Higgs Mass Predictions in the CP-Violating High-Scale NMSSM

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Together with: Thi Nhung Dao, Martin Gabelmann, Margarete Mühlleitner, Heidi Rzehak
(NMSSMCALC Collaboration)

Collaborative Research Center TRR 257
P³H
Particle Physics Phenomenology after the Higgs Discovery

Young Scientists Meeting of the CRC TRR 257
Heidelberg – 21/07/2025

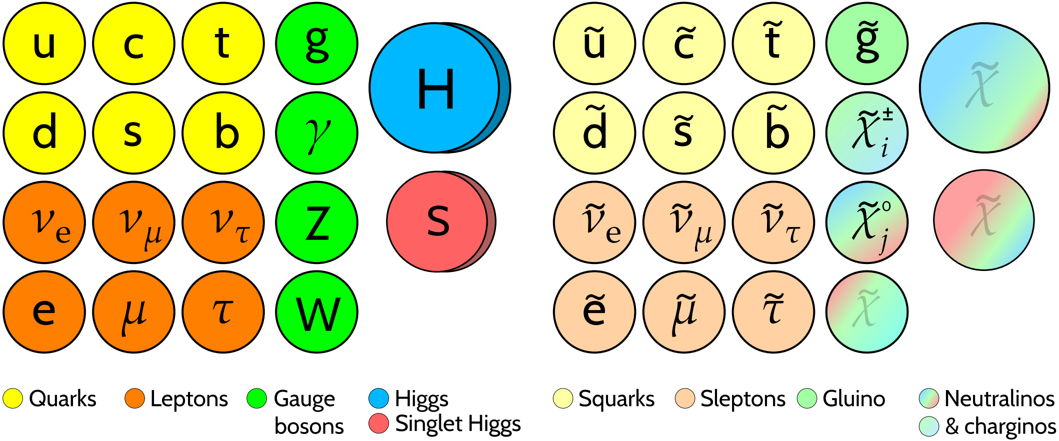
Outline

1. Next-to-Minimal Supersymmetric Standard Model
2. Calculating M_h^{SM} in the high-scale NMSSM
3. Numerical analysis
4. Summary

The Next-to-Minimal Supersymmetric Standard Model

Standard Model particles**

Supersymmetric partners



The Next-to-Minimal Supersymmetric Standard Model

Standard Model particles**

Supersymmetric partners

Superpotential of the $\mathbb{Z}_3$-symmetric NMSSM

$$\mathcal{W}_{\text{NMSSM}} = \left[y_e \hat{H}_d \cdot \hat{L}\hat{E}^c + y_d \hat{H}_d \cdot \hat{Q}\hat{D}^c - y_u \hat{H}_u \cdot \hat{Q}\hat{U}^c\right] - \lambda \hat{S}\hat{H}_d \cdot \hat{H}_u + \frac{1}{3}\kappa \hat{S}^3$$

- Complex scalar singlet extension of the MSSM (λ, κ complex, e.g. $\lambda = |\lambda| e^{i\varphi_\lambda}$)
- Supersymmetry softly broken via addition of $\mathcal{L}_{\text{soft}}$ terms to Lagrangian

Quarks | Leptons | Gauge bosons | Higgs | Singlet Higgs | Squarks | Sleptons | Gluino | Neutralinos & charginos

The CP-violating NMSSM Higgs sector

Mass basis

$$H_d = \begin{pmatrix} \frac{v_d + h_d + i a_d}{\sqrt{2}} \\ h_d^- \end{pmatrix}, \quad H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{v_u + h_u + i a_u}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{e^{i\varphi_S}}{\sqrt{2}}(v_S + h_S + i a_S)$$

$$\tan\beta = \frac{v_u}{v_d}$$

$$v = \sqrt{v_u^2 + v_d^2} = 246\,\mathrm{GeV}$$

$h_d, h_u, h_S, a_d, a_u, a_S$ and $h_d^\pm, h_u^\pm$

mixing to

$h_1, h_2, h_3, h_4, h_5, G^0$ and $h^\pm, G^\pm$

- $m_{h_1} < m_{h_2} < m_{h_3} < m_{h_4} < m_{h_5}$, G^0 and $G^\pm$ would-be Goldstone bosons
- h_1 **or** h_2 is the “SM-like” (h_u-like) Higgs, $m^2_{h^{\text{“SM”}}} \leq m_Z^2 \cos^2 2\beta + |\lambda|^2 v^2 \sin^2 2\beta$
- CP violation possible at tree level (unlike in the MSSM) via $\varphi_u, \varphi_S, \varphi_\lambda, \varphi_\kappa$

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< 125 GeV! Higher orders δm_h required!

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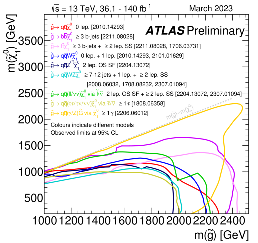
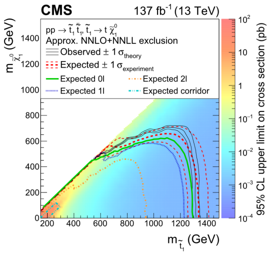
Supersymmetry – out of reach?





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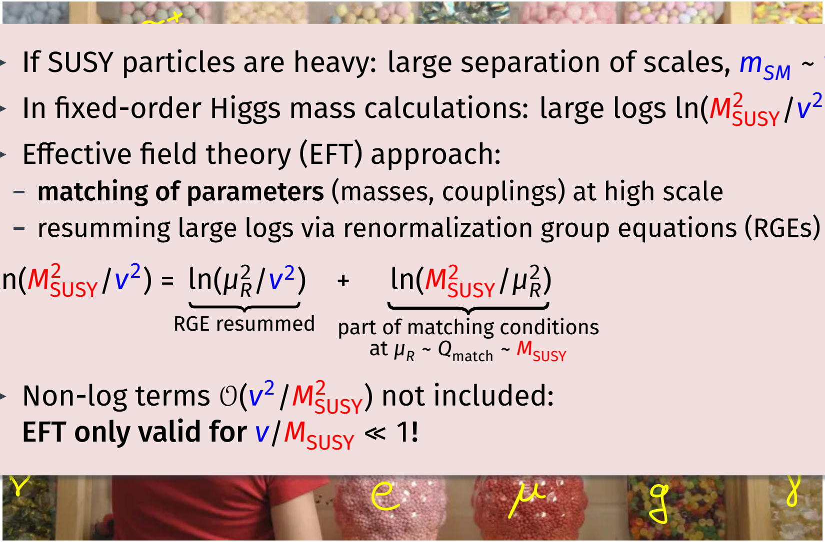
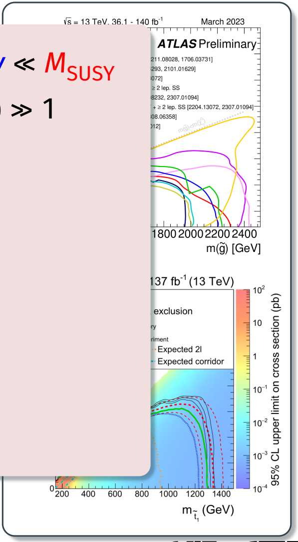


- If SUSY particles are heavy: large separation of scales, $m_{SM} \sim v \ll M_{SUSY}$
- In fixed-order Higgs mass calculations: large logs $\ln(M_{SUSY}^2/v^2) \gg 1$
- Effective field theory (EFT) approach:
  - **matching of parameters** (masses, couplings) at high scale
  - resumming large logs via renormalization group equations (RGEs)

$$\ln(M_{SUSY}^2/v^2) = \underbrace{\ln(\mu_R^2/v^2)}_{\text{RGE resummed}} + \underbrace{\ln(M_{SUSY}^2/\mu_R^2)}_{\text{part of matching conditions at } \mu_R \sim Q_{match} \sim M_{SUSY}}$$

- Non-log terms $\mathcal{O}(v^2/M_{SUSY}^2)$ not included:
  EFT only valid for $v/M_{SUSY} \ll 1$!



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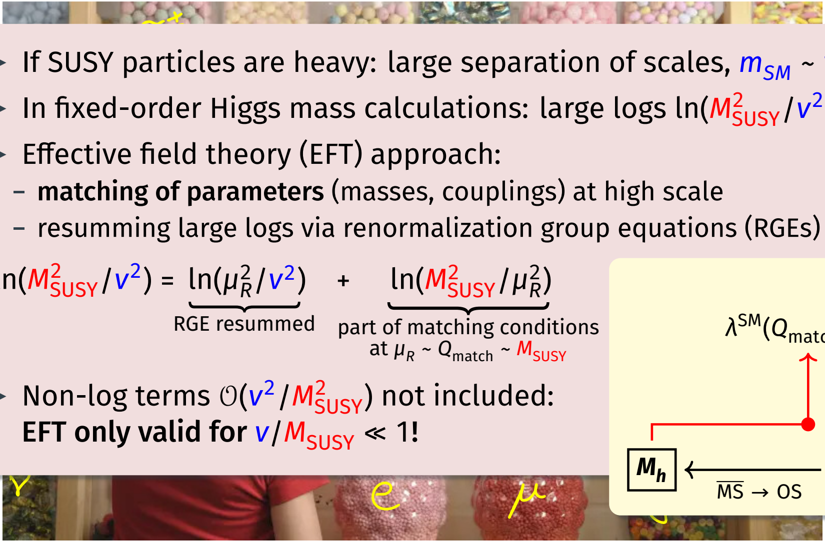

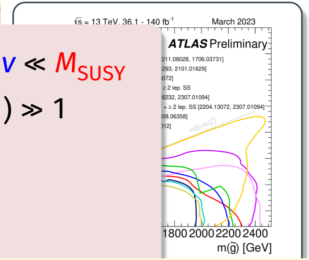

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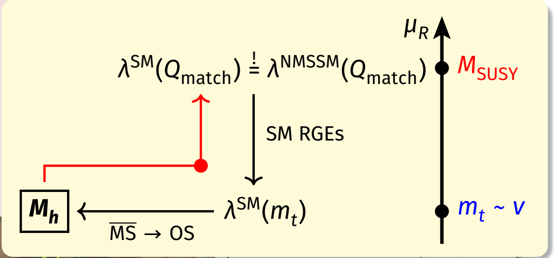

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Higgs mass calculations in the NMSSM

Constraining the NMSSM parameter space with the M_h^{SM} measurement

Fixed-order status:

- Full 1L, 2L, in different renormalisation schemes ($\overline{\text{DR}}$, mixed OS-$\overline{\text{DR}}$)
  [Ellwanger et al. '93, '05][Elliot et al. '93][Pandita '93][King, White '95][Degrassi, Slavich '10][Staub et al. '10][Drechsel et al. '17][Ham et al. '01-'07][Funakubo, Tao '04][Cheung et al. '10][Goodsell, Staub '17][Domingo et al. '17][Goodsell et al. '15][Ender et al. '12][Graf et al. '12][Mühlleitner et al. '14][Dao et al. '19-'21]
- Tools: `FlexibleSUSY` *[Athron et al.]*, `NMSSMCALC` *[Baglio et al.]*, `NMSSMTools` *[Ellwanger et al.]*, `SOFTSUSY` *[Allanach et al.]*, `SARAH/Spheno` *[Porod, Staub]*

Status of effective field theory approach:

- Pole-mass matching in `FlexibleEFTHiggs` *[Athron et al. '17-'25]*, `SARAH/Spheno` *[Staub, Porod '17]*
- Automated full 1L EFT matching in `SARAH` *[Gabelmann et al. '18-'19]*
- Full 1L + (NMSSM-specific) 2L EFT matching in the real NMSSM *[Bagnaschi, Goodsell, Slavich '22]*

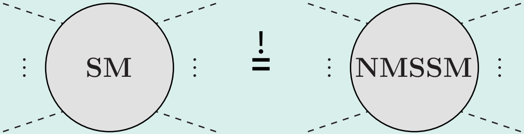
Matching the NMSSM parameters to the SM

Matching conditions relate the SM and NMSSM couplings such that both theories describe the same physics at the high scale $Q = Q_{\mathrm{match}}$

$V^{\mathrm{SM}} \supset \lambda^{\mathrm{SM}}|H|^4$

$$\lambda^{\mathrm{SM}}(Q) \leftrightarrow \lambda^{\mathrm{NMSSM}}(Q), \quad Y_i^{\mathrm{SM}}(Q) \leftrightarrow Y_i^{\mathrm{NMSSM}}(Q), \quad g_j^{\mathrm{SM}}(Q) \leftrightarrow g_j^{\mathrm{NMSSM}}(Q), \quad \ldots$$

In general (at the scale Q_{match}):



n-loop *m*-point amplitudes with the same external (light) states should yield the same results

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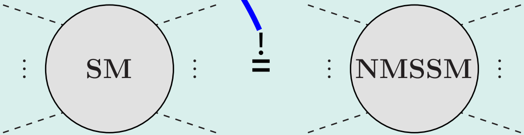

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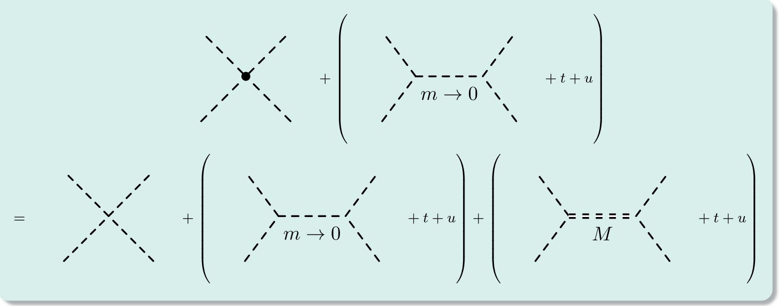
m light scale

M heavy scale



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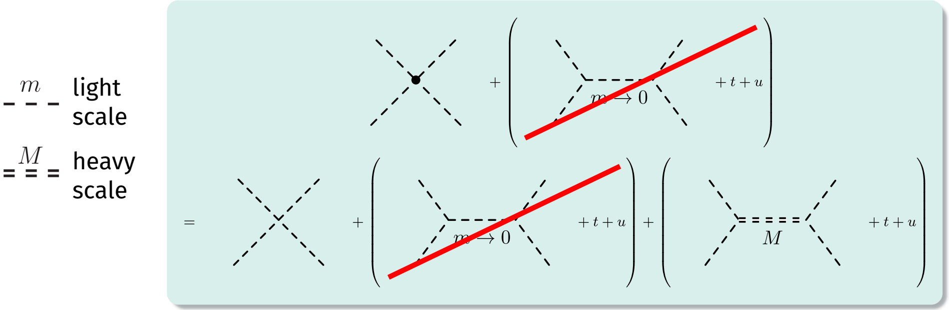

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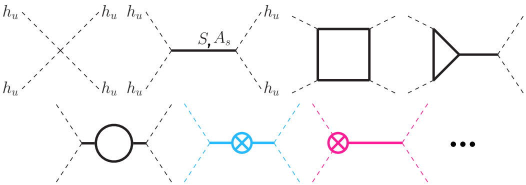
Quartic-coupling matching M_h^{IV}

"$\lambda^{\mathrm{SM}} = \lambda^{\mathrm{NMSSM}}$"



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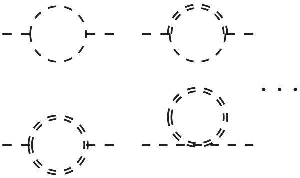

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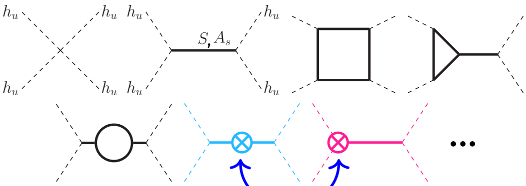

Careful treatment of tadpoles in counterterms!

Pole-mass matching M_h^{II}

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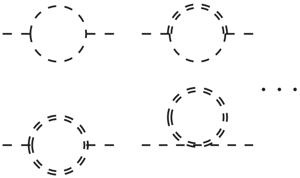

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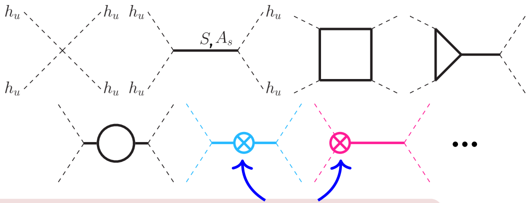

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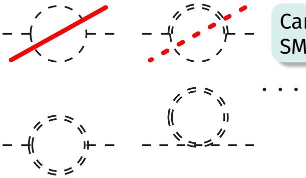

Cancellation of SM contributions

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Quartic-coupling matching M_h^{IV}

"$\lambda^{\text{SM}} = \lambda^{\text{NMSSM}}$"

- Matching of 4-point functions
- Evaluate directly in $v \to 0$ limit

→ **Analytical expressions**

Pole-mass matching M_h^{II}

"$M_h^{\text{SM}} = M_h^{\text{NMSSM}}$"

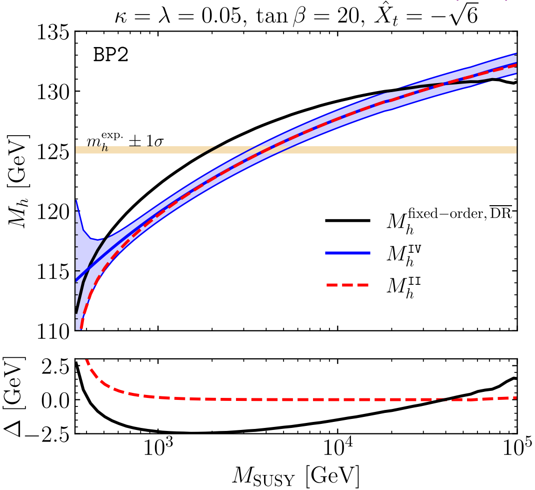
- Matching of 2-point functions
- $\mathcal{O}(v^2/M_{\text{SUSY}}^2)$ terms included

→ **Numerical expressions**

⇒ Compare both approaches, estimate size of $\mathcal{O}(v^2/M_{\text{SUSY}}^2)$ terms

Dependence on M_{SUSY}

[CB, Dao, Gabelmann, Mühlleitner, Rzehak '25]



BP2: MSSM-like point from KUTS report *[Slavich, Heinemeyer et al. '20]*

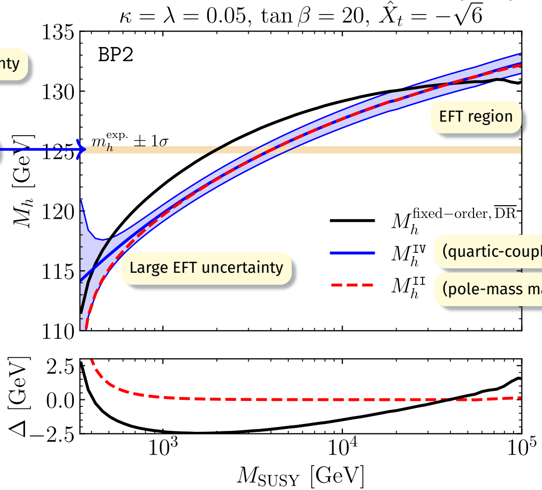
Dependence on M_{SUSY}

[CB, Dao, Gabelmann, Mühlleitner, Rzehak '25]

Blue band: total SM + SUSY + EFT uncertainty

Exp. allowed region

Δ: difference to M_h^{IV}



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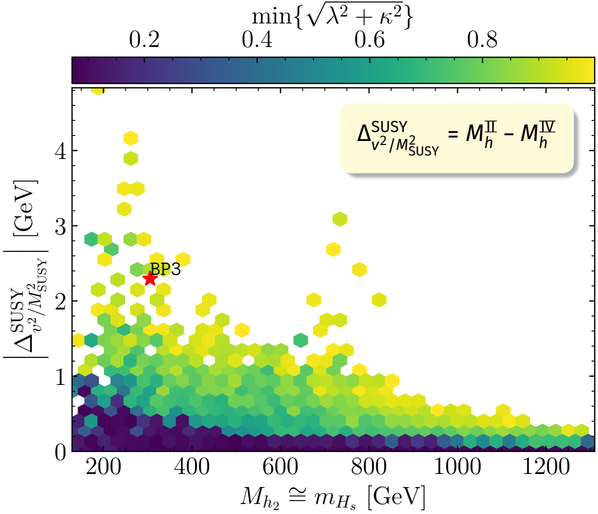
v/M_{SUSY} effects

[CB, Dao, Gabelmann, Mühlleitner, Rzehak '25]



Parameter scan:

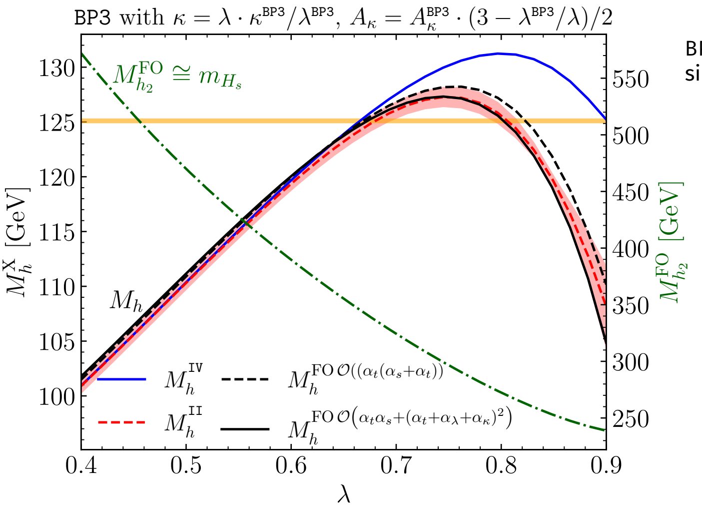
$$100\text{ GeV} \le M_1, M_2 \le 1.5\text{ TeV}, \quad 100\text{ GeV} \le \mu_{eff} \le 1.5\text{ TeV},$$
$$1\text{ TeV} \le m_{\tilde{Q}_{L_3}}, m_{\tilde{t}_{R_3}} \le 2.5\text{ TeV}, \quad M_{SUSY} = \sqrt{m_{\tilde{Q}_{L_3}} m_{\tilde{t}_{R_3}}},$$
$$M_3 = \max\{M_{SUSY}, 2.3\text{ TeV}\}, \quad -2.5\text{ TeV} \le A_\kappa \le 100\text{ GeV},$$
$$-2.5\text{ TeV} \le A_\lambda \le 2.5\text{ TeV}, \quad -\sqrt{6} \le \hat{X}_t \le \sqrt{6},$$
$$1 \le \tan\beta \le 20, \quad 0.05 \le \lambda, \kappa \le 1.0$$

Bounds and constraints:

- $M_{\tilde{t}_1} > 1310$ GeV, $M_{\tilde{g}} > 2.3$ GeV, $M_{H^+} > 500$ GeV
- $122\text{ GeV} \le M_h^{II} \le 128$ GeV
- Using HiggsTools *[Bahl et al. '22]* to check for compatibility with data from LEP, Tevatron, LHC

The case of a light singlet

[CB, Dao, Gabelmann, Mühlleitner, Rzehak '25]



BP3: point with small singlet-like mass

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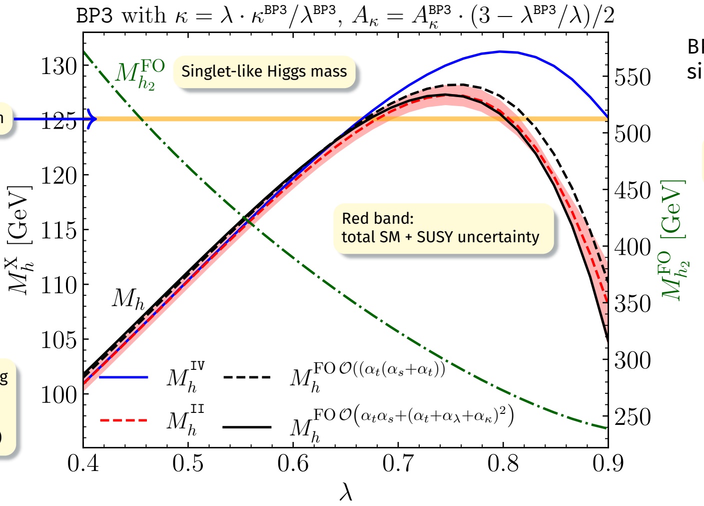

BP3: point with small singlet-like mass

Small M_S: large impact of v/M_{SUSY} terms

M_h^{IV}: quartic-coupling matching

M_h^{II}: pole-mass matching (contains v/M_{SUSY} terms)

Summary

Calculation of the SM-like Higgs mass for the CP-violating high-scale NMSSM

- Applicable for heavy SUSY masses ($v/M_{SUSY} \ll 1$), estimate of v/M_{SUSY} terms
- Implementation at full 1L (+2L MSSM) via quartic-coupling & pole-mass matching
  → Excellent agreement found for CPC and CPV case in $v \to 0$ limit ✓

✍ **Outlook:** NMSSM-specific 2L-$\mathcal{O}(\alpha_s\alpha_t)$ corrections to both matching approaches
[CB, Dao, Fontes, Gabelmann, Mühlleitner, Rzehak; work in progress]

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[Baglio, CB, Dao, Gabelmann, Gröber, Krause, Mühlleitner, Le, Rzehak, Spira, Streicher, Walz]

- Spectrum calculator of 1L & 2L & EFT Higgs masses, self couplings, decay widths
- For the CP-conserving and CP-violating NMSSM
- ...and more: electron EDMs, muon $g-2$, ρ parameter, M_W

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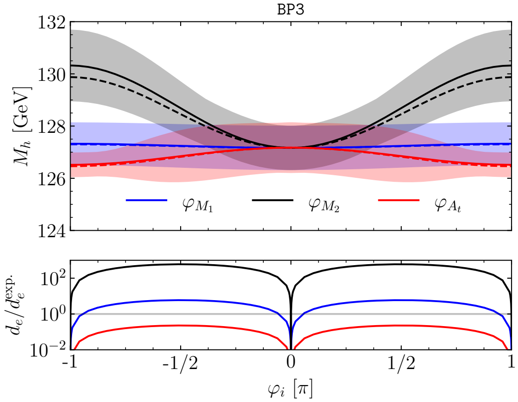
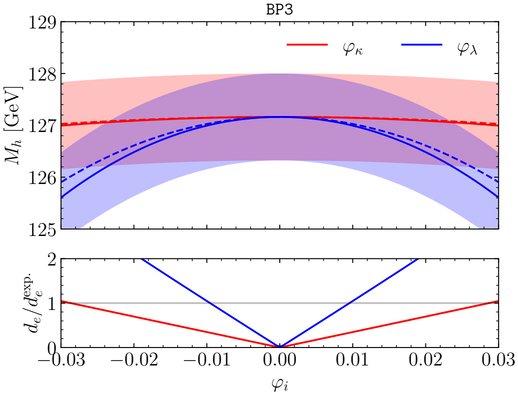
THANK YOU FOR YOUR ATTENTION! ☺

Backup

Effects of CP-violating phases

[CB, Dao, Gabelmann, Mühlleitner, Rzehak '25]

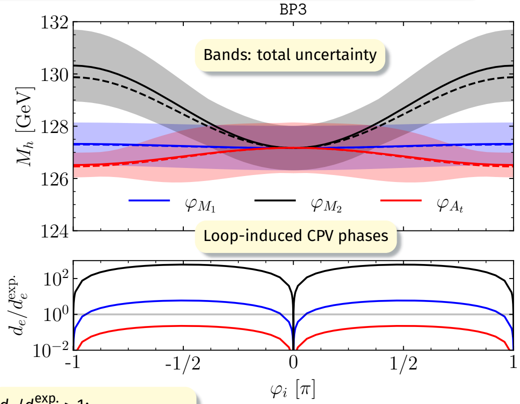


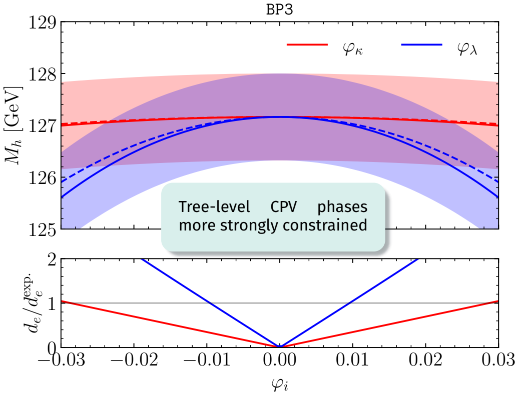


Effects of CP-violating phases

Solid: pole-mass matching; dashed: quartic-coupling matching (shifted)
Solid vs. dashed: v/M_{SUSY} effects from CPV phases

[CB, Dao, Gabelmann, Mühlleitner, Rzehak '25]





$d_e/d_e^{\text{exp.}} > 1$:
excluded by electron EDMs

The Next-to-Minimal Supersymmetric Standard Model

Complex Next-to-Minimal Supersymmetric Standard Model

Superpotential of the $\mathbb{Z}_3$-symmetric NMSSM

$$\mathcal{W}_{\text{NMSSM}} = \left[y_e \hat{H}_d \cdot \hat{L}\hat{E}^c + y_d \hat{H}_d \cdot \hat{Q}\hat{D}^c - y_u \hat{H}_u \cdot \hat{Q}\hat{U}^c\right] - \lambda \hat{S}\hat{H}_d \cdot \hat{H}_u + \frac{1}{3}\kappa \hat{S}^3$$

- Complex scalar singlet extension of the MSSM (λ, κ complex, e.g. $\lambda = |\lambda| e^{i\varphi_\lambda}$)

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Higgs sector

$$H_d = \begin{pmatrix} \frac{v_d + h_d + i a_d}{\sqrt{2}} \\ h_d^- \end{pmatrix}, \quad H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{v_u + h_u + i a_u}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{e^{i\varphi_s}}{\sqrt{2}}(v_S + h_S + i a_S)$$

$$\tan\beta = \frac{v_u}{v_d}$$

$$v = \sqrt{v_u^2 + v_d^2} = 246\,\text{GeV}$$

$h_d, h_u, h_S, a_d, a_u, a_S$ and $h_d^\pm, h_u^\pm$ mixing to $h_1, h_2, h_3, h_4, h_5, G^0$ and $h^\pm, G^\pm$

NMSSM tree-level Higgs masses in the unbroken phase limit

Higgs mass matrix $\mathcal{M}_{ij}$ becomes block-diagonal for $v \to 0$, easy to diagonalise analytically (after applying tadpole equations):

$$h_1 \sim h_u : \quad m_{h_1}^2 \equiv m_{h_{SM}}^2 = 0 \quad (\text{since it is } \propto v^2)$$

$$h_2 \sim a_s : \quad m_{h_2}^2 \equiv m_{A_s}^2 = -\frac{3|\kappa| \operatorname{Re} A_\kappa v_S}{\sqrt{2}\cos\varphi_w}$$

$$h_3 \sim a_d : \quad m_{h_3}^2 = m_{H^\pm}^2 = \frac{|\lambda| v_S \left(\sqrt{2}\operatorname{Re} A_\lambda + |\kappa| v_S \cos\varphi_w\right)}{\sin\beta\cos\beta\cos(\varphi_w - \varphi_y)}$$

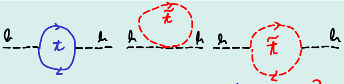
$$h_4 \sim h_d : \quad m_{h_4}^2 = m_{h_3}^2$$

$$h_5 \sim h_s : \quad m_{h_5}^2 \equiv m_{H_s}^2 = \frac{|\kappa| v_S \left(4|\kappa| v_S + \sqrt{2}\frac{\operatorname{Re} A_\kappa}{\cos\varphi_w}\right)}{2}$$

Higgs mass calculations at higher orders

Fixed-order calculations for the Higgs mass:

- Full perturbative series truncated at fixed order



- Reliable for not too high SUSY masses
- Dominant corrections from top/stop sector, e.g. at 1-loop: $\Delta M_h^2 \sim \frac{m_t^4}{v^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2}$

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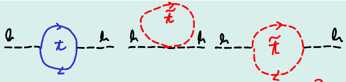

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If SUSY masses (e.g. stops) are heavy: large separation of scales

EW scale: $m_t \sim v \quad \ll \quad$ SUSY scale: $m_{\tilde{t}} \sim M_{\text{SUSY}} \quad \Rightarrow \quad \ln \frac{M_{\text{SUSY}}^2}{v^2} \gg 1$

Large logs $\ln \frac{M_{\text{SUSY}}^2}{v^2}$ from higher orders are relevant and **need to be resummed!**

Effective field theory approach to calculating M_h

Assuming **all** SUSY particles are heavy:

Consider the SM as a (renormalisable) effective field theory valid at the EW scale $\sim m_t \sim v$, and the NMSSM as its UV completion at the high scale $\sim M_{\text{SUSY}}$

Effective field theory calculations:

- Full SUSY theory matched to low-energy EFT at high matching scale Q_{match}
- RGE running from high down to EFT scale: resummation of large logarithms

$$\ln(M_{\text{SUSY}}^2/v^2) = \underbrace{\ln(\mu_R^2/v^2)}_{\text{resummed by RGEs}} + \underbrace{\ln(M_{\text{SUSY}}^2/\mu_R^2)}_{\substack{\text{part of matching conditions} \\ \text{at } \mu_R \sim Q_{\text{match}} \sim M_{\text{SUSY}}}}$$

→ Non-log terms $\mathcal{O}(v^2/M_{\text{SUSY}}^2)$ not included: **EFT only valid for $v/M_{\text{SUSY}} \ll 1$!**

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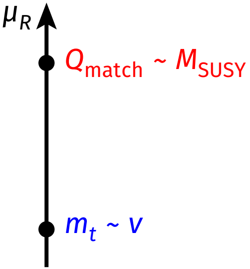

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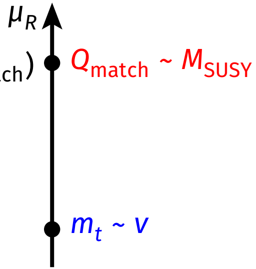

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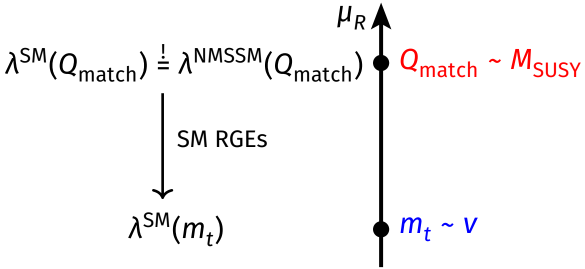

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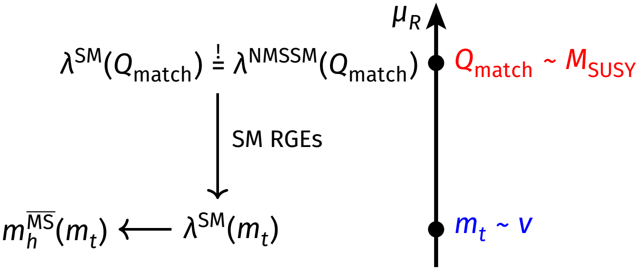

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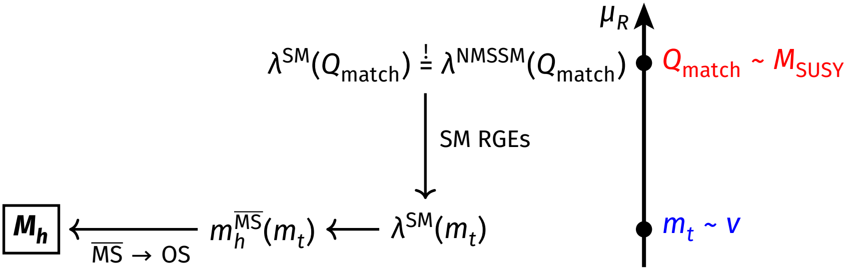

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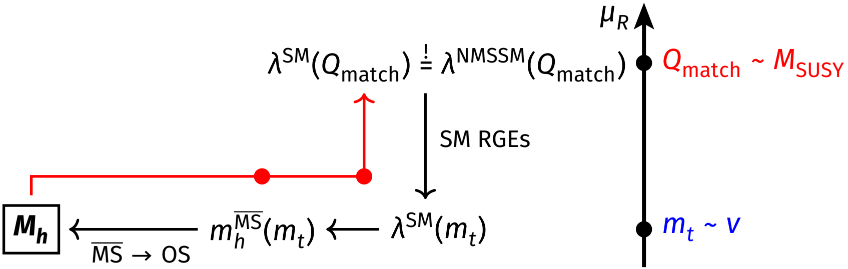

Matching the NMSSM parameters to the SM

Matching conditions relate the SM and NMSSM couplings such that both theories describe the same physics at the high scale $Q = Q_{\text{match}}$

$V^{\text{SM}} \supset \lambda^{\text{SM}} |H|^4$

$$\lambda^{\text{SM}}(Q) \leftrightarrow \lambda^{\text{NMSSM}}(Q), \quad Y_i^{\text{SM}}(Q) \leftrightarrow Y_i^{\text{NMSSM}}(Q), \quad g_j^{\text{SM}}(Q) \leftrightarrow g_j^{\text{NMSSM}}(Q), \quad \ldots$$

Yukawa couplings only appear starting from one-loop corrections

$\Rightarrow$ For one-loop matching of λ^{SM}, match Yukawa couplings at tree level, e.g.:

$$Y_t^{\text{NMSSM},\overline{\text{DR}}} = Y_t^{\text{SM},\overline{\text{MS}}} / \sin\beta$$

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Gauge couplings already appear at tree level

⇒ For one-loop matching of λ^{SM}, match gauge couplings at one loop:

$$g_i^{\text{NMSSM},\overline{\text{DR}}} = g_i^{\text{SM},\overline{\text{MS}}} + \delta g_i^{\text{reg}} + \delta g_i^{\text{thr}}$$

- δg_i^{reg}: one-loop regularisation scheme shifts ($\overline{\text{DR}} \to \overline{\text{MS}}$)
- δg_i^{thr}: one-loop gauge threshold corrections

Quartic-coupling matching

Evaluated in limit of unbroken EW symmetry (i.e. $v_u, v_d \to 0$) and vanishing ext. momentum, but keeping $\tan\beta$ = const., $v_S \neq 0$:

$$\lambda^{\mathrm{SM}}(Q_{\mathrm{match}}) \stackrel{!}{=} \lambda^{\mathrm{NMSSM}}(Q_{\mathrm{match}})$$

with

[SusyHD: Pardo Vega, Villadoro '15]
[Bagnaschi et al. '19]

$$\lambda^{\mathrm{NMSSM},\overline{\mathrm{MS}}}(Q_{\mathrm{match}}) = \lambda_h^{\mathrm{NMSSM,tree}} + \Delta\lambda_h^{\mathrm{NMSSM,1l}} + \Delta\lambda_h^{\mathrm{MSSM,2l}}$$

Note: $\Delta\lambda_h^{\mathrm{MSSM,2l}}$: 2L QCD and mixed QCD-EW corrections in the limit of the CP-conserving MSSM (not sensitive to the CPV phases in $\lambda_h^{\mathrm{NMSSM,tree}}$ and $\Delta\lambda_h^{\mathrm{NMSSM,1l}}$)

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expressed in terms of $\overline{\text{MS}}$ parameters

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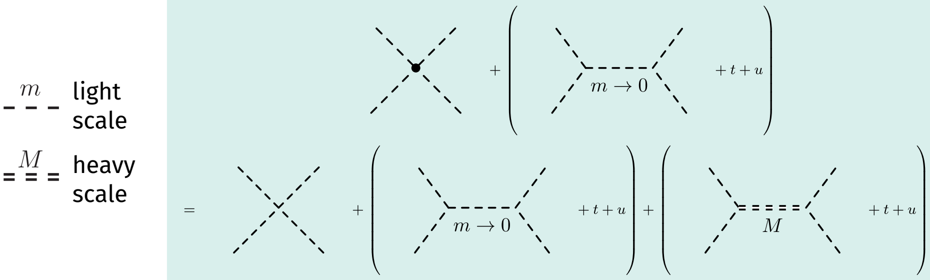

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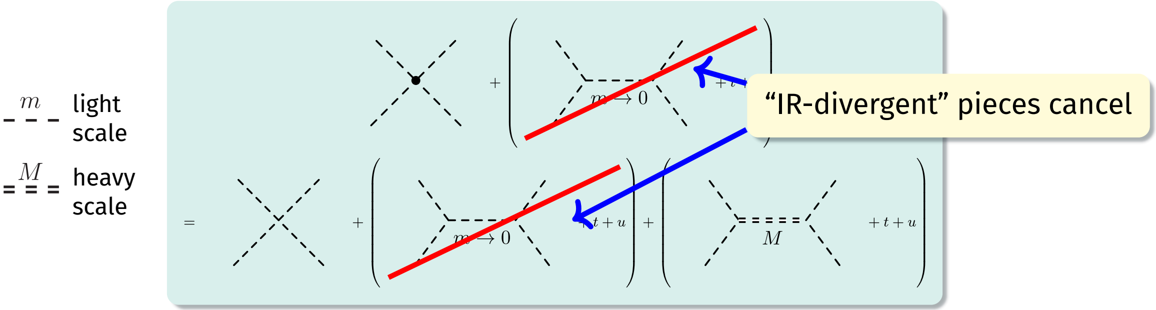

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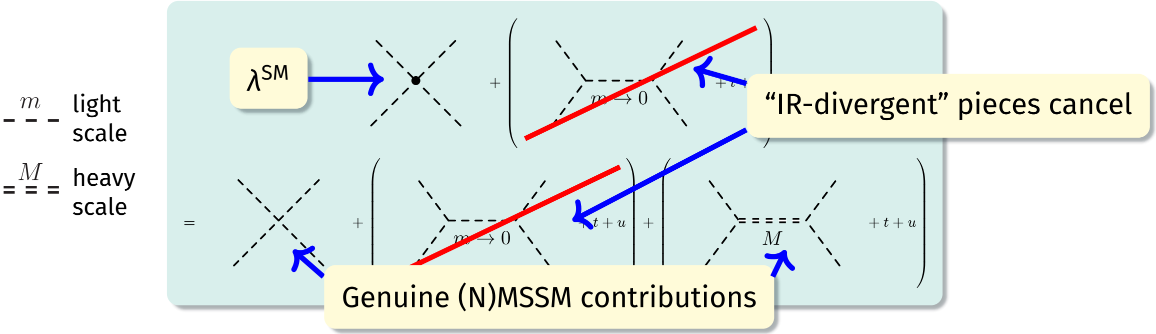

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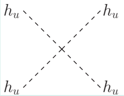
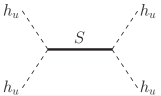
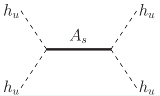
$$\lambda_h^{\mathrm{NMSSM,tree}} = \underbrace{\frac{1}{8}(g_1^2+g_2^2)\cos^2 2\beta}_{\text{MSSM-like terms}} + \underbrace{\frac{1}{4}|\lambda|^2\sin^2 2\beta}_{\text{NMSSM-like terms}}$$



$$\underbrace{- \frac{1}{48|\kappa|^2 m_{H_s}^2(3m_{H_s}^2+m_{A_s}^2)}\Big(3|\kappa|^2 m_{H^\pm}^2(1-\cos 4\beta) + (3m_{H_s}^2+m_{A_s}^2)\big(|\kappa||\lambda|\cos\varphi_y\sin 2\beta - 2|\lambda|^2\big)\Big)^2}_{s/t/u\text{-channel } S}$$



$$\underbrace{- \frac{3}{16m_{A_s}^2}|\lambda|^2(3m_{H_s}^2+m_{A_s}^2)\sin^2 2\beta\sin^2\varphi_y}_{s/t/u\text{-channel } A_s}$$



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Quartic-coupling matching: one-loop contribution

$$\lambda^{\text{NMSSM},\overline{\text{MS}}}(Q_{\text{match}}) = \lambda_h^{\text{NMSSM,tree}} + \Delta\lambda_h^{\text{NMSSM,1l}} + \Delta\lambda_h^{\text{MSSM,2l}}$$

with

$$\Delta\lambda_h^{\text{NMSSM,1l}} = \Delta\lambda_{\square} + \Delta\lambda_{\triangle} + \Delta\lambda_{\text{SE}} + \Delta\lambda_{\text{CT}} + \Delta\lambda_{\text{reg}} + \Delta\lambda_{\text{gauge-thr}}$$

- $\overline{\text{DR}} \to \overline{\text{MS}}$ shifts: $\Delta\lambda_{\text{reg}} = \frac{1}{64\pi^2}\left[\frac{g_2^4}{3}\cos^2 2\beta - \frac{1}{2}\left(g_1^4 + 2g_1^2 g_2^2 + 3g_2^4\right)\right]$
- 1L gauge thresholds: $\Delta\lambda_{\text{gauge-thr}} = \frac{1}{4}\left(g_1\delta g_1^{\text{thr}} + g_2\delta g_2^{\text{thr}}\right)\cos^2 2\beta + \mathcal{O}\left((\delta g_i)^2\right)$

(same as in the MSSM)

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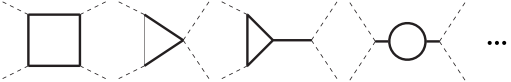

- box contributions: $\Delta\lambda_{\Box}$
- vertex contributions: $\Delta\lambda_{\triangle} = -\frac{g_{hhS}}{m_{H_s}^2}\Delta g_{hhS} - \frac{g_{hhA_s}}{m_{A_s}^2}\Delta g_{hhA_s}$
- self-energy contributions: $\Delta\lambda_{\text{SE}} = -\frac{1}{2}\left(\frac{g_{hhS}}{m_{H_s}^2}\right)^2\Sigma_{SS}(0) - \frac{1}{2}\left(\frac{g_{hhA_s}}{m_{A_s}^2}\right)^2\Sigma_{A_sA_s}(0) - \frac{g_{hhA_s}g_{hhS}}{m_{A_s}^2 m_{H_s}^2}\Sigma_{SA_s}(0)$

1L amplitudes from SARAH with SM part already subtracted *[Gabelmann et al. '18]*

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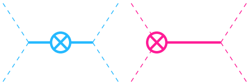
WFR + 

$$\delta^{(1)}Z_h = -\frac{d\Sigma_{hh}}{dp^2}\bigg|_{p^2=0}$$

Contributions from **singlet tadpoles**

$$\Delta\lambda_{\text{CT}} = 2\lambda_h^{\text{NMSSM,tree}}\delta^{(1)}Z_h - \frac{\delta^{(1)}t_{h_S}}{2v_S}\left(\frac{g_{hhS}^2}{m_{H_S}^4} + \frac{g_{hhA_S}^2}{m_{A_S}^4}\right) + \frac{\delta^{(1)}t_{a_S}}{2v_S}\left(3\frac{g_{hhA_S}^2}{m_{A_S}^4}\tan\varphi_\omega - 4\frac{g_{hhA_S}g_{hhS}}{m_{A_S}^2 m_{H_S}^2} - \frac{g_{hhS}^2}{m_{H_S}^4}\tan\varphi_\omega\right)$$
$$+\,\delta^{(1)}\operatorname{Im}A_\lambda\left(\frac{g_{hhA_S}}{m_{A_S}^2}\frac{\partial g_{hhA_S}}{\partial \operatorname{Im}A_\lambda} + \frac{g_{hhS}}{m_{H_S}^2}\frac{\partial g_{hhS}}{\partial \operatorname{Im}A_\lambda}\right)\bigg|_{\text{min}}$$

evaluated at the minimum of the potential

Quartic-coupling matching: one-loop contribution

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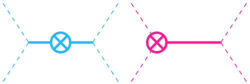
WFR +



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$$\left. + \delta^{(1)}\operatorname{Im}A_\lambda\left(\frac{g_{hhA_S}}{m_{A_S}^2}\frac{\partial g_{hhA_S}}{\partial \operatorname{Im}A_\lambda} + \frac{g_{hhS}}{m_{H_S}^2}\frac{\partial g_{hhS}}{\partial \operatorname{Im}A_\lambda}\right)\right|_{\min}$$

evaluated at the minimum of the potential

a_d tadpole: **non-vanishing** contribution for $v \to 0$ and CPV!

Quartic-coupling matching: couplings

$$g_{hhS} = \frac{1}{2v_s}\Big(|\kappa||\lambda|v_s^2 \sin(2\beta)\cos\varphi_y - 2|\lambda|^2 v_s^2 + m_{H^\pm}^2 \sin^2(2\beta)\Big)\,,$$

$$g_{hhA_S} = -\frac{3}{2}|\kappa||\lambda|v_s \sin(2\beta)\sin\varphi_y\,,$$

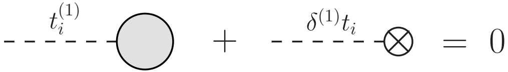
$$\frac{\partial g_{hhS}}{\partial\,\mathrm{Im}\,A_\lambda} = -\frac{|\lambda|}{\sqrt{2}}\sin(2\beta)\sin(\varphi_\omega - \varphi_y)\,,$$

$$\frac{\partial g_{hhA_S}}{\partial\,\mathrm{Im}\,A_\lambda} = -\frac{|\lambda|}{\sqrt{2}}\sin(2\beta)\cos(\varphi_\omega - \varphi_y)$$

Intermezzo: treatment of tadpoles and counterterms

Work in $\overline{\text{DR}}$ or $\overline{\text{MS}}$ scheme → no finite counterterm (CT) contributions

Exception: tadpoles are calculated in a "tadpole-on-shell scheme":



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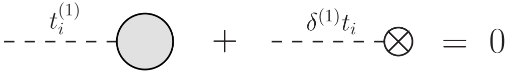
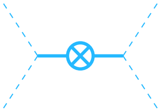
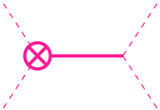


such that $\delta^{(1)}t_i = -t_i^{(1)}$

- Minimum of V_H does not move: tree-level VEV corresponds to the true VEV
- No explicit tadpoles in loop diagrams (exactly cancelled by the tadpole CTs)
- **However:** tadpole CTs $\delta^{(1)}t_i$ appearing in mass and vertex CTs





Intermezzo: tadpole equation for a_d

Tree-level tadpole equation for the a_d component (CP-odd doublet):

$$\frac{t_{a_d}}{v\sin\beta} = \frac{1}{2}|\lambda|v_S\Big(-|\kappa|v_S\sin\varphi_y + \sqrt{2}\,\mathrm{Im}\,A_\lambda\cos(\varphi_\omega - \varphi_y) + \sqrt{2}\,\mathrm{Re}\,A_\lambda\sin(\varphi_\omega - \varphi_y)\Big)$$

$$\Rightarrow \mathrm{Im}\,A_\lambda = \frac{\sqrt{2}}{|\lambda|v_S\cos(\varphi_\omega - \varphi_y)\sin\beta}\frac{t_{a_d}}{v} + \frac{|\kappa|v_S}{\sqrt{2}}\frac{\sin\varphi_y}{\cos(\varphi_\omega - \varphi_y)} - \mathrm{Re}\,A_\lambda\tan(\varphi_\omega - \varphi_y)$$

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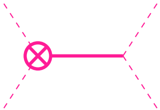
→ Finite CT contribution from $t_{a_d} \to t_{a_d}^{(0)} + \delta^{(1)} t_{a_d}$ (all other pars. renormalised $\overline{\mathrm{DR}}$):

$$\delta^{(1)}\,\mathrm{Im}\,A_\lambda = \frac{\sqrt{2}}{|\lambda| v_S \cos(\varphi_\omega - \varphi_y) \sin\beta} \frac{\delta^{(1)} t_{a_d}}{v}$$



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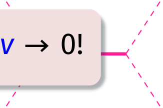
$\rightarrow$ Finite CT contribution from $t_{a_d} \rightarrow t_{a_d}^{(0)} + \delta^{(1)} t_{a_d}$ (all other pars. renormalised $\overline{\mathrm{DR}}$):

$\delta^{(1)}$

Non-vanishing contribution from $\delta^{(1)} t_{a_d}/v$ for $v \rightarrow 0$!



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Contribution of a_d tadpole for $v \to 0$

One-loop contribution from stops (after expanding stop mixing matrices to $\mathcal{O}(v)$):

$$\delta^{(1)} t_{a_d} = v \frac{3|\lambda||A_t| v_S Y_t^2}{16\pi^2 \sqrt{2} \sin\beta} B(m_{\tilde{Q}_3}, m_{\tilde{U}_{R,3}}) \sin(\varphi_\omega - \varphi_y + \varphi_{A_t})$$

with $B(x,y) = \big(A(x) - A(y)\big)/(x^2 - y^2)$ and $A(x) = x^2 \left(1 + \ln(\mu^2/x^2)\right)$

General way of computing the contributions: one-loop effective potential:

$$\delta^{(1)} t_{a_d} = v \cdot \frac{\partial t_{a_d}}{\partial v}\bigg|_{v=0} + \mathcal{O}(v^2) = v \cdot \frac{\partial^2 V(h, a_d, \dots)}{\partial a_d \partial h}\bigg|_{h=a_d=0; v=0} + \mathcal{O}(v^2)$$

$$= v \cdot \Sigma_{h\to a_d}(p^2 = 0)\Big|_{v=0} + \mathcal{O}(v^2)$$

$$= v \cdot \big(\sin\beta \Sigma_{h\to G}(p^2 = 0) - \cos\beta \Sigma_{h\to A}(p^2 = 0)\big)\Big|_{v=0} + \mathcal{O}(v^2)$$

Pole-mass matching

Demand that the pole masses M_h^X (X = SM, NMSSM) of the SM-like Higgs states are the same:

$$(M_h^{\text{SM}})^2 \stackrel{!}{=} (M_h^{\text{NMSSM}})^2$$

e.g. [Athron et al. '16], [Braathen et al. '18]

with $(M_h^X)^2 = (m_h^X)^2 - \text{Re}\,\hat{\Sigma}_h^X\left(p^2 = (M_h^X)^2\right)$

- m_h^X: SM(-like) $\overline{\text{MS}}$ ($\overline{\text{DR}}$) Higgs mass in the SM (NMSSM)
- $\hat{\Sigma}_h^X$: $\overline{\text{MS}}$ ($\overline{\text{DR}}$) renormalised self energies in the SM (NMSSM)

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Expansion of self energy around small external momenta:

$$\mathrm{Re}\,\hat{\Sigma}_h^X\big((m_h^X)^2\big) = \hat{\Sigma}_h^X(0) + (m_h^X)^2\,\hat{\Sigma}_h^{X\prime}(0) + \mathcal{O}\big((m_h^X)^4\big)$$

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Gauge thresholds at one loop:

$$\begin{aligned}(m_h^{\text{NMSSM}})^2 &\equiv \Big(m_h^{\text{NMSSM}}\big(g_i^{\text{NMSSM},\overline{\text{DR}}} \to g_i^{\text{SM},\overline{\text{MS}}} + \delta g_i\big)\Big)^2 \\ &= \Big(m_h^{\text{NMSSM}}\big(g_i^{\text{SM},\overline{\text{MS}}}\big)\Big)^2 + \delta^{\text{gauge}} m_h^2 + \mathcal{O}\big((\delta g_i)^2\big)\end{aligned}$$

Pole-mass matching: tree-level contribution

At tree level, pole masses = tree-level masses:

$$(M_h^{\text{SM}})^2 \overset{!}{=} (M_h^{\text{NMSSM}})^2 \quad \Rightarrow \quad (m_h^{\text{SM}})^2 \overset{!}{=} (m_h^{\text{NMSSM}})^2$$

Use $\overline{\text{MS}}$ relation $(m_h^{\text{SM}})^2 = 2(v^{\text{SM}})^2\lambda_h^{\text{SM}}$ and solve for λ_h^{SM}:

$$\lambda_h^{\text{SM}} = \frac{(m_h^{\text{NMSSM}})^2}{2(v^{\text{NMSSM}})^2}$$

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where $v^{\text{NMSSM}} = v^{\text{SM}}$ and $\delta g_i = 0$ (tree-level matching of the VEV and the g_i)

Analytical diagonalisation of tree-level mass matrix & expansion in v/M_{SUSY}: obtained same expression as for tree-level quartic-coupling matching!

Pole-mass matching: one-loop contribution

At one loop, take into account 1L self energies (reuse from implementation for fixed-order calculations in NMSSMCALC):

$$(M_h^{\text{SM}})^2 \stackrel{!}{=} (M_h^{\text{NMSSM}})^2 \quad \Rightarrow \quad (m_h^{\text{SM}})^2 - \text{Re}\,\hat{\Sigma}_h^{\text{SM}}\big((m_h^{\text{SM}})^2\big) \stackrel{!}{=} (m_h^{\text{NMSSM}})^2 - \text{Re}\,\hat{\Sigma}_h^{\text{NMSSM}}\big((m_h^{\text{NMSSM}})^2\big)$$

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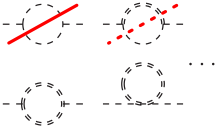
Expand for small ext. moms., again use $(m_h^{\text{SM}})^2 = 2(v^{\text{SM}})^2\lambda_h^{\text{SM}}$, and solve for λ_h^{SM}:

$$\lambda_h^{\text{SM}} = \frac{1}{2(v^{\text{NMSSM}})^2}\left[(m_h^{\text{NMSSM}})^2\left(1 - 2\Delta\hat{\Sigma}_h'\right) - \Delta\hat{\Sigma}_h\right]$$



with $\Delta\hat{\Sigma}_h^{(\prime)} \equiv \hat{\Sigma}_h^{\text{NMSSM}(\prime)}(0) - \hat{\Sigma}_h^{\text{SM}(\prime)}(0)$

$\Rightarrow$ Consistent expansion at 1L, get v/M_{SUSY} corrections for free!

Pole-mass matching: one-loop contribution

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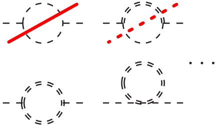
$$(M_h^{\rm SM})^2 \stackrel{!}{=} (M_h^{\rm NMSSM})^2 \quad \Rightarrow \quad (m_h^{\rm SM})^2 \stackrel{!}{=} (m_h^{\rm NMSSM})^2 - \mathrm{Re}\,\hat{\Sigma}_h^{\rm NMSSM}\big((m_h^{\rm NMSSM})^2\big) + \mathrm{Re}\,\hat{\Sigma}_h^{\rm SM}\big((m_h^{\rm SM})^2\big)$$

Expand for small ext. moms., again use $(m_h^{\rm SM})^2 = 2(v^{\rm SM})^2\lambda_h^{\rm SM}$, and solve for $\lambda_h^{\rm SM}$:

$$\lambda_h^{\rm SM} = \frac{1}{2(v^{\rm NMSSM})^2}\left[(m_h^{\rm NMSSM})^2\left(1 - 2\Delta\hat{\Sigma}'_h\right) - \Delta\hat{\Sigma}_h\right]$$



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⇒ Consi … 1L, get $v/M_{\rm SUSY}$ corrections for free!

Large cancellations!

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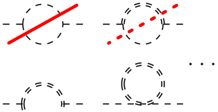

with $\Delta\hat{\Sigma}_h^{(\prime)} \equiv \hat{\Sigma}_h^{NMSSM(\prime)}(0) - \hat{\Sigma}_h^{SM(\prime)}(0)$

$\Rightarrow$ Consi... 1L, get v...

Large cancellations!

$$(v^{SM})^2 = (v^{NMSSM})^2 + \delta v^2; \quad \frac{\delta v^2}{v^2} = \Delta\hat{\Sigma}_h' + \mathcal{O}\left(\frac{v^2}{M_{SUSY}^2}\right)$$

(One-loop matching of the VEV, e.g. *[Braathen et al. '18]*)

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Pole-mass matching: one-loop contribution

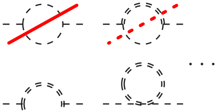
At one loop, take into account 1L self energies (reuse from implementation for fixed-order calculations in NMSSMCALC):

$$(M_h^{SM})^2 \stackrel{!}{=} (M_h^{NMSSM})^2 \Rightarrow (m_h^{SM})^2 \stackrel{!}{=} (m_h^{NMSSM})^2 - \mathrm{Re}\,\hat{\Sigma}_h^{NMSSM}\big((m_h^{NMSSM})^2\big) + \mathrm{Re}\,\hat{\Sigma}_h^{SM}\big((m_h^{SM})^2\big)$$

Numerical limit of $v \to 0$: excellent agreement with λ_h^{SM} from quartic-coupling matching!

$= 2(v^{SM})^2\lambda_h^{SM}$, and solve for λ_h^{SM}:

$$\lambda_h^{SM} = \frac{1}{2(v^{NMSSM})^2}\left[(m_h^{NMSSM})^2\left(1 - 2\Delta\hat{\Sigma}_h'\right) - \Delta\hat{\Sigma}_h\right]$$



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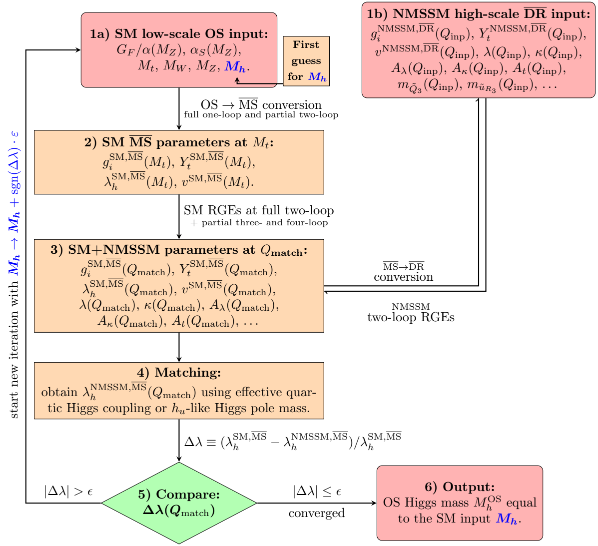
One-loop gauge thresholds

e.g. *[Bagnaschi, Giudice, Slavich, Strumia '14]*

$$\delta g_1^{\text{thr}} = -\frac{g_1^3}{512\pi^2}\left[12\ln\frac{|\mu_{\text{eff}}|^2}{Q^2} + 3\ln\frac{m_{H^\pm}^2}{Q^2} + \sum_{i=3}\left(3\ln\frac{m_{\tilde{L}_i}^2}{Q^2} + 6\ln\frac{m_{\tilde{e}_{R,i}}^2}{Q^2}\right)\right.$$
$$\left. + \sum_{i=1}^{3}\left(\ln\frac{m_{\tilde{Q}_i}^2}{Q^2} + 8\ln\frac{m_{\tilde{u}_{R,i}}^2}{Q^2} + 2\ln\frac{m_{\tilde{d}_{R,i}}^2}{Q^2}\right)\right],$$
$$\delta g_2^{\text{thr}} = -\frac{g_2^3}{192\pi^2}\left[8\ln\frac{M_2^2}{Q^2} + 4\ln\frac{|\mu_{\text{eff}}|^2}{Q^2} + \ln\frac{m_{H^\pm}^2}{Q^2} + \sum_{i=3}\left(\ln\frac{m_{\tilde{L}_i}^2}{Q^2} + 3\ln\frac{m_{\tilde{Q}_i}^2}{Q^2}\right)\right],$$
$$\delta g_3^{\text{thr}} = -\frac{g_3^3}{192\pi^2}\left[12\ln\frac{M_3^2}{Q^2} + \sum_{i=1}^{3}\left(2\ln\frac{m_{\tilde{Q}_i}^2}{Q^2} + \ln\frac{m_{\tilde{u}_{R,i}}^2}{Q^2} + \ln\frac{m_{\tilde{d}_{R,i}}^2}{Q^2}\right)\right]$$

NMSSMCALC algorithm to determine SM Higgs mass prediction M_h



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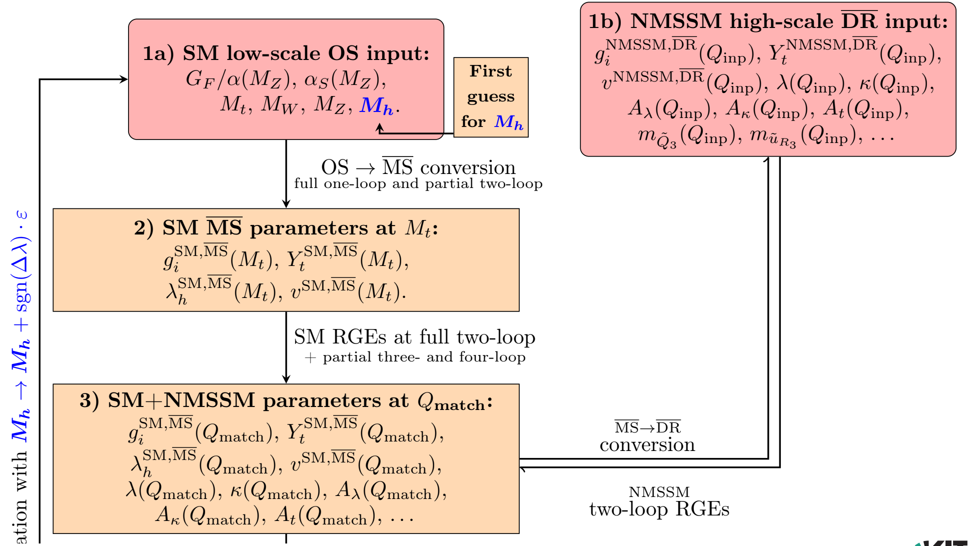

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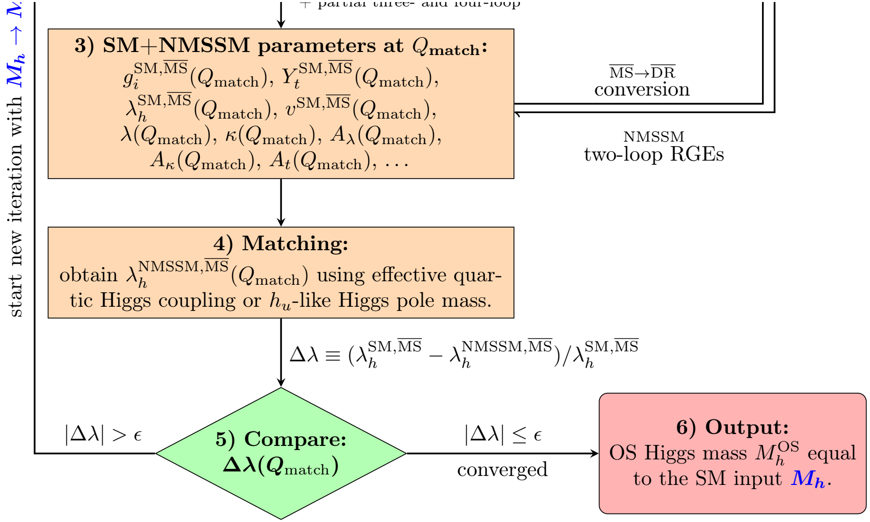

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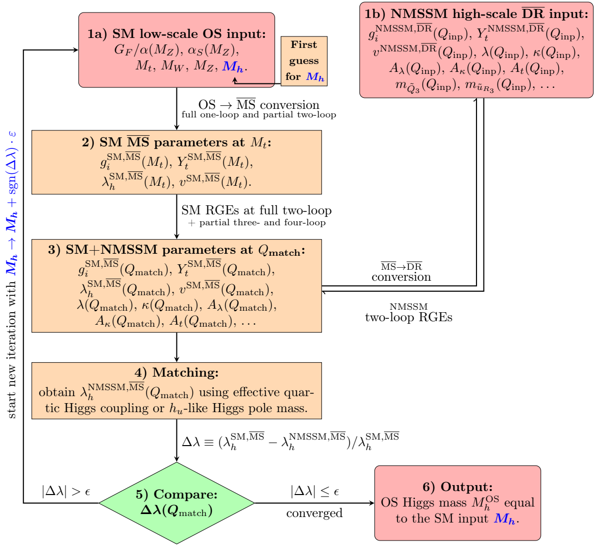

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Uncertainty estimate (I)

- **SM uncertainties:**
  - missing EW corrections (from choice of input scheme): $\Delta^{\text{SM}}_{G_F/\alpha_{M_Z}} = |M_h^{G_F} - M_h^{\alpha_{M_Z}}|$
  - missing h.o. terms in $\overline{\text{MS}}$-OS conversion between $\lambda_h^{\text{SM},\overline{\text{MS}}}$ and M_h:

$$0 \stackrel{!}{=} p^2 - 2\lambda_h^{\text{SM},\overline{\text{MS}}}(Q_{\text{EW}})v^2(Q_{\text{EW}}) + \text{Re}\,\hat{\Sigma}_h^{\text{SM},\overline{\text{MS}}}(p^2, Q_{\text{EW}})$$

Solve iteratively for $p^2 = M_h^{\overline{\text{MS}},\text{pole}}(Q_{\text{EW}})$, vary Q_{EW}:

$$\Delta^{\text{SM}}_{Q_{\text{EW}}} = \max\left\{|M_h^{\text{OS}} - M_h^{\overline{\text{MS}},\text{pole}}(2M_t)|, |M_h^{\text{OS}} - M_h^{\overline{\text{MS}},\text{pole}}(M_t/2)|\right\}$$

  - missing 3L top corrections: $\Delta^{\text{SM}}_{Y_t} = M_h(Y_t^{\mathcal{O}(\alpha_s^2)}) - M_h(Y_t^{\mathcal{O}(\alpha_s^3)})$

Uncertainty estimate (II)

- **SUSY uncertainties:**
  - variation of the matching scale Q_match with respect to the SUSY input scale M_SUSY:

$$\Delta^\text{SUSY}_{Q_\text{match}} = \max\left\{|M_h^{M_\text{SUSY}/2} - M_h^{M_\text{SUSY}}|, |M_h^{2M_\text{SUSY}} - M_h^{M_\text{SUSY}}|\right\}$$

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  - variation of the matching scale Q_{match} with respect to the SUSY input scale M_{SUSY}:

$$\Delta^{\text{SUSY}}_{Q_{\text{match}}} = \max\left\{ \left| M_h^{M_{\text{SUSY}}/2} - M_h^{M_{\text{SUSY}}} \right|, \left| M_h^{2M_{\text{SUSY}}} - M_h^{M_{\text{SUSY}}} \right| \right\}$$

Combined uncertainty:

For pole-mass matching M_h^{II}:

$$\Delta M_h^{\text{II}} = \left[\left(\Delta^{\text{SM}}_{G_F/\alpha_{M_Z}} \right)^2 + \left(\Delta^{\text{SM}}_{Q_{\text{EW}}} \right)^2 + \left(\Delta^{\text{SM}}_{Y_t} \right)^2 + \left(\Delta^{\text{SUSY}}_{Q_{\text{match}}} \right)^2 \right]^{\frac{1}{2}}$$

For quartic-coupling matching M_h^{IV}:

$$\Delta M_h^{\text{IV}} = \left[\left(\Delta M_h^{\text{II}} \right)^2 + \Big(\underbrace{M_h^{\text{II}} - M_h^{\text{IV}}}_{\equiv \Delta^{\text{SUSY}}_{v^2/M^2_{\text{SUSY}}}} \Big)^2 \right]^{\frac{1}{2}}$$

"EFT uncertainty"

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SM input parameters

$$
\begin{aligned}
G_F &= 1.1663788 \times 10^{-5}\,\mathrm{GeV}^{-2}, & \alpha(M_Z) &= 1/127.955\,, \\
\alpha_s(M_Z) &= 0.1181\,, & M_t &= 172.69\,\mathrm{GeV}\,, \\
m_b^{\overline{\mathrm{MS}}} &= 4.18\,\mathrm{GeV}\,, & M_\tau &= 1.77682\,\mathrm{GeV}\,, \\
M_W &= 80.377\,\mathrm{GeV}\,, & M_Z &= 91.1876\,\mathrm{GeV}\,,
\end{aligned}
$$

Benchmark points (I)

BP1: *[Bagnaschi et al. '22]*
BP2: *[Slavich, Heinemeyer et al. '20]*
BP3: our own scan

	$\tan\beta$	λ	κ	M_1	M_2	M_3	A_t	A_λ	A_κ	$\mu_{eff.}$	$m_{\tilde{Q}_{L_3}}$	$m_{\tilde{t}_{R_3}}$
BP1	3.0	0.6	0.6	1.0	2.0	2.5	12.75	0.3	-2.0	1.5	5.0	5.0
BP2	20.0	0.05	0.05	4.5	4.5	4.5	-10.79	-4.28	-1.5	4.5	4.5	4.5
BP3	1.27	0.73	0.62	0.24	1.18	2.3	-0.39	0.06	-1.44	0.49	1.79	1.51

	M_h^{II}	M_h^{IV}	m_{h_2}	m_{h_3}	m_{A_1}	m_{A_2}	m_{H^+}
BP1	124.29 (h_u)	124.31 (h_u)	2407.6 (h_s)	2971.8 (h_d)	2905.7 (a)	3000.2 (a_s)	2967.1
BP2	125.15 (h_u)	125.15 (h_u)	4486.6 (h_d)	8616.7 (h_s)	4510.9 (a_s)	4984.0 (a)	4995.0
BP3	127.16 (h_u)	129.46 (h_u)	305.5 (h_s)	659.5 (h_d)	663.8 (a)	1308.7 (a_s)	658.4

Benchmark points (II)

Other SUSY masses:

	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\chi_1^0}$	$m_{\chi_2^0}$	$m_{\chi_3^0}$	$m_{\chi_4^0}$	$m_{\chi_5^0}$	$m_{\chi_1^+}$	$m_{\chi_2^+}$
BP1	4829.6	5168.2	997.2	1491.5	1502.4	2010.5	3003.3	1490.2	2010.5
BP2	4335.2	4662.4	4432.6	4500.0	4500.0	4567.8	9000.0	4407.2	4559.9
BP3	1514.2	1799.1	232.8	484.1	498.2	835.4	1192.7	477.3	1192.6

Uncertainties:

	$\Delta^{SM}_{Y_t}$	$\Delta^{SM}_{Q_{EW}}$	$\Delta^{SM}_{G_F/\alpha_{M_Z}}$	$\Delta^{SUSY}_{Q_{match}}$	$\Delta^{SUSY}_{v^2/M^2_{SUSY}}$	ΔM_h^{II}	ΔM_h^{IV}
BP1	-738	208	-19	376	-21	854	836
BP2	-685	208	-69	189	-5	743	743
BP3	-401	198	20	694	-2294	826	2415

Quartic-coupling vs. pole-mass matching for $v \to 0$