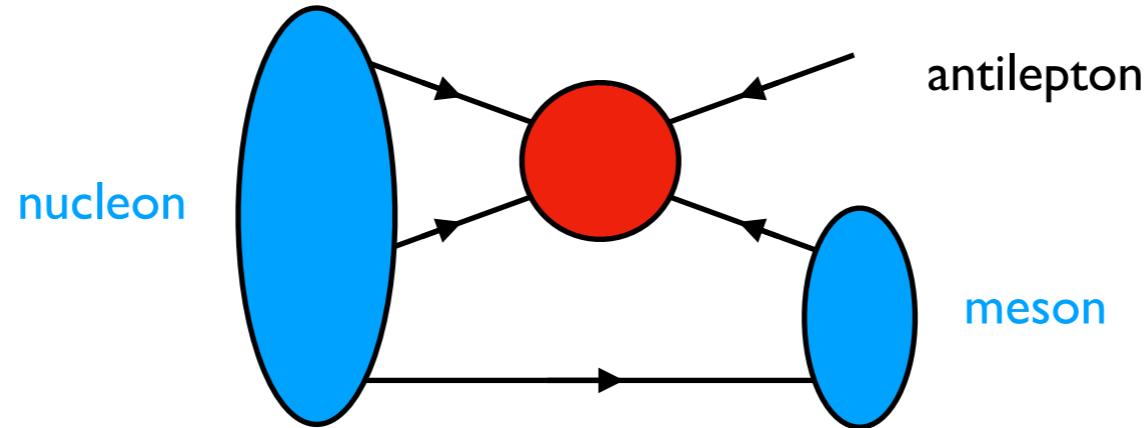
$$\frac{f_1}{M^2} \bar{Q} u^c \bar{Q} e^c, \quad \frac{f_2}{M^2} u^c \bar{Q} d^c \bar{L}$$

$$\frac{f_3}{M^2} Q Q Q L, \quad \frac{f_4}{M^2} u^c u^c d^c e^c$$

for details see e.g. L. Abbott and M. B. Wise, Phys. Rev. D22 (1980) 2208
or B. Grzadkowski et al., JHEP 10 (2010) 085, arXiv: 1008.4884

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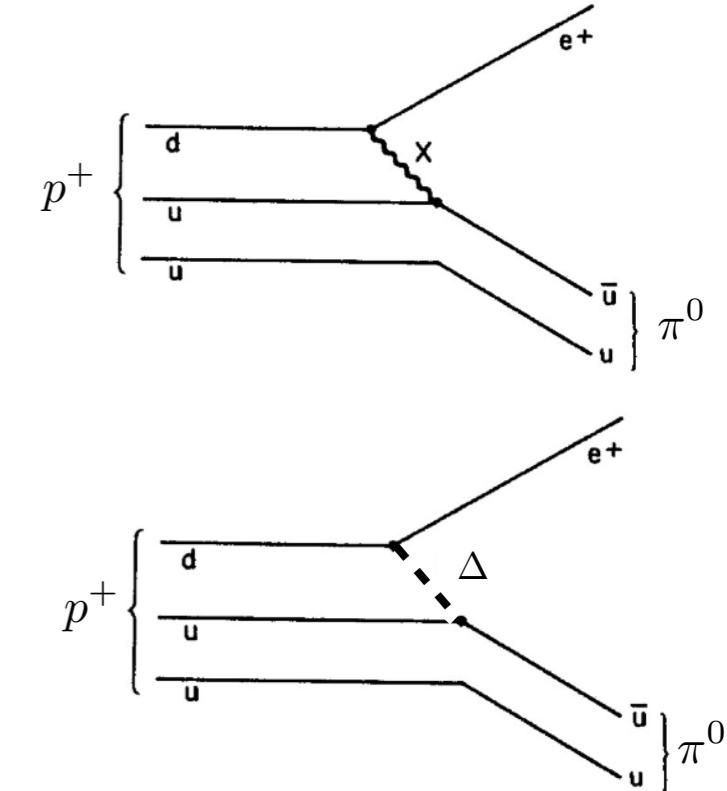


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mediator (vector or scalar) mass

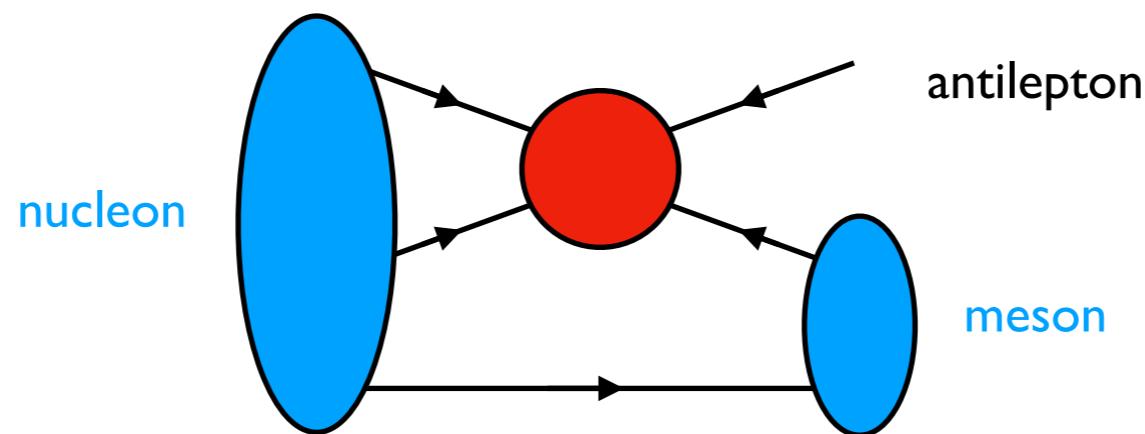
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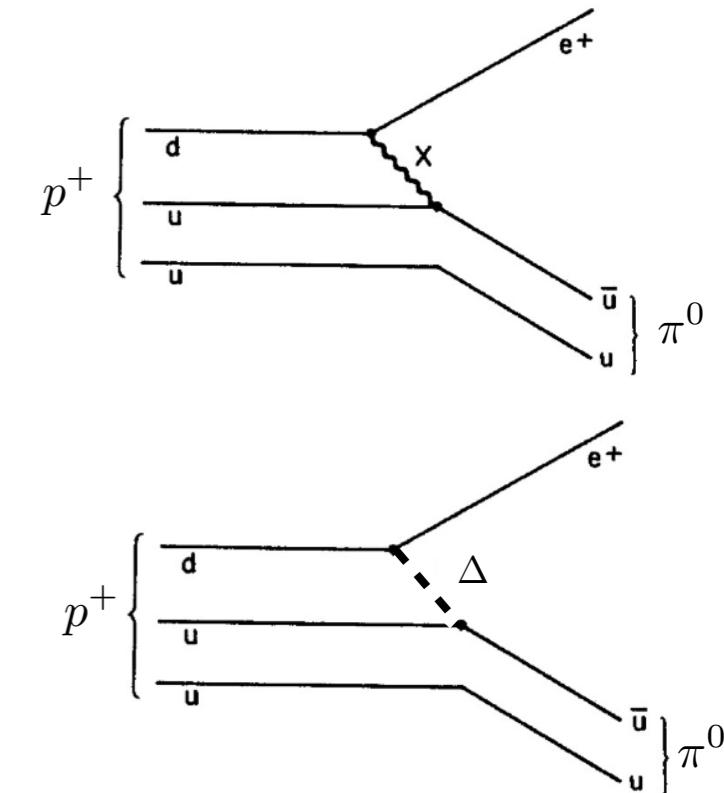
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gauge + flavour structure

$$\frac{f_3}{M^2} Q Q Q L, \quad \frac{f_4}{M^2} u^c u^c d^c e^c$$

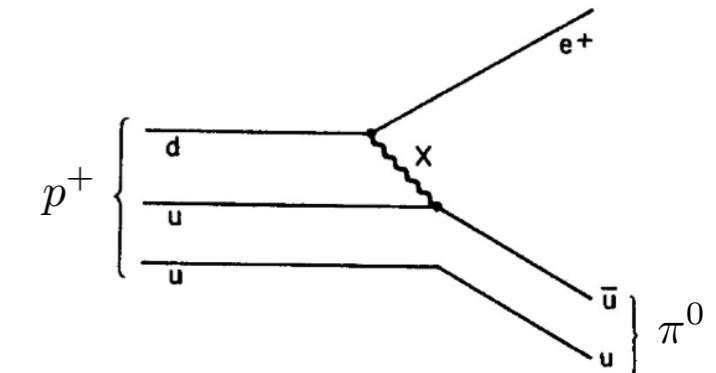
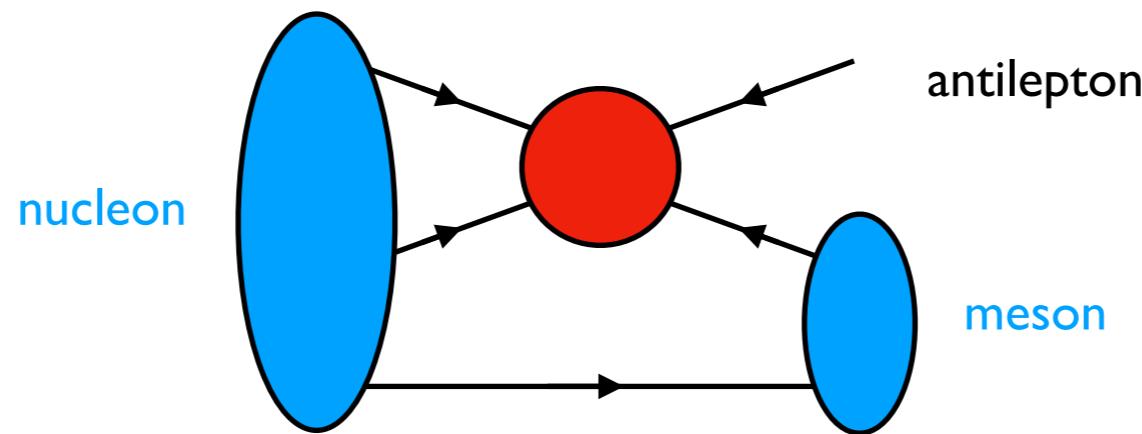
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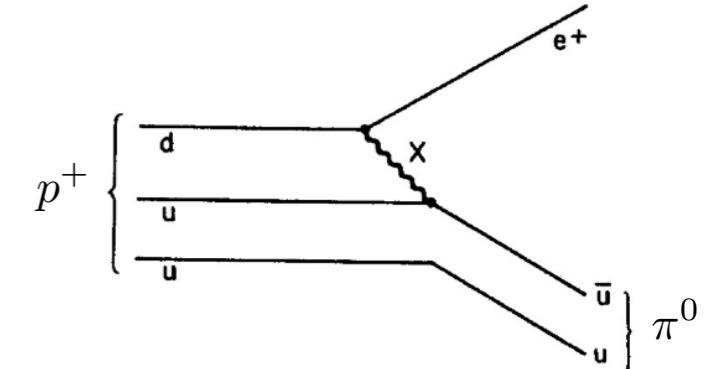
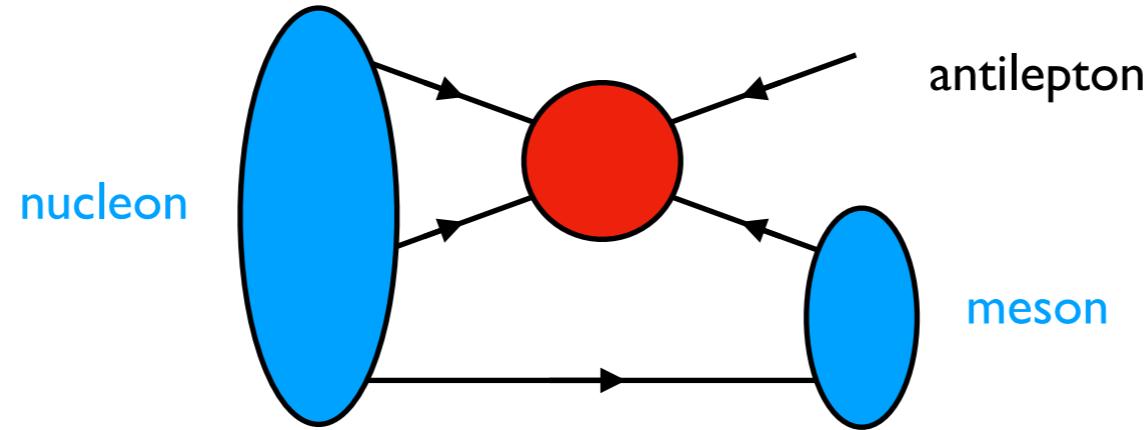
Decay rates (single vector mediator example):

$$\Gamma(p \rightarrow \pi^0 e_\beta^+) = \frac{g^4}{4M_X^4} \frac{m_p}{16\pi f_\pi^2} A_L^2 |\alpha|^2 (1 + D + F)^2 \{ |c(e_\beta, d^C)|^2 + |c(e_\beta^C, d)|^2 \}$$

$$\Gamma(p \rightarrow K^+ \bar{\nu}) = \frac{g^4}{4M_X^4} \frac{(m_p^2 - m_K^2)^2}{8\pi m_p^3 f_\pi^2} A_L^2 |\alpha|^2 \sum_{i=1}^3 \left| \frac{2m_p}{3m_B} D c(\nu_i, d, s^C) + [1 + \frac{m_p}{3m_B} (D + 3F)] c(\nu_i, s, d^C) \right|^2$$

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Flavour factors: $c(e_\alpha^C, d_\beta) = (U_c^\dagger U)_{11} (E_c^\dagger D)_{\alpha\beta} + (U_c^\dagger D)_{1\beta} (E_c^\dagger U)_{\alpha 1}$ $F_c^T Y_F F = Y_F^{\text{diag.}}$

$$c(e_\alpha, d_\beta^C) = (U_c^\dagger U)_{11} (D_c^\dagger E)_{\beta\alpha}$$

$$c(\nu_l, d_\alpha, d_\beta^C) = (U_c^\dagger D)_{1\alpha} (D_c^\dagger N)_{\beta l}$$

d=6 BNV calculability options

Non-GUTs

p-decay in models of leptons:

J. Heeck, J. Heisig and A. Thapa, Phys. Rev. D 108 (2023) 035014 - Dirac leptogenesis with leptoquarks
T. Nomura, O. Popov, arXiv:2406.0065 - loop suppressed p-decay in scotogenic model of neutrinos

kinematically allowed low-scale BNV:

M. Beneke, G. Finauri and A. A. Petrov, JHEP09 (2024) 090 - low-scale leptoquark coupled to 3rd gen.

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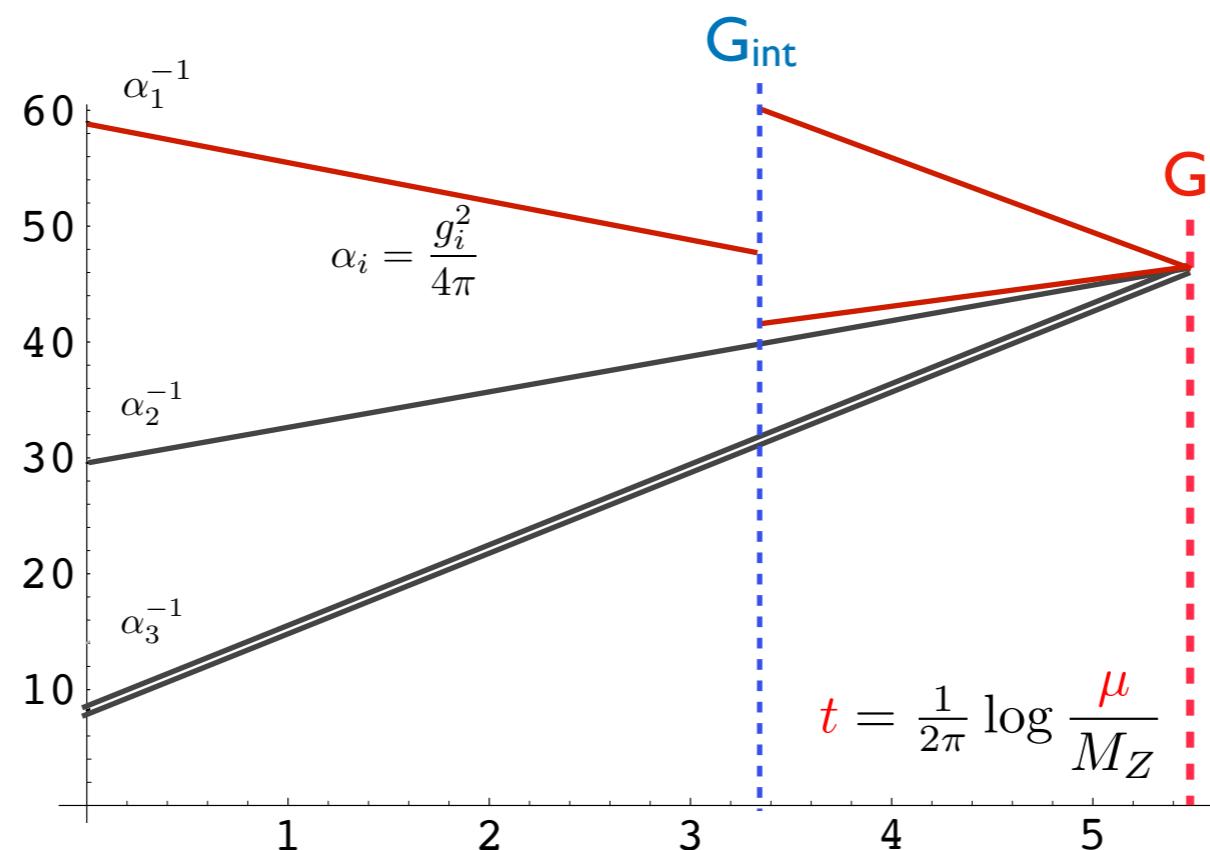
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GUTs

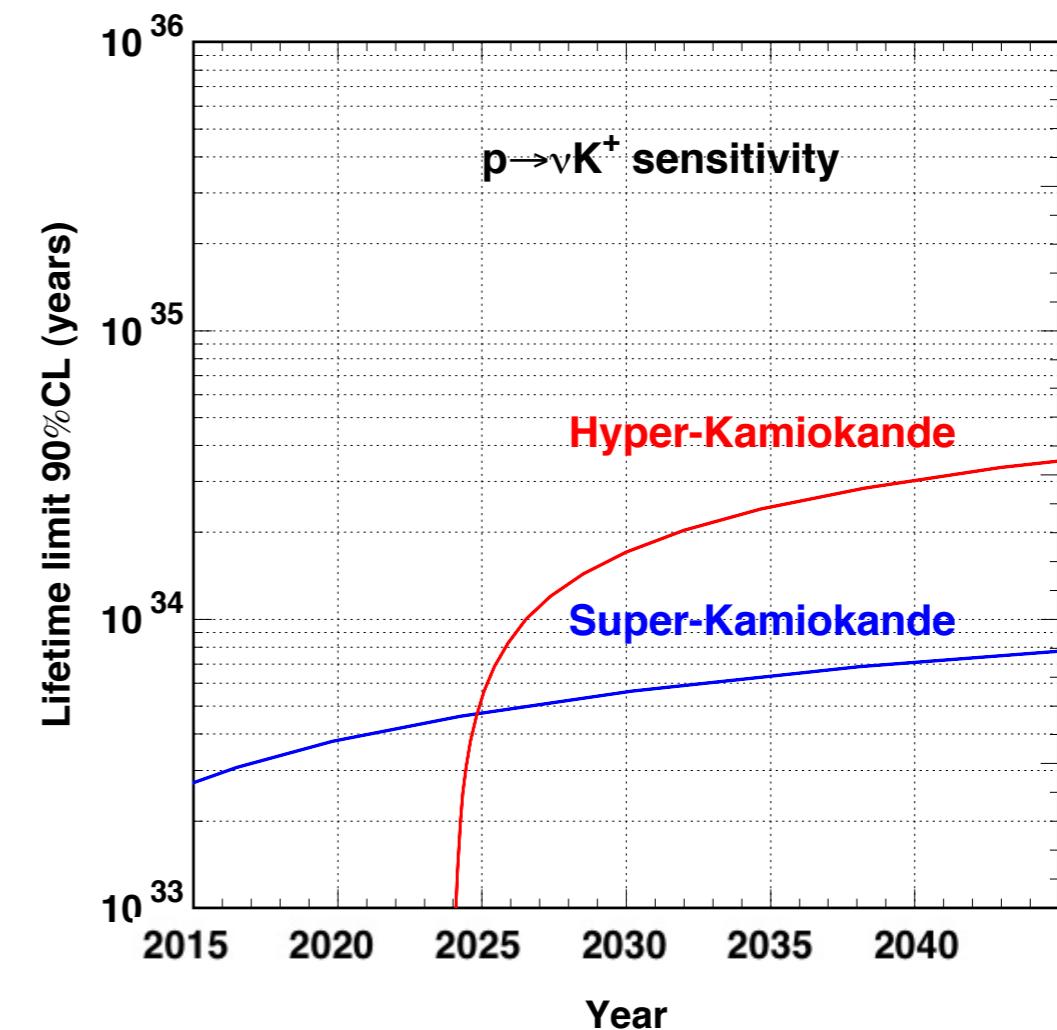
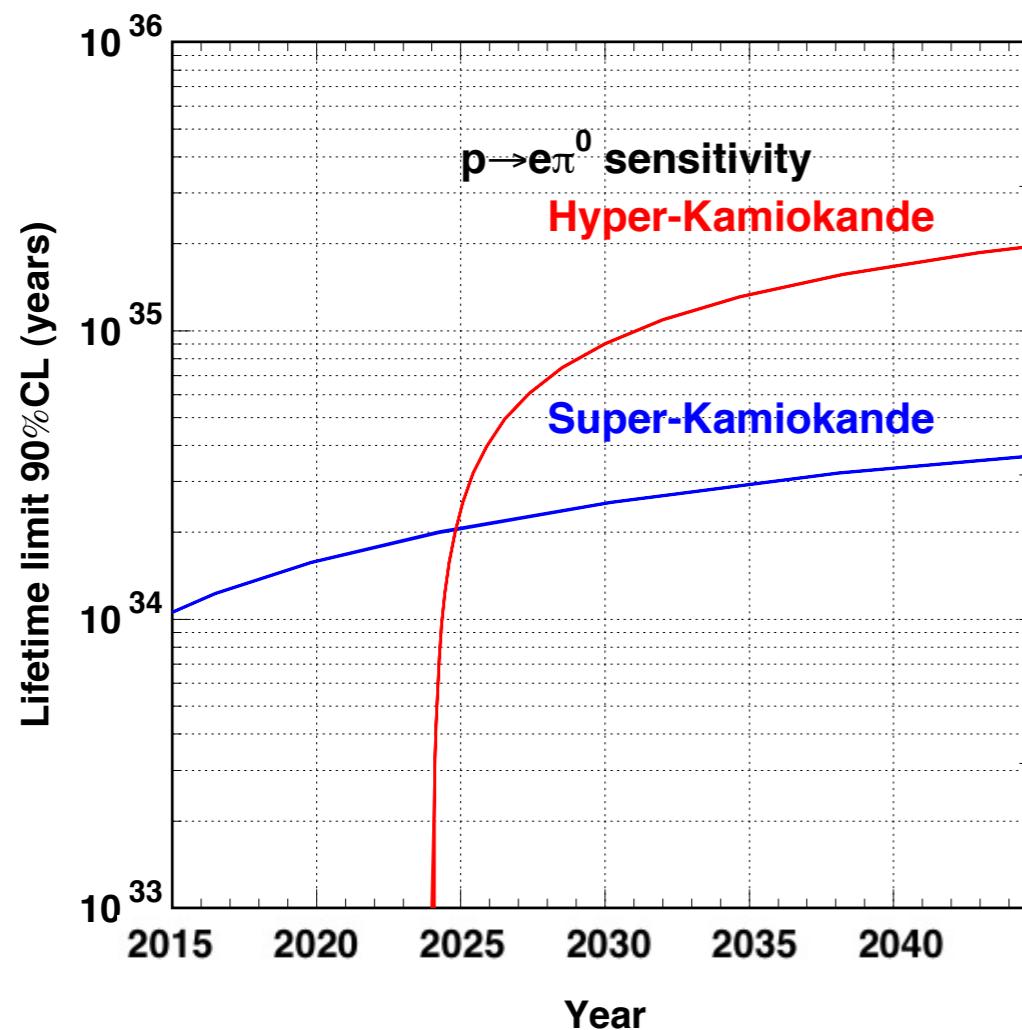
SU(5), SO(10),...



gauge and Yukawa
unification
spectrum and
flavour structure
constrained

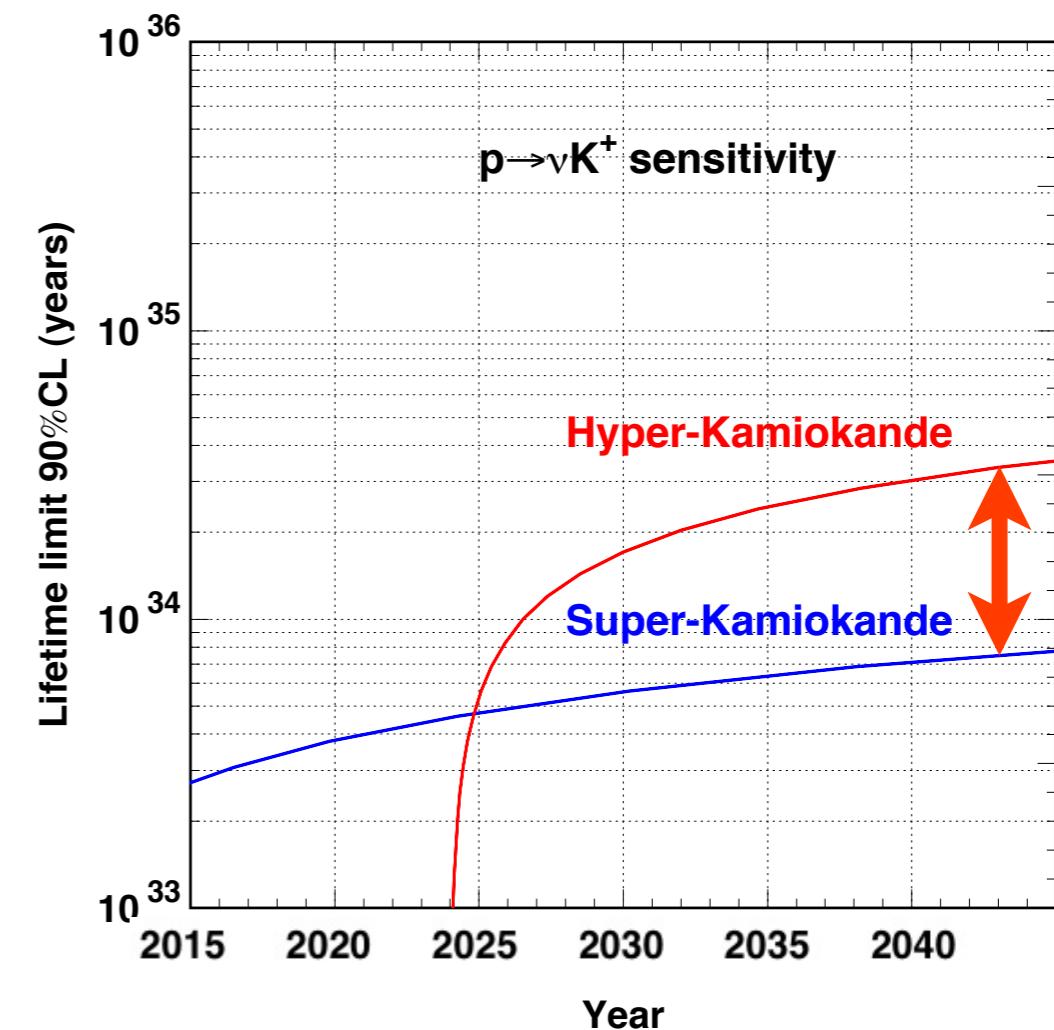
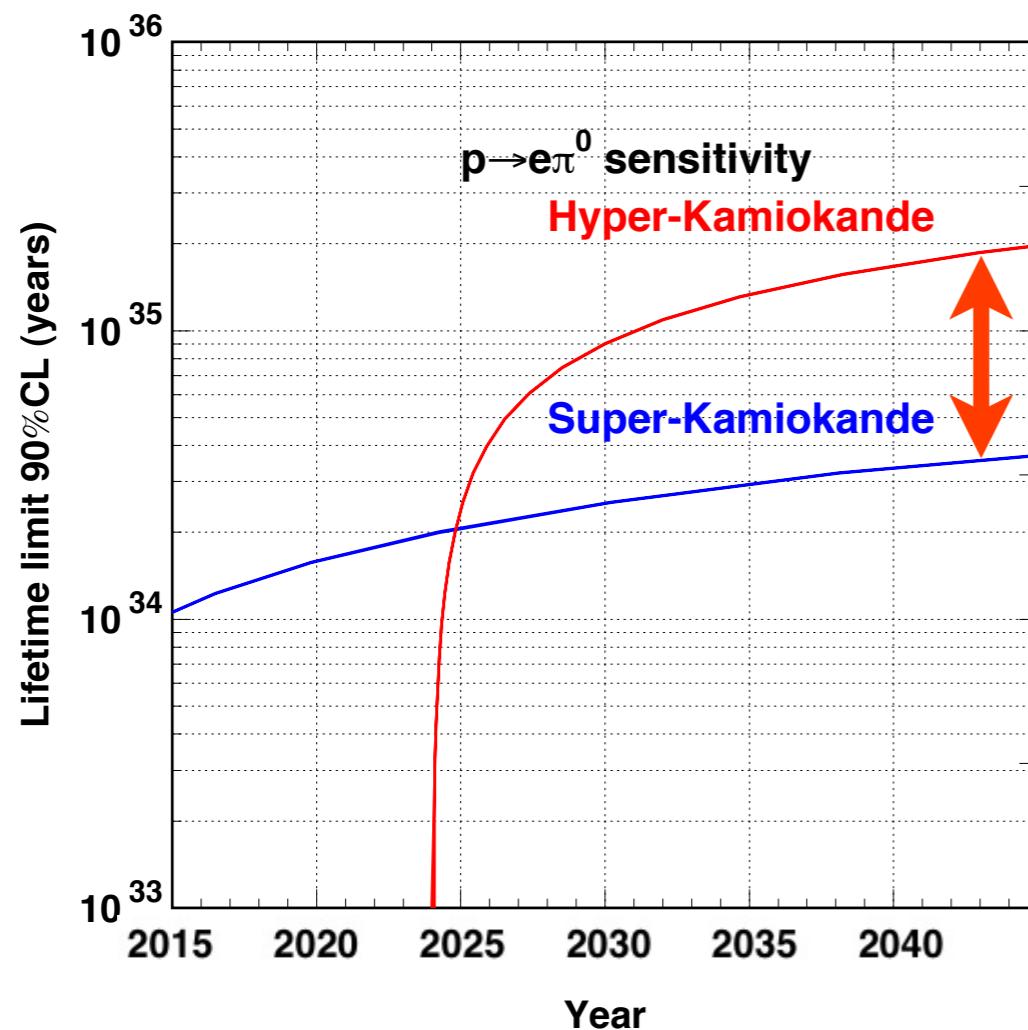
d=6 BNV calculability scope

Hyper-K, Abe et al., arXiv:1109.3262 [hep-ex]



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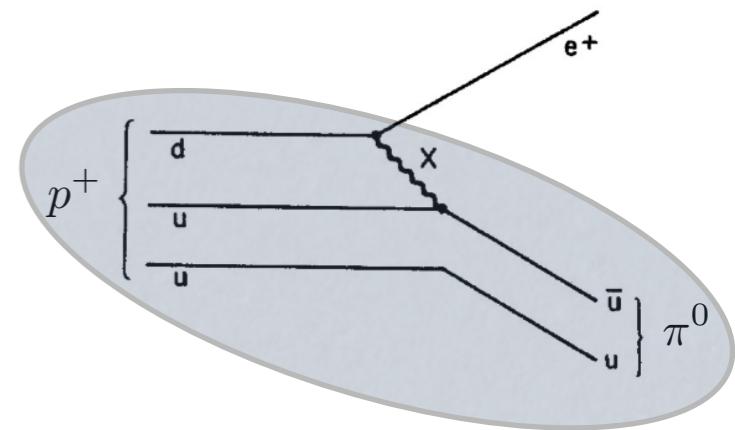
Hyper-K, Abe et al., arXiv:1109.3262 [hep-ex]



d=6 BNV calculability prerequisites

(model independent)

Hadronic / nuclear matrix elements



Lattice:

N.Tsutsui et al.Phys. Rev. D70 (2004) 111501

Y.Aoki, P. Boyle, P. Cooney et al., Phys. Rev. D78 (2008) 054505

Y.Aoki, T. Izubuchi, E. Shintani, and A. Soni., Phys. Rev. D96 (2017) 014506

J.-S.Yoo, Y.Aoki, P. Boyle, T. Izubuchi, A. Soni, and S. Syritsyn, Phys. Rev. D, 105(2022) 074501

see also

Z. Davoudi, W. Detmold, K. Orginos et al., Phys.Rept. 900 (2021) 1 - recent review

NB U. Haisch and A. Hala, arXiv:JHEP 05 (2021) 258 - light-cone sum-rules in QCD

d=6 BNV calculability prerequisites (model independent)

from Y.Aoki, E. Shintani, and A. Soni. Phys. Rev. D, 89(2014) 014505

Matrix element $W_0(\mu = 2\text{GeV}) \text{ GeV}^2$	(%)	Total error
$\langle \pi^0 (ud)_R u_L p \rangle$ -0.103 (23) (34)	40	
$\langle \pi^0 (ud)_L u_L p \rangle$ 0.133 (29) (28)	30	
$\langle \pi^+ (ud)_R d_L p \rangle$ -0.146 (33) (48)	40	
$\langle \pi^+ (ud)_L d_L p \rangle$ 0.188 (41) (40)	30	
$\langle K^0 (us)_R u_L p \rangle$ 0.098 (15) (12)	20	
$\langle K^0 (us)_L u_L p \rangle$ 0.042 (13) (8)	36	
$\langle K^+ (us)_R d_L p \rangle$ -0.054 (11) (9)	26	
$\langle K^+ (us)_L d_L p \rangle$ 0.036 (12) (7)	39	
$\langle K^+ (ud)_R s_L p \rangle$ -0.093 (24) (18)	32	
$\langle K^+ (ud)_L s_L p \rangle$ 0.111 (22) (16)	25	
$\langle K^+ (ds)_R u_L p \rangle$ -0.044 (12) (5)	30	

Parametrization: $\bar{v}_{\ell\alpha}^C(\vec{q}) \langle \Pi(\vec{p}) | \mathcal{O}_\alpha^{\chi\chi'}(q) | N(\vec{k}) \rangle \approx [\bar{v}_\ell^C(\vec{q}) P_{\chi'} u_N(\vec{k})] W_0(-m_\ell^2) + O(m_\ell/m_N)$

d=6 BNV calculability prerequisites (model independent)

from J.-S. Yoo, Y. Aoki, P. Boyle, T. Izubuchi, A. Soni, and S. Syritsyn, Phys. Rev. D, 105(2022) 074501

	$W_0[\text{GeV}^2]$		
	24ID	32ID	Cont.
$\langle \pi^+ (ud)_L d_L p \rangle$	0.1032(86)(26) 0.1050(87)(36)	0.1252(48)(50) 0.1271(49)(50)	0.151(14)(8)(26) 0.153(14)(7)(26)
$\langle \pi^+ (ud)_L d_R p \rangle$	-0.1125(78)(41) -0.1139(78)(45)	-0.134(5)(11) -0.136(5)(12)	-0.159(15)(20)(25) -0.161(15)(20)(26)
$\langle K^0 (us)_L u_L p \rangle$	0.0395(22)(36) 0.0397(22)(36)	0.0411(13)(25) 0.0411(13)(25)	0.0430(38)(12)(19) 0.0427(37)(12)(16)
$\langle K^0 (us)_L u_R p \rangle$	0.0688(37)(19) 0.0693(36)(20)	0.0764(17)(36) 0.0769(17)(36)	0.0854(57)(55)(90) 0.0860(56)(55)(91)
$\langle K^+ (us)_L d_L p \rangle$	0.0263(19)(6) 0.0266(19)(6)	0.0273(9)(11) 0.0278(9)(11)	0.0284(30)(17)(12) 0.0293(30)(18)(15)
$\langle K^+ (us)_L d_R p \rangle$	-0.0301(21)(10) -0.0307(21)(10)	-0.0345(9)(14) -0.0351(8)(15)	-0.0398(31)(20)(52) -0.0403(31)(20)(52)
$\langle K^+ (ud)_L s_L p \rangle$	0.0923(48)(35) 0.0932(47)(37)	0.0961(26)(46) 0.0972(26)(48)	0.1006(80)(60)(46) 0.1019(79)(60)(47)
$\langle K^+ (ud)_L s_R p \rangle$	-0.0835(58)(3) -0.0846(58)(6)	-0.0954(32)(39) -0.0964(32)(40)	-0.109(10)(8)(14) -0.110(10)(8)(14)
$\langle K^+ (ds)_L u_L p \rangle$	-0.0651(33)(26) -0.0658(32)(28)	-0.0681(18)(33) -0.0686(18)(34)	-0.0717(54)(41)(35) -0.0720(53)(40)(34)
$\langle K^+ (ds)_L u_R p \rangle$	-0.0394(22)(20) -0.0393(21)(21)	-0.0417(11)(23) -0.0416(11)(23)	-0.0443(35)(26)(27) -0.0444(35)(26)(27)

Uncertainties shrunk enormously in the last decade!

d=6 BNV calculability prerequisites

(model dependent)

Optimistic case: good grip on both mediator masses and couplings

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Typical case: just part of the necessary information available

- overly complicated scalar sector - spectrum (& thresholds) out of control
- overly complicated Yukawa sector - flavour structure underconstrained
- proximity of the Planck scale
- ...

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Not necessarily sterile!

Example 1: SU(5) with a symmetric u-quark mass matrix

$$\Gamma(p \rightarrow \pi^+ \bar{\nu}) \propto k_1^4 |(V_{CKM})_{11}|^2$$

$$\Gamma(p \rightarrow K^+ \bar{\nu}) \propto k_1^4 \left(B_1^2 |(V_{CKM})_{12}|^2 + B_2^2 |(V_{CKM})_{11}|^2 \right)$$

$$B_1 = \frac{2m_p}{3m_B} D$$

$$B_2 = \frac{m_p}{3m_B} (D + 3F) + 1$$

$$k_1 \equiv \frac{g}{M_X}$$

$$k_2 \equiv \frac{g}{M'_X}$$

Example 2: SO(10) with both quark mass matrices symmetric

$$\Gamma(p \rightarrow \pi^+ \bar{\nu}) \propto k_1^4 |(V_{CKM})_{11}|^2 + k_2^4 + 2k_1^2 k_2^2 |(V_{CKM})_{11}|^2$$

$$\Gamma(p \rightarrow K^+ \bar{\nu}) \propto k_1^4 \left(B_1^2 |(V_{CKM})_{12}|^2 + B_2^2 |(V_{CKM})_{11}|^2 \right)$$

Nath, Fileviez-Perez, Phys.Rept.441 (2007)
Dorsner, Fileviez-Perez, PLB605 (2005)

The game of scales

Absolute mass scale of BLNV mediators

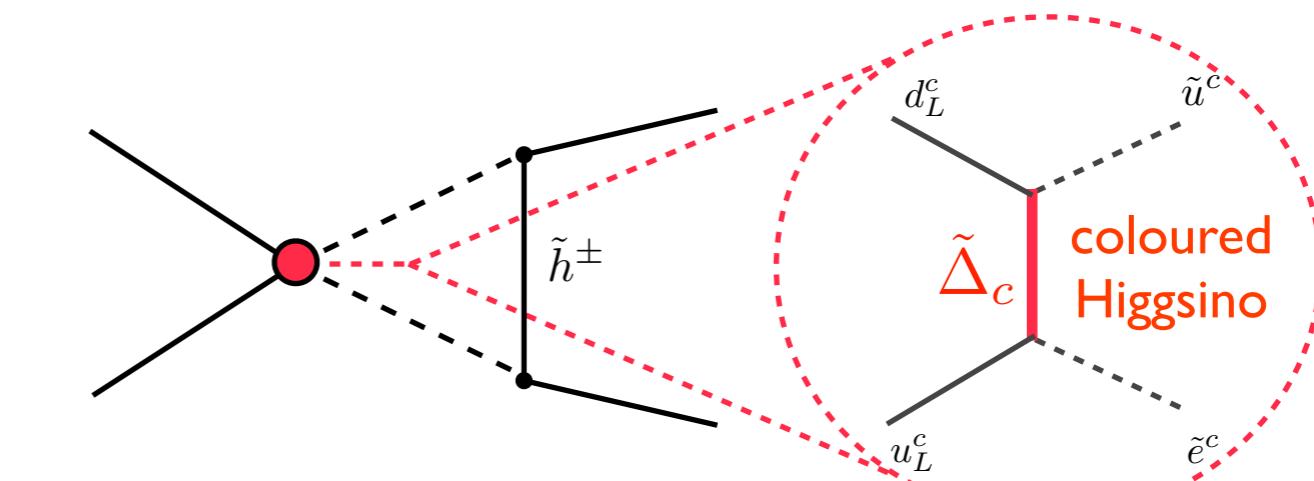
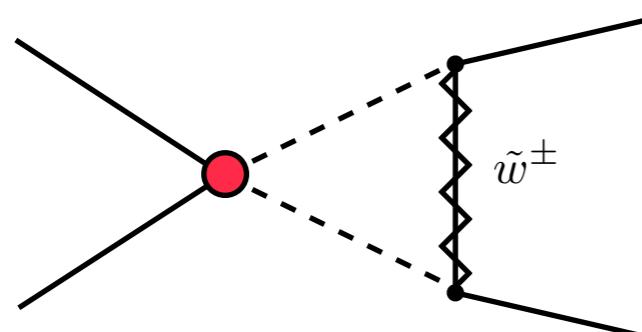
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SUSY: unknown soft spectrum shape

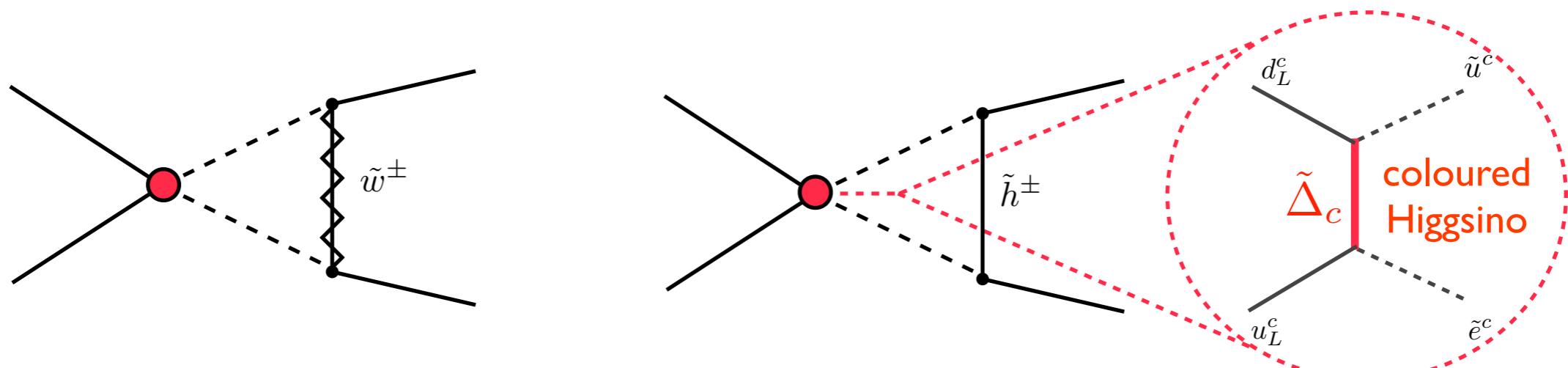


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recent SU(5) gauge mediation p-decay papers:

J. Evans, Y. Shigekami, arXiv:2409.06239 - p-decay as primary motivation!

S.-S. Kim, H. M. Lee, S.-B. Sim, Phys.Rev.D 109 (2024) 075035 - low-scale phenomenology focused

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Planck-scale-induced unification-smearing effects

$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle \Phi \rangle F_{\mu\nu}$$

Larsen, Wilczek, NPB 458 (1996) 249
G. Veneziano, JHEP 06 (2002) 051
Calmet, Hsu, Reeb, PRD 77 (2008) 125015
G. Dvali, Fortsch. Phys. 58 (2010) 528

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- uncontrolled **inhomogeneous** shifts in the gauge matching

$$\Delta \alpha_i^{-1} \sim 1$$

$$\Delta \log_{10}(M_X/M_Z) \sim 1$$

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SU(5): $(24 \otimes 24)_{sym} = 24 \oplus 75 \oplus 200$

difficult to avoid!!!

SO(10): $(45 \otimes 45)_{sym} = 54 \oplus 210 \oplus 770$

The game of scales

Absolute mass scale of BLNV mediators

Recent spectral / SSB studies within “safe” settings:

SO(10) broken by 45:

K. Jarkovská, MM, T. Mede, V. Susič, PhysRevD 105 (2022) 095003 - spectrum perturbativity study

K. Jarkovská, MM, V. Susič, PhysRevD 108 (2023) 055003 - perturbative no-go

A. Preda, G. Senjanovic, M. Zantedeschi, Phys.Lett.B 838 (2023) 137746 - several light scalars
partly also in S. K. Shukla, arXiv:2403.14331

E6 broken by 78:

Ch. Dash, S. Mishra, S. Patra and P. Sahu, Int.J.Mod.Phys.A 39 (2024) 2450018

The game of scales

Qualitative constraints on symmetry breaking patterns

novel SSB chains in E6 broken by 650:

K.S. Babu, B. Bajc and V. Susič in JHEP 07 (2023) 011 and JHEP 06 (2024) 018

E8 (extreme) case:

K.V. Stepanyantz, Teor.Mat.Fiz. 218 (2024) 2, 341, arXiv:2305.01295

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New inputs from cosmology (GWs) !

NB A great review talk by Jessica Turner at BLV 2022 based on

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Recent developments:

NANOgrav 15 years data analysis in Astrophys.J.Lett. 951 (2023) 1

B. Fu, S.F. King, L. Marsili, S. Pascoli, J. Turner, Y.-L. Zhou, Phys.Rev.D 109 (2024) 5

- complete SUSY SO(10) model, flavour study, scales, p-decay estimates etc.

S.Antusch, K. Hinze, S. Saad, JCAP 10 (2024) 007 - inflationary aspects

R. Maji, Q. Shafi, arXiv:2408.14350 - correlating cosmic strings' formation as the source of the PTA signal with p-decay accessibility at HK in a special SO(10) with d=5 kinetic mixing of U(1)'s

The game of flavour

non-minimal SU(5) studies:

T. Goto, S. Mishima, T. Shindou, PRD 108 (2023) 095012 - 45_H , flavour, light scalar triplet pheno

S. Antusch, I. Dorsner, K. Hinze, S. Saad, Phys.Rev.D 108 (2023) 015025 - 15_F , axion DM

S. Antusch, K. Hinze, S. Saad Phys.Rev.D 108 (2023) 095010 - follow-up incl. leptogenesis

G. Senjanovic, M. Zantedeschi, Phys.Rev.D 109 (2024), 095009 - effective SU(5), flavour-spectrum corrs.

G.-X. Fang, Y.-L. Zhou, arXiv:2406.06861 - 24_F , type I+III seesaw, light fermionic triplet

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recent studies of flavour in SO(10):

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$$M_d = \textcolor{blue}{Y_{10}} v_d^{10} + \textcolor{blue}{Y_{126}} v_d^{126}$$

$$m_\nu^{II} \propto \textcolor{blue}{Y_{126}} \langle \Delta_L^0 \rangle$$

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NB 19 parameters (6 compact) , 3+3+4 (quarks) + 3+2+3 (leptons) masses+mixings!!!

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large leptonic Dirac CP phase!

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p-decay and/or leptogenesis in flipped SU(5)xU(1)

R. Fonseca, MM, V. Miřátský, M. Zdráhal, Phys.Rev.D 110 (2024) 015030

S. F. King, G, K. Leontaris, L. Marsilic, and Y.-L. Zhou, arXiv:2407.02701

see also K. Hamaguchi, S. Hor, N. Nagata, H. Takahashi, arXiv:2407.11892

The game of scales & flavour

Leptogenesis in the minimal potentially realistic ren. flipped SU(5) UT

R. Fonseca, MM, V. Miřátský, M. Zdráhal, Phys.Rev.D 110 (2024) 015030

Perturbative demise of the minimal potentially realistic ren. SO(10) GUT

K. Jarkovská, MM, V. Susič, PhysRevD 108 (2023) 055003

LO Planck-scale-induced unification-smearing effects absent in both cases!

BLNV nucleon decays in flipped SU(5) - one U_V rules them all

C.Arbelaez-Rodriguez, H. Kolešová, MM, Phys. Rev. D89 (2014) 055003

$$\Gamma(p \rightarrow \pi^0 \ell_\alpha^+) \quad \Gamma(p \rightarrow \pi^+ \bar{\nu}) \quad \Gamma(n \rightarrow \pi^- \ell_\alpha^+) \quad \Gamma(n \rightarrow \pi^0 \bar{\nu})$$

$$\Gamma(p \rightarrow K^0 \ell_\alpha^+) \quad \Gamma(p \rightarrow K^+ \bar{\nu}) \quad \Gamma(n \rightarrow K^- \ell_\alpha^+) \quad \Gamma(n \rightarrow K^0 \bar{\nu})$$

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Charged mesons:
(no flavour ambiguity!)

$$\Gamma(p \rightarrow K^+ \bar{\nu}) = 0$$

$$\Gamma(p \rightarrow \pi^+ \bar{\nu}) = \left(\frac{g_G}{M_G} \right)^4 \frac{m_p}{8\pi f_\pi^2} A_L^2 |\alpha|^2 (1 + D + F)^2$$

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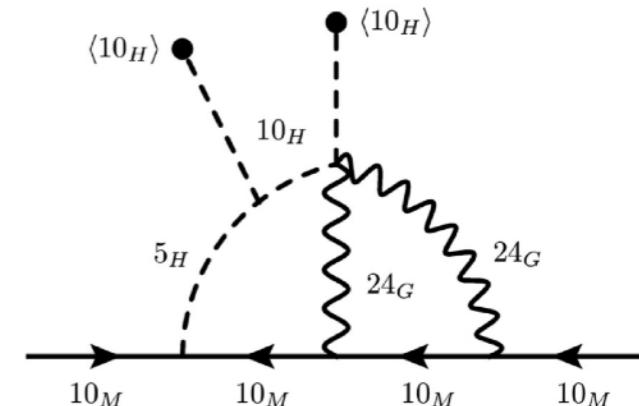
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M_R generated at two loops

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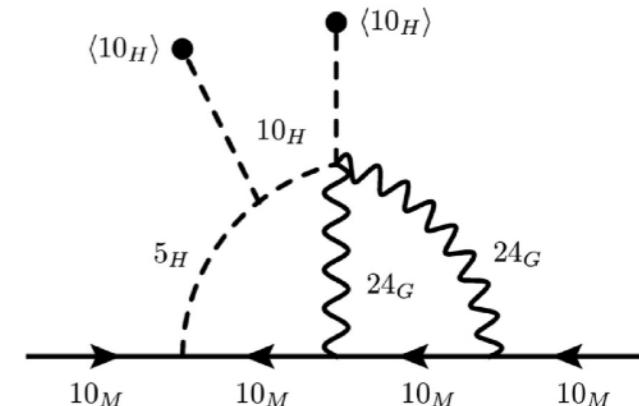
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Constraining U_ν yields constraints for all 2-body BNV channels !!!



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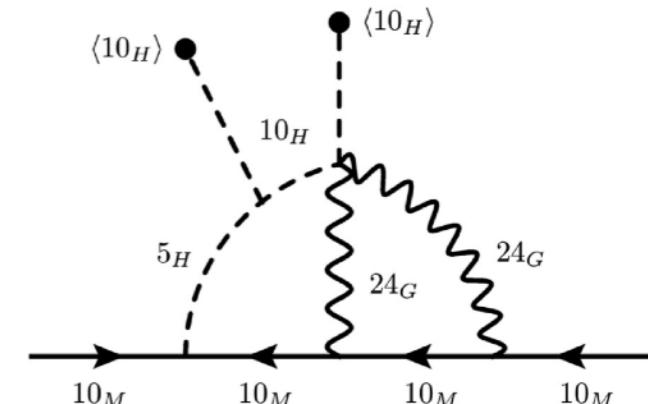
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Constraining U_ν yields constraints for all 2-body BNV channels !!!

$m_1 < 0.03$ eV - constraints from perturbativity: $|D_u U_\nu^\dagger D_\nu^{-1} U_\nu^* D_u| \lesssim 10^{14}$ GeV



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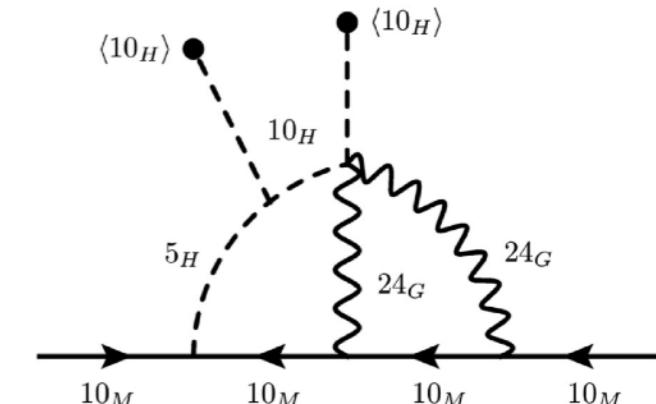
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$\text{BR}(p \rightarrow \pi^0 e^+)$ or $\text{BR}(p \rightarrow \pi^0 \mu^+)$ significant



M_R generated at two loops

Thermal leptogenesis in the minimal flipped SU(5) à la Witten

Baryon asymmetry generation may be problematic for $m_1 > 0.03$ eV !

$D_u U_\nu^\dagger D_\nu^{-1} U_\nu^* D_u = M_M \implies$ heavy neutrino spectrum hierarchical and suppressed!

Detailed numerical analysis:

MM, V. Miřátský, R. Fonseca, M. Zdráhal, PRD 110, 015030 (2024)

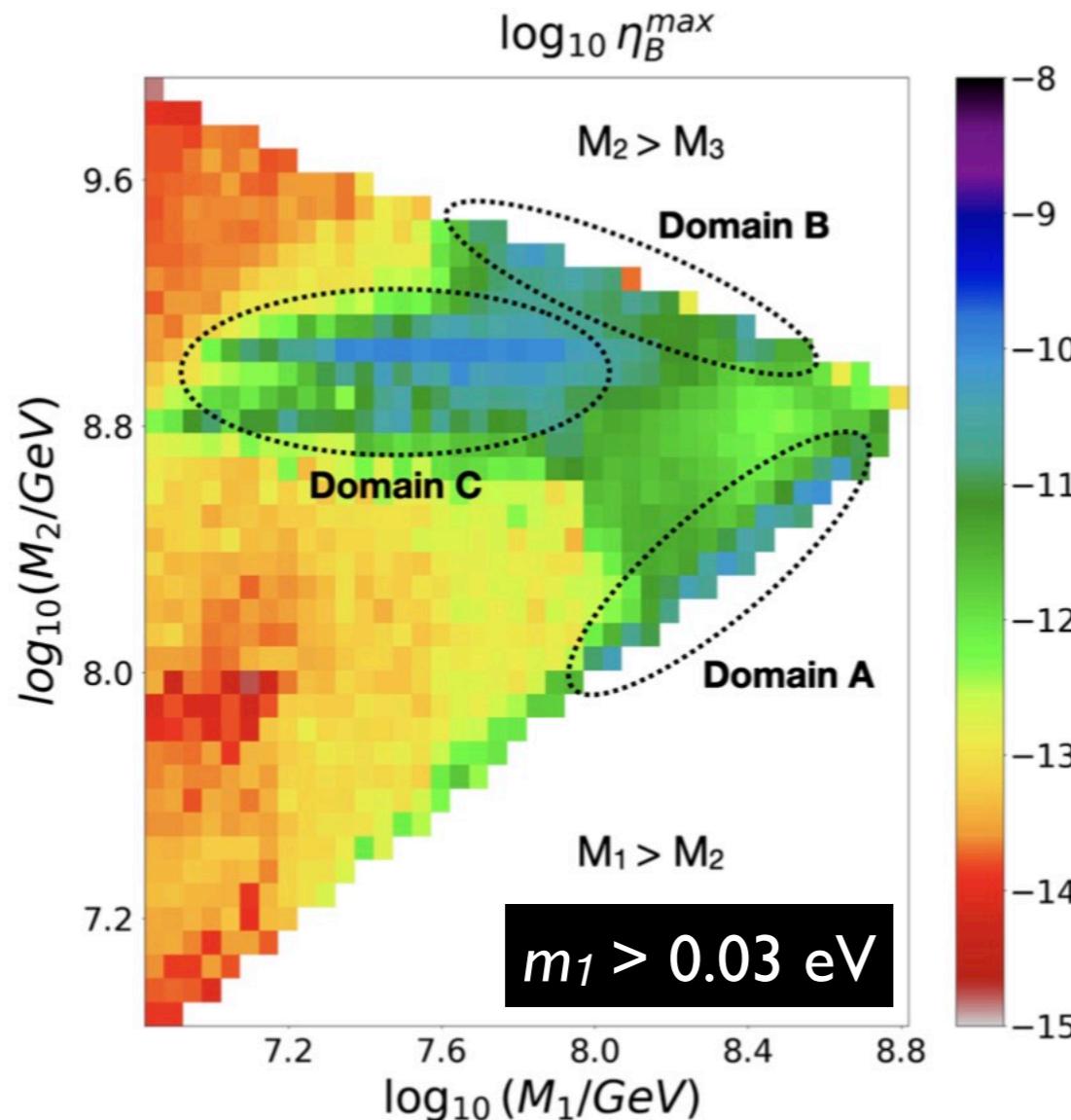
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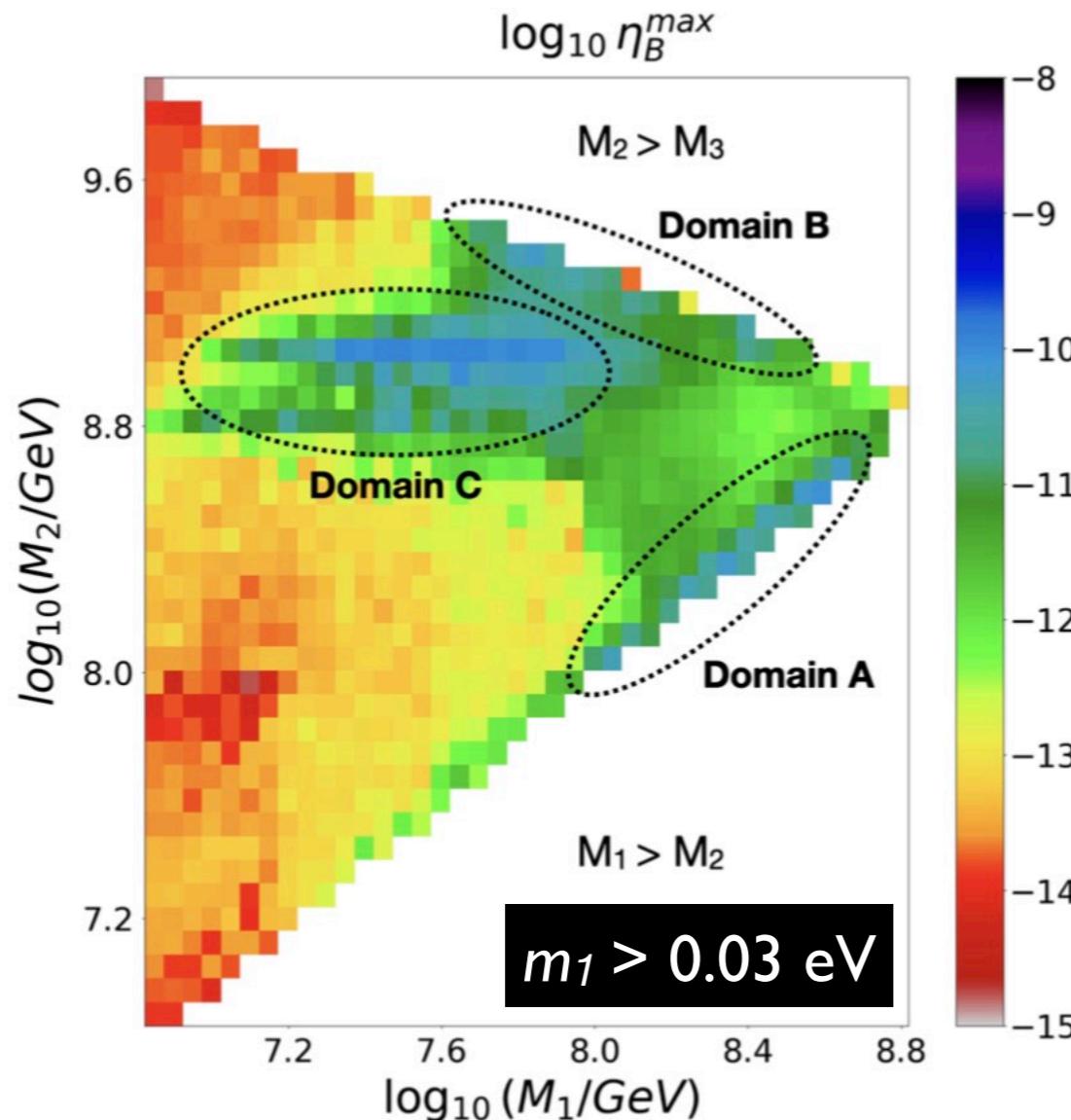
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No-go for $m_1 > 0.03$ eV! No signal in KATRIN!

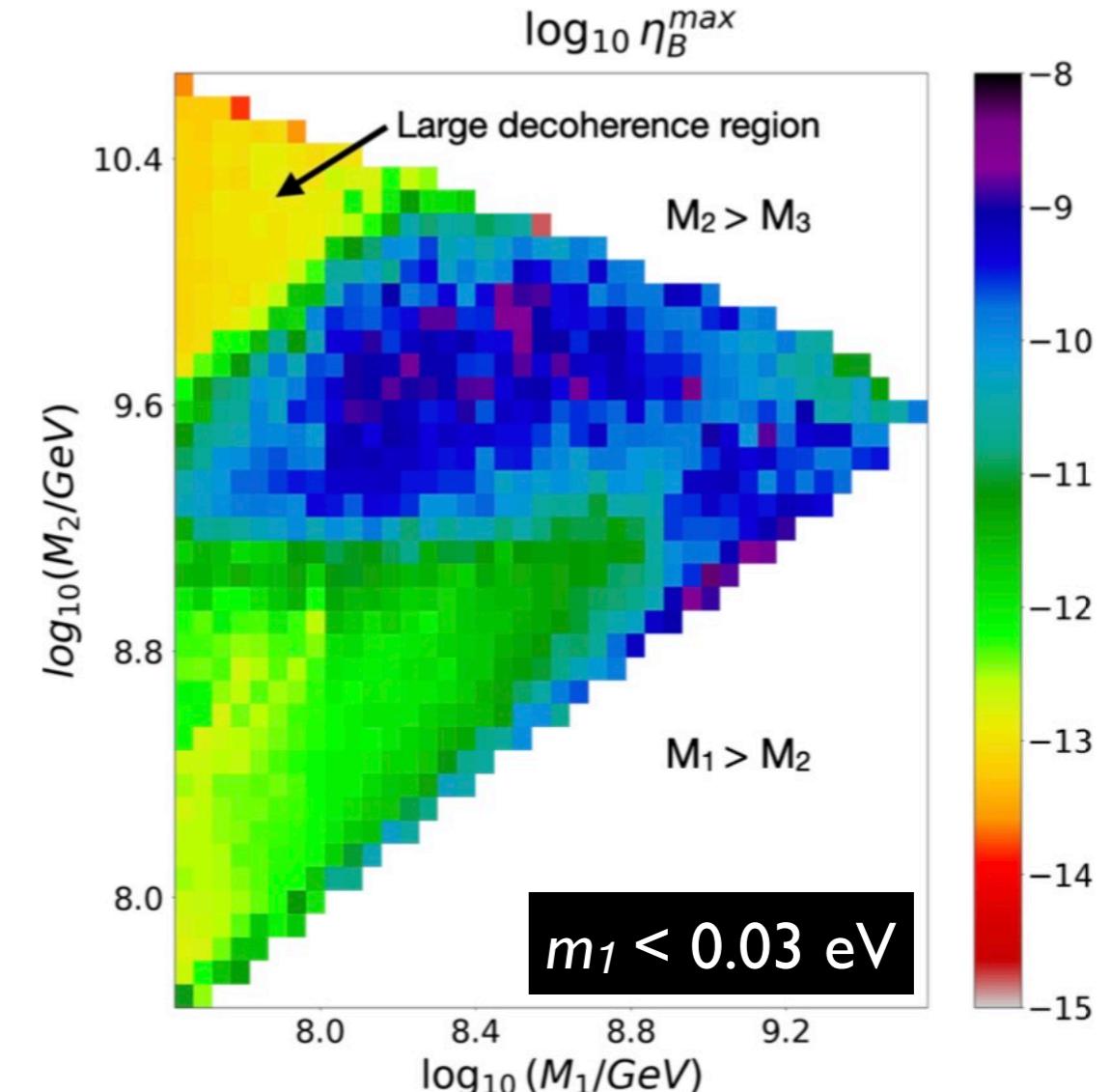
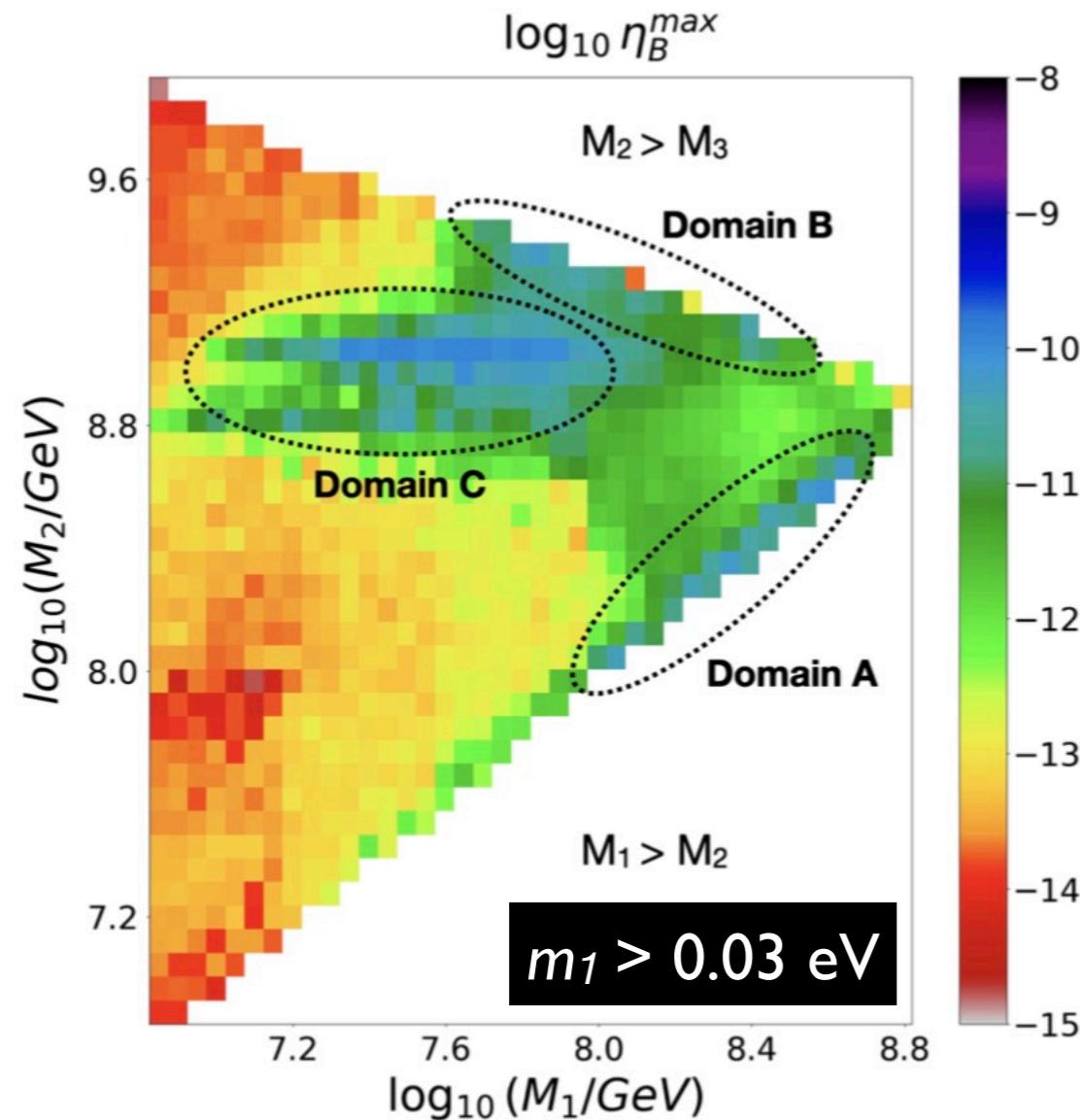
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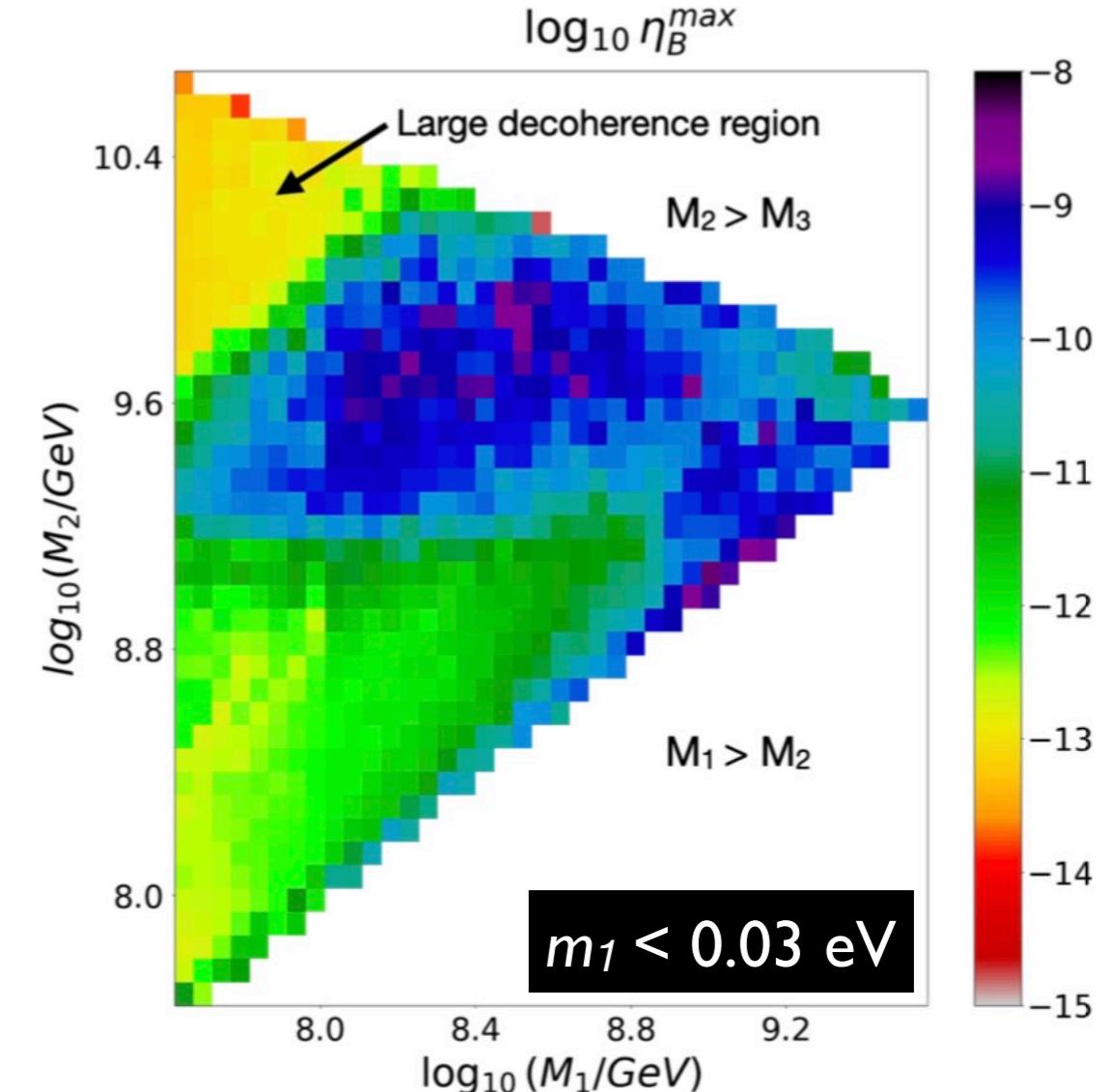
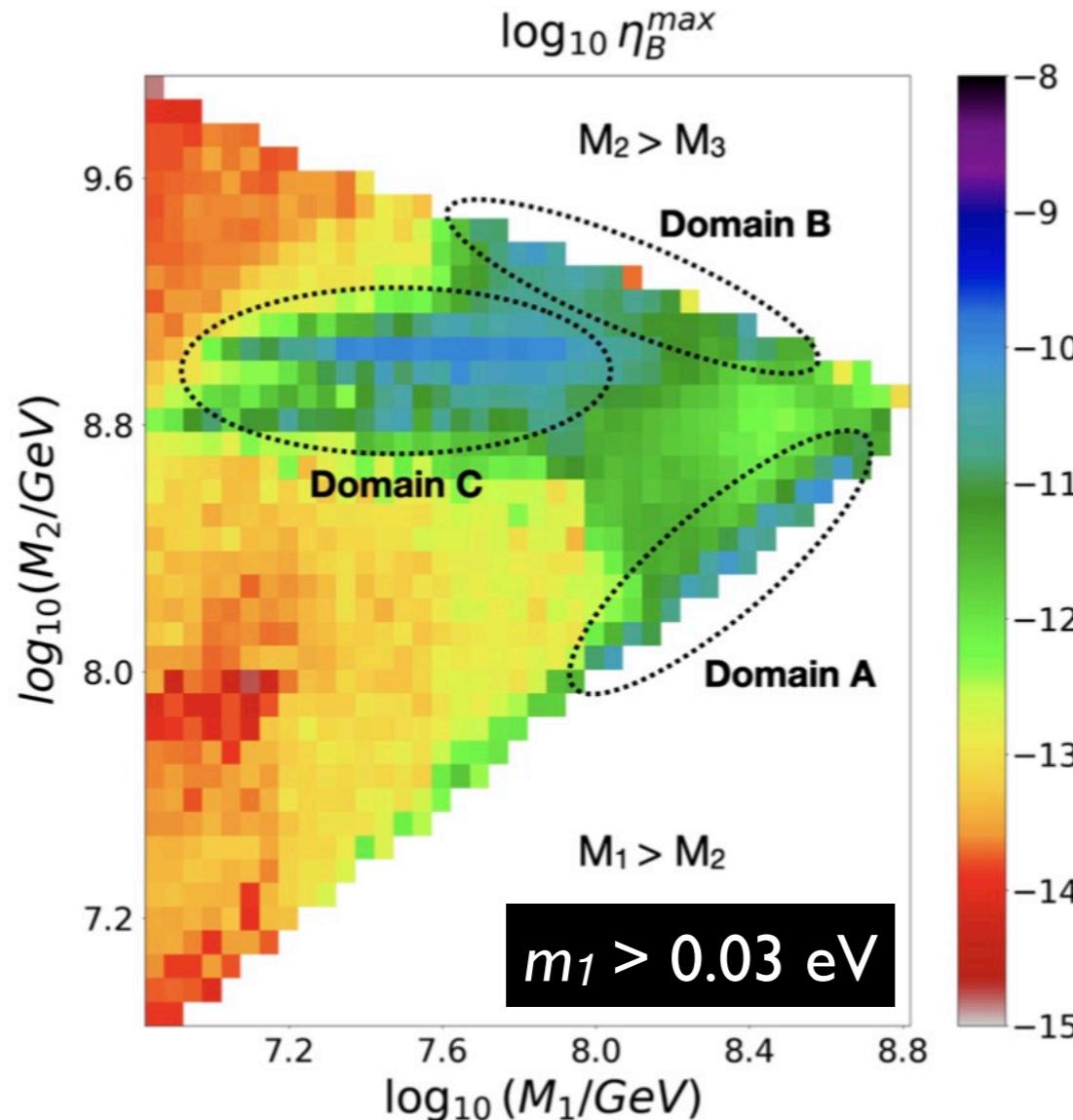
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OK for $m_1 < 0.03$ eV, $\text{BR}(p \rightarrow \pi^0 \mu^+) < 0.09$

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K. Jarkovská, MM, V. Susič, PhysRevD108 (2023) 055003

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SO(10) broken by 45

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- unification scale not smeared (!)

$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle 45 \rangle F_{\mu\nu} = 0$$

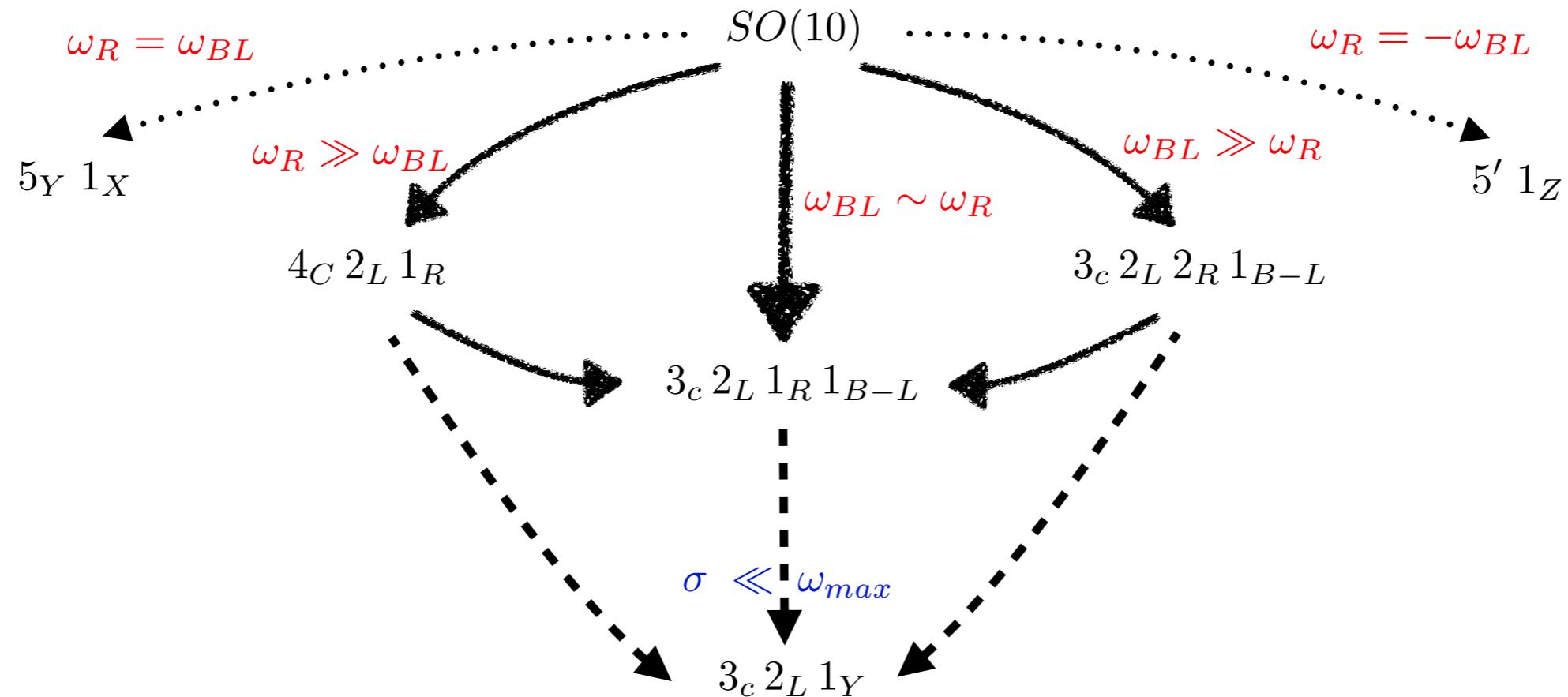
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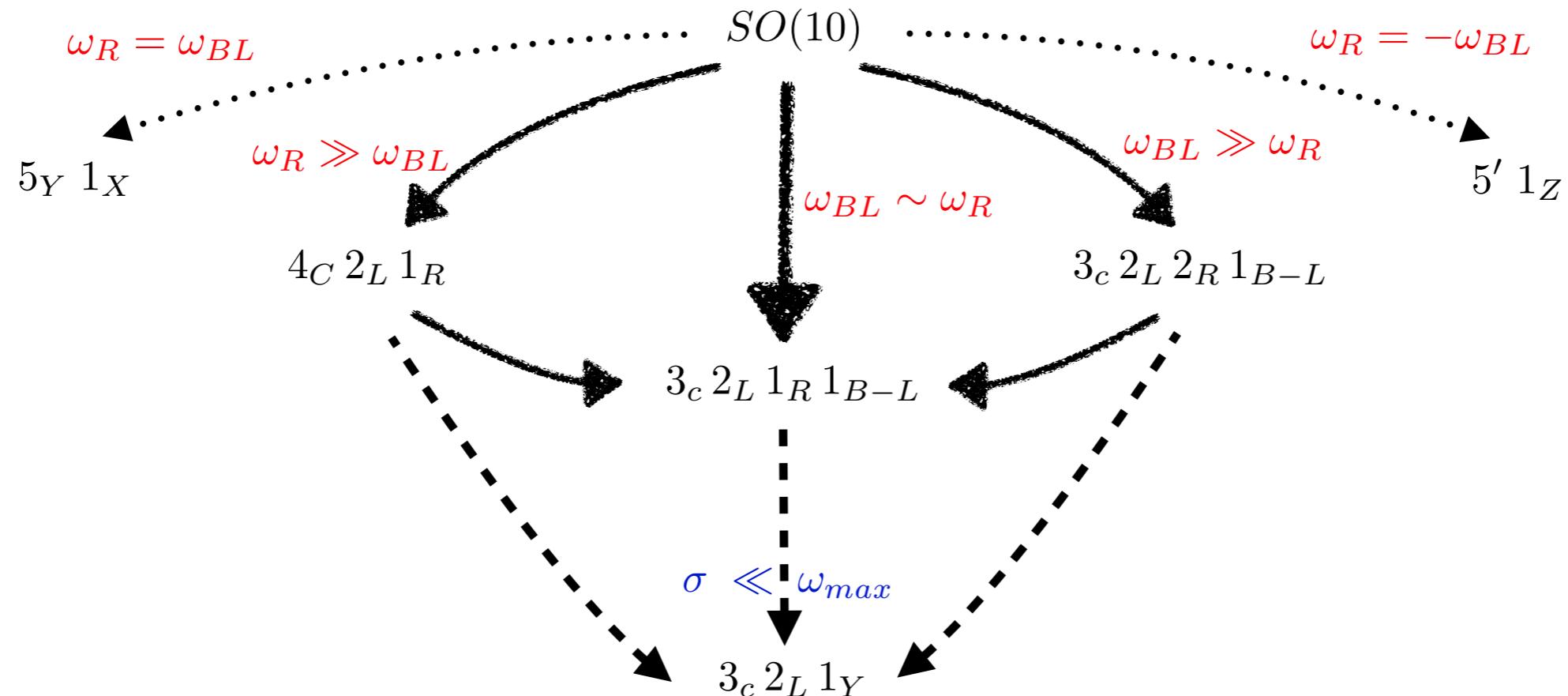
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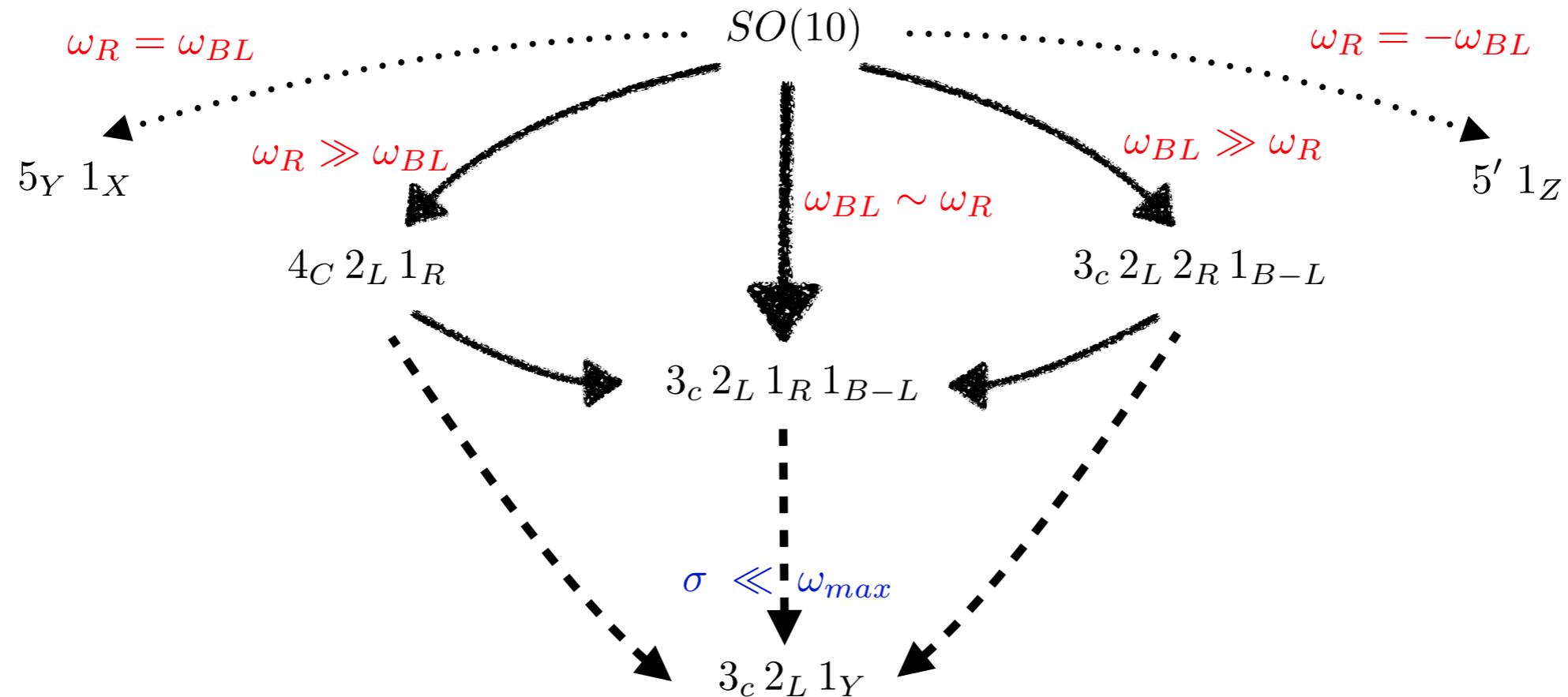
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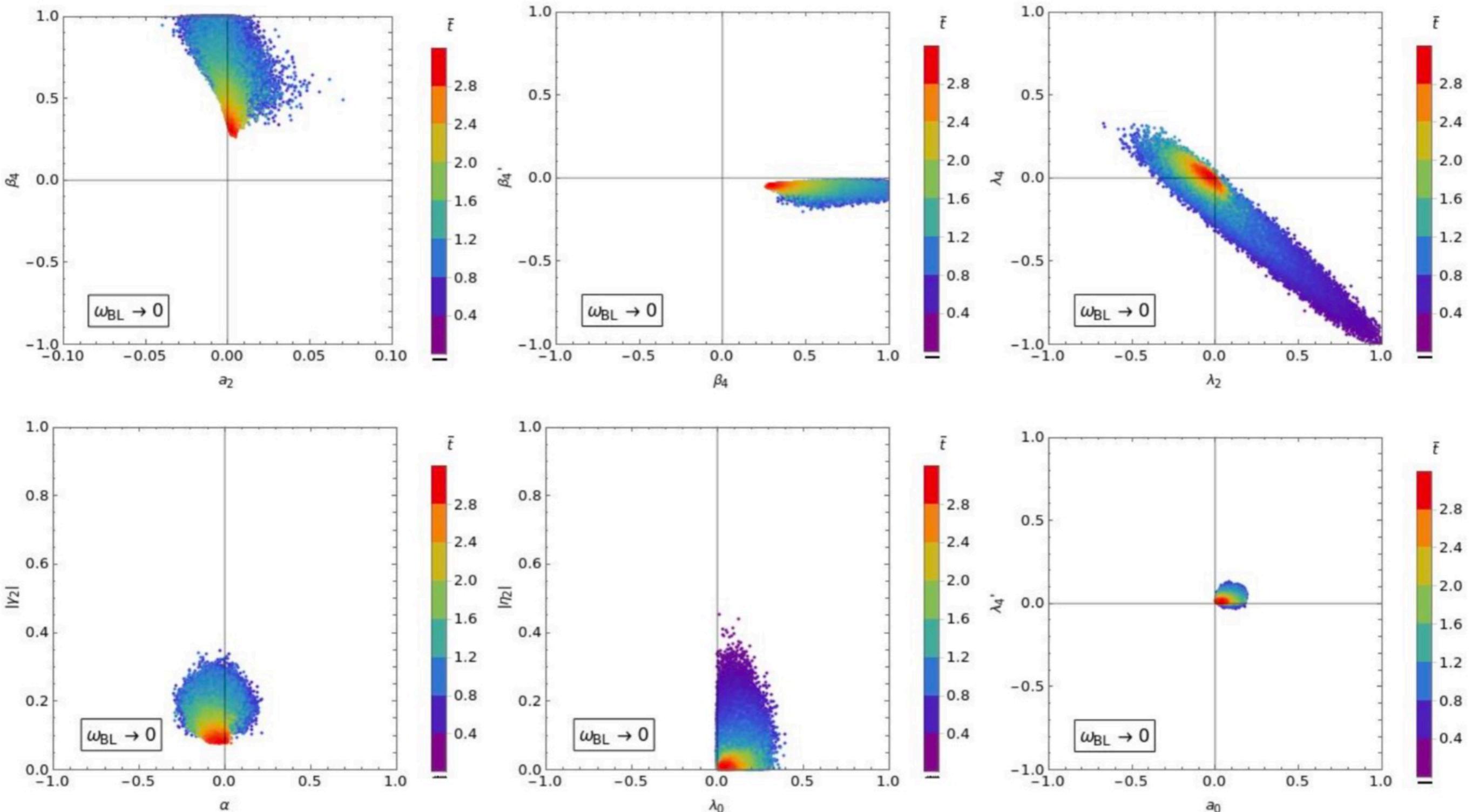
Only the $SU(4) \times SU(2) \times U(1)$ chain is potentially realistic and “reasonably perturbative”

K. Jarkovská, MM, T. Mede, V. Susič, PRD 105, 095003 (2022)

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$SO(10) \rightarrow 4_C 2_L 1_R \rightarrow \text{SM}$

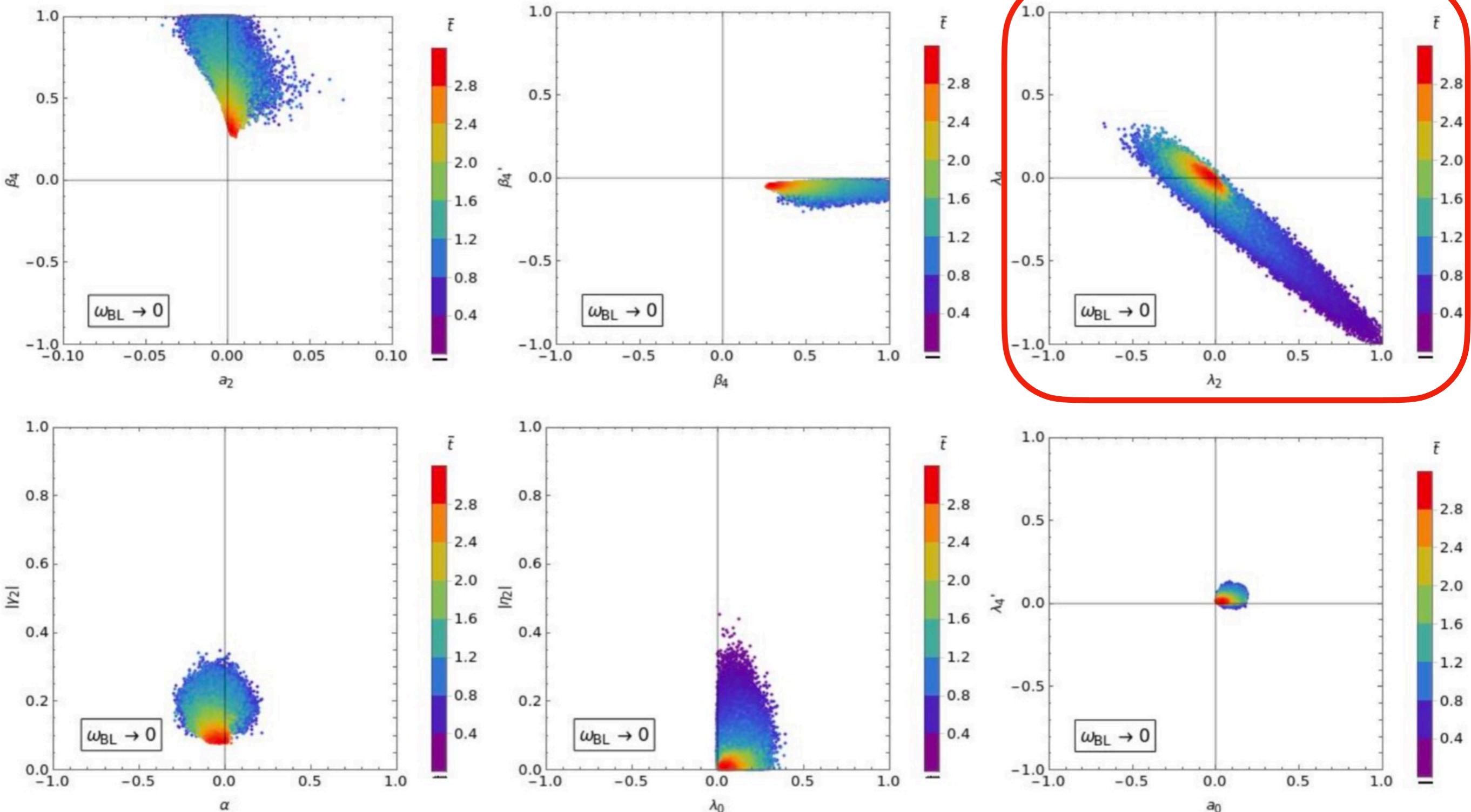
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Effective flavour structure:

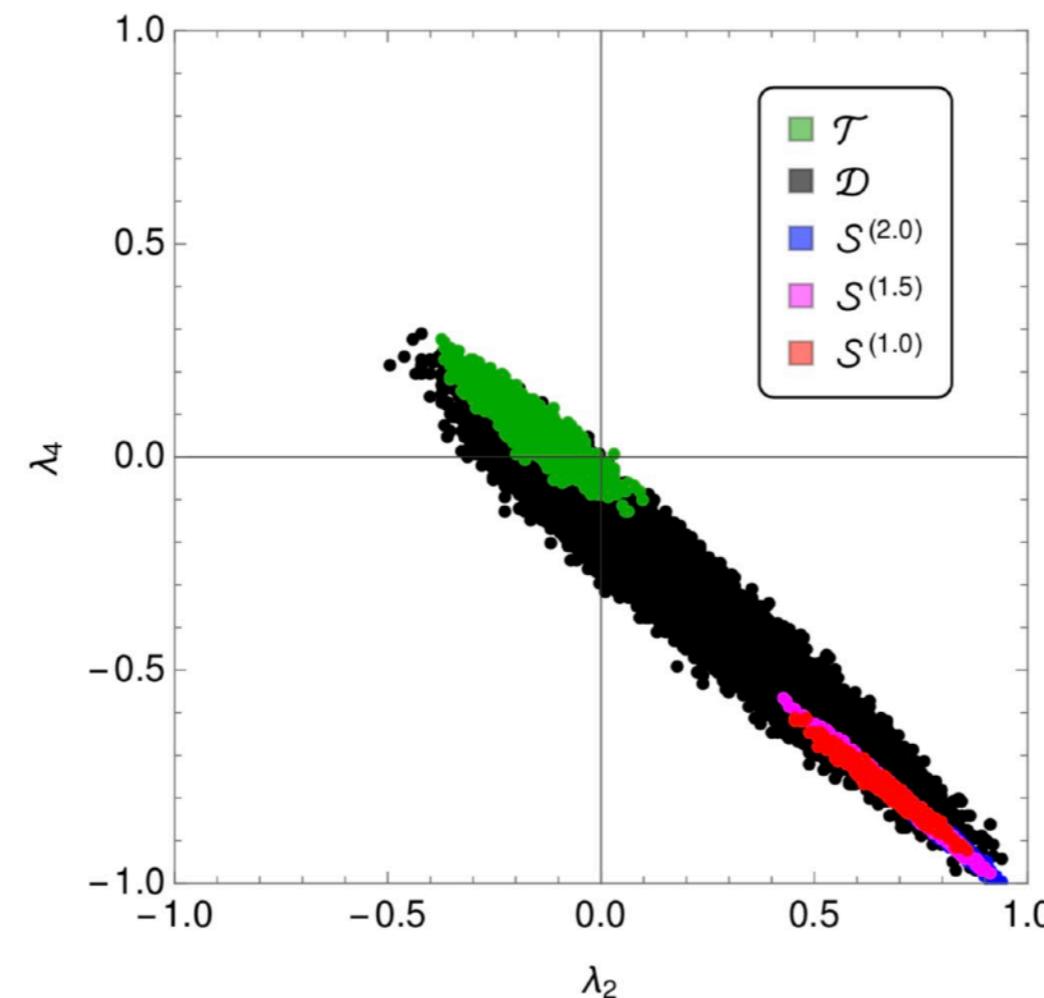
$$\begin{aligned} M_d &\sim Y_{10} v_d^{10} + Y_{126} v_d^{126} \\ M_e &\sim Y_{10} v_d^{10} - 3Y_{126} v_d^{126} \\ &\vdots \quad \vdots \end{aligned}$$

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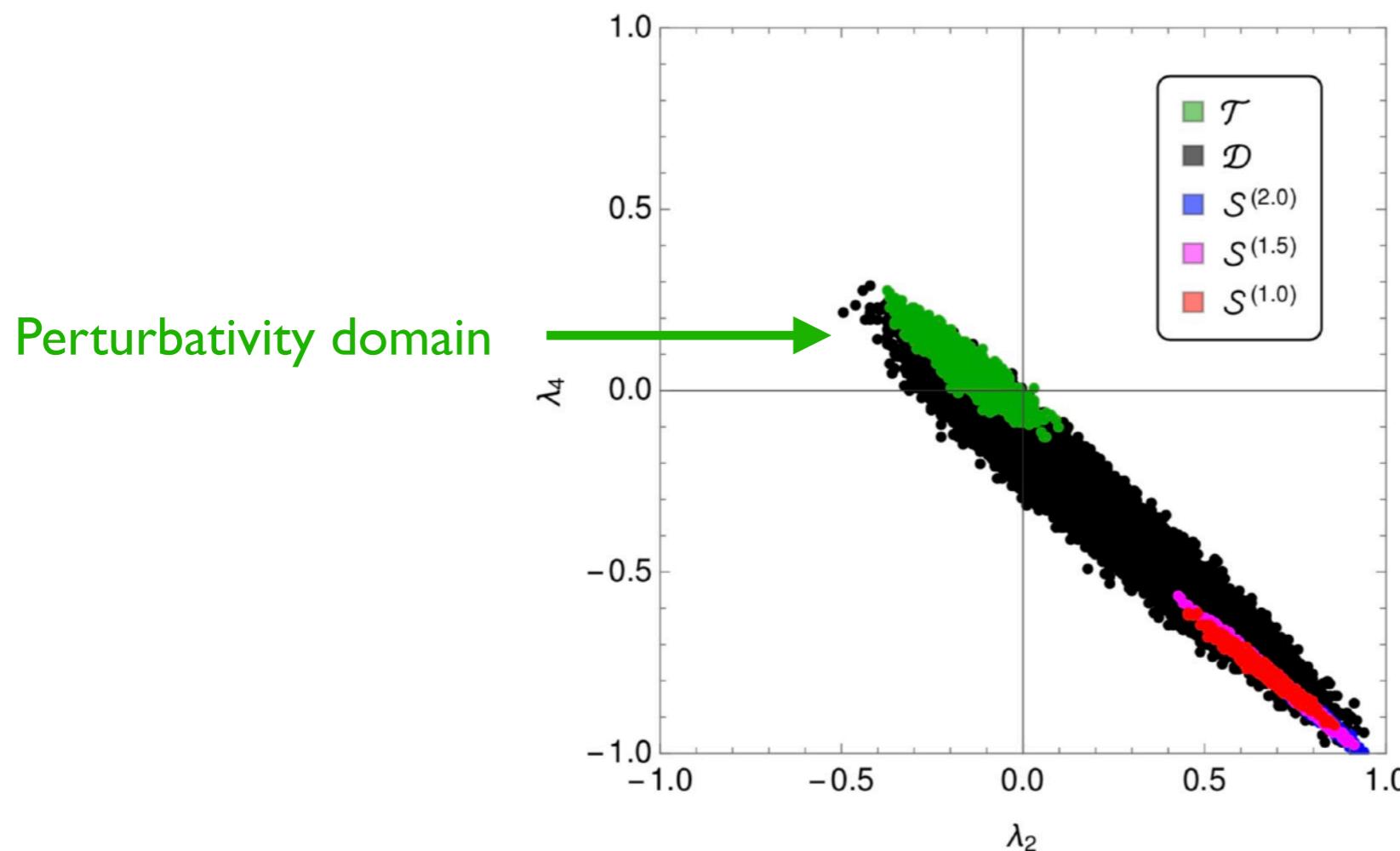


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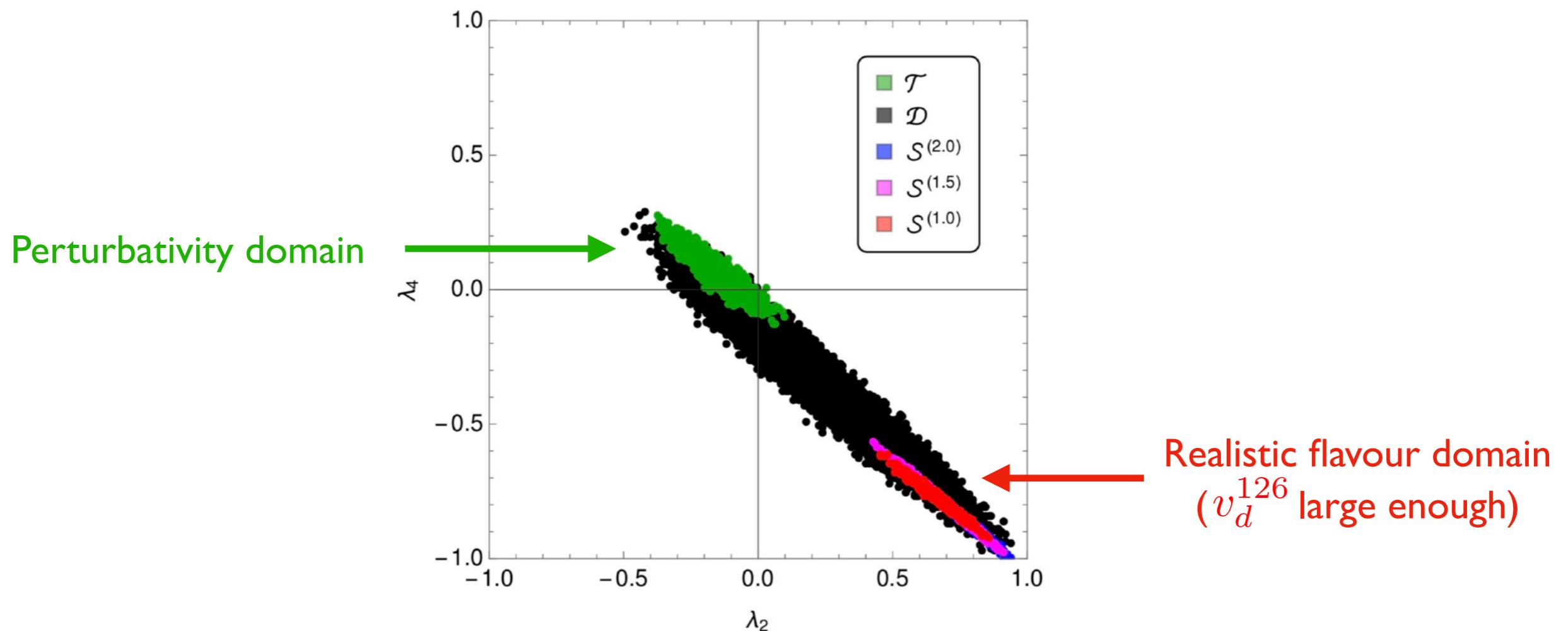


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$$M_d \sim Y_{10} v_d^{10} + Y_{126} v_d^{126}$$
$$M_e \sim Y_{10} v_d^{10} - 3Y_{126} v_d^{126}$$
$$\vdots$$
$$\vdots$$
$$\vdots$$



Concluding remarks

- Despite its longevity the field of GUTs is still thriving
- New constraints from cosmology & gravitational waves
- Ever improving inputs vs. running out of calculable models

Addendum

Interesting recent studies that, unfortunately, did not fit elsewhere:

asymptotic safety in GUTs

B. Bajc, M. Del Piano, F. Sannino, Phys.Rev.D 109 (2024) 055043 - UV finite SUSY GUTs

A. Eichhorn, S. Ray, Phys.Lett.B 850 (2024) 138529 - p-decay suppression in quantum gravity

Radiative BNV nucleon decay

S. Fajfer, M. Sadl, Phys.Rev.D 108 (2023), 015011

Loop-level p-decay in a leptoquark model

I. Dorsner, S. Fajfer, O. Sumensari, JHEP 05 (2022) 183

fermion-monopole scattering amplitudes

V. Khoze, JHEP 11 (2023) 214

doublet-triplet splitting

S. Antusch, K. Hinze, S. Saad, J. Steiner, Phys.Rev.D 109 (2024) 055031

I. Dorsner, S. Saad, arXiv:2404.09021

GUT scale from EW-scale lagrangians

H. Georgi, Phys.Lett.B 853 (2024) 138703

... and others

Thanks for your kind attention!