Spontaneous CP violation and a new Higgs doublet below 500 GeV

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Why new physics? (via direct/indirect search)

- **Naturalness**: A long-standing guiding principle.
- Field theory solution to the hierarchy puzzle
  = new physics not much heavier than TeV.
- A benchmark example: SUSY.
- Is naturalness **physically satisfying**?

Motivation and Introduction

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SUSY is a decoupling theory

Question: **Can** the LHC rule out supersymmetry?

Answer: No. Supersymmetry is an example (one of many!) of a decoupling theory; the more you raise the masses of the new particles, the better it agrees with the Standard Model.

From S. Martin
pre-SUSY 2021

Naturalness require:

$$\Lambda_{\mathrm{SUSY}} < 500\ \mathrm{GeV}, 1\ \mathrm{TeV}, 3\ \mathrm{TeV}, 10\ \mathrm{TeV}, \ldots ?$$

Not too heavy is a vague statement.

Motivation for NP other than naturalness?

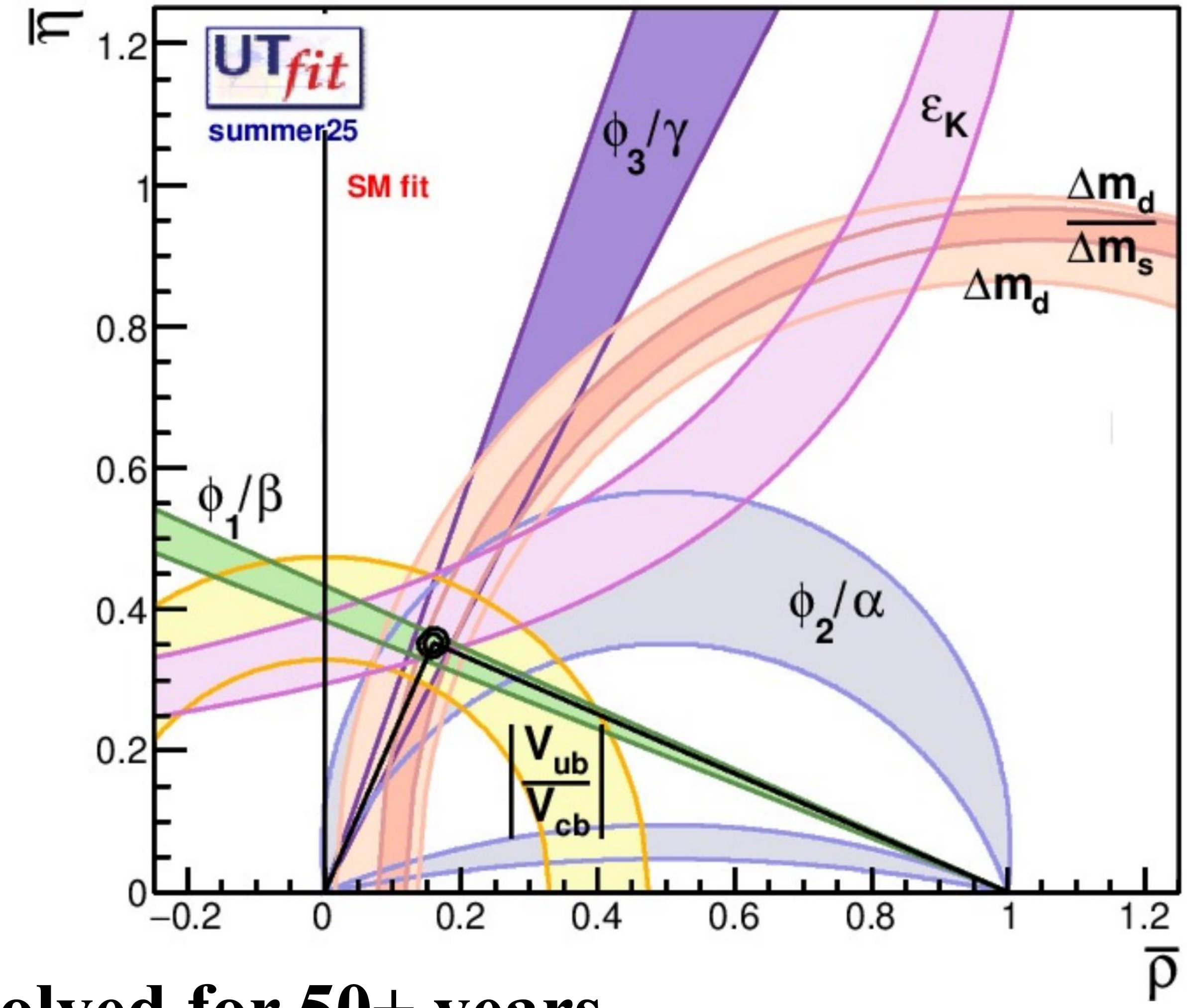
Spontaneous CP violation

Stand origin of CPV: KM phase

Kobayashi, Maskawa. ’72

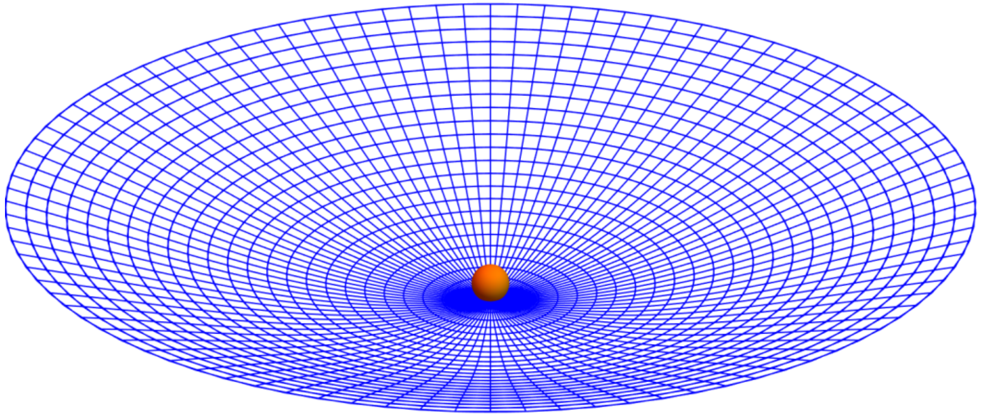
However:

- CP can serve as a fundamental symmetry.
- Vacuum solution leads to CPV.
- KM phase may arise from complex VEVs. T. D. Lee ’73.

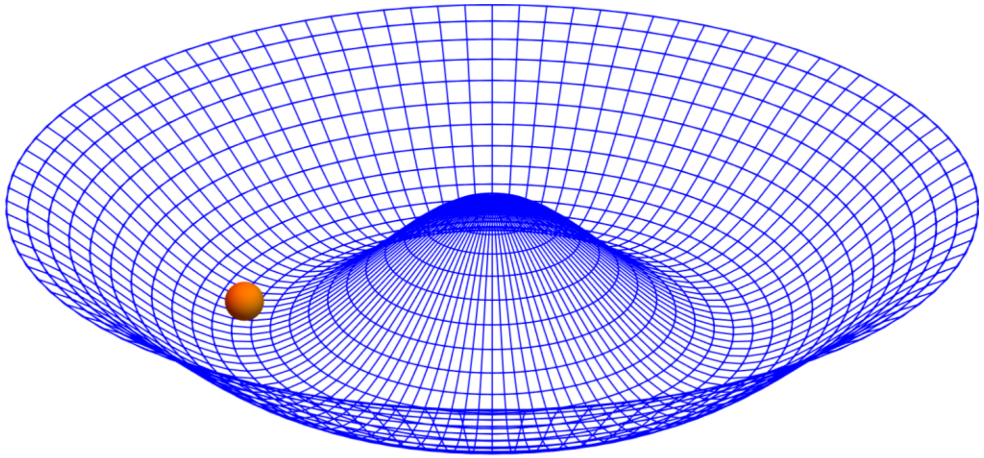


Explicit or Spontaneous CPV? Unsolved for 50+ years.

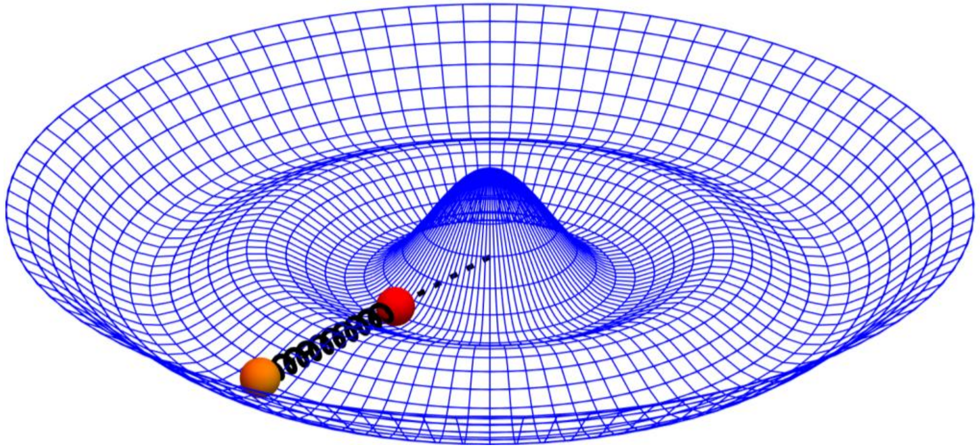
Spontaneous CP violation



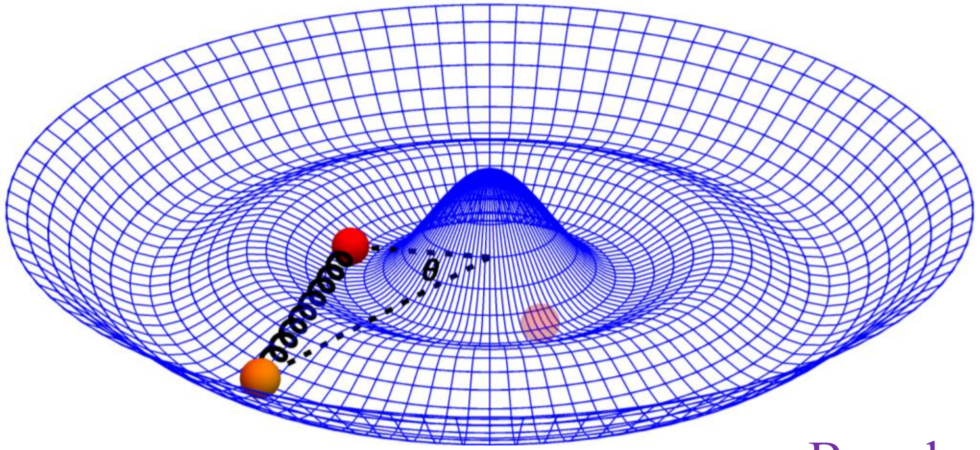
(a) No SSB



(b) SM SSB: Mexican Hat



(c) Unstable vacuum



(d) Spontaneous CPV

Based on the planar diagrams in Lee ’73

Spontaneous CP violation

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The initial proposal of T. D. Lee ’73: 2HDM

Two degenerate EW vacua v_i, v_i^*

→ Two light Higgs Doublets ≲545 GeV.

→ SM-like fine-tuning once again.

Mohapatra, Senjanovic. ’83;
Nebot, Botella, Branco. ’19;
Nierste, Tabet, Ziegler, ’20;
Miró, Nebot, Queiroz. ’24.

$$m_h^2 + m_H^2 + m_A^2 \lesssim M_{\mathrm{LQT}}^2 = (700 \sim 800\ \mathrm{GeV})^2.$$

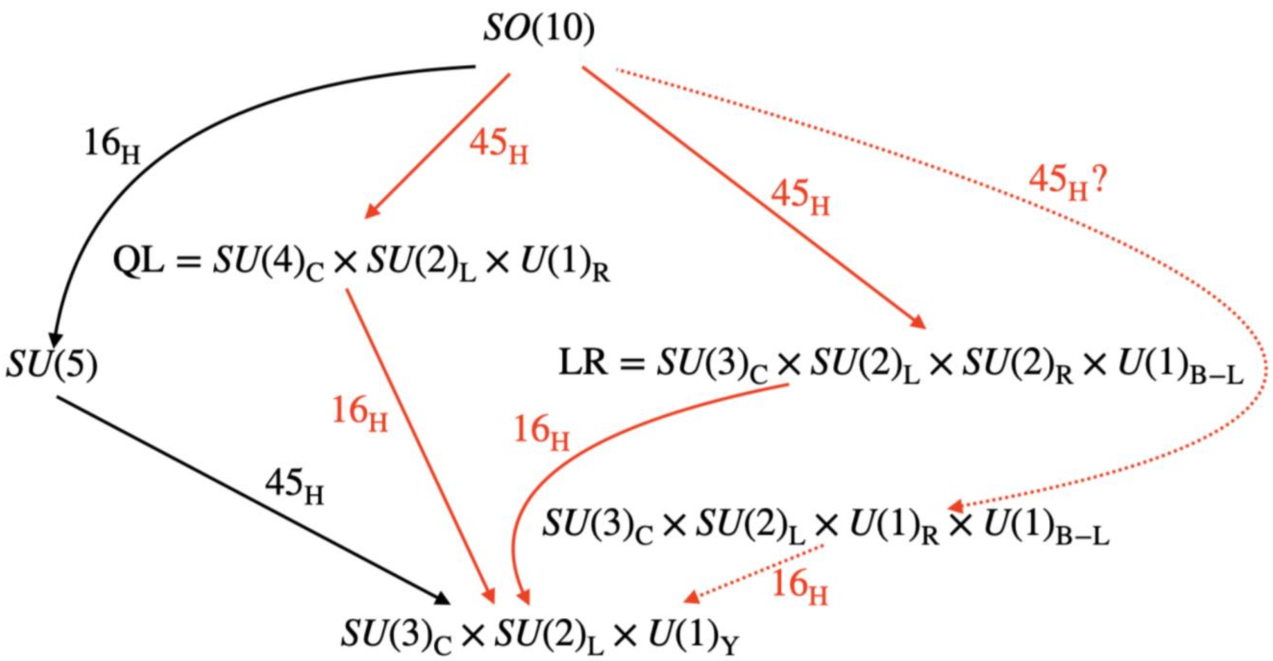
No decoupling limit
Even if $\delta_{CP} \ll 1$!

A good guiding principle for NP? Still needs:

- To ensure no SCPV at higher scale.
- To constrain the couplings.

The theory: Minimal SO(10)×CP

- Why SO(10) GUT
  - $Q_i \equiv 0, \pm\frac{1}{3}e, \pm\frac{2}{3}e, \pm e, \ldots$ **Quantized!**
  - Only exp show $Q_n \lesssim 10^{-20}e$.
  - Quantization is necessary for unification:
    - $(Q_L, u_R, d_R) + (l_L, \nu_R, e_R) = 16_F$



Babu, Mohapatra. ’89;
Foot, Lew, Volkas. ’93;
Bressi, et al. ’11;
Herms, Ruhdorfer. ’24.

Georgi, Glashow. ’74;
Fritzsch, Minkowski. ’75
Bertolini, Di Luzio, Malinsky.’10 ’12;
Preda, Senjanovic,
Zantedeschi.’22 ’24 (figure above);

The theory: Minimal SO(10)×CP

$$SO(10)\times CP \xrightarrow[M_{\rm GUT}]{\langle(1,1,1,0)\rangle\in 45_H} SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times CP$$
$$\xrightarrow[M_R]{\langle(1,1,3,1)\rangle\in 126_H} SU(3)_C\times SU(2)_L\times U(1)_Y\times CP$$
$$\xrightarrow[M_W]{\langle(1,2,2,0)\rangle\in 126_H,10_H} SU(3)_c\times U(1)_{\rm EM}.$$

$$45_H\to 45_H,\quad 126_H\to\overline{126}_H,\quad 10_H\to 10_H^*$$

- Only real couplings in the limit of CP invariance.
- 45_H real, 126_H phase unphysical: No high-scale SCPV

The theory: Minimal SO(10)×CP

Yukawa Sector: constrained by SO(10):

$$-\mathcal{L}_Y = Y_{10} 16_F 10_H 16_F + \widetilde{Y}_{10} 16_F 10^*_H 16_F + Y_{126} 16_F \overline{126}_H 16_F + \text{h.c.}$$

Successful numerical fit: Patel '23.

$Y_{10}, \tilde{Y}_{10}, Y_{126}$ are real, symmetric, linear combinations of M_U, M_D, M_E.

$$M_D = D^* m_D D^\dagger, \quad M_U = U^* m_U U^\dagger, \quad M_E = E^* m_E E^\dagger;$$
$$V_{\text{CKM}} = U^\dagger D, \qquad V_E = E^\dagger D.$$

The theory: Minimal SO(10)×CP

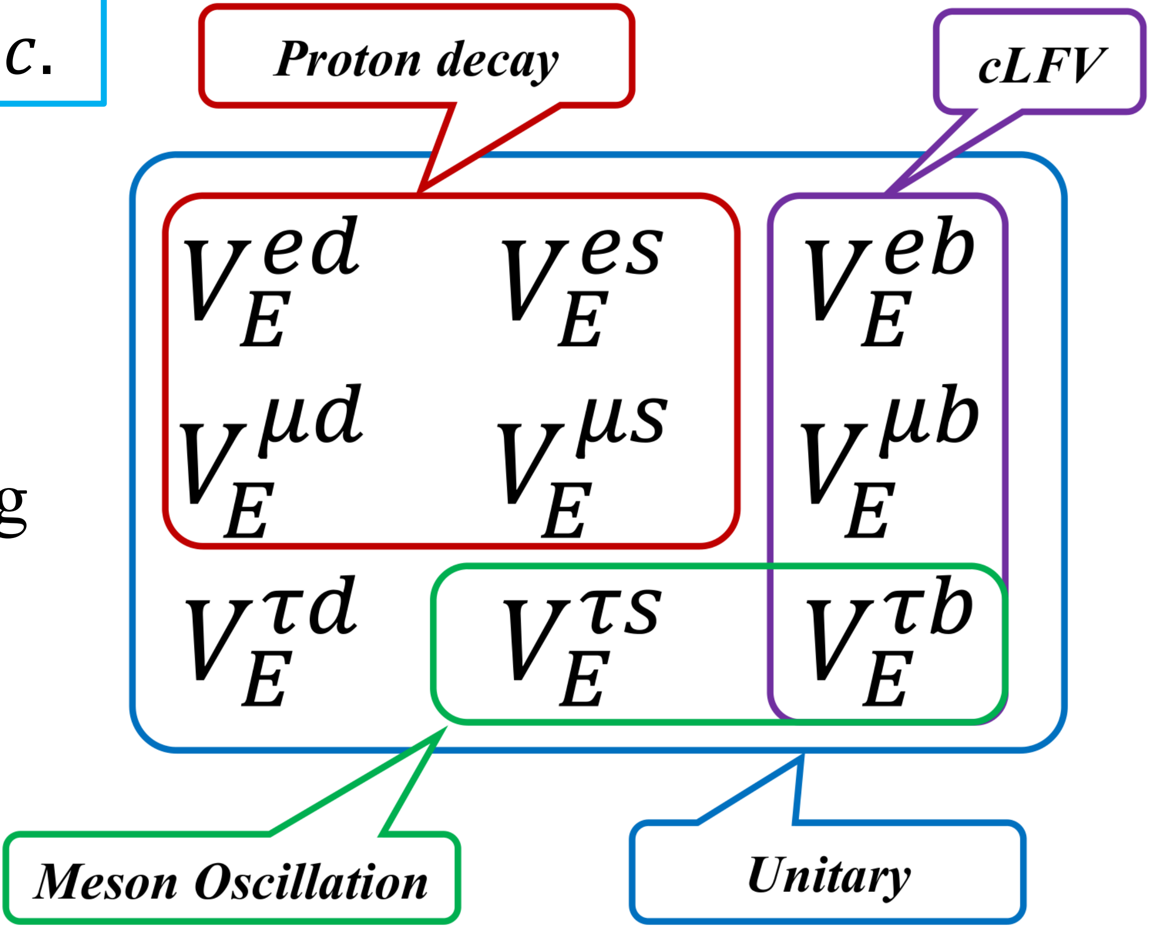
The low energy theory: Flavor violating 2HDM

$$-\mathcal{L}_Y \supset Y_E \overline{L_L} H_2 e_R + Y_D \overline{Q_L} H_2 d_R + h.c.$$

- NMFV-like, absolute strength swamped by fitting.
- Neglecting 1,2 gen quark masses.
- Neglecting small 1-3, 2-3 gen mixing angles.

$$Y_E^{\ell\ell'} \propto V_E^{\ell b} V_E^{\ell' b}, \quad \ell \neq \ell';$$

$$Y_D^{qq'} \propto V_E^{\tau q} V_E^{\tau q'}, \quad q \neq q'.$$



Phenomenology: A Window to Check SO(10)

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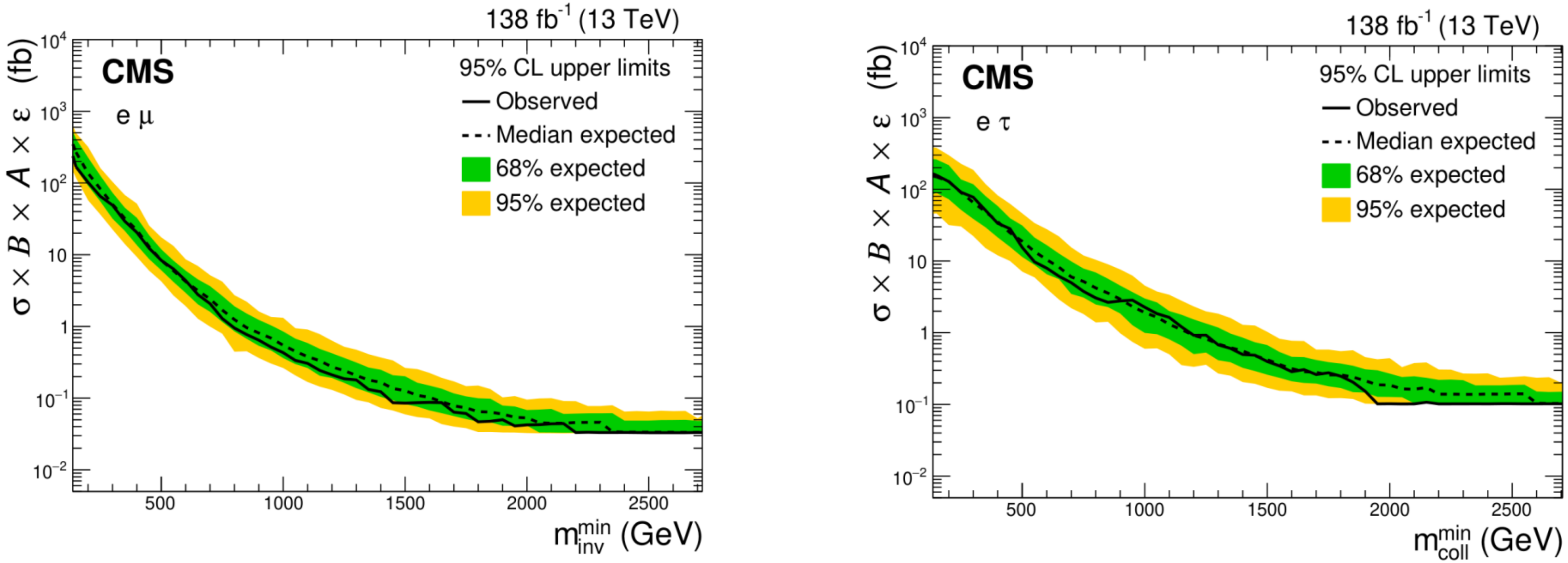
LFV decay of H and A (third column of V_E)

$$\sigma(pp \to H, A) \times \mathrm{Br}(H, A \to \ell\ell') \;\propto\; \left|Y_E^{\ell\ell'}\right|^2 \;\propto\; |V_E^{\ell b} V_E^{\ell' b}|^2$$

$$\frac{N_{e\mu}}{N_{\tau\mu}} = \frac{|V_E^{eb}|^2}{|V_E^{\tau b}|^2}, \qquad \frac{N_{e\mu}}{N_{e\tau}} = \frac{|V_E^{\mu b}|^2}{|V_E^{\tau b}|^2}.$$

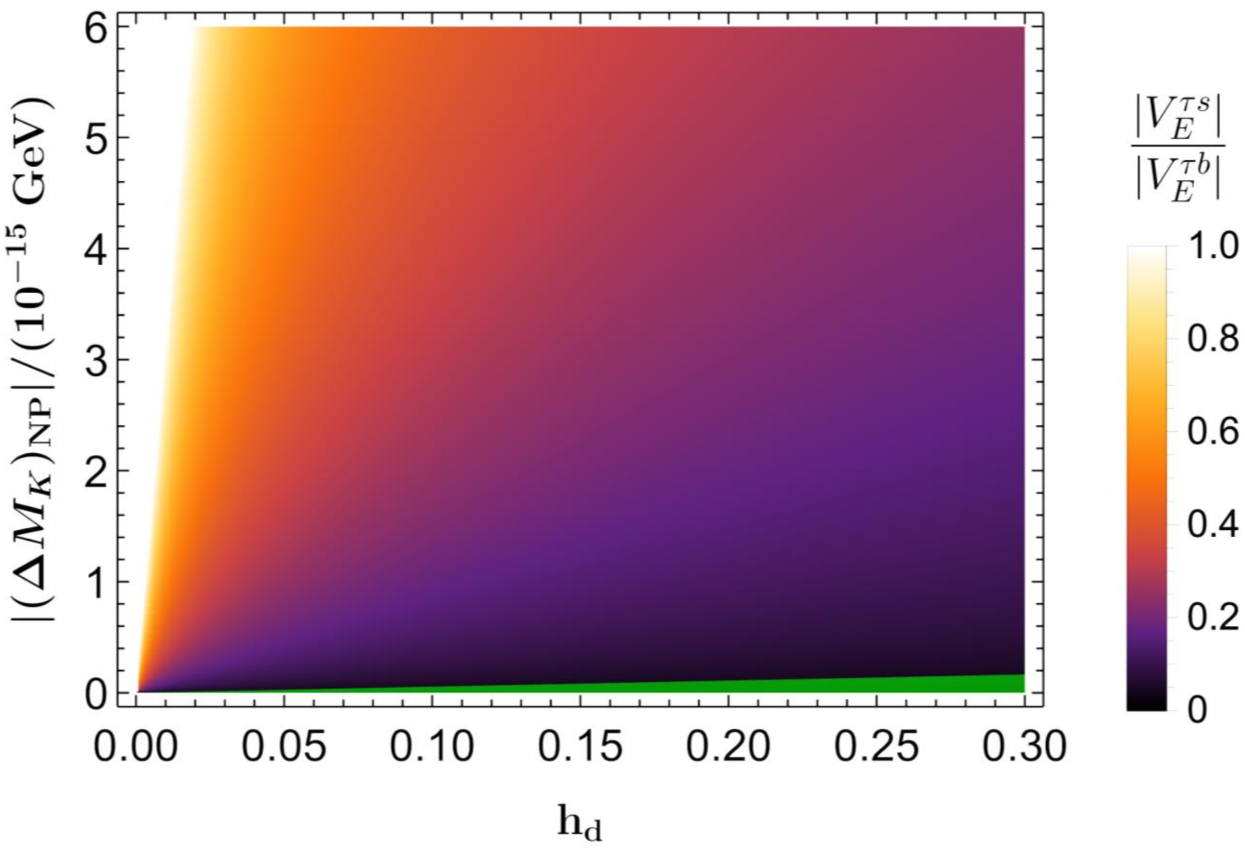
$N_{ll'}$, total # of excess evens, normalized by detection efficiency.

Limits: CMS. 2205.06709



Phenomenology: A Window to Check SO(10)

Neutral meson mixing (third row of V_E)



MFV only for the narrow green band.

MFV:

$$\frac{|(\Delta M_K)_{\rm NP}|m_K}{\langle K^0|\overline{d_L}s_R\overline{d_R}s_L|\overline{K^0}\rangle} \approx \lambda^4\cdot\frac{2\mathbf{h_d}|M_{12}^{d\rm SM}|m_{B_d}}{\langle B_d^0|\overline{b_L}d_R\overline{b_R}d_L|\overline{B_s^0}\rangle}$$
$$\approx \lambda^6\cdot\frac{2\mathbf{h_s}|M_{12}^{s\rm SM}|m_{B_s}}{\langle B_s^0|\overline{b_L}s_R\overline{b_R}s_L|\overline{B_s^0}\rangle}.$$

NMFV:

$$\frac{|(\Delta M_K)_{\rm NP}|}{2\mathbf{h_d}|M_{12}^{d\rm SM}|\xi_B} = \frac{|V_E^{\tau s}|^2}{|V_E^{\tau b}|^2},$$

$$\xi_B = \frac{m_B\langle K^0|\overline{d_L}s_R\overline{d_R}s_L|\overline{K^0}\rangle}{m_K\langle B_d^0|\overline{b_L}d_R\overline{b_R}d_L|\overline{B_d^0}\rangle}.$$

Phenomenology: A Window to Check SO(10)

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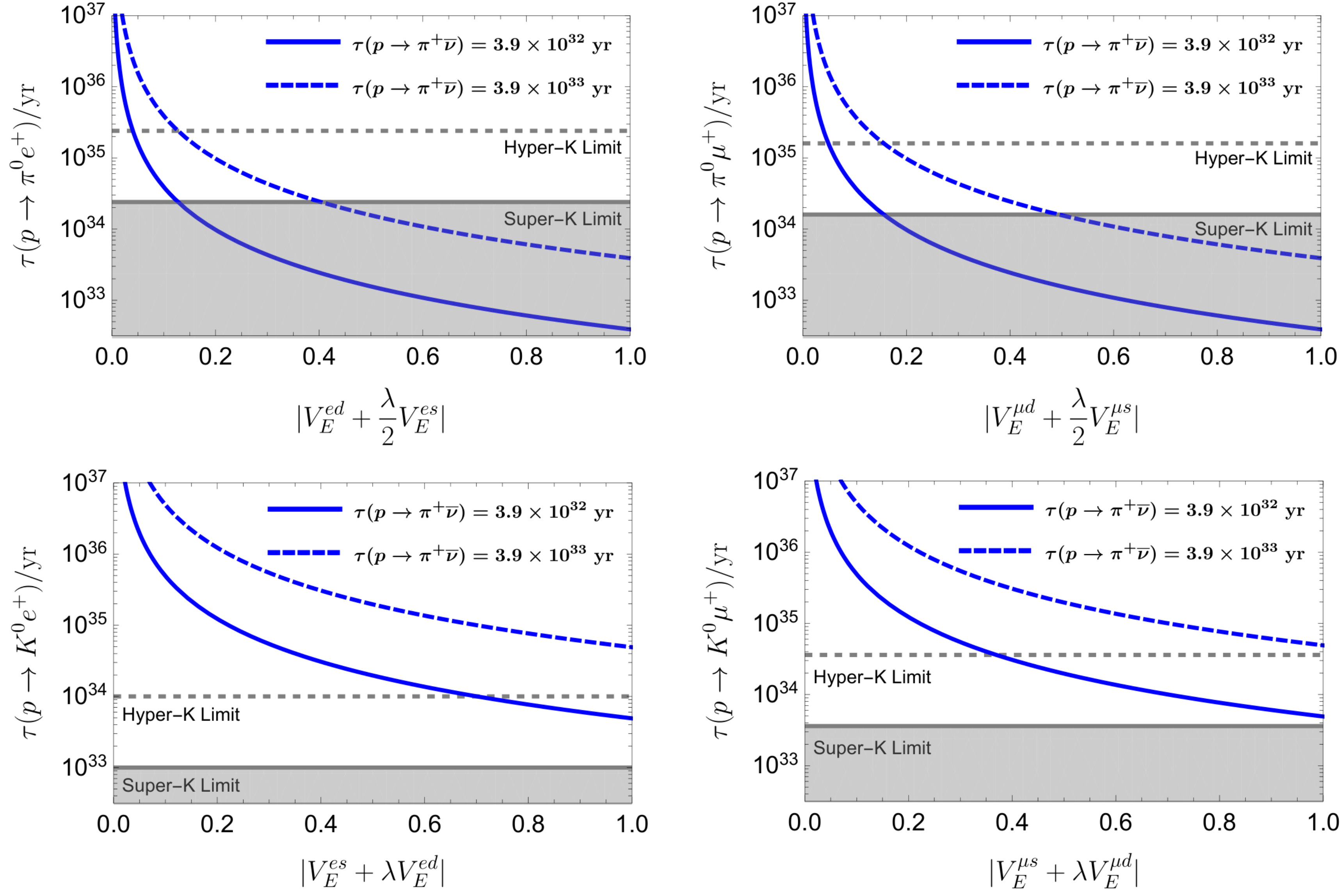

Combining π and K modes, λ goes away.

$$\frac{2\Gamma(p \to \pi^0 e^+)}{\Gamma(p \to \pi^+\overline{\nu})} - \frac{\Gamma(p \to K^0 e^+)}{\xi_K \Gamma(p \to K^+\overline{\nu})}$$

$$= 2|V_E^{ed}|^2 - |V_E^{es}|^2,$$

$$\frac{2\Gamma(p \to \pi^0 \mu^+)}{\Gamma(p \to \pi^+\overline{\nu})} - \frac{\Gamma(p \to K^0 \mu^+)}{\xi_K \Gamma(p \to K^+\overline{\nu})}$$

$$= 2|V_E^{\mu d}|^2 - |V_E^{\mu s}|^2.$$

Conclusion and Outlook

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Prediction: a concrete flavor **sum rule**:

$$\frac{2\Gamma(p\to\pi^0\ell^+)}{\Gamma(p\to\pi^+\overline{\nu})}-\frac{\Gamma(p\to K^0\ell^+)}{\xi_K\Gamma(p\to K^+\overline{\nu})}$$
$$=\left(\frac{3|(\Delta M_K)_{\rm NP}|}{2\mathbf{h_d}|M_{12}^{d\rm SM}|\xi_B}-\frac{N_{e\mu}}{N_{\tau\mu}}-\frac{N_{e\mu}}{N_{e\tau}}+1\right)\left(\frac{N_{e\mu}}{N_{\tau\mu}}+\frac{N_{e\mu}}{N_{e\tau}}+1\right)^{-1}$$

- Waiting for Hyper-K, HL-LHC, and more lattice QCD results.
- Hopefully, a hint for SO(10) in near future.
- What we need most? Patience!

Thanks

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Part of the scalar potential

ϕ: 45_H Σ: 126_H

Bertolini, Di Luzio, Malinsky. '12

$$\langle(1,1,1,0)\rangle \equiv \omega_{BL},\ \langle(1,1,3,0)\rangle \equiv \omega_R,$$
$$\langle(1,1,3,+1)\rangle \equiv \sigma,$$

$$\begin{aligned}\langle V\rangle = & -\mu^2\left(3\omega_{BL}^2+2\omega_R^2\right)+a_0\left(12\omega_R^2\omega_{BL}^2+9\omega_{BL}^4+4\omega_R^4\right)\\ & +\frac{a_2}{2}\left(3\omega_{BL}^4+2\omega_R^4\right)-2\nu^2|\sigma|^2+4\lambda_0|\sigma|^4\\ & +2\tau\left(3\omega_{BL}+2\omega_R\right)|\sigma|^2+2\alpha\left(3\omega_{BL}^2+2\omega_R^2\right)|\sigma|^2\\ & -4\beta_4'\left(6\omega_R\omega_{BL}+3\omega_{BL}^2+\omega_R^2\right)|\sigma|^2\,. \end{aligned} \tag{B1}$$

$$V = V_{45}+V_{126}+V_{\rm mix}\,, \tag{1}$$

where

$$V_{45} = -\frac{\mu^2}{2}(\phi\phi)_0+\frac{a_0}{4}(\phi\phi)_0(\phi\phi)_0+\frac{a_2}{4}(\phi\phi)_2(\phi\phi)_2\,, \tag{2}$$

$$\begin{aligned}V_{126} = & -\frac{\nu^2}{5!}(\Sigma\Sigma^*)_0\\ & +\frac{\lambda_0}{(5!)^2}(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0+\frac{\lambda_2}{(4!)^2}(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2\\ & +\frac{\lambda_4}{(3!)^2(2!)^2}(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4+\frac{\lambda_4'}{(3!)^2}(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'}\\ & +\frac{\eta_2}{(4!)^2}(\Sigma\Sigma)_2(\Sigma\Sigma)_2+\frac{\eta_2^*}{(4!)^2}(\Sigma^*\Sigma^*)_2(\Sigma^*\Sigma^*)_2\,,\end{aligned} \tag{3}$$

$$\begin{aligned}V_{\rm mix} = & \frac{i\tau}{4!}(\phi)_2(\Sigma\Sigma^*)_2+\frac{\alpha}{2\cdot 5!}(\phi\phi)_0(\Sigma\Sigma^*)_0\\ & +\frac{\beta_4}{4\cdot 3!}(\phi\phi)_4(\Sigma\Sigma^*)_4+\frac{\beta_4'}{3!}(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'}\\ & +\frac{\gamma_2}{4!}(\phi\phi)_2(\Sigma\Sigma)_2+\frac{\gamma_2^*}{4!}(\phi\phi)_2(\Sigma^*\Sigma^*)_2\,.\end{aligned} \tag{4}$$

The theory: Minimal SO(10)×CP

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The low energy theory: Flavor violating 2HDM

$$-\mathcal{L}_{\Phi\overline{F}F} \supset (\frac{m_E}{v} + \epsilon Y_E^{\ell\ell'})h\overline{\ell_L}\ell'_R + (\frac{m_D}{v} + \epsilon Y_D^{qq'})\overline{d_L^q}d_R^{q'}h + (\frac{m_U}{v} + \epsilon Y_U^{qq'})\overline{u_L^q}u_R^{q'}h$$
$$+ \mathcal{Y}_E^{\ell\ell'}(H+iA)\overline{\ell_L}\ell'_R + \mathcal{Y}_D^{qq'}(H+iA)\overline{d_L^q}d_R^{q'} + \mathcal{Y}_U^{qq'}(H+iA)\overline{u_L^q}u_R^{q'} + \text{h.c.}$$

$$\begin{aligned} Y_E &= \mathcal{C}_{EE}\frac{m_E}{v} + \mathcal{C}_{ED}V_E^*\frac{m_D}{v}V_E^\dagger + \mathcal{C}_{EU}V_E^*V_{\text{CKM}}^T\frac{m_U}{v}V_{\text{CKM}}V_E^\dagger, \\ Y_D &= \mathcal{C}_{DE}V_E^T\frac{m_E}{v}V_E + \mathcal{C}_{DD}\frac{m_D}{v} + \mathcal{C}_{DU}V_{\text{CKM}}^T\frac{m_U}{v}V_{\text{CKM}}, \\ Y_U &= \mathcal{C}_{UE}V_{\text{CKM}}^*V_E^T\frac{m_E}{v}V_EV_{\text{CKM}}^\dagger + \mathcal{C}_{UD}V_{\text{CKM}}^T\frac{m_D}{v}V_{\text{CKM}} + \mathcal{C}_{UU}\frac{m_U}{v}, \end{aligned}$$

where $\mathcal{Y}_F = e^{i\alpha_c}Y_F,\ \ell,\ell' = e,\mu,\tau,\ q,q' = d,s,b$ or u,c,t

- Flavor structure: only V_E unknown.
- $\mathcal{C}_{FF'}$: free, not predicted.

Phenomenology: Is the Theory Safe?

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In which sense FCNC is not ruled out?

$$\begin{pmatrix} \mathcal{C}_{EE} & \mathcal{C}_{ED} & \mathcal{C}_{EU} \\ \mathcal{C}_{DE} & \mathcal{C}_{DD} & \mathcal{C}_{DU} \\ \mathcal{C}_{UE} & \mathcal{C}_{UD} & \mathcal{C}_{UU} \end{pmatrix} = \frac{v}{u} \begin{pmatrix} u_1 & -u_2 & -3u_3 \\ u_1 & -u_2 & u_3 \\ u_2^* & u_1^* & u_4^* \end{pmatrix} \begin{pmatrix} v_{10}^d & -v_{10}^{u*} & -3v_{126}^d \\ v_{10}^d & -v_{10}^{u*} & v_{126}^d \\ v_{10}^u & v_{10}^{d*} & v_{126}^u \end{pmatrix}^{-1}$$

- Naively, all $\mathcal{C}_{FF'}$ at $o(1)$. Strictly, not predicted.
- $\mathcal{C}_{EU} = \mathcal{C}_{DU}$. No other correlations.
- B_s mixing constrains $\mathcal{C}_{DU}$.
- MFV: B_d, K mixing less constrained.
- $\Delta F = 1$ processes: suppressed by loop factor or additional chirality flipping.

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ (v_i + \rho_i + i\eta_i)/\sqrt{2} \end{pmatrix},$$

$$u_i = v_i^* - \left(\sum_{j=1}^{4} v_j^{*2}\right) \frac{v_i}{v}, \quad u = \sum_{i=1}^{4} |u_i|^2.$$

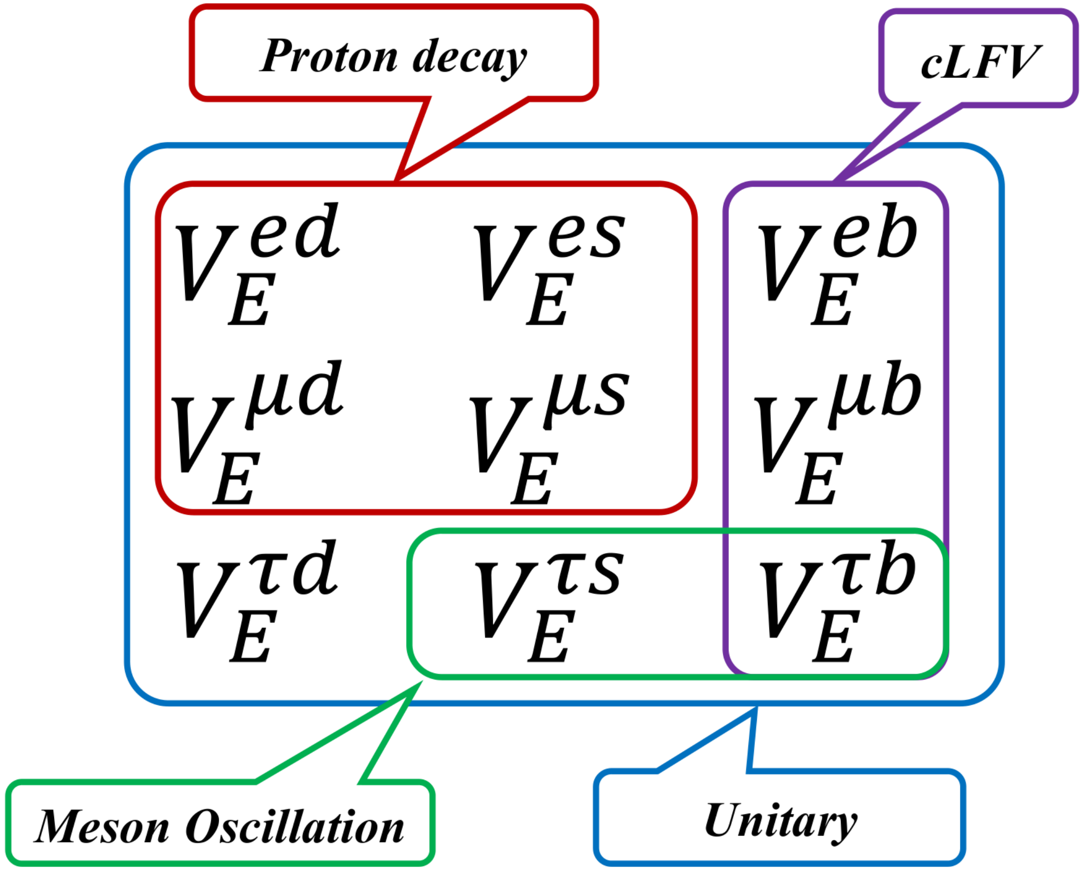
$$|\mathcal{C}_{DU}| \lesssim \left(\frac{v}{m_t} \cdot \frac{1}{|V_{\rm CKM}^{ts} V_{\rm CKM}^{tb}|}\right) \times \frac{m_H/\sqrt{2}}{10^3 \text{ TeV}}$$

$$\approx 0.013 \times \frac{m_H}{500 \text{ GeV}}$$

Understand the flavor structure:

- Charged Leptons: NMFV
- Down-type Quarks: MFV or NMF
- Up-type quarks: $o(\lambda)$ corrections.

  λ~0.2, the Cabbio angle



$$Y_E^{\ell\ell'} \propto (V_E^{\ell b} V_E^{\ell' b})^* + o(\lambda^2)(V_E^{\ell b} + V_E^{\ell' b})^*, \quad \ell \neq \ell',$$

$$Y_D^{qq'} \propto V_E^{\tau q} V_E^{\tau q'} + \frac{c_{DU} m_t}{c_{DE} m_\tau} V_{\rm CKM}^{tq} V_{\rm CKM}^{tq'}, \quad q \neq q',$$

$$Y_U^{qq'} \propto V_E^{\tau q} V_E^{\tau q'} + \frac{c_{UD} m_b}{c_{UE} m_\tau} V_{\rm CKM}^{tq} V_{\rm CKM}^{tq'} + o(\lambda), \quad q \neq q',$$

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Neutral meson mixing (third row of V_E)

$$
\begin{aligned}
H^q_{\rm NP} &= -\frac{1}{2m_H^2}\left(Y_D^{bq}\overline{b_L}q_R + Y_D^{qb*}\overline{b_R}q_L\right)^2 - \frac{1}{2m_A^2}\left(iY_D^{bq}\overline{b_L}q_R - iY_D^{qb*}\overline{b_R}q_L\right)^2 \\
&\approx -\frac{2|Y_D^{bq}|^2}{m_H^2}\overline{b_L}q_R\overline{b_R}q_L, \quad q = d, s.
\end{aligned}
$$

$\overline{b_L}q_R\overline{b_L}q_R$ and $\overline{b_R}q_L\overline{b_R}q_L$ vanish in the limit $m_H = m_A$

Observable	Current limit	Future sensitivity
$\mathbf{h_d}$	0.26	0.049 (0.038)
$\mathbf{h_s}$	0.12	0.044 (0.031)
$\lvert(\Delta M_K)_{\rm NP}\rvert$	5.2×10^{-15} GeV	0.2×10^{-15} GeV

$$
\frac{\langle B_q^0|H^q_{\rm NP}|\overline{B_q^0}\rangle}{\langle B_q^0|H^q_{\rm SM}|\overline{B_q^0}\rangle} = \mathbf{h_q}e^{i\sigma_q}
$$

$$
(\Delta M_K)_{\rm NP} = \frac{1}{m_K}\langle K^0|H^K_{\rm NP}|\overline{K^0}\rangle
$$

CKMfit, 2006.04824; KTEV, 1011.0127;
Bai, et al. '14; Wang '23.

Phenomenology: A Window to Check SO(10)

Proton decay (2×2 top-left submatrix of V_E)

$$\frac{\Gamma(p \to \pi^+\overline{\nu})}{\Gamma(p \to K^+\overline{\nu})} = \frac{4\left(1 - m_K^2/m_p^2\right)^{-2} \langle \pi^+|(du)_R d_L|p\rangle^2}{\langle K^+|(us)_R d_L|p\rangle^2 + \lambda^2 \langle K^+|(ud)_R s_L|p\rangle^2}$$

$$\frac{\Gamma(p \to \pi^0 e^+)}{\Gamma(p \to \pi^+\overline{\nu})} = |V_E^{ed} + \frac{\lambda}{2} V_E^{es}|^2, \qquad \frac{\Gamma(p \to \pi^0 \mu^+)}{\Gamma(p \to \pi^+\overline{\nu})} = |V_E^{\mu d} + \frac{\lambda}{2} V_E^{\mu s}|^2,$$

$$\frac{\Gamma(p \to K^0 e^+)}{\xi_K \Gamma(p \to K^+\overline{\nu})} = |V_E^{es} + \lambda V_E^{ed}|^2, \qquad \frac{\Gamma(p \to K^0 \mu^+)}{\xi_K \Gamma(p \to K^+\overline{\nu})} = |V_E^{\mu s} + \lambda V_E^{\mu d}|^2,$$

$$\text{with} \quad \xi_K = \frac{2\langle K^0|(us)_R u_L|p\rangle^2}{\langle K^+|(us)_R d_L|p\rangle^2 + \lambda^2 \langle K^+|(ud)_R s_L|p\rangle^2} \approx 6.4.$$

Symmetric Yukawa couplings simplifies a lot.

Perez '04; Nath, Perez '07.

Decay Mode	$\ell = e^+$	$\ell = \mu^+$	$\ell = \bar{\nu}$
$p \to \pi\ell$	$> 2.4 \times 10^{34}$ yr	$> 1.6 \times 10^{34}$ yr	$> 3.9 \times 10^{32}$ yr
$p \to K\ell$	$> 1.0 \times 10^{33}$ yr	$> 3.6 \times 10^{33}$ yr	$> 5.9 \times 10^{33}$ yr

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RGE towards M_I changes:

The IR theory (M_{EW}): SM → 2HDM

Just estimate an upper limit here.
Long way to the complete GUT Yukawa texture.

$$(Y_E^{\tau\ell} - Y_E^{\ell\tau})|_{M_{\rm EW}} = (Y_E^{\tau\ell} - Y_E^{\ell\tau})|_{M_I}, (Y_U^{tq'} - Y_U^{q't})|_{M_{\rm EW}} = (Y_U^{tq'} - Y_U^{q't})|_{M_I},$$

$$(Y_D^{bq} - Y_D^{qb})|_{M_{\rm EW}} = (Y_D^{bq} - Y_D^{qb})|_{M_I} + Y_D^{bq} \times \frac{y_t^2}{16\pi^2} \log\left(M_I/M_{\rm EW}\right)\left(\frac{3}{2}|\mathcal{C}_{UU}|^2 - \frac{1}{2}\right),$$

$$(M_D^{\tau\ell} - M_D^{\ell\tau})|_{M_{\rm EW}} = (M_D^{\tau\ell} - M_D^{\ell\tau})|_{M_I}, (M_D^{tq'} - M_D^{q't})|_{M_{\rm EW}} = (M_D^{tq'} - M_D^{q't})|_{M_I},$$

$$(M_D^{bq} - M_D^{qb})|_{M_{\rm EW}} = (M_D^{bq} - M_D^{qb})|_{M_I} + (m_b V_E^{\tau b} V_E^{\tau q}) \times \frac{y_t^2}{16\pi^2} \log\left(M_I/M_{\rm EW}\right)\left(\mathcal{C}_{UU}\mathcal{C}_{\rm DE}\frac{2m_\tau}{m_b}\right).$$

$$\frac{y_t^2}{16\pi^2} \log\left(M_I/M_{\rm EW}\right) \approx 3\%.$$

More on V_E:

$$V_E^T(V^*_{\rm PMNS} m_{\nu_L}^{\rm diag} V^\dagger_{\rm PMNS} - k_1 m_E^{\rm diag})V_E \;=\; -k_1 m_D^{\rm diag} + k_2 (V^T_{\rm CKM} m_U^{\rm diag} V_{\rm CKM}) + k_3 M_T,$$

$$k_1 \;=\; \frac{8(v_4^*)^2}{v_3\langle\Delta_R^0\rangle} - \frac{\langle\Delta_L\rangle}{8v_3}, \quad k_2 \;=\; \frac{16 v_4^*}{\langle\Delta_R^0\rangle}, \quad k_3 \;=\; \frac{8v_3}{\langle\Delta_R^0\rangle}.$$

$$M_T \;=\; V^T_{\rm CKM} m_U^{\rm diag} V_{\rm CKM}(V_E^T m_E^{\rm diag} V_E - m_D^{\rm diag})^{-1} V^T_{\rm CKM} m_U^{\rm diag} V_{\rm CKM}.$$

M_T is ‘diagonal’ when $m_t \to \infty$ and $V_{\rm CKM} \to \mathbf{1}$

- $k_1 m_\tau \gg m_{\nu_L} \sim 0.1$ eV, $V_E \sim \mathbb{1}$.
- $k_1 m_\tau \lesssim m_{\nu_L} \sim 0.1$ eV, $V_E \sim V_{\rm PMNS}$.