

The $B^+ - B_d^0$ Lifetime Differences @ NNLO

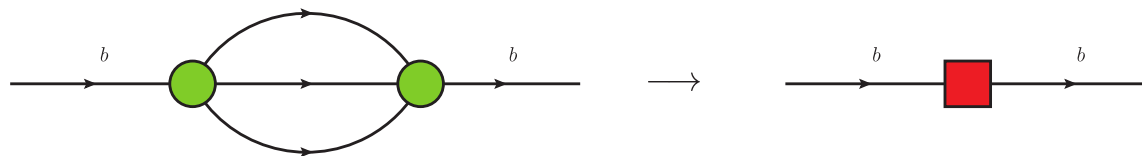
Francesco Moretti
CRC YS Meeting – Heidelberg
21 Jul 2025

Outline

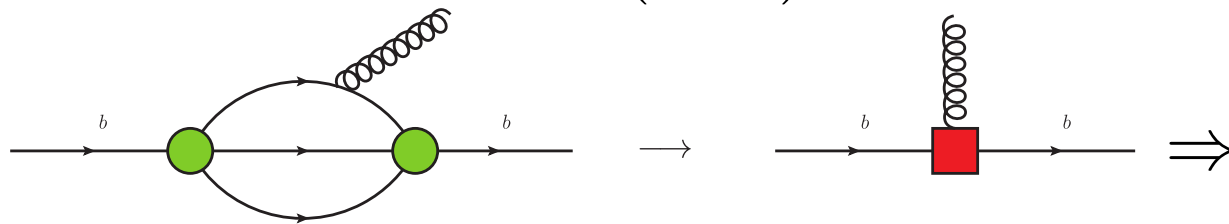
- Preliminaries
- Heavy Quark Expansion
- Next-to-Next-to-Leading Order Calculation
- Summary & Conclusions

Preliminaries

- Inclusive decay rates of beauty hadrons can be computed by expanding the amplitudes in increasing powers of $\frac{\Lambda_{\text{QCD}}}{m_b}$
- At leading order, $\Gamma(H_b) = \Gamma(b) \Rightarrow$ Lifetimes of all b-flavoured hadrons are of same order



- First corrections arise at $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2$ from chromomagnetic interactions



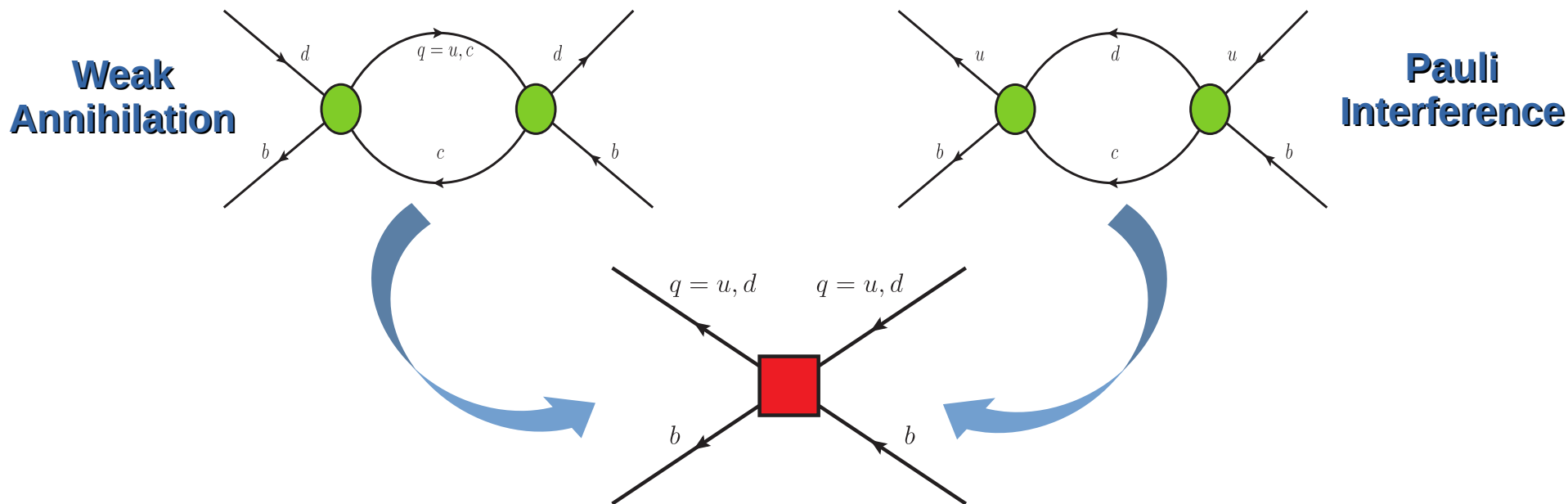
Negligible contributions to lifetime differences

$$\frac{\Gamma(B^+)}{\Gamma(B_d^0)} \approx 1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^3$$

Preliminaries

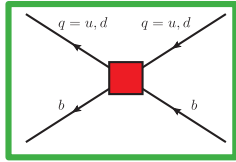
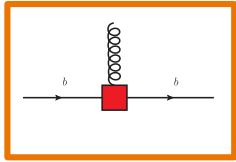
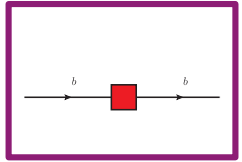
- At $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^3$ weak interaction of b-quarks with (light) valence quarks

Different contributions to B^+ and B_d^0 $\longrightarrow$ Lifetime difference of b-flavoured hadrons



Heavy Quark Expansion

- Optical theorem:** $\Gamma(H_b) = \frac{1}{2M_{H_b}} \langle H_b | \mathcal{T} | H_b \rangle$ $\mathcal{T} = \text{Im } i \int d^4x \text{ T } \{ H(x) H(0) \}$



Non Local Correlator !

$$m_b \gg \Lambda_{\text{QCD}}$$

Effective $|\Delta B| = 1$ local Hamiltonian

$$\mathcal{T} = \boxed{\mathcal{T}^0} + \boxed{\mathcal{T}^2} + \boxed{\mathcal{T}^3} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^4$$

$$H = \frac{G_F}{\sqrt{2}} V_{cb}^* \sum_{\substack{u'=u,c \\ d'=d,s}} V_{u'd'} \left[C_1(\mu_1) Q_1^{u'd'}(\mu_1) + C_2(\mu_1) Q_2^{u'd'}(\mu_1) \right]$$

μ_1 renormalisation scale

- We focus on contributions from dimension-6 operators $\Rightarrow \mathcal{T}^3 = \mathcal{T}^u + \mathcal{T}^d$

$$\bullet \mathcal{T}^u = \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi} \left[|V_{ud}|^2 (F^u Q^d + F_S^u Q_S^d + G^u T^d + G_S^u T_S^d) + |V_{cd}|^2 (F^c Q^d + F_S^c Q_S^d + G^c T^d + G_S^c T_S^d) \right]$$

$$\bullet \mathcal{T}^d = \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi} \left[(F^d Q^u + F_S^d Q_S^u + G^d T^u + G_S^d T_S^u) \right]$$

“Historical” basis:

$$Q_1^{u'd'} = (\bar{b}_i c_j)_{V-A} (\bar{u}'_j d'_i)_{V-A}$$

$$Q_2^{u'd'} = (\bar{b}_i c_i)_{V-A} (\bar{u}'_j d'_j)_{V-A}$$

Heavy Quark Expansion

- Effective local $|\Delta B|=0$ Hamiltonian

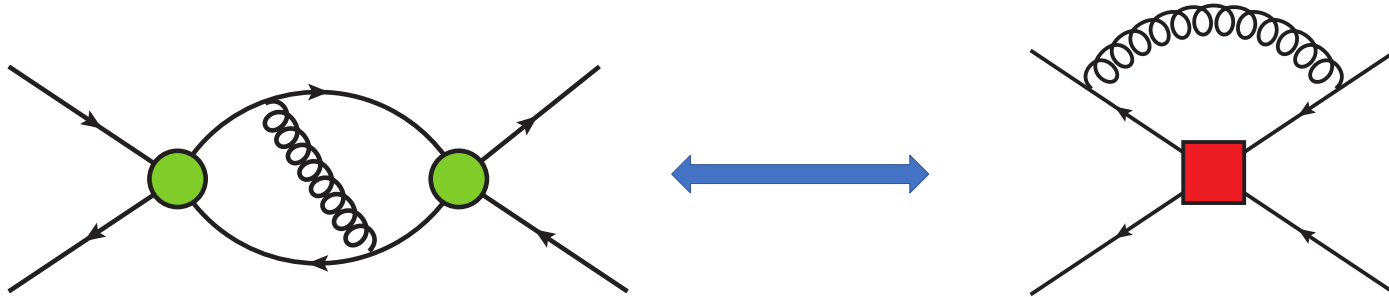
$$Q^q = (\bar{b} q)_{V-A} (\bar{q} b)_{V-A}$$

$$T^q = (\bar{b} T^a q)_{V-A} (\bar{q} T^a b)_{V-A}$$

$$Q_S^q = (\bar{b} q)_{S-P} (\bar{q} b)_{S+P}$$

$$T_S^q = (\bar{b} T^a q)_{S-P} (\bar{q} T^a b)_{S+P}$$

- We obtain the Wilson coefficients F^u , G^u , $\dots$ by matching the two effective theories



Heavy Quark Expansion

- Hadronic matrix elements

$$\langle B_d^0 | Q^{u,d} | B_d^0 \rangle = \langle B^+ | Q^{d,u} | B^+ \rangle$$

$$\langle B^+ | (Q^u - Q^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 B_1(\mu_0)$$

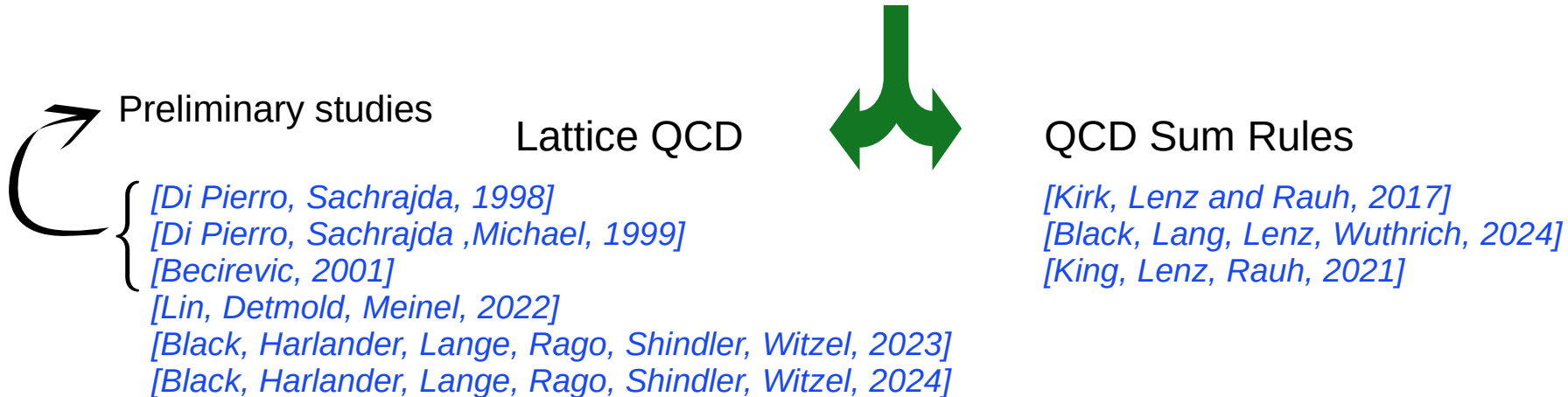
$$\langle B^+ | (Q_S^u - Q_S^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 B_2(\mu_0)$$

$$\langle B^+ | (T^u - T^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 \epsilon_1(\mu_0)$$

$$\langle B^+ | (T_S^u - T_S^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 \epsilon_2(\mu_0)$$

$|\Delta B| = 0$ renormalisation scale

- Non-perturbative evaluation of the bag parameters



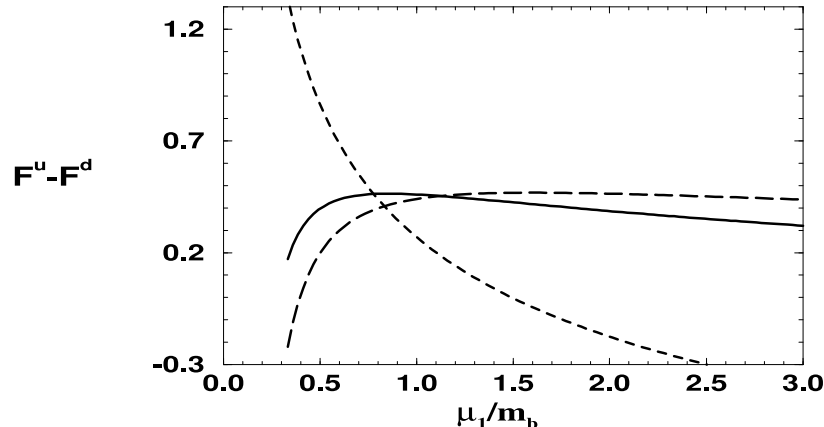
State Of The Art

- Next-to-Leading order calculation of the Wilson coefficients

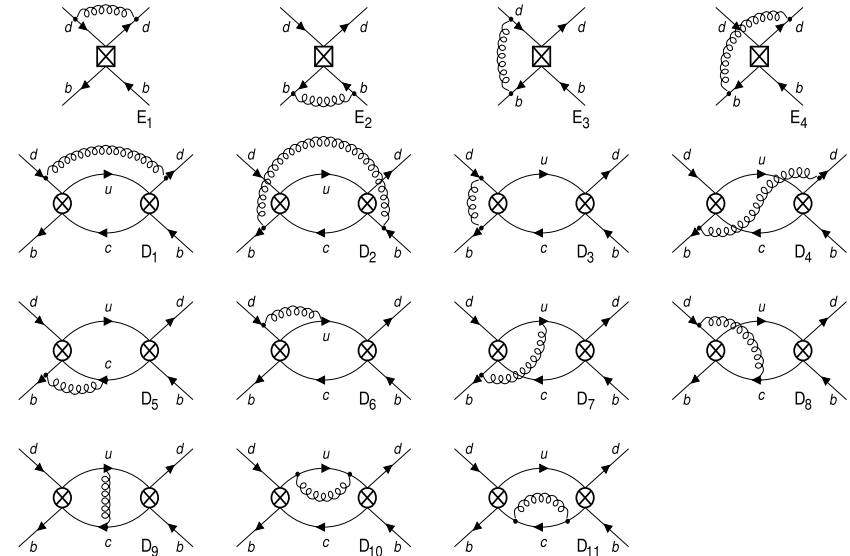
$$m_c = 0$$

- [Ciuchini, Franco, Lubicz, Mescia, 2001]
- [Franco, Lubicz, Mescia, Tarantino, 2002]
- [Beneke, Buchalla, Greub, Lenz, Nierste, 2002]

No contribution from $c\bar{c}$ intermediate states at NLO



Feynman diagrams



NNLO QCD Calculations

Workflow

[Nogueira, 1991]



Diagram generation based on model files

[Gerlach, Herren, Lang, 2022]
[Seidensticker, 1999]



Implementation of Feynman rules & mapping over topologies

[Vermaseren, 2000]
[Reeck, Shtabovenko, Steinhauser, 2024]



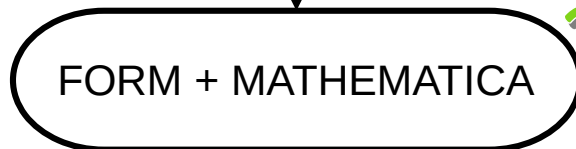
Reduction of Feynman amplitudes in terms of scalar integrals

[Klappert, Lange, Maierhöfer, Usovitsch, 2023]



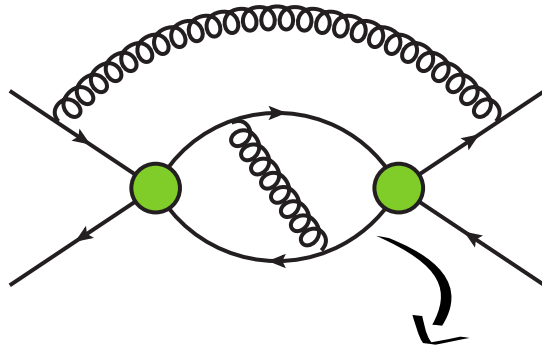
Integration-by-parts reduction to master integrals

[Wolfram Research, 2024]



Implementation of master integrals & renormalisation

$|\Delta B|=1$ Renormalisation

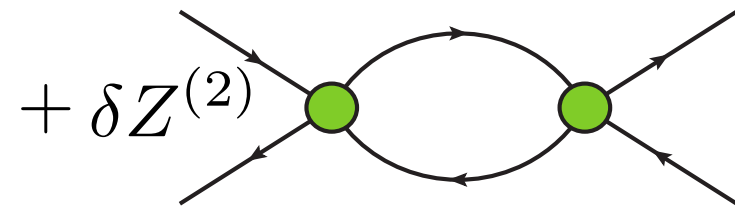
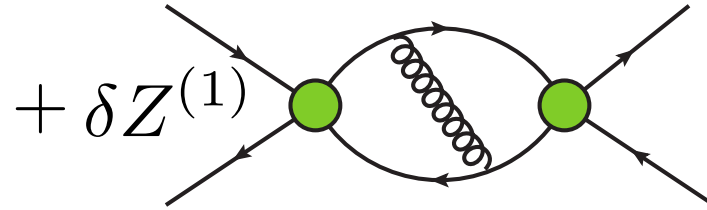


• $\mathcal{O}(2k)$ diagrams

- MIs from recent projects:

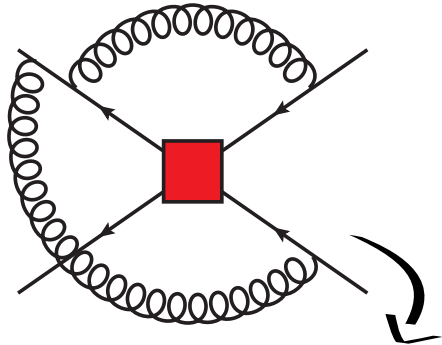
[Reeck, Shtabovenko, Steinhauser, 2024]

$$E[Q_1] = (\bar{q}_1^i \gamma^{\mu_1 \mu_2 \mu_3} P_L b^j) (\bar{q}_2^j \gamma_{\mu_1 \mu_2 \mu_3} P_L q_3^i) - (16 - 4\epsilon - 4\epsilon^2) Q_1$$



- Lower order counter-terms
- Mixing matrix up to $\mathcal{O}(\alpha_s^2)$ and appropriate choice of **Evanescence** operators:
[Egner, Fael, Schönwald, Steinhauser, 2024]
- **MS** renormalisation of charm quark mass
- on-shell renormalisation of bottom quark mass

$|\Delta B|=0$ Renormalisation

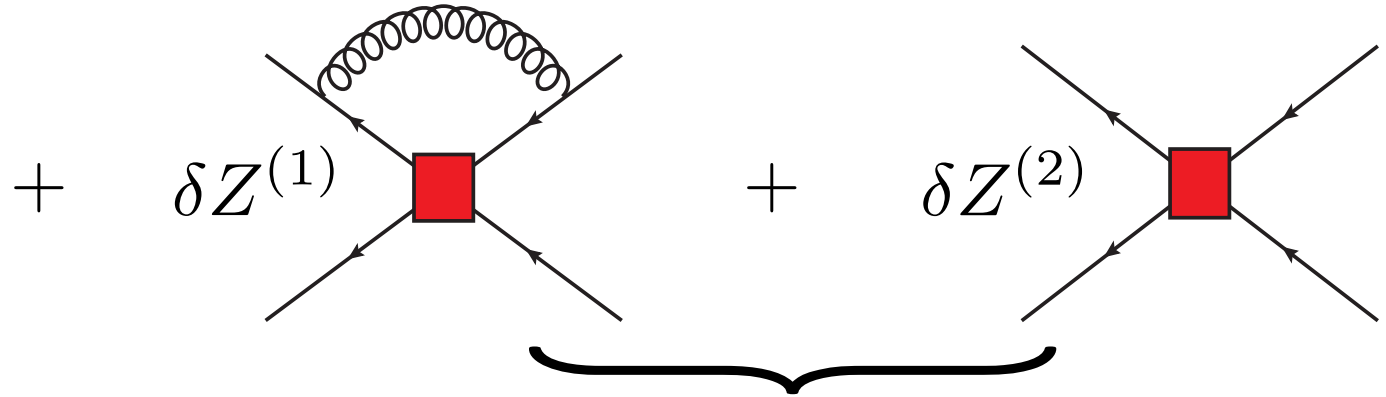


• $\mathcal{O}(1k)$ diagrams

- MIs from recent projects:

[Reeck, Shtabovenko, Steinhauser, 2024]

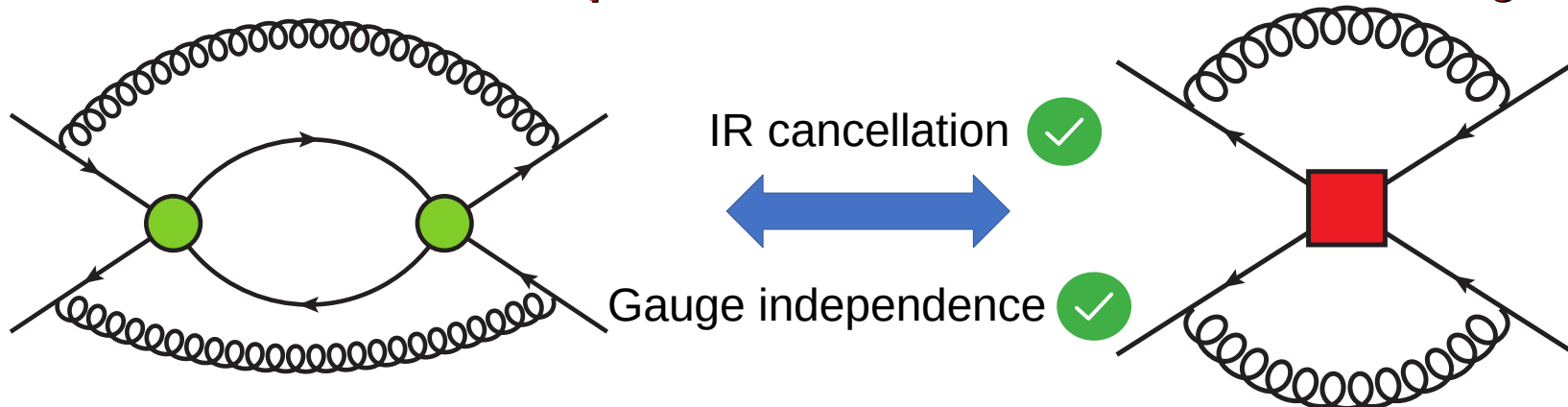
$$E[Q] = (\bar{b}\gamma^{\mu_1\mu_2\mu_3}P_L d)(\bar{d}\gamma_{\mu_3\mu_2\mu_1}P_L b) - (8 - 4\epsilon - A\epsilon^2)Q$$



- Lower order counter-terms
- Mixing matrix known up to $\mathcal{O}(\alpha_s)$. We derived the renormalisation of the $|\Delta B|=0$ basis at $\mathcal{O}(\alpha_s^2)$.
- $\overline{\text{MS}}$ renormalisation of charm quark mass
- on-shell renormalisation of bottom quark mass

Matching & Wilson Coefficients

- We perform the matching between the “full” $|\Delta B|=1$ theory and the effective $|\Delta B|=0$ theory in $d=4-2\epsilon$
- We set $p_{d,u}=0$ and $p_b^2=m_b^2 \Rightarrow$ We generate **IR spurious divergences**
- The IR behaviour of the two theories is the same $\Rightarrow$ cancellation of $1/\epsilon_{\text{IR}}$ poles
- Yet, spurious poles give finite contributions when they multiply $\mathcal{O}(\epsilon)$ parts of evanescent structures $\Rightarrow$ **Evanescent operators must be included in the matching**



Summary & Conclusions

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- HQE can be used to obtain theoretical predictions in QCD of lifetimes ratios of b-flavoured hadrons
- At leading order, we have $\Gamma(H_b)/\Gamma(H_b') = 1$. First corrections to the ratio comes from phase-space enhanced interactions of the heavy b-quarks with light spectator quarks
- In the framework of HQE, we need to match the non-local correlators in the $|\Delta B|=1$ side onto a set of local $|\Delta B|=0$ operators
- So far, this has been done only at $\mathcal{O}(\alpha_s)$. We expand the results in the literature and compute the matching at $\mathcal{O}(\alpha_s^2)$
- We checked the correct IR cancellation in the matching of some particular diagrams and the gauge independence of the final results
- Future outlooks: check the residual scale dependence at NNLO; implement the updated determinations of the non-perturbative bag parameters and check the impacts on $\Gamma(B^+)/\Gamma(B_d^0)$

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Thank You!