Power corrections in N-jettiness slicing scheme

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N-jettiness Slicing

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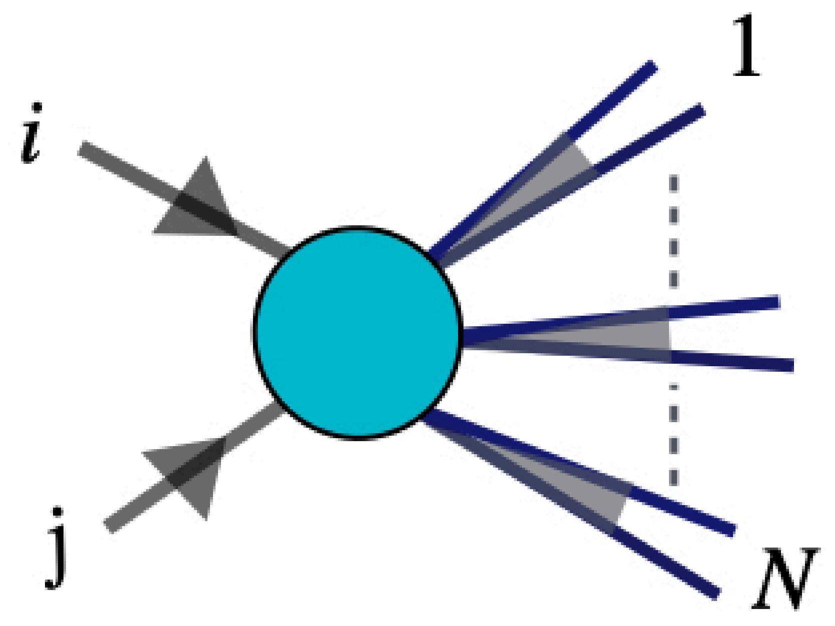
- **N-jettiness variable:** $\tau = \sum_k \min\left(\frac{2p_i \cdot p_k}{P_i}, \frac{2p_j \cdot p_k}{P_j}, \ldots\right)$ [Stewart, Tackmann, Waalewijn ' 10]

- τ can be used to slice the phase space in the following way:



$$\sigma = \underbrace{\int_0^{\tau_o} d\tau \frac{d\sigma}{d\tau}}_{\text{singular}} + \int_{\tau_o} d\tau \frac{d\sigma}{d\tau}$$

— τ_o : Imposed cut on τ

- The SCET factorization theorem makes the above slicing a convenient choice:

$$\int_0^{\tau_o} d\tau \frac{d\sigma}{d\tau} = \int B_\tau \otimes B_\tau \otimes S_\tau \otimes H_\tau \otimes \prod^N J_{i,\tau} + \mathcal{O}(\tau_o)$$

No such theorems or generalizations even at NLO!

Why Power Corrections are required?

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Problem: Slicing schemes show the absence of the cancelation of power suppressed terms. To minimize it, one needs to choose the slicing parameter to be very small, which leads to large numerical cancellations.

$$\sigma = \underbrace{\int_0^{\tau_o} d\tau \frac{d\sigma}{d\tau}}_{\log\tau} + \underbrace{\int_{\tau_o} d\tau \frac{d\sigma}{d\tau}}_{\log\tau,\, \tau\log\tau,\, \tau} = \mathcal{O}(1) + \mathcal{O}(\tau_o)$$

Solution: Include more power suppressed terms in the calculation of the singular terms.

$$\sigma = \underbrace{\int_0^{\tau_o} d\tau \frac{d\sigma}{d\tau}}_{\log\tau,\, \tau\log\tau,\, \tau} + \underbrace{\int_{\tau_o} d\tau \frac{d\sigma}{d\tau}}_{\log\tau,\, \tau\log\tau,\, \tau} = \mathcal{O}(1) + \mathcal{O}(\tau_o^2)$$

Requires us to go beyond the well-understood soft and collinear expansions.

- Power corrections to cross sections have been calculated for relatively simple $2 \to 1$ processes in recent years, mostly at NLO. [Moult, Rothen et. al' 16] [Ebert , Moult, et. al' 18] [Boughezal et. al '18] [Boughezal et. al '19]

Power Corrections to 0-jettiness slicing

- We wish to calculate the power corrections for an arbitrary of color singlet production: $f_a(p_a) + f_b(p_b) \to X(P_X)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = N \int [\mathrm{d}\tilde{P}_X]_m [\mathrm{d}k] (2\pi)^d \, \delta(p_a + p_b - \tilde{P}_X - k) \, \delta(\tau - T_0(p_a, p_b, k)) \, \mathcal{O}(\tilde{P}_X) \sum_{\mathrm{col,pol}} |M|^2 (p_b, p_a, k, \tilde{P}_X) \, .$$

- The power corrections to the above equation can be written in terms of two distinct regions: Soft and Collinear

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim \tau^{-1-2\epsilon} f_s + \tau^{-1-\epsilon} f_c$$

The jettiness function constraints the gluon's energy or it's transverse momentum k_T to be $\mathcal{O}(\tau)$. It defines the two regions which result from expanding in τ.

$$T_0(p_a, p_b, k) = \min\left\{\frac{2\mathrm{p_a}\,.\,\mathrm{k}}{\mathrm{Q}}, \frac{2\mathrm{p_b}\,.\,\mathrm{k}}{\mathrm{Q}}\right\}$$

- If the gluon energy $\sim \mathcal{O}(\tau)$, $k_T \sim \mathcal{O}(1)$: Soft Expansion
- If the gluon energy $\sim \mathcal{O}(1)$, $k_T \sim \mathcal{O}(\tau)$: Collinear Expansion

General Construct

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$$\frac{d\sigma}{d\tau} = N\int [d\tilde{P}_X]_m [dk](2\pi)^d\, \delta(p_a + p_b - \tilde{P}_X - k)\, \delta(\tau - T_0(p_a, p_b, k)) \times \mathcal{O}(\tilde{P}_X) \times \sum_{\text{col,pol}} |M|^2(p_b, p_a, k, \tilde{P}_X)\,.$$

Phase Space — Observable — Matrix Element

- The building blocks to obtain power corrections by expanding in τ are:

Phase Space	Matrix Element	Observable
Can be expressed in a process independent manner using momenta redefinitions.	LBK theorem can be used to calculate next-to-leading power soft contributions. No such theorem for the collinear contribution.	Function of final state momenta. NLP corrections can be calculated using expansions of the final state momenta.

The Soft Contribution

- We get rid of the gluon momenta using a combination of rescaling and Lorentz boost:

$$P_{ab}^{\mu} = \lambda^{-1} \left[\Lambda_s\right]^{\mu}_{\nu}(P_{ab}^{\nu} - k^{\nu}) \qquad \lambda = \sqrt{1 - \frac{2P_{ab} \cdot k}{P_{ab}^2}} \approx 1 - \frac{P_{ab} \cdot k}{P_{ab}^2} + \mathcal{O}(k^2)\,.$$

Phase Space: $d\Phi_m(p_a, p_b, \tilde{P}_X, k) \approx d\Phi_m(p_a, p_b, P_X)\,[dk] \left(1 - \kappa_m \frac{P_{ab} \cdot k}{P_{ab}^2}\right)$ Phase space scales with the rescaling parameter λ

Matrix Element: $|M|^2(p_b, p_a, k, \lambda\Lambda_s^{-1}P_X)\,.$ Using the explicit form of the boost and the LBK theorem, the subleading terms of the matrix element can be calculated.

Observable: $O(\lambda\Lambda_s^{-1}P_X)$ The observable can be expanded in gluon momenta.

The Soft Contribution

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- The final result for the Soft contribution is quite compact:

$$\frac{d\sigma^{(s)}}{d\tau} = N\int [d\Phi_m(p_a, p_b, P_X)] \Bigg\{ \mathcal{O}(P_X) \Bigg[I_1 - \kappa_m I_2 - I_2 \sum_{i\in L_f} p_i^\mu \frac{\partial}{\partial p_i^\mu} \Bigg] |M|^2(p_b, p_a, P_X) - I_2\, |M|^2(p_b, p_a, P_X) \sum_{i=1}^{m} p_i^\mu \frac{\partial}{\partial p_i^\mu} \mathcal{O}(P_X) \Bigg\}$$

Derivatives of Born

Observable contribution in terms of its derivatives

$$I_1 = [\alpha_s]\left(\frac{Q}{\sqrt{s}}\right)^{-2\epsilon} \frac{4}{\epsilon\, \tau^{1+2\epsilon}}, \qquad I_2 = [\alpha_s]\left(\frac{Q\tau}{\sqrt{s}}\right)^{-2\epsilon} \frac{4Q}{s}\left(\frac{1}{2\epsilon} - \frac{1}{2} - \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2)\right)$$

The Collinear contribution

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Momenta Redefinition:

$$k = \frac{k \cdot P_{ab}}{p_a \cdot p_b} p_a + \tilde{k}_a = (1-x) p_a + \tilde{k}_a \longrightarrow \tilde{k}_a = \frac{2kp_a}{s}(p_b - p_a)^\mu + k_\perp^\mu$$

$$k_\perp \sim \sqrt{\tau}$$

- The collinear contribution after incorporating the Lorentz boosts:

$$\frac{\mathrm{d}\sigma^{ca}}{\mathrm{d}\tau} \approx \frac{C_F\,[\alpha_s] Q^{1-\epsilon}}{2\tau^\epsilon} N \int_0^1 \mathrm{d}x\, \mathrm{d}\Phi_m^{xa} \left[\mathrm{d}\Omega_k^{(d-2)}\right] (1-x)^{-\epsilon} \left(1 + \frac{\epsilon \rho_{ak}^*}{2}\right) \mathcal{O}(\Lambda_a^{-1} Q_X) \sum_{\mathrm{pol,col}} |M(p_b, p_a, k, \Lambda_a^{-1} Q_X)|^2$$

– Expand in $\rho_{ak}^* \sim k_\perp^2 \sim \tau$

$$\Lambda_a : \Lambda(Q_x^\mu + \tilde{k}^\mu, P_x^\mu)$$

The Collinear contribution

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— No subleading theorem for collinear expansion

— The $\sqrt{\tau}$ terms in the expansion makes things worse.

— Very tedious expansion for complicated processes.

The Collinear contribution

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Momenta Redefinition:

$$k = \frac{k \cdot P_{ab}}{p_a \cdot p_b} p_a + \tilde{k}_a = (1-x) p_a + \tilde{k}_a \longrightarrow \tilde{k}_a = \frac{2kp_a}{s}(p_b - p_a)^\mu + k_\perp^\mu$$

$$k_\perp \sim \sqrt{\tau}$$

- The collinear contribution after incorporating the Lorentz boosts:

$$\frac{d\sigma^{ca}}{d\tau} \approx \frac{C_F\,[\alpha_s] Q^{1-\epsilon}}{2\tau^\epsilon} N \int_0^1 dx\; d\Phi_m^{xa} \left[d\Omega_k^{(d-2)}\right] (1-x)^{-\epsilon} \left(1 + \frac{\epsilon \rho_{ak}^*}{2}\right) \mathcal{O}(\Lambda_a^{-1} Q_X) \sum_{\text{pol,col}} |M(p_b, p_a, k, \Lambda_a^{-1} Q_X)|^2$$

- To target process independence, we use the Lorentz invariance of the matrix element:

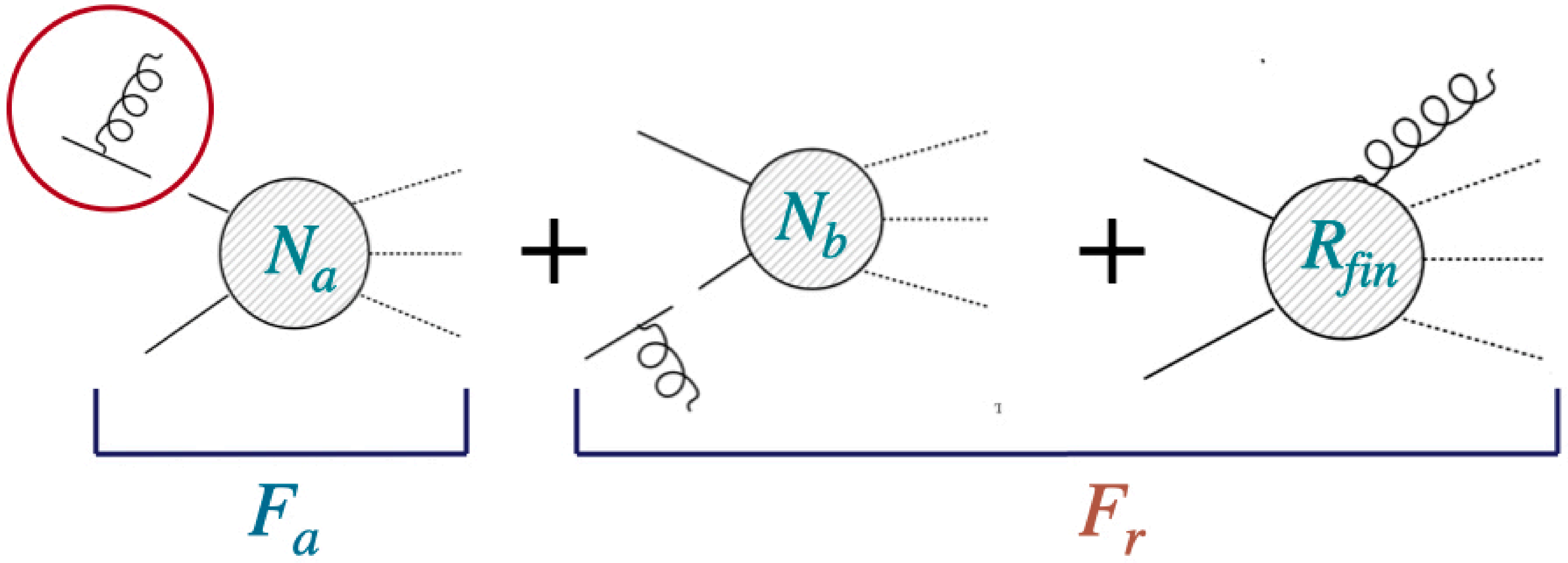
$$\sum_{\text{pol}} |M|^2 (p_b, p_a, k, \Lambda_a^{-1} Q_X) = \sum_{\text{pol}} |M|^2 (\Lambda_a p_b, \Lambda_a p_a, \Lambda_a k, Q_X)$$

The Collinear contribution

$$M = -\, g_s T^a \epsilon_\nu^* \bar{v}_b \left[N_a \frac{(\hat{p}_a - \hat{k})\gamma^\nu}{(-2p_a \cdot k)} + R_{\text{fin}}^\nu(p_b, p_a, k, Q_X) + \gamma^\nu \frac{(\hat{p}_b - \hat{k})}{2p_b \cdot k} N_b \right] u_a \,.$$

$$|M|^2 = F_{aa} + F_{ar} + F_{rr}$$

Subleading in τ



- F_{aa} and F_{ar} contain logarithmic and power-like soft singularities.
- We need to expand each term systematically in $k_\perp^2$ to isolate the poles and obtain the subleading contribution.

The Collinear contribution

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$$\frac{2P_{qq}(x)}{2p_a\cdot k}\mathrm{Tr}\left[N_a\hat{p}_a N_a^+\hat{p}_b\right]_{\Lambda_a} \qquad \mathrm{Tr}\left[\frac{N_a\hat{k}_a\gamma_\nu\hat{p}_a N_{\mathrm{fin},a}^{+,\nu}\hat{p}_b}{(-2p_a\cdot k)}\right]_{\Lambda_a}$$

Their expansions are rather complicated

Requires us to calculate the derivatives of the Green's functions N_a, N_b and R_{fin}.

All such terms are systematically expanded and combined to get a finite result for both sectors.

The master formula for the 'sector a' collinear expansion :

$$\begin{aligned}
\mathrm{C}^{\mathrm{NLP},a} = & -2\int \mathrm{d}\Phi_m\, |\mathcal{M}(p_b,p_a,P_X)|^2\, \mathcal{O}(P_X) + \int \mathrm{d}x\, \mathrm{d}\Phi_m^{xa}\Bigg\{\frac{\bar{P}_{qq}(x)}{x}\Bigg[W_a(x) \\
& +\frac{s}{4}(1-x)g_\perp^{\rho\alpha}\left(D_\rho^{xa,b}\,|\mathcal{M}|^2(p_b,xp_a,\ldots)-2\mathrm{Tr}\left[N_a\gamma_\rho N_a^+\hat{p}_b\right]\right)b_{a_\alpha}^{\ \mu\nu}L_{\mu\nu} \\
& +|\mathcal{M}|^2(p_b,xp_a,\ldots)\, l_a^{\mu\nu}(x)L_{\mu\nu}-\frac{s\,(1-x)}{4}|\mathcal{M}|^2(p_b,xp_a,\ldots)t_a^{\mu\mu_1,\nu\nu_1}L_{\mu\mu_1}L_{\nu\nu_1}\Bigg] \\
& -\frac{1}{(1-x)_+}\left(\kappa_m+2p_a^\mu\frac{\partial}{\partial p_a^\mu}+\left(g^{\rho\sigma}+\omega_{ab}^{\rho\sigma}\right)L_{\rho\sigma}\right)|\mathcal{M}|^2(p_b,xp_a,\ldots) \\
& -\frac{2p_b^\nu}{(1-x)_+}\left(\mathrm{Tr}\left[N_a\hat{p}_a R_{\mathrm{fin},\nu}^+\hat{p}_b\right]+\mathrm{c.c.}\right)+F_{\mathrm{fin},a} \\
& +\frac{s}{4}(1-x)g_\perp^{\alpha\beta}\Bigg[-2\mathrm{Tr}\left[N_a\gamma_\beta N_a^+\hat{p}_b\right] \\
& +\mathrm{Tr}\left[N_a\gamma_\beta\gamma_\rho\hat{p}_a\left(R_{\mathrm{fin}}^{\rho,+}+\frac{N_b^+(\hat{p}_b-(1-x)\hat{p}_a)\gamma^\rho}{(1-x)s}\right)\hat{p}_b\right]+\mathrm{c.c.} \\
& +\frac{2x}{1-x}\mathrm{Tr}\left[N_a\hat{p}_a\left(R_{\mathrm{fin},\beta}^+-\frac{N_b^+\hat{p}_a\gamma_\beta}{s}\right)\hat{p}_b\right]+\mathrm{c.c.}\Bigg]b_{a_\alpha}^{\ \mu\nu}L_{\mu\nu}\Bigg\}\,\mathcal{O}(P_X),
\end{aligned}$$

The Collinear contribution: Master Formula

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The master formula for the collinear expansion:

$$
\begin{aligned}
C^{\mathrm{NLP},a} = & -2\int \mathrm{d}\Phi_m\, |M(p_b,p_a,P_X)|^2\, \mathcal{O}(P_X) + \int \mathrm{d}x\, \mathrm{d}\Phi_m^{xa} \Bigg\{ \frac{\bar{P}_{qq}(x)}{x} \Bigg[W_a(x) \\
& + \frac{s}{4}(1-x) g_\perp^{\rho\alpha} \left(D_\rho^{xa,b} |M|^2(p_b, xp_a, \ldots) - 2\mathrm{Tr}\left[N_a \gamma_\rho N_a^+ \hat{p}_b \right] \right) b_{a_\alpha}^{\mu\nu} L_{\mu\nu} \\
& + |M|^2(p_b, xp_a, \ldots)\, l_a^{\mu\nu}(x) L_{\mu\nu} - \frac{s\,(1-x)}{4} |M|^2(p_b, xp_a, \ldots) t_a^{\mu\mu_1,\nu\nu_1} L_{\mu\mu_1} L_{\nu\nu_1} \Bigg] \\
& - \frac{1}{(1-x)_+} \left(\kappa_m + 2p_a^\mu \frac{\partial}{\partial p_a^\mu} + \left(g^{\rho\sigma} + \omega_{ab}^{\rho\sigma} \right) L_{\rho\sigma} \right) |M|^2(p_b, xp_a, \ldots) \\
& - \frac{2p_b^\nu}{(1-x)_+} \left(\mathrm{Tr}\left[N_a \hat{p}_a R_{\mathrm{fin},\nu}^+ \hat{p}_b \right] + \mathrm{c.c.} \right) + F_{\mathrm{fin},a} \\
& + \frac{s}{4}(1-x) g_\perp^{\alpha\beta} \Bigg[-2\mathrm{Tr}\left[N_a \gamma_\beta N_a^+ \hat{p}_b \right] \\
& + \mathrm{Tr}\left[N_a \gamma_\beta \gamma_\rho \hat{p}_a \left(R_{\mathrm{fin}}^{\rho,+} + \frac{N_b^+ (\hat{p}_b - (1-x)\hat{p}_a)\gamma^\rho}{(1-x)s} \right) \hat{p}_b \right] + \mathrm{c.c.} \\
& + \frac{2x}{1-x} \mathrm{Tr}\left[N_a \hat{p}_a \left(R_{\mathrm{fin},\beta}^+ - \frac{N_b^+ \hat{p}_a \gamma_\beta}{s} \right) \hat{p}_b \right] + \mathrm{c.c.} \Bigg] b_{a_\alpha}^{\mu\nu} L_{\mu\nu} \Bigg\}\, \mathcal{O}(P_X),
\end{aligned}
$$

Can be fully calculated using Green's functions and their derivatives.

The Final result

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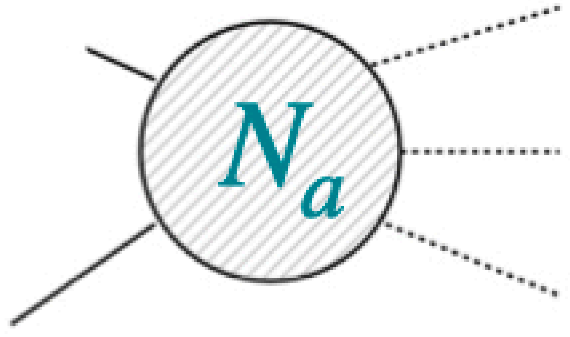
$$\frac{d\sigma^{\text{NLP}}}{d\tau} = \frac{[\alpha_s] C_F\, Q}{s} N \left\{ 2 \left[\ln\left(\frac{Q\tau}{s}\right) + 1 \right] \mathrm{C}^{\text{NLP},s} + \mathrm{C}^{\text{NLP},a} + \mathrm{C}^{\text{NLP},b} \right\}.$$

Soft Contribution: $\mathrm{C}^{\text{NLP},s} = \int d\Phi_m(p_a, p_b, P_X) \left(\kappa_m + \sum_{i \in L_f} p_i^\mu \frac{\partial}{\partial p_i^\mu} \right) |M(p_b, p_a, P_X)|^2\, \mathcal{O}(P_X),$

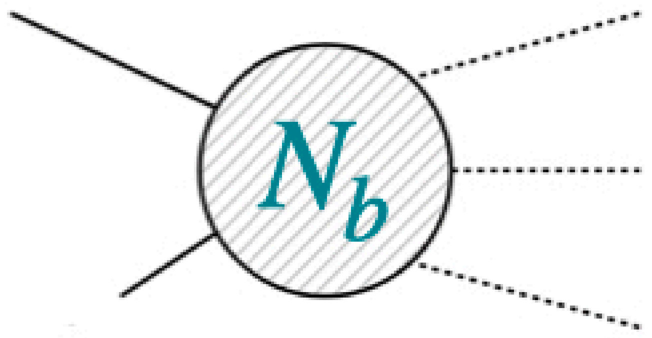
Collinear Contribution: $C_{NLP,a}$ already presented in the previous slide. $C_{NLP,b}$ is the collinear contribution from the other quark line and can be calculated in a similar way.

The methodology for calculating the required Green's functions for a generic process is already present!

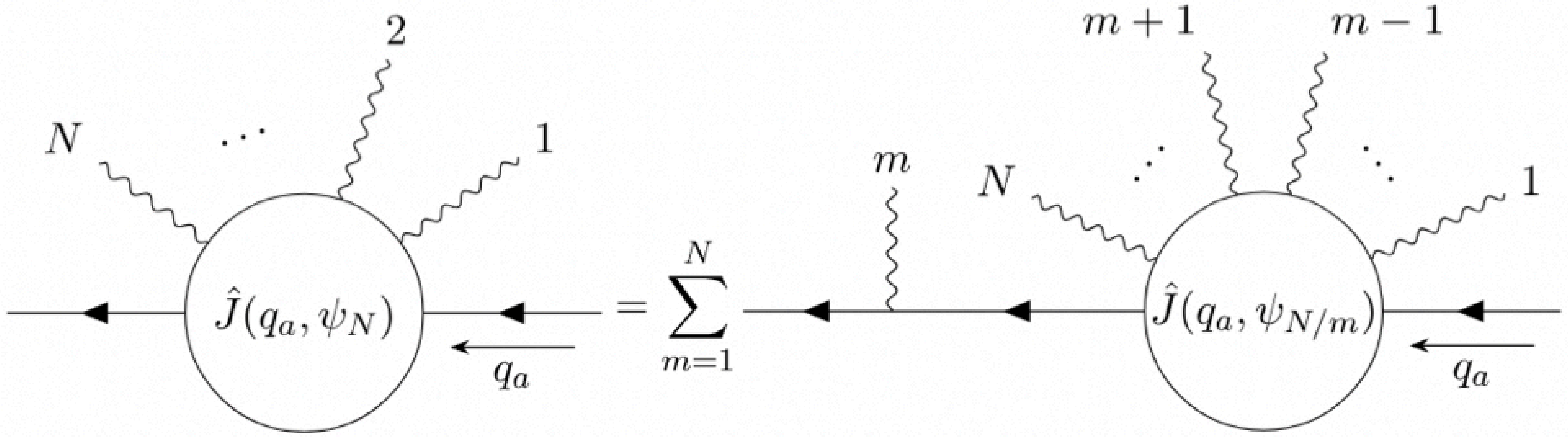
Berends-Giele recursions for N photons



$N(xp_a, P_X)$



$N(p_a, P_X)$



$$\hat{J}(q_a, \psi_N) = \frac{i}{\hat{q}_a - \hat{Q}_N} \sum_{m=1}^{N} \left(ie_q \hat{\epsilon}_m \right) \hat{J}(q_a, \psi_{N/m})$$

$$N_{a,b} = \sum_{m=1}^{N} \left(ie_q \hat{\epsilon}_m \right) \hat{J}\left(q_{a,b}, \psi_{N/m}\right)$$

Berends-Giele recursions for N photons

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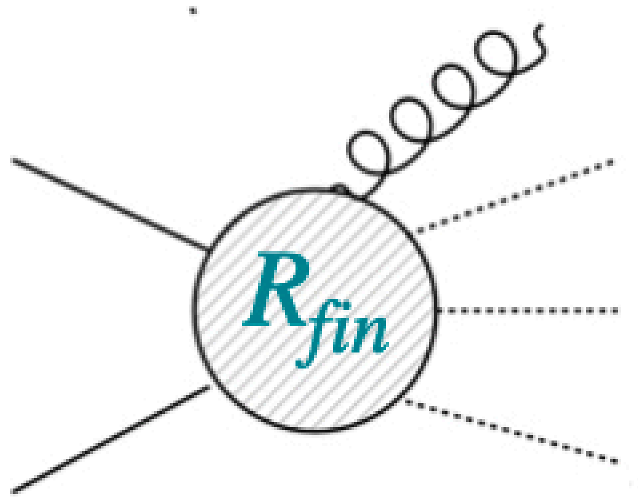
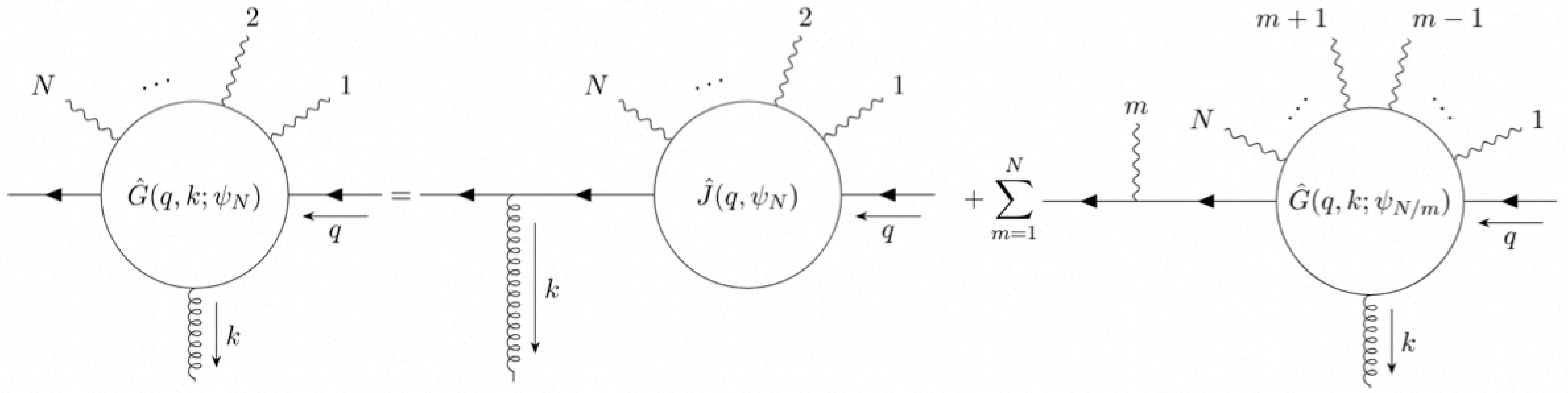


$R^{\nu}_{\text{fin}}(q, k, P_X)$

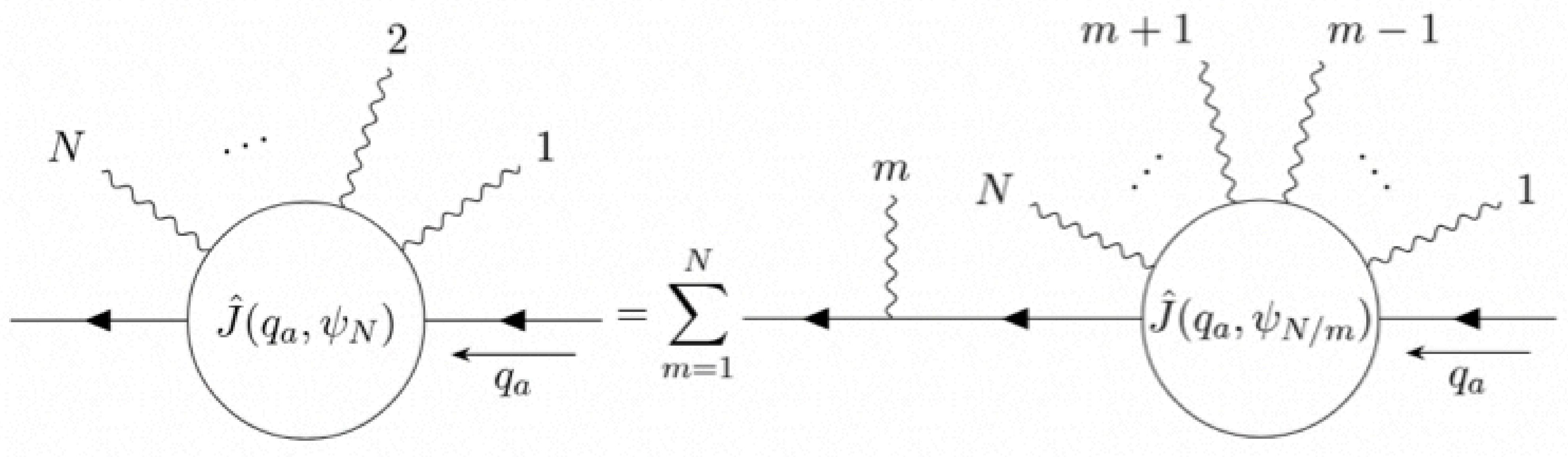
Recursively using a combination of J and G currents.



$$\hat{G}^{\nu}(q, k; \psi_N) = \frac{i}{\hat{q} - \hat{k} - \hat{Q}_N} \left[i\gamma^{\nu} \hat{J}(q, \psi_N) + \sum_{m=1}^{N} \left(ie_q \hat{\epsilon}_m \right) \hat{G}^{\nu} \left(q, k; \psi_{N/m} \right) \right]$$

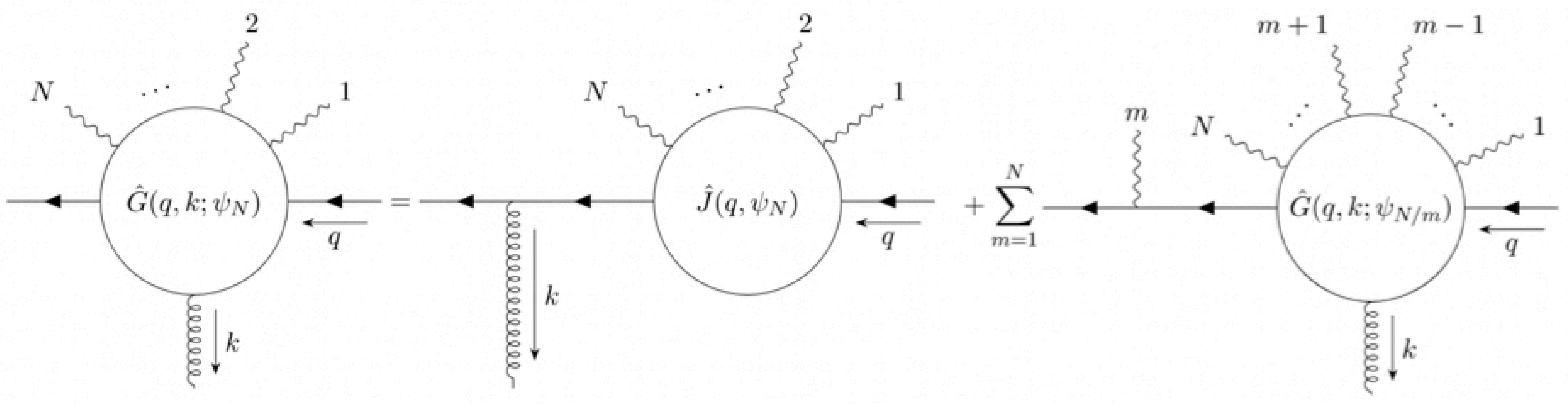
$$R^{\nu}_{\text{fin}}(q, k, \psi_N) = \sum_{m=1}^{N} \left(ie_q \hat{\epsilon}_m \right) \hat{G}^{\nu} \left(q, k; \psi_{N/m} \right) .$$

Berends-Giele recursions for N photons



$$\hat{J}(q_a, \psi_N) = \frac{i}{\hat{q}_a - \hat{Q}_N} \sum_{m=1}^{N} \left(ie_q \hat{\epsilon}_m\right) \hat{J}(q_a, \psi_{N/m}),$$

$$N_{a,b} = \sum_{m=1}^{N} \left(ie_q \hat{\epsilon}_m\right) \hat{J}\left(q_{a,b}, \psi_{N/m}\right),$$



$$\hat{G}^{\nu}(q, k; \psi_N) = \frac{i}{\hat{q} - \hat{k} - \hat{Q}_N} \left[i\gamma^{\nu} \hat{J}(q, \psi_N) + \sum_{m=1}^{N} \left(ie_q \hat{\epsilon}_m\right) \hat{G}^{\nu}\left(q, k; \psi_{N/m}\right) \right]$$

$$R^{\nu}_{\text{fin}}(q, k, \psi_N) = \sum_{m=1}^{N} \left(ie_q \hat{\epsilon}_m\right) \hat{G}^{\nu}\left(q, k; \psi_{N/m}\right).$$

Derivatives of N_a, N_b and R_{fin} can also be calculated in a recursive manner.

Applications and Checks

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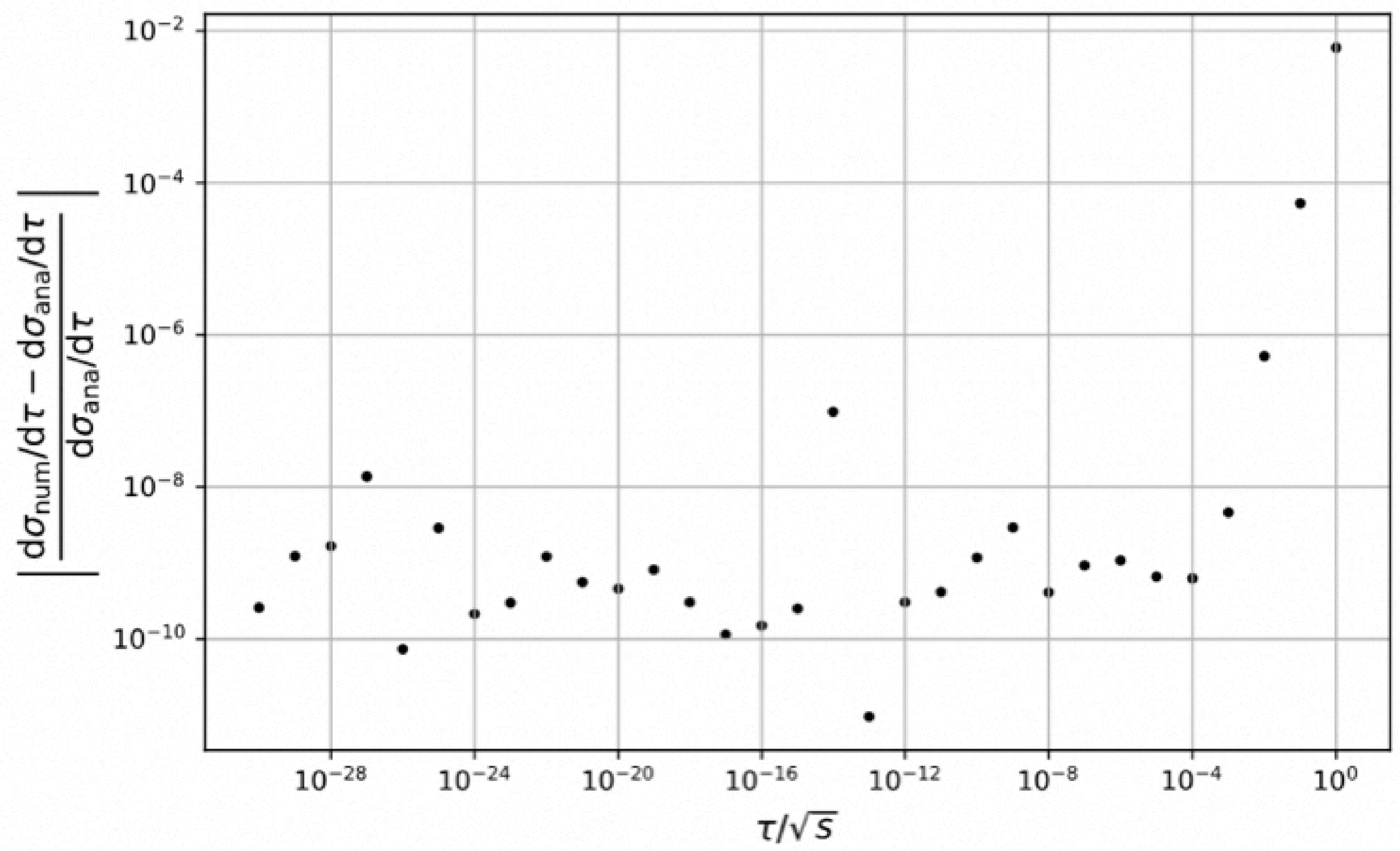
Drell Yan: $q\bar{q} \to \gamma^* \to l^+ l^-$

- Analytical check versus the naive expansion of the DY matrix element.
- Numerical checks using a χ^2 fit .

$$\mathcal{O}(p_1, p_2) = \theta\left((p_1 + p_2)^2 - s_0\right)$$

- Comparison against the DY results presented in *[Ebert, Moult et. al.' 10]* using the relevant observable and adapting to their definition of the jettiness function.



coefficient	fit	analytic
$C_{\mathrm{LP,LL}}$	$-4.740\,740\,718$	$-4.740\,740\,741$
$C_{\mathrm{LP,NLL}}$	$13.741\,118\,266$	$13.741\,118\,217$
$C_{\mathrm{NLP,LL}}$	$0.000\,17$	$0.000\,00$
$C_{\mathrm{NLP,NLL}}$	-1.0710	-1.0725

Applications and Checks

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2-photon production:

- Application and check against the naive expansion of its matrix element.

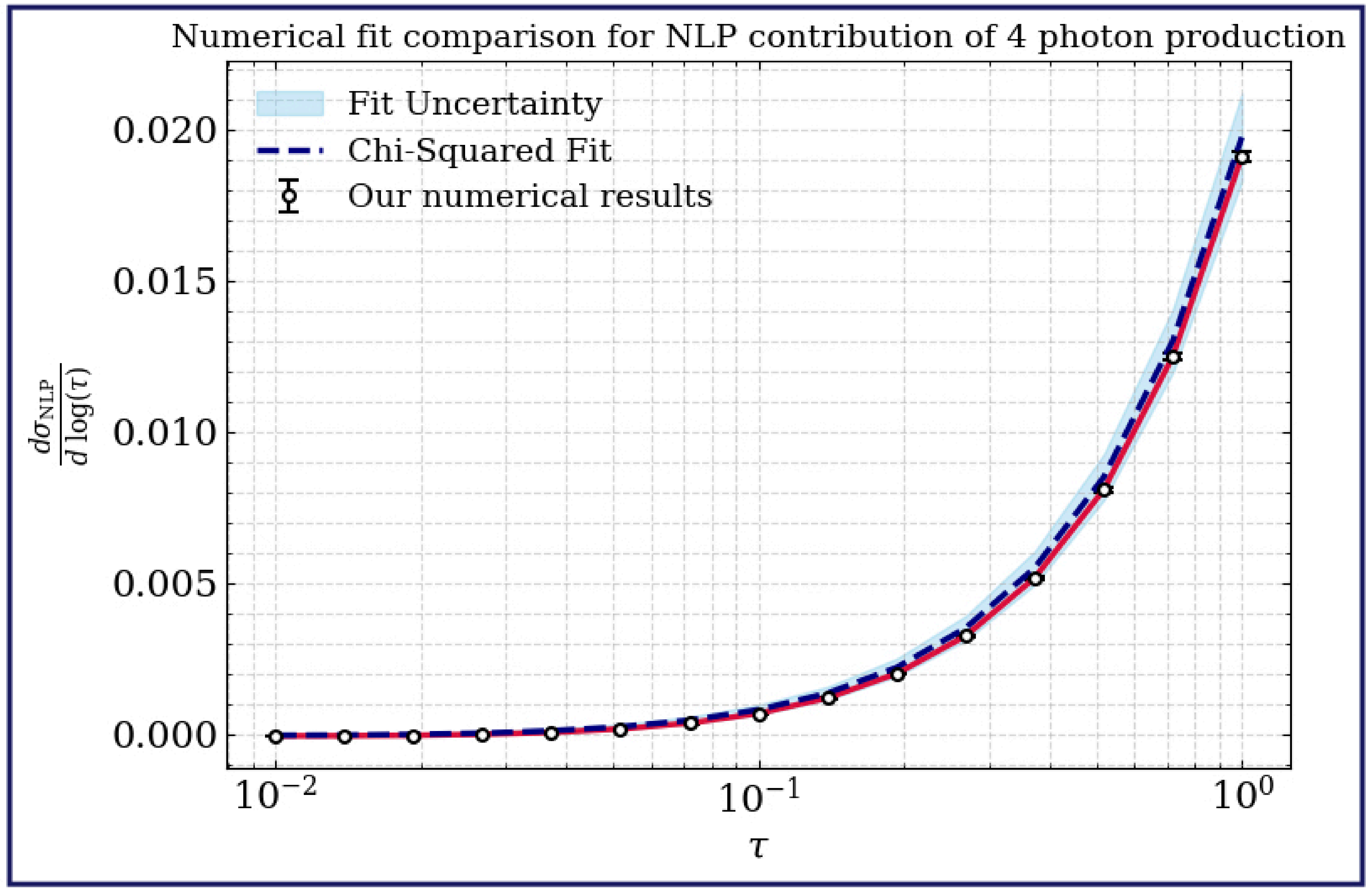
  (Complete check of the master formula.)

4-photon production:

- To show that the master formula can be implemented and used to calculate any generic 0-jettiness process.

$$\mathcal{O}(P_{N\gamma}) = \frac{p_{1,\perp}^2\, p_{2,\perp}^2\, \cdots\, p_{N,\perp}^2}{s^N}$$

- Checked against a χ^2 fit analysis and find good agreement.



$\sqrt{s} = 100$ GeV

Conclusions and Outlook

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- We calculated the power corrections to zero-jettiness cross sections for arbitrary number of massless and colorless final states at NLO.
- We show that momenta redefinitions, a familiar concept in fixed order calculations, can be used to express the phase space in a generic manner.
- We investigate the major challenge regarding the collinear expansion of a generic matrix element at subleading powers, as factorization breaks down at this level. While this still remains a challenge, a master formula has been derived to make the calculation simpler.

Future Works:

- The first complete calculation of power corrections to 1-jettiness processes at NLO.

Already In Progress ! Stay Tuned !

Thank You for your attention!

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Backup Slides

Backup: Collinear Expansion

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$$\frac{2P_{qq}(x)}{2p_a\cdot k}\mathrm{Tr}\left[N_a\hat{p}_aN_a^+\hat{p}_b\right]_{\Lambda_a}$$

- $\mathrm{Tr}\left[N_a(p_b,p_a-k,Q_X)\,x\hat{p}_a\,N_a^+(p_b,p_a-k,Q_X)\,\hat{p}_b)\right]_{\Lambda_a} = |M|^2(p_b,xp_a,Q_X)$
- $-\frac{k_\perp^\mu}{2}\left(D_\mu^{xa,b}|M|^2(p_b,xp_a,Q_X)-2\mathrm{Tr}\left[N_a\gamma_\mu N_a^+\hat{p}_b\right]\right)+\frac{2kp_a}{s}W_a(x)\,.$

$$W_a(x) = -p_{b,\mu}W_{a1}^\mu(x)+(1-x)W_{a2}(x),$$

$$W_{a1}^\mu(x) = \mathrm{Tr}\left[N_a^{(1),\mu}\,x\hat{p}_a\,N_a^+\hat{p}_b\right]+\mathrm{c.c.},$$

$$W_{a2}(x) = -\frac{1}{x}p_{b,\mu}W_{a1}^\mu+\frac{1}{4x}\left(4+(p_b^\mu-xp_a^\mu)D_\mu^{xa,b}\right)|M|^2(p_b,xp_a,Q_X)$$
$$-\frac{s}{16}g_\perp^{\mu\nu}\left(D_\nu^{xa,b}D_\mu^{xa,b}\,|M|^2(p_b,xp_a,Q_X)-4D_\mu^{xa,b}\,\mathrm{Tr}\left[N_a\gamma_\nu N_a^+\hat{p}_b\right]\right).$$

The derivatives of the Green's function start appearing here, and some contributions can be written as the derivatives of the Born matrix element.

Backup: Collinear Expansion

The intermediate expression we get after expanding each contribution of the matrix element. The next step is to apply the boost and expand each term in the correct power of $k_\perp$.

$$
\begin{aligned}
|M|^2 = &\frac{2P_{qq}(x)}{2p_a\cdot k}\mathrm{Tr}\left[N_a\hat{p}_aN_a^+\hat{p}_b\right] + \frac{2}{s}\frac{(1+x+x^2-x^3)}{(1-x)^2}\mathrm{Tr}\left[N_a\hat{p}_aN_a^+\hat{p}_b\right] \\
&-\frac{4p_b^\nu}{s(1-x)}\mathrm{Tr}\left[N_a\hat{p}_aR^+_{\mathrm{fin},\nu}\hat{p}_b\right] + \mathrm{c.c.} - \frac{2x}{s(1-x)}\mathrm{Tr}\left[N_a\hat{p}_aN_a^+\hat{p}_b\right] + \mathrm{c.c.} \\
&+\frac{1}{sx}\mathrm{Tr}\left[N_a\hat{p}_b\gamma^\nu\hat{p}_aN_b^+\frac{\hat{p}_a\gamma_\nu\hat{p}_b}{s}\right] + \mathrm{c.c.} - \frac{1}{sx}\mathrm{Tr}\left[N_a\hat{p}_b\gamma_\nu\hat{p}_aR^{\nu,+}_{\mathrm{fin}}\hat{p}_b\right] + \mathrm{c.c.} \\
&-\frac{2}{s}\mathrm{Tr}\left[N_a\hat{p}_aN_a^+\hat{p}_b\right] + \frac{2}{sx}\mathrm{Tr}\left[N_a\hat{p}_bN_a^+\hat{p}_b\right] + F_{rr,a} \\
&+\frac{2}{2p_a\cdot k}\mathrm{Tr}\left[N_a\hat{\kappa}_aN_a^+\hat{p}_b\right] + \mathrm{Tr}\left[\frac{N_a\hat{\kappa}_a\gamma_\nu\hat{p}_aN^{+,\nu}_{\mathrm{fin},a}\hat{p}_b}{(-2p_a\cdot k)}\right] + \mathrm{c.c.} \\
&+\frac{2\kappa_{a,\nu}}{(1-x)(-2p_a\cdot k)}\mathrm{Tr}\left[N_ax\hat{p}_a\left(R^{\nu,+}_{\mathrm{fin}} - N_b^+\frac{\hat{p}_a\gamma^\nu}{s}\right)\hat{p}_b\right] + \mathrm{c.c.}.
\end{aligned}
$$

Backup: Collinear Expansion

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- The derivative of the currents are given by:

For $N_{a,b}$ currents:

$$\hat{J}^{(1),\mu}(q_a,\psi_N) = -\frac{1}{\hat{q}_a - \hat{Q}_N}\gamma^\mu J^{(0)}(q_a,\psi_N) + \frac{i}{\hat{q}_a - \hat{Q}_N}\sum_{m=1}^{N}\left(ie_q\hat{\epsilon}_m\right)\hat{J}^{(1),\mu}(q_a,\psi_{N/m}),$$

$$\hat{J}^{(2),\mu\nu}(q_a,\psi_N) = -\frac{1}{\hat{q}_a - \hat{Q}_N}\left[\gamma^\mu J^{(1),\nu}(q_a,\psi_N) + \gamma^\nu J^{(1),\mu}(q_a,\psi_N)\right] + \frac{i}{\hat{q}_a - \hat{Q}_N}\sum_{m=1}^{N}\left(ie_q\hat{\epsilon}_m\right)\hat{J}^{(2),\mu\nu}(q_a,\psi_{N/m}).$$

For R_{fin} current:

$$\hat{G}^{(1),\nu\mu}(p_a,(1-x)p_a,\psi_N) = \frac{1}{2}\frac{1}{x\hat{p}_a - \hat{Q}_N}\gamma^\mu \hat{G}^{(0),\nu}(p_a,(1-x)p_a,\psi_N) - \frac{1}{x\hat{p}_a - \hat{Q}_N}\left[\frac{1}{2x}\gamma^\nu \hat{J}^{(1),\mu}(p_a,\psi_N) + \sum_{m=1}^{N}(e_q\hat{\epsilon}_m)\hat{G}^{(1),\nu\mu}(p_a,(1-x)p_a,\psi_{N/m})\right],$$