

Top-Yukawa-Induced Corrections to Higgs Pair Production

with A. Bhattacharya, F. Campanario, J. Chang, J. Mazzitelli, M. M. Mühlleitner, J. Ronca and M. Spira

Sauro Carlotti, Institute for Theoretical Physics, KIT | Young Scientists Meeting of the CRC TRR 257, July 22, 2025

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Motivation

The Standard Model (SM) has been tested to highest accuracy, but there are open questions that call for new physics.

There are different ways to search for new physics:

- **Direct searches** of new phenomena, e.g. new particles
- **Indirect effects** by computing observables at high precision

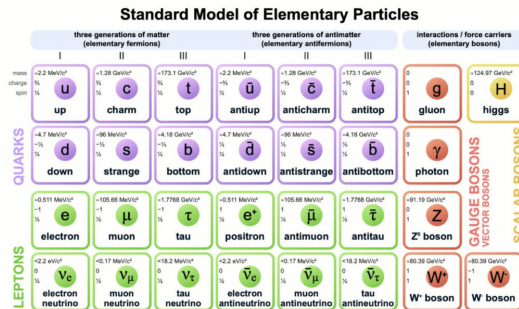


Figure: [\[Wikimedia\]](#)

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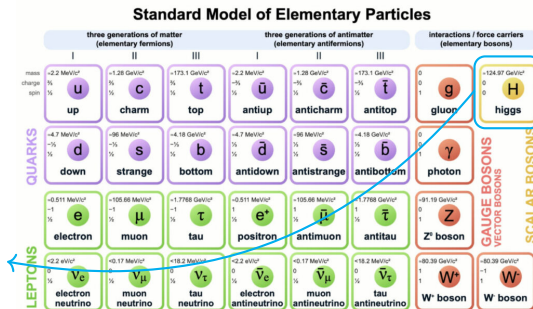


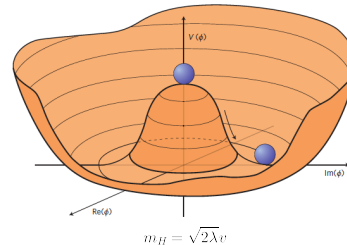
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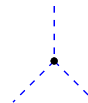
Scalar Higgs Potential in the Standard Model

After the electroweak symmetry breaking (EWSB) the Higgs potential in the SM becomes:


$$V_{\text{SM}} = \frac{1}{2}m_H^2 H^2 + \frac{1}{3!}\lambda_{HHH}H^3 + \frac{1}{4!}\lambda_{HHHH}H^4$$

Figure: [J.Ellis, 2013]





$$\lambda_{HHH} = \frac{3m_H^2}{v}$$



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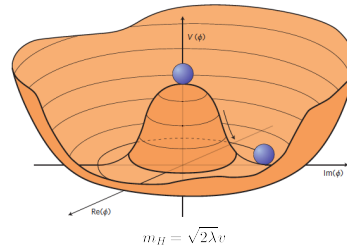
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- Measurement of the Higgs boson mass [ATLAS, CMS, 2012]

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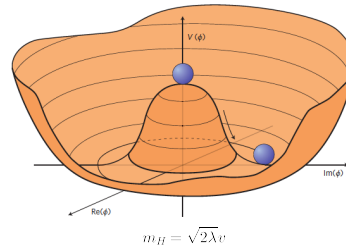
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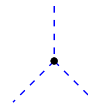
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
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- Test the shape of the Higgs potential

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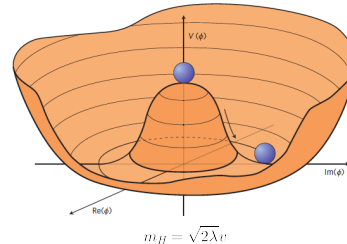
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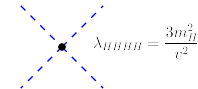
λ_{HHH} is accessible through the Higgs pair production via gluon fusion at hadron colliders

Figure: [J.Ellis, 2013]





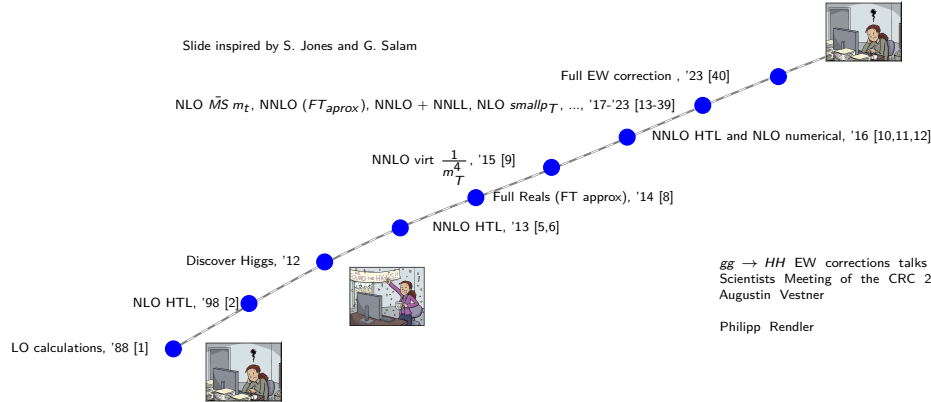
$$\lambda_{HHH} = \frac{3m_H^2}{v}$$



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Evolution of Theoretical Calculations for $gg \rightarrow HH$

Slide inspired by S. Jones and G. Salam



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 96; [3] Shao, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirkle 16; [12] Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirkle 16; [13] Ferrera, Pires 16; [14] Heinrich, Jones, Kerner, Luisoni, Vryonidou 17; [15] Jones, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrandi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafante, Degrandi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagdasarian, Degrandi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23; [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, Jones, Kerner, Stone, Vestner; [39] Li, Si, Wang, Zhang, Zhao 24; [40] Bi, Huang, Huang, Ma, Yu; [41] M. Bonetti, P. Rendler, W.J. Torres; [42] J. Davies, K. Schönwald, M. Steinhauser, D. Stremmer.

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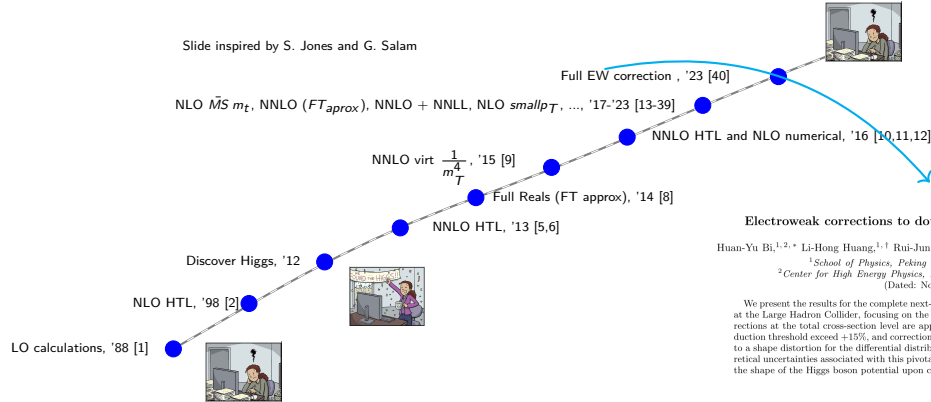
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Top-Yukawa-Induced Corrections

$gg \rightarrow HH$ is a loop-induced process where we have two types of diagrams



Sizeable contributions of the EW corrections come from the top-Yukawa coupling

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Our framework

- Gaugeless limit \rightarrow presence of the Goldstone bosons

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Sizeable contributions of the EW corrections come from the top-Yukawa coupling

Our framework

- Gaugeless limit \rightarrow presence of the Goldstone bosons
- Massive top and bottom quarks, light quarks massless
- Only top-Yukawa-induced corrections + light quark loops

Top-Yukawa-Induced Corrections

Matrix element split into tensor structures

$$\mathcal{M}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$$

The cross section can be written as

$$d\sigma \sim \text{Re} \left[(C_{\Delta} F_{1\Delta} + F_{1\Box})^* (C_{\Delta} F_{1\Delta} + F_{1\Box}) + F_{2\Box}^* F_{2\Box} \right]$$

Since only virtual corrections are involved, the corrections can be added by shifting the LO terms

$$C_{\Delta} F_{1\Delta} \rightarrow C_{\Delta} F_{1\Delta} (1 + \delta_1 + \Delta_{HHH})$$

$$F_{1\Box} \rightarrow F_{1\Box} (1 + \Delta_{1\Box})$$

$$F_{2\Box} \rightarrow F_{2\Box} (1 + \Delta_{2\Box})$$

→ Expand $d\sigma$ properly to next-to-leading order (NLO)

$$\begin{aligned} T_1^{\mu\nu} &= g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2} \quad \text{with } p_T = \sqrt{\frac{tu - m_H^4}{s}} \\ T_1^{\mu\nu} &= g^{\mu\nu} + \frac{m_H^2 p_1^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2p_1 \cdot p_3 p_3^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} \\ &\quad - \frac{2p_2 \cdot p_3 p_1^\nu p_3^\mu}{p_T^2 p_1 \cdot p_2} + \frac{2p_3^\nu p_3^\mu}{p_T^2} \end{aligned}$$

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Contributions computed

$$\begin{aligned}
C_\Delta F_{1\Delta} &\rightarrow C_\Delta F_{1\Delta} (1 + \delta_1 + \Delta_{HHH}) \\
F_{1\Box} &\rightarrow F_{1\Box} (1 + \Delta_{1\Box}) \\
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Relevant Diagrams

How many diagrams need to be considered?

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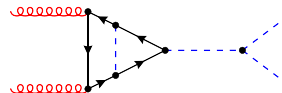
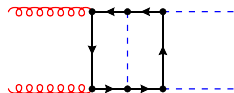
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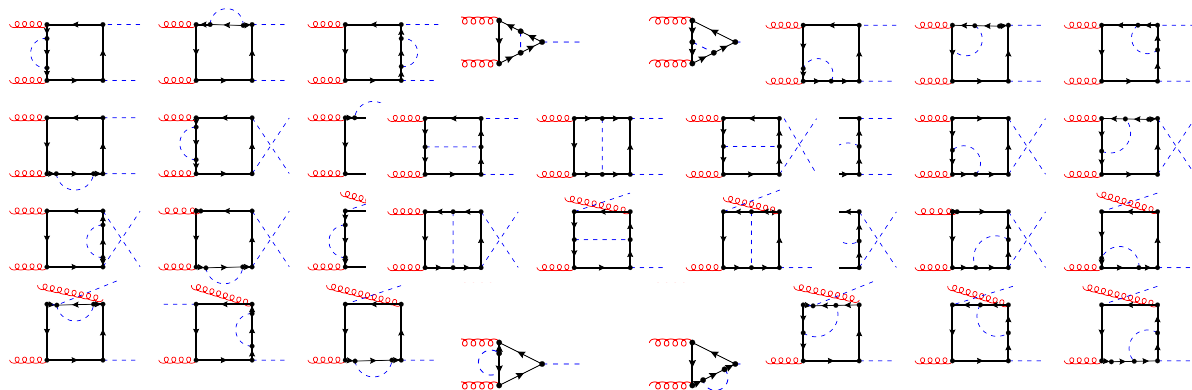
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- Identify all the Feynman diagrams that contribute to the process for the considered correction

Workflow

- Identify all the Feynman diagrams that contribute to the process for the considered correction
- Apply Feynman parametrization to all the diagrams

$$\frac{1}{a_1^{\alpha_1} a_2^{\alpha_2} \dots a_n^{\alpha_n}} = \frac{\Gamma(\alpha_1 + \dots + \alpha_n)}{\Gamma(\alpha_1) \dots \Gamma(\alpha_n)} \int_0^1 dx_1 \dots \int_0^{1-x_1-\dots-x_{n-2}} dx_{n-1} \frac{(1-x_1-\dots-x_{n-1})^{\alpha_1-1} x_1^{\alpha_2-1} \dots x_{n-1}^{\alpha_n-1}}{(a_1(1-x_1-\dots-x_{n-1}) + a_2 x_1 + \dots a_n x_{n-1})^{\sum \alpha_i}}$$

→ up to $(n-1)$ -dimensional integral

Workflow

- Identify all the Feynman diagrams that contribute to the process for the considered correction
- Apply Feynman parametrization to all the diagrams
- Small mass regulator for the internal particles + endpoint subtraction \rightarrow isolate the UV singularities

$$m^2 \rightarrow m^2 (1 - i\delta)$$

$$\int_0^1 dx \frac{f(x)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \frac{f(x) - f(1)}{1-x} + \mathcal{O}(\epsilon)$$

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- Instability? → Integration by parts

$$\int_0^1 dx \frac{f(x)}{(a+bx)^3} = \frac{f(0)}{2a^2b} - \frac{f(1)}{2b(a+b)^2} + \int_0^1 \frac{dx}{2b} \frac{f'(x)}{(a+bx)^2}$$

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 - Richardson extrapolation to do the narrow-width limit $\delta \rightarrow 0$
- ... and if is it not enough? \rightarrow choice of a new parametrization

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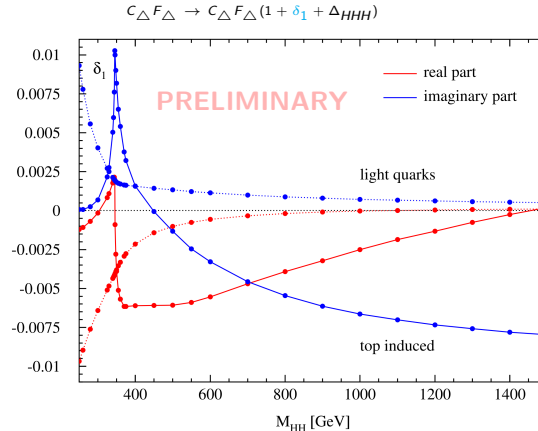
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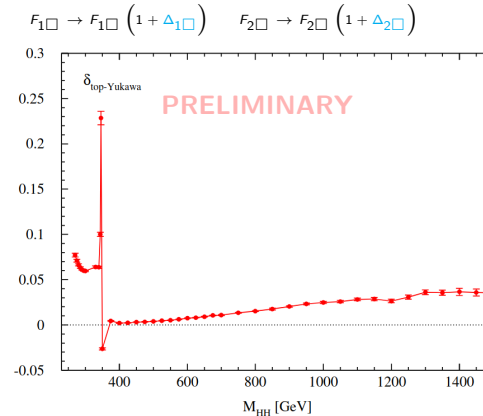
Triangle-like Corrections

- Top-Yukawa-induced corrections to \mathcal{M} as a function of the invariant Higgs-pair mass M_{HH}
- Solid lines represent the top-Yukawa-induced corrections
- Dashed lines represent the light quark loops contributions
- Large contribution around $t\bar{t}$ threshold



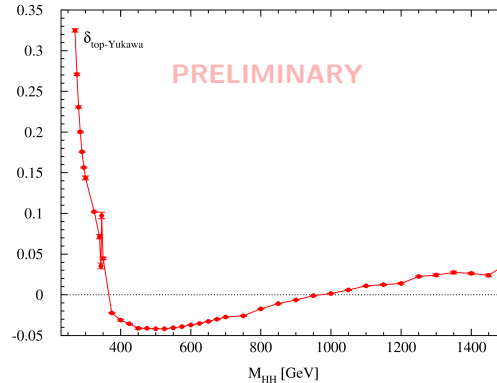
Box-like Corrections

- Top-Yukawa-induced corrections to σ as a function of the invariant Higgs-pair mass M_{HH}
- $\delta_{top-Yukawa}$: corrections proportional to $\Delta_{1\Box}$, $\Delta_{2\Box}$, normalized to σ_{LO}
- Large contributions around the $t\bar{t}$ threshold
- Few percent corrections at the high energy tail



Full Top-Yukawa-induced corrections

- Full Top-Yukawa-induced corrections to σ as a function of the invariant Higgs-pair mass M_{HH} , normalized to σ_{LO}
- Large contributions around the $t\bar{t}$ threshold and below (Δ_{HHH})



Light Quark Loops Corrections

- Projector 1 light quark loops corrections to σ as a function of the invariant Higgs-pair mass M_{HH} , normalized to σ_{LO}
- Sizeable contributions below $M_{HH} = 400$ GeV
- Small contributions at high energies

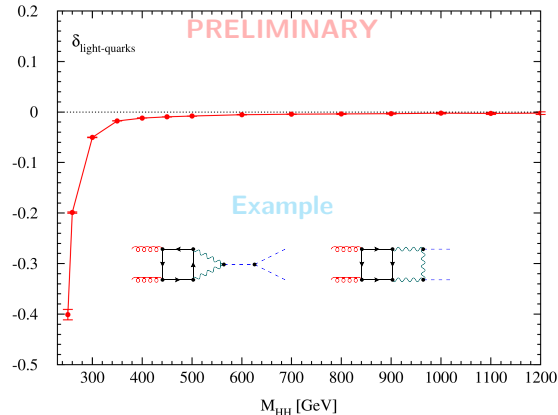


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Summary and Conclusions

- We calculated the next-to-leading order top-Yukawa-induced corrections and the light quark loop induced corrections to $gg \rightarrow HH$ production
- Full symbolic dependencies on kinematical variables and masses kept
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 - $\sim 2\text{--}4\%$ contribution from top-Yukawa-induced corrections in the high-energy tail

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What's next?

- Complete the calculations and perform further cross-checks

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What's next?

- Complete the calculations and perform further cross-checks
- Full EW corrections to double Higgs production
- Investigate the residual theoretical uncertainties

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Summary and Conclusions

- We calculated the next-to-leading order top-Yukawa-induced corrections and the light quark loop induced corrections to $gg \rightarrow HH$ production
- Full symbolic dependencies on kinematical variables and masses kept
- Preliminary results suggest:
 - no sizeable corrections at the high-energy tail from the light quark loops
 - $\sim 2\text{--}4\%$ contribution from top-Yukawa-induced corrections in the high-energy tail

What's next?

- Complete the calculations and perform further cross-checks
- Full EW corrections to double Higgs production
- Investigate the residual theoretical uncertainties

Thank You for your Attention!

Motivation
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Top-Yukawa-Induced Corrections: Computation
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Results
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Summary and Conclusions
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BACKUP

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Projectors

Matrix element can be written as:

$$M^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$$

The Form factors can be obtained via the projectors

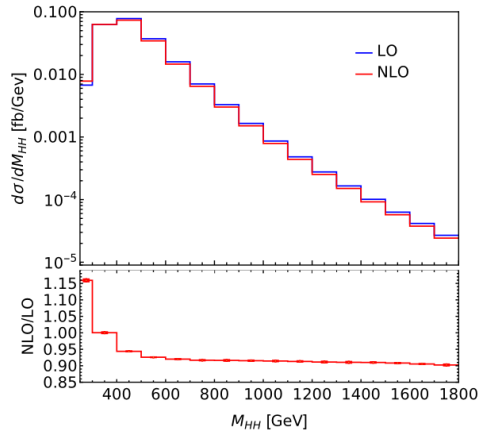
$$P_i^{\mu\nu} T_{j,\mu\nu} = \delta_{ij}$$

$$T_1^{\mu\nu} = g_{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g_{\mu\nu} + \frac{m_H^2 p_1^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2p_1 \cdot p_3 p_3^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2p_2 \cdot p_3 p_1^\nu p_3^\mu}{p_T^2 p_1 \cdot p_2} + \frac{2p_3^\nu p_3^\mu}{p_T^2}$$

with $p_T = \sqrt{\frac{tu - m_H^4}{s}}$

Full EW cross section



Bi, Huang, Huang, Ma, Yu 24.

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Integration by parts

$$\int_0^1 dx \frac{f(x)}{N^3(x)} \stackrel{1'IBP}{=} -\frac{1}{2b} \frac{f(x)}{N^2(x)} \Big|_0^1 + \frac{1}{2b} \int_0^1 dx \frac{f'(x)}{N^2(x)} \stackrel{2'IBP}{=} \frac{1}{2b} \left[-\frac{f(x)}{N^2(x)} \Big|_0^1 + \frac{1}{b} \left(\frac{f'(x)}{N(x)} \Big|_0^1 + \int_0^1 dx \frac{f''(x)}{N^2(x)} \right) \right]$$

Richardson Extrapolation

Let a function $I(\epsilon)$ behave for small ϵ as:

$$I(\epsilon) = I(0) + \mathcal{O}(\epsilon)$$

If we know $I(\epsilon)$ from two different values of ϵ , we can construct the new function:

$$R_n(\epsilon, t) = \frac{t^n I(\epsilon) - I(t\epsilon)}{t^n - 1}$$

where $I(\epsilon) = I(0) + \mathcal{O}(\epsilon^{n+1})$ for small regulator.

Thus, the new extrapolation function will be:

$$R_1(\epsilon, t) = \frac{tI(\epsilon) - I(t\epsilon)}{t - 1}$$

Richardson Extrapolation

Iteratively we have:

$$R_1(\epsilon, t) = 2I(\epsilon) - I(2\epsilon)$$

$$R_2(\epsilon, t) = \frac{1}{3} [8I(\epsilon) - 6I(2\epsilon) + I(4\epsilon)]$$

$$R_3(\epsilon, t) = \frac{1}{21} [64I(\epsilon) - 56I(2\epsilon) + 14I(4\epsilon) - I(8\epsilon)]$$

$$\vdots$$

where $t = 2$.

Richardson Extrapolation

Improve accuracy by combining approximations with different step sizes.

- 1 Compute the approximation S_h for step size h .
- 2 Compute the approximation S_{2h} for step size $2h$.
- 3 Extrapolated solution:

$$S_{\text{extr}} = \frac{2S_h - S_{2h}}{1}$$

- 4 First four polynomials of Richardson extrapolation:

$$S_h = p_0 + p_1 h + p_2 h^2 + p_3 h^3 + \dots$$

$$S_{2h} = p_0 + p_1(2h) + p_2(2h)^2 + p_3(2h)^3 + \dots$$

- 5 Applying Richardson's method, we obtain:

$$S_{\text{extr}} = \frac{2(p_0 + p_1 h + p_2 h^2 + p_3 h^3) - (p_0 + p_1(2h) + p_2(2h)^2 + p_3(2h)^3)}{1}$$

Higgs Potential in the Gaugeless Limit

$$\begin{aligned}
 \mathcal{V}(\phi) = -\mu|\phi|^2 + \frac{\lambda}{2}|\phi|^4 = & -\frac{m_H^2}{8}v^2 + \frac{m_H^2}{2}H^2 + \frac{m_H^2}{v} \left[\frac{H^3}{2} + \frac{H}{2}(G^0)^2 + HG^+G^- \right] \\
 & + \frac{m_H^2}{2v^2} \left[\frac{H^4}{4} + \frac{H^2}{2}(G^0)^2 + H^2G^+G^- + (G^+G^-)^2 + (G^0)^2G^+G^- + \frac{(G^0)^4}{4} \right] |
 \end{aligned}$$

Counter Terms

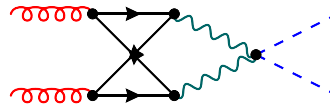
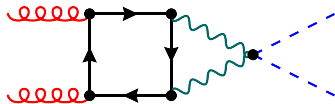
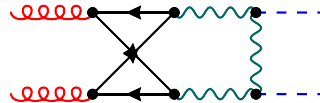
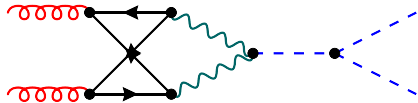
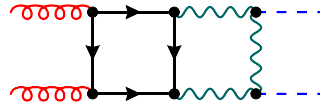
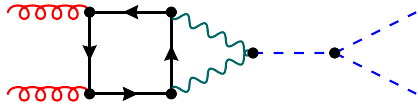
$$CT_1 = \frac{1}{2} \Sigma'_H(Q^2) = 3x_t [(4m_t^2 - Q^2) B'_0(Q^2, m_t, m_t) - B_0(Q^2, m_t, m_t)]$$

$$CT_2 = -\frac{\delta v}{v} = -\frac{1}{2} \frac{\delta m_W^2}{m_W^2} = \frac{1}{2} \frac{\Sigma_W(0)}{m_W^2} = \frac{T_1}{vm_H^2} + x_t [B_0(0, m_t, m_b) + 2B_0(0, m_t, m_t) + m_t^2 B'_0(0, m_t, m_b)]$$

$$CT_3 = -\frac{\delta m_t}{m_t} \left(\frac{m_t \partial}{\partial m_t} F_{LO}^{(n)} \right) \frac{1}{F_{LO}}$$

where $x_t = \frac{G_F m^2}{8\sqrt{2}\pi^2}$

Light quark diagrams



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Cross section

The cross section can be decomposed as follows

$$d\sigma = d\sigma_{LO} + d\sigma_{virt}$$

$$d\sigma_{virt} \sim Re \left[(C_{\Delta} F_{1\Delta}^{LO} + F_{1\Box}^{LO})^* (C_{\Delta} F_{1\Delta} + F_{1\Box}) + F_{2\Box}^{LO*} F_{2\Box} \right]$$

$$\delta_{top-yukawa-\Delta} \sim Re \left| (C_{\Delta} F_{1\Delta}^{LO})^* C_{\Delta} F_{1\Delta}^{LO} \right|$$

$$\delta_{top-Yukawa} \sim Re \left| \left[(C_{\Delta} F_{1\Delta}^{LO} + F_{1\Box}^{LO})^* (C_{\Delta} F_{1\Delta} + F_{1\Box}) + F_{2\Box}^{LO*} F_{2\Box} \right] \right| - \delta_{top-yukawa-\Delta}$$

Heavy Top-quark Limit (HTL)

In the HTL, the top-Yukawa-induced electroweak corrections to the effective Hgg and $HHgg$ couplings can be obtained as

$$\mathcal{L}_{eff} = C_1 \frac{\alpha_s}{12\pi} G^{a\mu\nu} G_{\mu\nu}^a \log \left(1 + C_2 \frac{H}{v} \right)$$

where $G_{\mu\nu}^a$ denotes the gluonic field-strength tensor, $C_1 = 1 - 3x_t + O(x_t^2)$, and $C_2 = 1 + \frac{7}{2}x_t + O(x_t^2)$. C_1 and C_2 yield the explicit effective Hgg and $HHgg$ couplings,

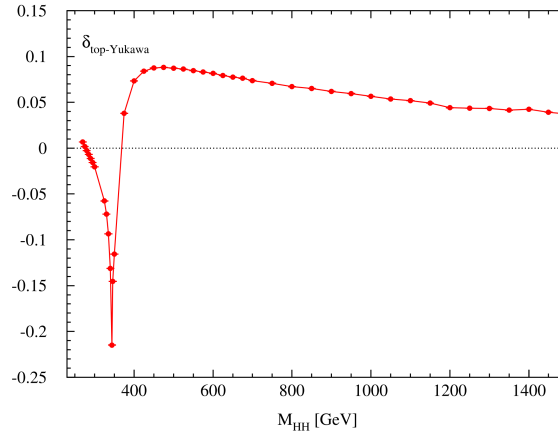
$$\mathcal{L}_{eff} = C_1 \frac{\alpha_s}{12\pi} G^{a\mu\nu} G_{\mu\nu}^a \log \left[\left(1 + \delta_1 \right) \frac{H}{v} + \left(1 + \eta_1 \right) \frac{H^2}{2v^2} + O(H^3) \right]$$

where

$$\delta_1 = \frac{x_t}{2} + O(x_t^2)$$

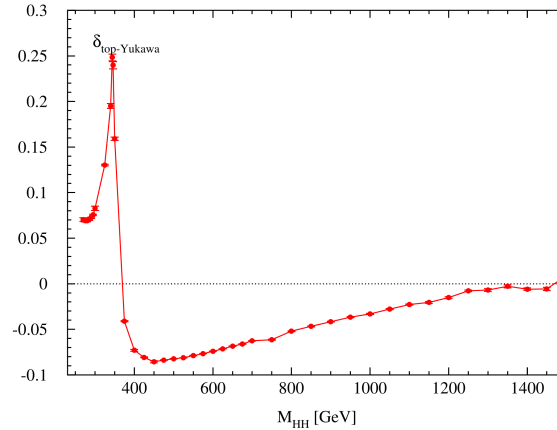
$$\eta_1 = 4x_t + O(x_t^2)$$

Counter Terms



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Box - like Contribution w/o CT



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