The $B^+ - B_d^0$ Lifetime Differences @ NNLO

Francesco Moretti

CRC YS Meeting – Heidelberg

21 Jul 2025

KIT 200 JAHRE

Institute for Theoretical Particle Physics

Collaborative Research Center TRR 257

P H

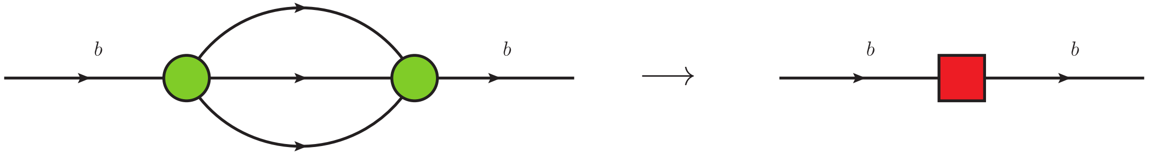
Particle Physics Phenomenology after the Higgs Discovery

Outline

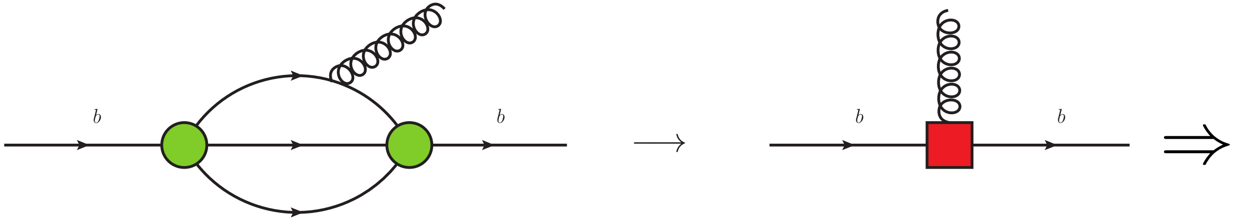
- Preliminaries
- Heavy Quark Expansion
- Next-to-Next-to-Leading Order Calculation
- Summary & Conclusions

Preliminaries

- Inclusive decay rates of beauty hadrons can be computed by expanding the amplitudes in increasing powers of $\frac{\Lambda_{\mathrm{QCD}}}{m_b}$
- At leading order, $\Gamma(H_b) = \Gamma(b) \Longrightarrow$ Lifetimes of all b-flavoured hadrons are of same order



- First corrections arise at $\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^2$ from chromomagnetic interactions



Negligible contributions to lifetime differences

$$\frac{\Gamma(B^+)}{\Gamma(B_d^0)} \approx 1 + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^3$$

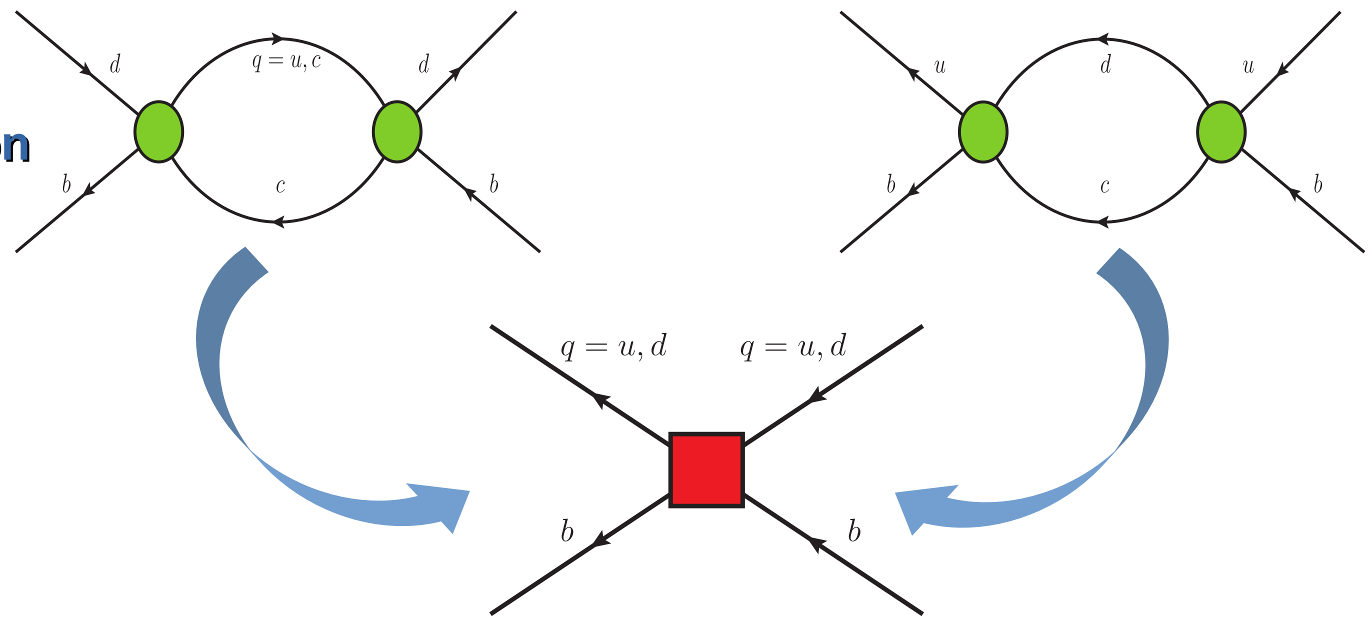
Preliminaries

- At $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3$ weak interaction of b-quarks with (light) valence quarks

Different contributions to B^+ and B_d^0 ⟶ Lifetime difference of b-flavoured hadrons

Weak Annihilation

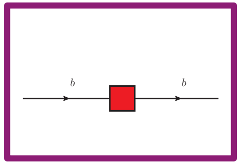
Pauli Interference

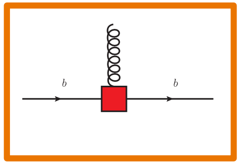


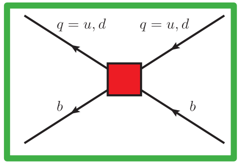
Heavy Quark Expansion

- **Optical theorem:** $\Gamma(H_b) = \frac{1}{2M_{H_b}} \langle H_b | \mathcal{T} | H_b \rangle$ $\qquad \mathcal{T} = \mathrm{Im}\, i \int d^4x \, \mathrm{T}\{H(x)\, H(0)\}$

Non Local Correlator !







$m_b \gg \Lambda_{\mathrm{QCD}}$

$$\mathcal{T} = \mathcal{T}^0 + \mathcal{T}^2 + \mathcal{T}^3 + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^4$$

Effective $|\Delta B| = 1$ local Hamiltonian

$$H = \frac{G_F}{\sqrt{2}} V_{cb}^* \sum_{\substack{u'=u,c \\ d'=d,s}} V_{u'd'} \left[C_1(\mu_1) Q_1^{u'd'}(\mu_1) + C_2(\mu_1) Q_2^{u'd'}(\mu_1) \right]$$

μ_1 renormalisation scale

- We focus on contributions from dimension-6 operators $\Rightarrow \mathcal{T}^3 = \mathcal{T}^u + \mathcal{T}^d$

"Historical" basis:
$$Q_1^{u'd'} = (\bar{b}_i c_j)_{V-A} (\bar{u}'_j d'_i)_{V-A}$$
$$Q_2^{u'd'} = (\bar{b}_i c_i)_{V-A} (\bar{u}'_j d'_j)_{V-A}$$

- $$\mathcal{T}^u = \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi} \left[|V_{ud}|^2 \left(F^u Q^d + F_S^u Q_S^d + G^u T^d + G_S^u T_S^d \right) + |V_{cd}|^2 \left(F^c Q^d + F_S^c Q_S^d + G^c T^d + G_S^c T_S^d \right) \right]$$

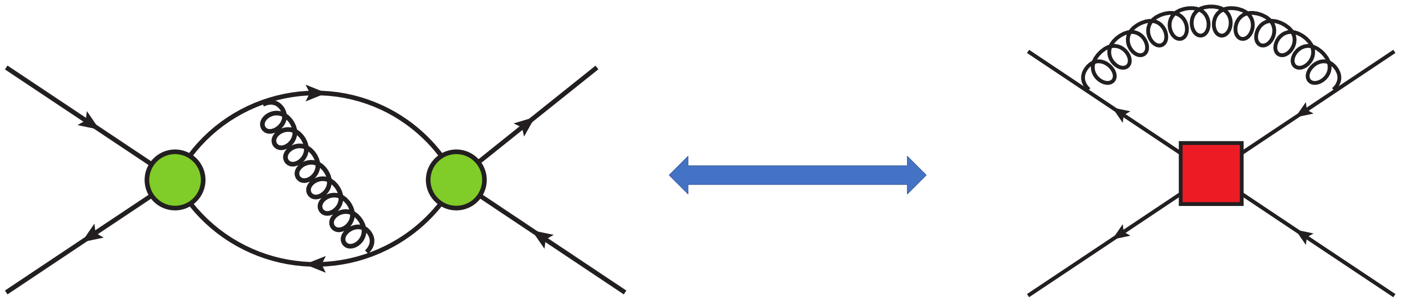
- $$\mathcal{T}^d = \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi} \left[\left(F^d Q^u + F_S^d Q_S^u + G^d T^u + G_S^d T_S^u \right) \right]$$

Heavy Quark Expansion

- Effective local $|\Delta B|=0$ Hamiltonian

$$Q^q = (\bar{b}\, q)_{V-A}(\bar{q}\, b)_{V-A} \qquad Q^q_S = (\bar{b}\, q)_{S-P}(\bar{q}\, b)_{S+P}$$
$$T^q = (\bar{b}\, T^a q)_{V-A}(\bar{q}\, T^a b)_{V-A} \qquad T^q_S = (\bar{b}\, T^a q)_{S-P}(\bar{q}\, T^a b)_{S+P}$$

- We obtain the Wilson coefficients $F^u,\ G^u,\ \cdots$ by matching the two effective theories



Heavy Quark Expansion

- Hadronic matrix elements

$$\langle B_d^0|Q^{u,d}|B_d^0\rangle = \langle B^+|Q^{d,u}|B^+\rangle$$

$$\langle B^+|(Q^u - Q^d)(\mu_0)|B^+\rangle = f_B^2 M_B^2 B_1(\mu_0) \qquad \langle B^+|(Q_S^u - Q_S^d)(\mu_0)|B^+\rangle = f_B^2 M_B^2 B_2(\mu_0)$$

$$\langle B^+|(T^u - T^d)(\mu_0)|B^+\rangle = f_B^2 M_B^2 \epsilon_1(\mu_0) \qquad \langle B^+|(T_S^u - T_S^d)(\mu_0)|B^+\rangle = f_B^2 M_B^2 \epsilon_2(\mu_0)$$

$|\Delta B| = 0$ renormalisation scale

- Non-perturbative evaluation of the bag parameters

Preliminary studies

Lattice QCD

[Di Pierro, Sachrajda, 1998]
[Di Pierro, Sachrajda ,Michael, 1999]
[Becirevic, 2001]
[Lin, Detmold, Meinel, 2022]
[Black, Harlander, Lange, Rago, Shindler, Witzel, 2023]
[Black, Harlander, Lange, Rago, Shindler, Witzel, 2024]

HQET Sum Rules

[Kirk, Lenz and Rauh, 2017]
[Black, Lang, Lenz, Wuthrich, 2024]
[King, Lenz, Rauh, 2021]

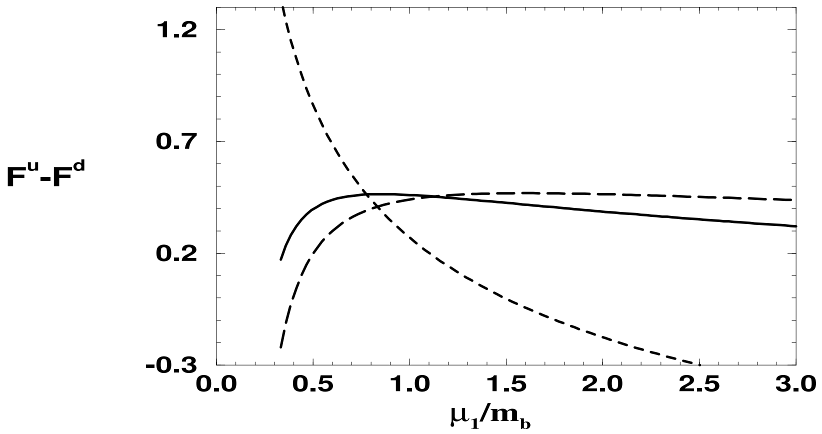
State Of The Art

- Next-to-Leading order calculation of the Wilson coefficients

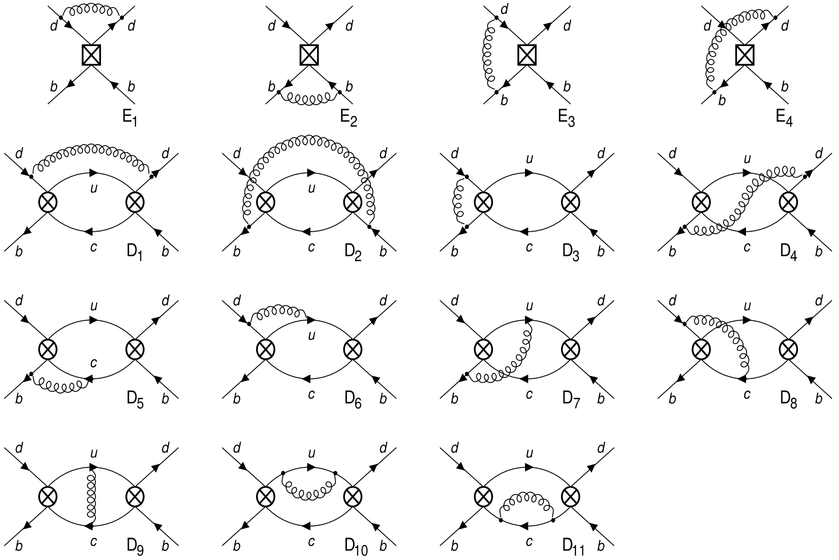
$m_c = 0$

  - *[Ciuchini, Franco, Lubicz, Mescia, 2001]*
  - *[Franco, Lubicz, Mescia, Tarantino, 2002]*
  - *[Beneke, Buchalla, Greub, Lenz, Nierste, 2002]*

No contribution from $\bar{c}c$ intermediate states at NLO



Feynman diagrams

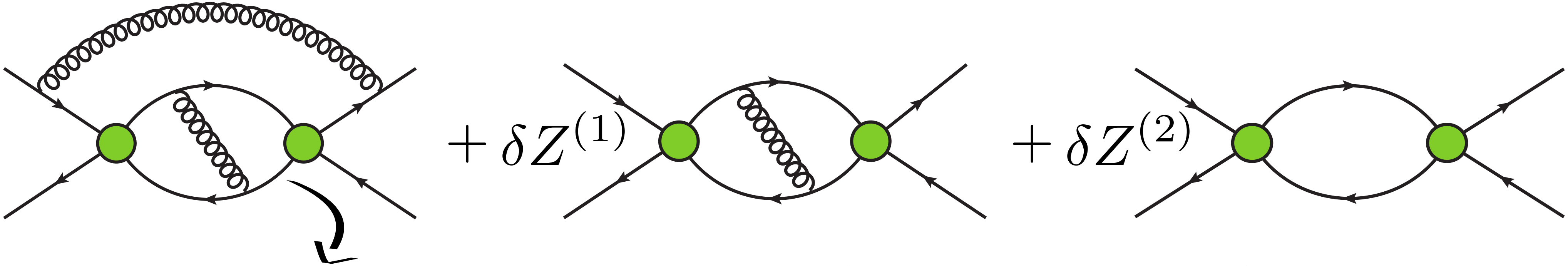


NNLO QCD Calculations

Workflow

[Nogueira, 1991]	QGRAF	Diagram generation based on model files
[Gerlach, Herren, Lang, 2022] [Seidensticker, 1999]	TAPIR + EXP	Implementation of Feynman rules & mapping over topologies
[Vermaseren, 2000] [Reeck, Shtabovenko, Steinhauser, 2024]	FORM	Reduction of Feynman amplitudes in terms of scalar integrals
[Klappert, Lange, Maierhöfer, Usovitsch, 2023]	KIRA	Integration-by-parts reduction to master integrals
[Wolfram Research, 2024]	FORM + MATHEMATICA	Implementation of master integrals & renormalisation

$|\Delta B|=1$ Renormalisation



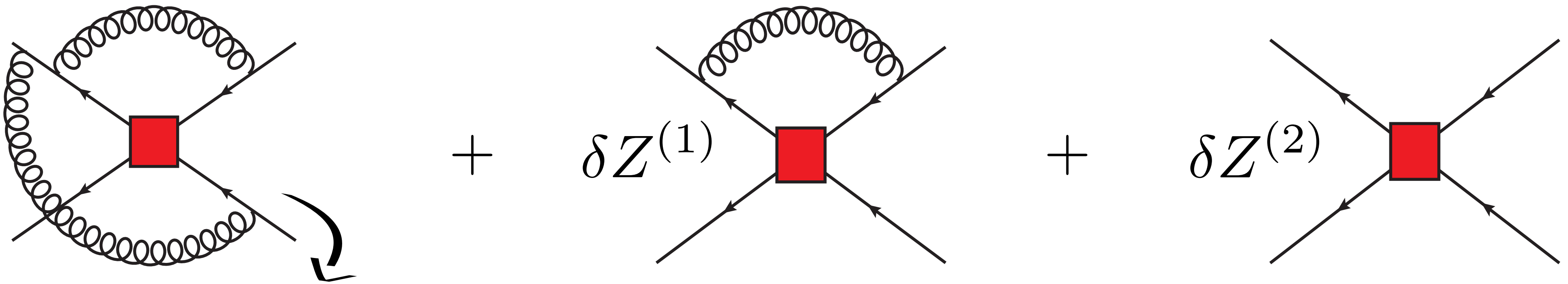
- $\mathcal{O}(2k)$ diagrams

- MIs from recent projects:
  [Reeck, Shtabovenko, Steinhauser, 2024]

$$E[Q_1] = (\bar{q}_1^i \gamma^{\mu_1\mu_2\mu_3} P_L b^j)(\bar{q}_2^j \gamma_{\mu_1\mu_2\mu_3} P_L q_3^i) - (16 - 4\epsilon - 4\epsilon^2) Q_1$$

- Lower order counter-terms
- Mixing matrix up to $\mathcal{O}(\alpha_s^2)$ and appropriate choice of Evanescent operators:
  [Egner, Fael, Schönwald, Steinhauser, 2024]
- $\overline{\textbf{MS}}$ renormalisation of charm quark mass
- on-shell renormalisation of bottom quark mass

$|\Delta B|=0$ Renormalisation



- $\mathcal{O}(1k)$ diagrams

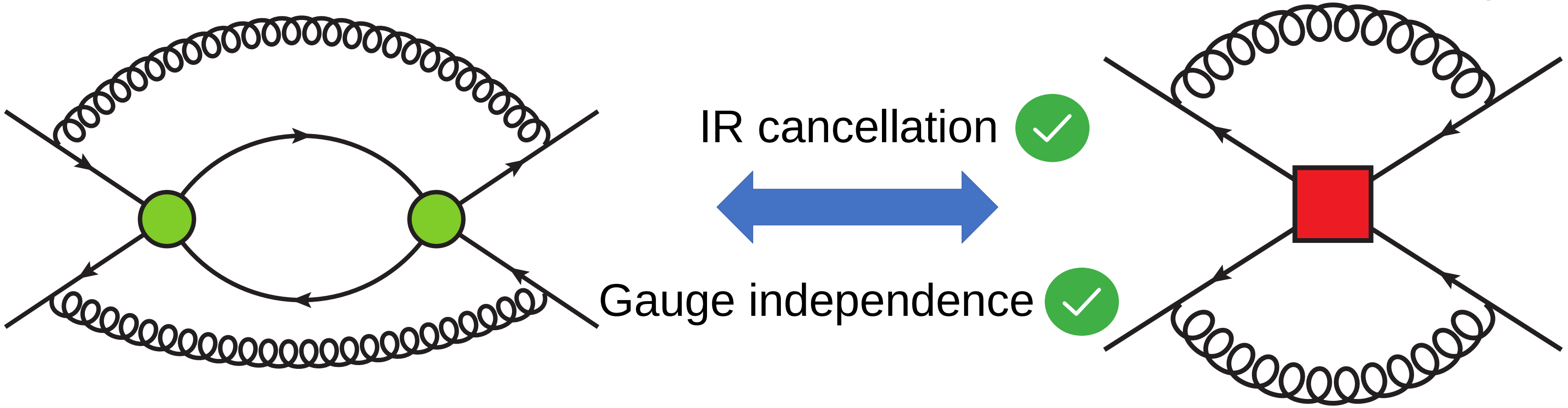
- MIs from recent projects:
  [Reeck, Shtabovenko, Steinhauser, 2024]

$$E[Q] = (\bar{b}\gamma^{\mu_1\mu_2\mu_3} P_L d)(\bar{d}\gamma_{\mu_3\mu_2\mu_1} P_L b) - (8 - 4\epsilon - A\epsilon^2)Q$$

- Lower order counter-terms
- Mixing matrix know up to $\mathcal{O}(\alpha_s)$. We derived the renormalisation of the $|\Delta B|=0$ basis at $\mathcal{O}(\alpha_s^2)$.
- $\overline{\textbf{MS}}$ renormalisation of charm quark mass
- on-shell renormalisation of bottom quark mass

Matching & Wilson Coefficients

- We perform the matching between the "full" $|\Delta B|=1$ theory and the effective $|\Delta B|=0$ theory in **$d=4-2\epsilon$**
- We set $p_{d,u}=0$ and $p_b^2=m_b^2$ $\Longrightarrow$ We generate **IR spurious divergences**
- The IR behaviour of the two theories is the same $\Longrightarrow$ cancellation of $1/\epsilon_{\rm IR}$ poles
- Yet, spurious poles give finite contributions when they multiply $\mathcal{O}(\epsilon)$ parts of evanescent structures $\Longrightarrow$ **Evanescent operators must be included in the matching**



Summary & Conclusions

Summary & Conclusions

- HQE can be used to obtain theoretical predictions in QCD of lifetimes ratios of b-flavoured hadrons
- At leading order, we have $\Gamma(H_b)/\Gamma(H_b') = 1$. First corrections to the ratio comes from phase-space enhanced interactions of the heavy b-quarks with light spectator quarks
- In the framework of HQE, we need to match the non-local correlators in the $|\Delta B| = 1$ side onto a set of local $|\Delta B| = 0$ operators
- So far, this has been done only at $\mathcal{O}(\alpha_s)$. We expand the results in the literature and compute the matching at $\mathcal{O}(\alpha_s^2)$
- We checked the correct IR cancellation in the matching of some particular diagrams and the gauge independence of the final results
- Future outlooks: check the residual scale dependence at NNLO; implement the updated determinations of the non-perturbative bag parameters and check the impacts on $\Gamma(B^+)/\Gamma(B_d^0)$

Summary & Conclusions

- HQE can be used to obtain theoretical predictions in QCD of lifetimes ratios of b-flavoured hadrons

- At leading order, we have $\Gamma(H_b)/\Gamma(H_b')=1$. First corrections to the ratio comes from phase-space enhanced interactions of the heavy b-quarks with light spectator quarks

- In the framework of HQE, we need to match the non-local correlators in the $|\Delta B|=1$ side onto a set of local $|\Delta B|=0$ operators

- So far, this has been done only at $\mathcal{O}(\alpha_s)$. We expand the results in the literature and compute the matching at $\mathcal{O}(\alpha_s^2)$

- We checked the correct IR cancellation in the matching of some particular diagrams and the gauge independence of the final results

- Future outlooks: check the residual scale dependence at NNLO; implement the updated determinations of the non-perturbative bag parameters and check the impacts on $\Gamma(B^+)/\Gamma(B_d^0)$

Thank You!