Karlsruhe Institute of Technology
Collaborative Research Center TRR 257
Particle Physics Phenomenology after the Higgs Discovery
Institute for Theoretical Physics

On the road to NLO electroweak corrections to $gg \to HH$ production

YSM 2025, based on 2407.04653

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ITP - KIT, IPPP

What it is all about

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There are 19 parameters in the SM:

- $\alpha_s, \bar{\theta}$
- g, g', μ, λ
- $Y_{u,d,c,s,t,b,e,\mu,\tau}$
- $\theta_{12}, \theta_{23}, \theta_{13}, \delta$

Higgs potential:

$$V(H) = \frac{m_H^2}{2} H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$$

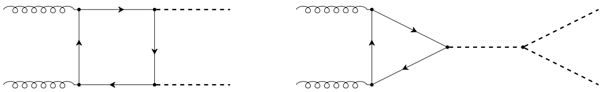
Experimental limits:

ATLAS: $-1.2 < \kappa_\lambda < 7.2$ (Aad, Aakvaag, et al. 2024)

CMS: $-1.39 < \kappa_\lambda < 7.02$ (CMS-PAS-HIG-20-011)

Why calculate higher orders to $gg \to HH$

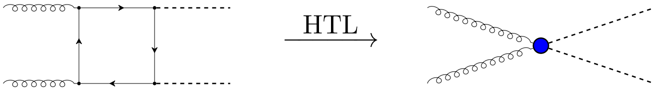
- Sensitivity to Higgs selfcoupling λ



- LO already calculated 1988 (Glover and van der Bij 1988)
- Match expected experimental uncertainty at (HL-)LHC, corrections impact the extracted constraints
- Sizeable effects on differential cross sections
- First full m_t dependent NLO QCD result from 2016 (Borowka, Greiner, et al. 2016), (Baglio, Campanario, et al. 2019)

Beyond NLO$_{\mathrm{QCD}}$

- Approximation of higher orders (restricted to certain kinematic regions) with
  - heavy top limit, (De Florian and Mazzitelli 2018; Florian, Grazzini, et al. 2016; Grigo, Hoff, and Steinhauser 2015)



  - expansions in kinematic parameters (Davies, Herren, et al. 2022)
- On the way to higher orders numerous combinations of these techniques are used, e.g. (Bagnaschi, Degrassi, and Gröber 2023; Grazzini, Heinrich, et al. 2018)
  - N^{3}LO (Chen, Li, et al. 2020a,b)
  - N^{3}LO + N^{3}LL (Ajjath and Shao 2023)
- Reaching a scale uncertainty of $\mathcal{O}(\%)$

Besides N^{n}LO$_{\mathrm{QCD}}$

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- EW corrections are at a similar order of magnitude and distort the distributions
- Les Houches Wishlist > 2015

Wishlist	known dσ	desired dσ
2016	N^{2}LO$_{\mathrm{HTL}}$, NLO$_{\mathrm{QCD}}$	N^{2}LO$_{\mathrm{HTL}}$+NLO$_{\mathrm{QCD}}$+NLO$_{\mathrm{EW}}$
2021	N^{3}LO$_{\mathrm{HTL}} \otimes$ NLO$_{\mathrm{QCD}}$	NLO$_{\mathrm{EW}}$

- Massive internal bosons
- Similar approximative methods can be employed, e.g. (Davies, Schönwald, et al. 2023)
- Several partial results (Borowka, Duhr, et al. 2019; Davies, Mishima, et al. 2022; Mühlleitner, Schlenk, and Spira 2022)
- First full NLO EW result from 2023 (Bi, Huang, et al. 2023)

Our higher order calculation toolchain

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1. Produce contributing diagrams (`QGRAF`)
2. Project onto form factors (`Mathematica, Form`)
3. Reduce the number of integrals (`kira`, `Reduze`, `Ratracer`)
4. Integrate the remaining master integrals (`pySecDec`)
5. Perform the Renormalization (blood, sweat and tears)
6. Crosschecks (`DiffExp`)
7. Put everything back together

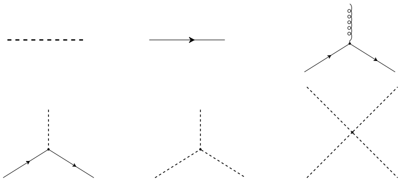
Yukawa-induced	done (unitary gauge)
λ induced	done (unitary gauge)
Vector induced	in progress (Feynman-'t Hooft gauge)
Light quark loops	see Philipp's talk tomorrow

λ and Yukawa corrections: The bare Lagrangian

- Gaugeless limit $\Rightarrow$ Weak bosons decouple
- Unitary gauge $\Rightarrow$ Goldstone bosons decouple

$$\mathcal{L} = -\frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu} + \frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) - \frac{m_{H,0}^2}{2}H_0^2 - \frac{m_{H,0}^2}{2v_0}H_0^3 - \frac{m_{H,0}^2}{8v_0^2}H_0^4$$
$$+ i\bar{t}_0 \not{D} t_0 - m_{t,0}\bar{t}_0 t_0 - \frac{m_{t,0}}{v_0}H_0\bar{t}_0 t_0 + \mathrm{constant}$$

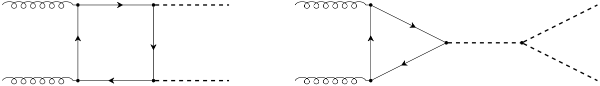
Yields Feynman rules for:



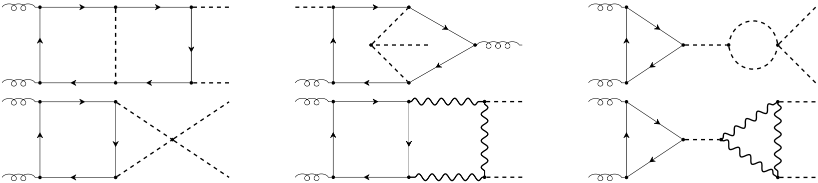
Reparametrized in terms of $m_{H,0}$, $m_{t,0}$ and v_0.

Contributing Diagrams

LO



NLO (examples)



Automated by the tool `QGRAF`. (Nogueira 1993)

Splitting and Projecting

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Each diagram is

- projected onto form factors F_i for two different tensor structures,
- sorted into classes, according to the occurring couplings, e.g.

$$g_{t,0} \equiv \frac{m_{t,0}}{v_0} \qquad g_{3,0} \equiv \frac{3m_{H,0}^2}{v_0} \qquad g_{4,0} \equiv \frac{3m_{H,0}^2}{v_0^2} \,,$$

- tagged as 1PI or 1PR contribution.

$$F_i|_{\mathrm{NLO}_{\lambda,\mathrm{Yuk}}} = g_{s,0}^2 \Big(g_{3,0}\, g_{4,0}\, g_{t,0}\; F_{i,g_3 g_4 g_t} + g_{3,0}^3\, g_{t,0}\; F_{i,g_3^3 g_t} + g_{4,0}\, g_{t,0}^2\; F_{i,g_4 g_t^2} + g_{3,0}^2\, g_{t,0}^2\; F_{i,g_3^2 g_t^2} + g_{3,0}\, g_{t,0}^3\; F_{i,g_3 g_t^3} + g_{t,0}^4\; F_{i,g_t^4} \Big)$$

Type	$g_3 g_4 g_t$	$g_3^3 g_t$	$g_4 g_t^2$	$g_3^2 g_t^2$	$g_3 g_t^3$	g_t^4
1PI	0	0	3	6	24	60
1PR	12	6	1	6	24	26
Total	12	6	4	12	48	86

IBP Reduction

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- Use integration by parts to relate different integrals to each other:

$$\int \prod_{\ell=1}^{L} \mathrm{d}^D k_\ell \frac{\mathrm{d}}{\mathrm{d}k_i^\mu} \left[\eta^\mu \mathcal{I}(\vec{\eta})\right] = 0$$

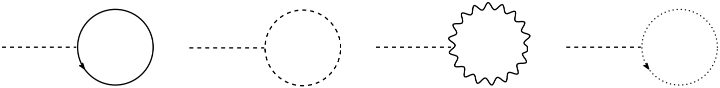
- Choose a suitable basis of master integrals M.I.:
  - prefer dots over numerators, i.e. modified propagator powers and dimension shifts
  - search for finite coefficients for top-level M.I. from non-planar sectors
- Yukawa-&λ-induced: fully symbolic reduction to M.I.s retaining dependence on s, t, m_t and m_H using `kira` with `ratracer` (Klappert, Lange, et al. 2021; Magerya 2022)
- Full EW: be thrilled for YSM 2026

The Master Integrals

- *d*-factorizing integrals, i.e. dimensionality *d* and kinematics dependent parts are separated
- Still, too many mass scales to solve analytically
- Numerical evaluation using `pySecDec` (Heinrich, Jones, et al. 2024)

	# M.I.	spur. poles
Yukawa & λ	492	$\mathcal{O}(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2})$
Full EW	ca. 1300	???

Tadpole Renormalization



- At higher orders the vev gets shifted.
- Fleischer-Jegerlehner tadpole scheme: (Fleischer and Jegerlehner 1981)

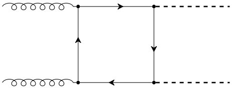
$$H + v \to H + v + \Delta v$$

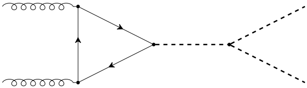
- Require the tadpole diagrams T_H to vanish also at NLO through the tadpole counterterm

$$\delta T = -T_H$$

- Identify $\delta T = -\Delta v \, m_H^2$
- This corresponds to a redistribution of tadpole contributions.

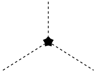
Counterterms





Introduce CTs:

$$
\begin{aligned}
H_0 &= \sqrt{Z_H} H = \sqrt{1+\delta_H} H \\
t_0 &= \sqrt{Z_t} t = \sqrt{1+\delta_t} t \\
m_{H,0}^2 &= m_H^2 (1 + \delta m_H^2) \\
m_{t,0} &= m_t (1 + \delta m_t) \\
v_0 + \Delta v &= v(1+\delta_v) + \Delta v
\end{aligned}
$$



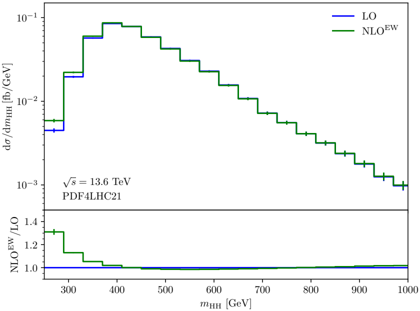
$$
= -i3\frac{m_H^2}{v}\left(\delta m_H^2 + \frac{3}{2}\delta_H - \delta_v - \frac{\delta T}{v m_H^2}\right)
$$

etc.

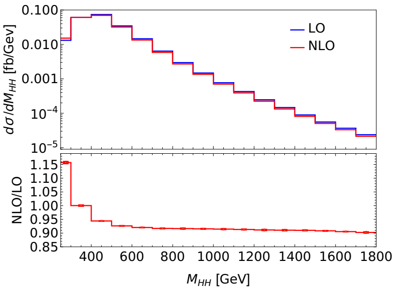
- $\delta_H, \delta_t, \delta m_H^2, \delta m_t$ fixed through on-shell renormalization conditions
- δ_v fixed in G_μ scheme according to (Biekötter, Pecjak, et al. 2023)

The Cross Section

Corrections		Yukawa		Full EW
$\sqrt{s}$	13 TeV	13.6 TeV	14 TeV	14 TeV
LO [fb]	16.45	18.26	19.52	19.96
$\mathrm{NLO}^{\mathrm{EW}}$ [fb]	16.69	18.52	19.79	19.12
$\mathrm{NLO}^{\mathrm{EW}}/\mathrm{LO}$	1.01	1.01	1.01	0.958



(Bi, Huang, et al. 2023)



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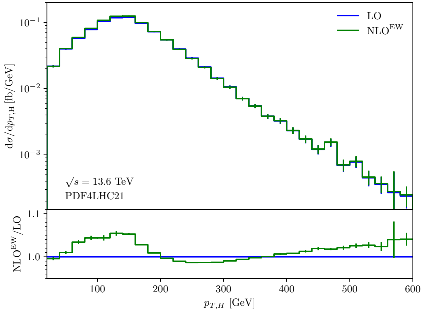

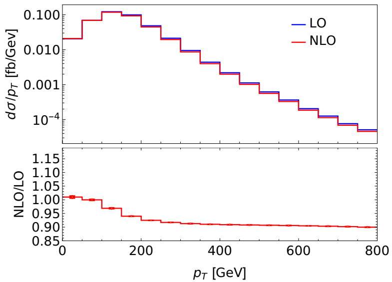

$\longrightarrow$ Considerable changes after including vector bosons!

Conclusion

Where we are:

- Achieved fully symbolic reduction for the gaugeless sector
- Crosschecked with (Davies, Schönwald, et al. 2024)
- Found $K = 1.01$
- Observations for invariant Higgs pair mass and transverse momentum distributions of the cross section
  - Quite large enhancement in low m_{HH} region
  - No Sudakov logs $\Rightarrow$ tail of distributions only slightly changed
  - Dominant contributions to the tail from vector bosons

Where to go:

- Include the full EW corrections and cross-check the result of (Bi, Huang, et al. 2023)
- Investigate the effects of the bottom quark
- Implement an EFT framework

Formfactors

Separate the matrix element into tensor structures and formfactors

$$\mathcal{M}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$$

Formfactors can be obtained by using projectors

$$\mathcal{P}_i^{\mu\nu} T_{j,\mu\nu} = \delta_{ij}$$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_1^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2p_2 \cdot p_3 p_1^\nu p_3^\mu}{p_T^2 p_1 \cdot p_2} + \frac{2p_3^\mu p_3^\nu}{p_T^2}$$

with

$$p_T = \sqrt{\frac{ut - m_H^4}{s}}$$

Deriving the Tensor structure

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General structure:

$$\begin{aligned}\mathcal{M}^{\mu\nu} = \quad & a_{00} g^{\mu\nu} + a_{21} p_2^\mu p_1^\nu + a_{31} p_3^\mu p_1^\nu + a_{23} p_2^\mu p_3^\nu + a_{33} p_3^\mu p_3^\nu \\ & + a_{11} p_1^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{12} p_1^\mu p_2^\nu + a_{13} p_1^\mu p_3^\nu + a_{32} p_3^\mu p_2^\nu\end{aligned}$$

Further constraints from Ward identities:

$$\epsilon_{1,\mu} p_1^\mu = 0 \qquad \epsilon_{2,\nu} p_2^\nu = 0$$

Basic example of Sector Decomposition

$$\mathfrak{I} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y\right)^{-1}$$

Diverging for $x \to 0$ and $y \to 0$

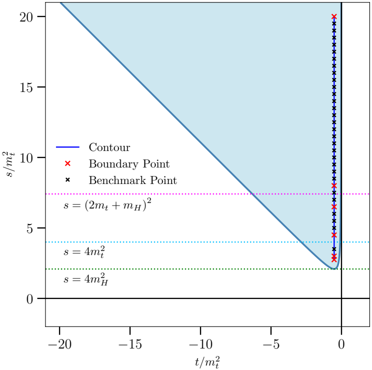
$$\mathfrak{I} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y\right)^{-1} \left[\Theta(x-y) + \Theta(y-x)\right]$$

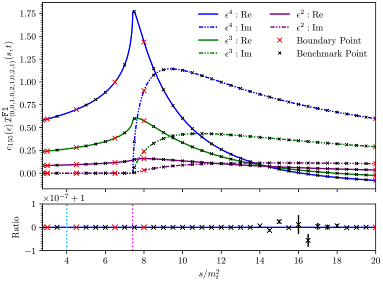
Variable transformation $y = xt$ and $x = yt$

$$\begin{aligned}\mathfrak{I} = &\int_0^1 \frac{\mathrm{d}x}{x^{1+(a+b)\epsilon}} \int_0^1 \frac{\mathrm{d}t}{t^{b\epsilon}\left(1 + (1-x)t\right)} \\ &+ \int_0^1 \frac{\mathrm{d}x}{y^{1+(a+b)\epsilon}} \int_0^1 \frac{\mathrm{d}t}{t^{1+a\epsilon}\left(1 + (1-y)t\right)}\end{aligned}$$

Both limits $x \to 0$ and $y \to 0$ are independent

Crosscheck with DiffExp





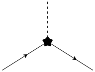
- Run contours in DiffExp between boundary points
- Check pySecDec vs DiffExp for benchmark points

On-Shell Renormalization

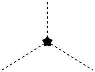
$$0 = \left[\Sigma_i(\hat{p})\right]_{\not{p}=m_i} \qquad 0 = \left[\frac{\mathrm{d}}{\mathrm{d}\not{p}}\Sigma_i(\not{p})\right]_{\not{p}=m_i}$$

$$= -i\left[(m_t - \not{p})\delta_t + m_t\delta m_t - \frac{m_t}{v m_H^2}\delta T\right]$$

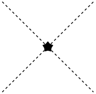
$$= -i\left[(m_H^2 - p^2)\delta_H + m_H^2\delta m_H^2 - 3\frac{\delta T}{v}\right]$$



$$= -i\frac{m_t}{v}\left(\delta m_t + \frac{\delta_H}{2} + \delta_t - \delta_v\right)$$



$$= -i3\frac{m_H^2}{v}\left(\delta m_H^2 + \frac{3}{2}\delta_H - \delta_v - \frac{\delta T}{v m_H^2}\right)$$



$$= -i3\frac{m_H^2}{v^2}(\delta m_H^2 + 2\delta_H - 2\delta_v)$$

Gaugeless Tadpole Renormalization I

- At higher orders the vev gets shifted.
- Fleischer-Jegerlehner tadpole scheme: (Fleischer and Jegerlehner 1981)

$$H + v \to H + v + \Delta v$$

- Require the tadpole diagrams T_H to vanish also at NLO through the tadpole counterterm

$$\delta T = -T_H$$

- Identify $\delta T = -\Delta v\, m_H^2$
- This corresponds to a redistribution of tadpole contributions.

Gaugeless Tadpole Renormalization II

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) + \frac{\mu_0^2}{2}(v_0 + H_0)^2 + \frac{\lambda}{16}(v_0 + H_0)^4$$

$$+ i\bar{t}_0 \not{D} t_0 - y_{t,0}\frac{v_0 + H_0}{\sqrt{2}}\bar{t}_0 t_0 - \frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu}$$

$$\to \frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) + \frac{\mu_0^2}{2}(v_0 + \Delta v + H_0)^2 + \frac{\lambda_0}{16}(v_0 + \Delta v + H_0)^4$$

$$+ i\bar{t}_0 \not{D} t_0 - y_{t,0}\frac{v_0 + \Delta v + H_0}{\sqrt{2}}\bar{t}_0 t_0 - \frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu}$$

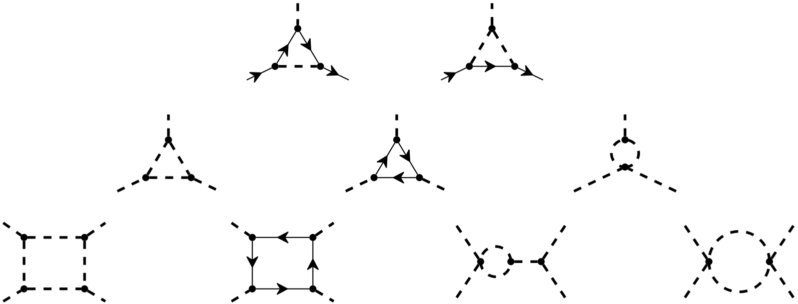
$$= \frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) + H_0\left(\mu_0^2 v_0 + \frac{\lambda_0 v_0^3}{4} + \Delta v(\mu_0^2 + \frac{3}{4}\lambda_0 v_0^2)\right)$$

$$+ H_0^2\left(\frac{\mu_0^2}{2} + \frac{3v_0^2\lambda_0}{8} + \frac{3}{4}\lambda_0 v_0 \Delta v\right) + H_0^3\left(\frac{\lambda_0 v_0}{4} + \Delta v\frac{\lambda_0}{4}\right) + H_0^4\frac{\lambda_0}{16}$$

$$+ i\bar{t}_0 \not{D} t_0 - m_{t,0}\bar{t}_0 t_0 - \frac{m_{t,0}}{v_0}\Delta v\bar{t}_0 t_0 - \frac{m_{t,0}}{v_0}H_0\bar{t}_0 t_0 - \frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu} + \ldots$$

δ_v Counterterm

$$\delta_v|_{\mathrm{UV}} = -\frac{3m_H^4 + 2m_H^2 m_t^2 N_c - 8m_t^4 N_c}{32\pi^2 m_H^2 v^2 \epsilon}$$



$$\delta_v|_{G_\mu} = \frac{1}{2^D \pi^{D/2}} \frac{1}{2v^2} \left(-\frac{m_H^2}{2} + N_c m_t^2 - 2N_c A_0(m_t^2) - 3A_0(m_H^2) + 8N_c \frac{m_t^2}{m_H^2} A_0(m_t^2) \right)$$