

ggxy – NLO QCD corrections to loop induced gg initiated processes

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Based on [arXiv:2506.04323](https://arxiv.org/abs/2506.04323)



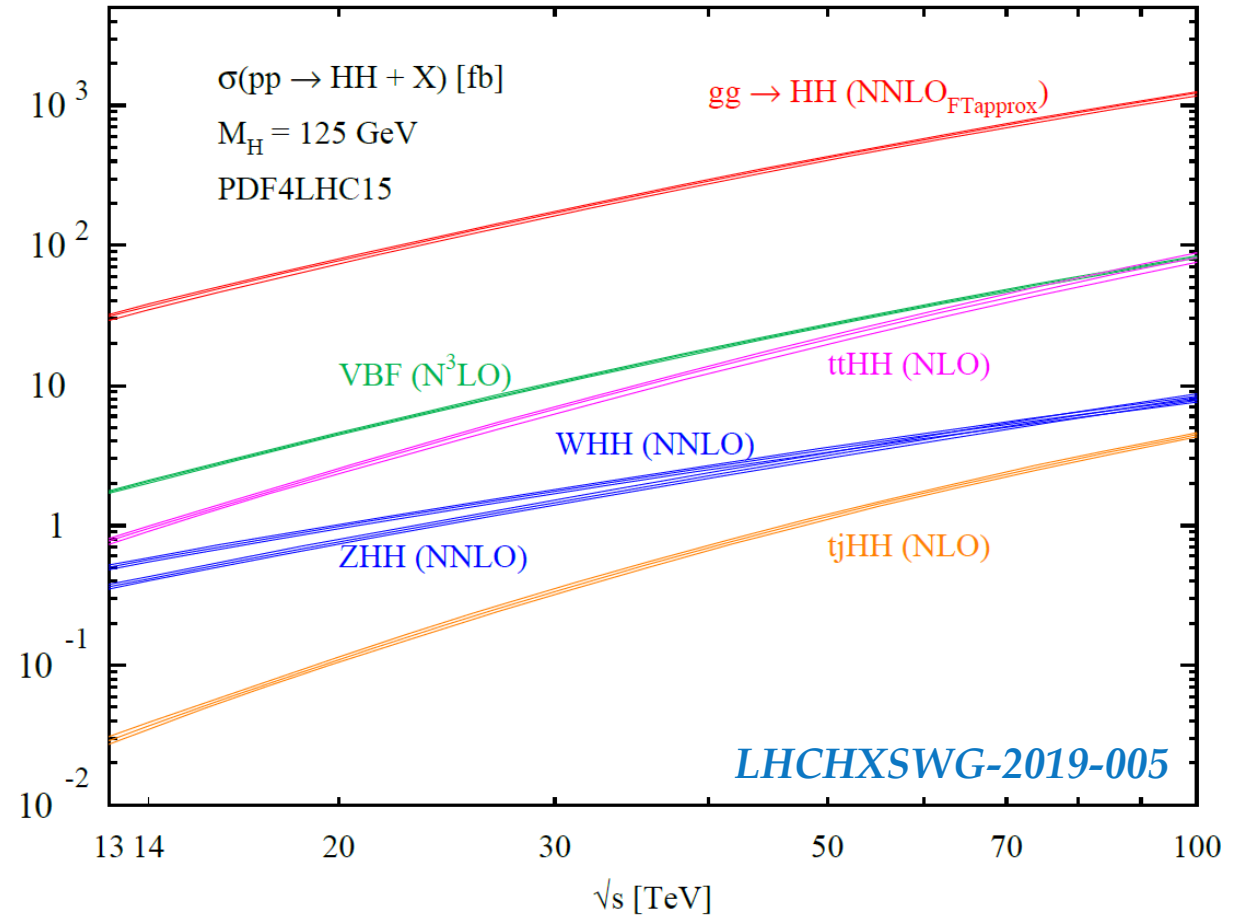
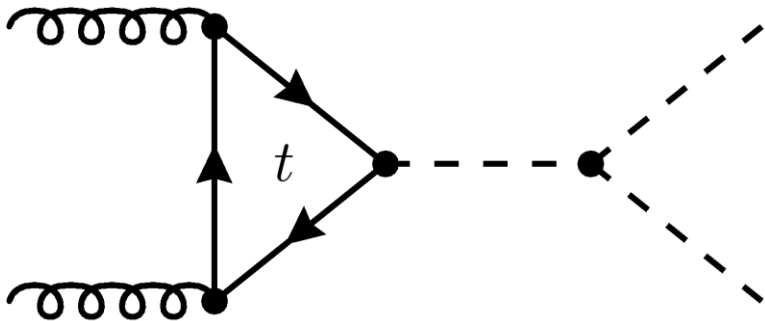
Young Scientists Meeting of the CRC TRR 257, Heidelberg, 22 July 2025

Motivation - HH production

- Standard Model Higgs potential:

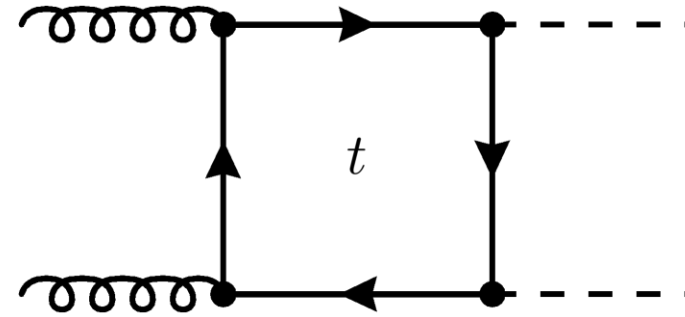
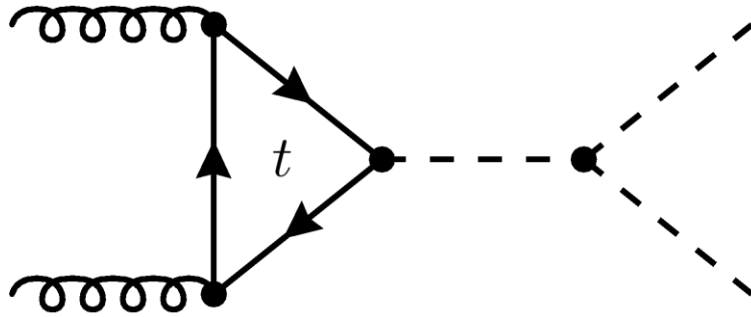
$$V(H) = \frac{m_H^2}{2} H^2 + \lambda_3 v H^3 + \frac{\lambda_4}{4} H^4, \quad \text{with} \quad \lambda_3^{\text{SM}} = \lambda_4^{\text{SM}} = m_H^2 / (2v^2)$$

- Often parametrized by: $\kappa_\lambda = \lambda_3 / \lambda_3^{\text{SM}}$
- Current limits:
 - $-1.2 < \kappa_\lambda < 7.2$ *ATLAS '24*
 - $-1.39 < \kappa_\lambda < 7.02$ *CMS '24*
- HH production dominated by gluon fusion



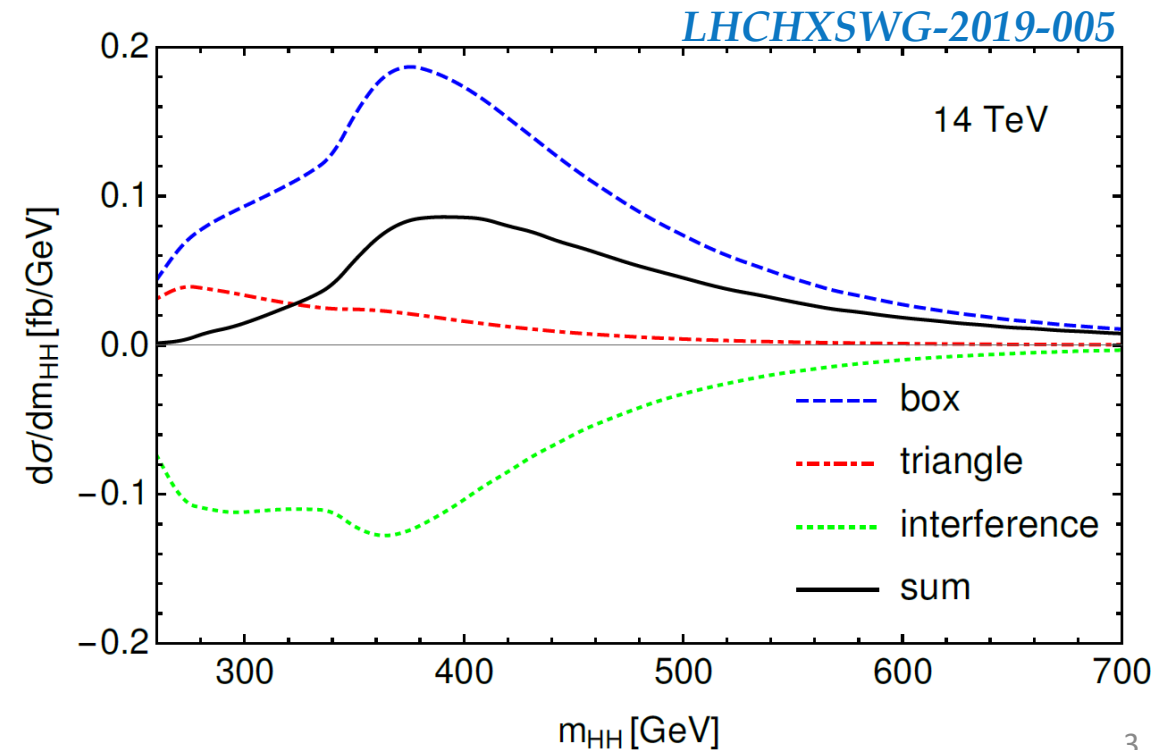
HH production in gluon fusion

- Exact LO (1-loop) results known for a long time:



Glover, van der Bij '88; Plehn, Spira, Zerwas '98

- Destructive interference between triangle and box contributions



Higher order QCD corrections to $gg \rightarrow HH$

■ NLO QCD

- large- m_t expansion
- numerical integration
- large- m_t + threshold exp. with Padé
- high-energy expansion
- small- p_T expansion
- small- t expansion
- high-energy + small- p_T

Dawson, Dittmaier, Spira '98; Grigo, Hoff, Melnikov, Steinhauser '13

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16

Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19

Gröber, Maier, Rauh '17

Davies, Mishima, Steinhauser, Wellman '18 '19

Bonciani, Degrassi, Giardino, Gröber '18

Davies, Mishima, Schönwald, Steinhauser '23

Bellafronte, Degrassi, Giardino, Gröber, Vitti '22; Bagnaschi, Degrassi, Gröber '23

■ NNLO QCD

- large- m_t expansion
- FTapprox
- small- t expansion $n_l n_h$ and n_h (LC)
- 1PR reducible

de Florian, Mazzitelli '13; Grigo, Melnikov, Steinhauser '14; Grigo, Hoff, Steinhauser '18

Davies, Herren, Mishima, Steinhauser '19 '21

Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18

Davies, Schönwald, Steinhauser '23 '25

Davies, Schönwald, Steinhauser, Vitti '24

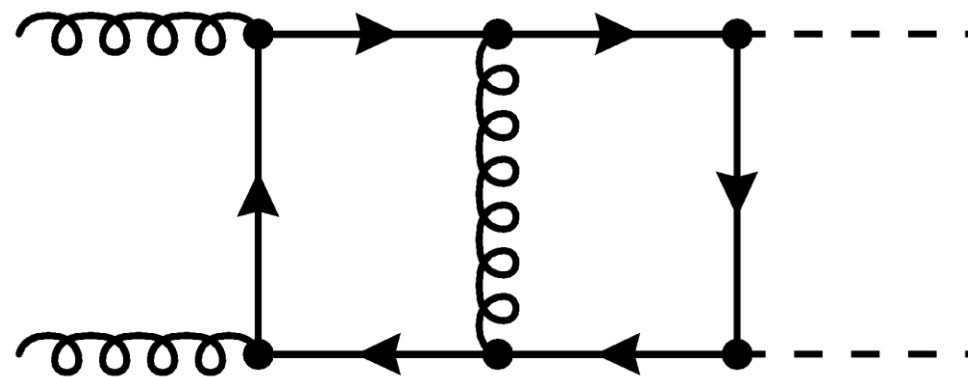
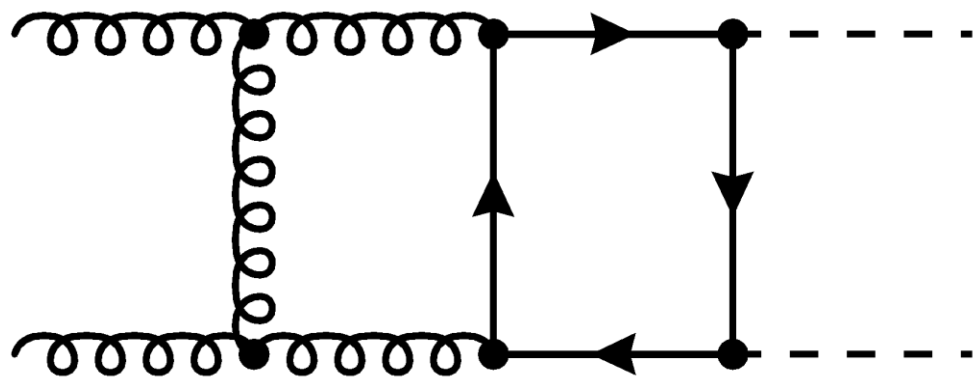
- NNNLO QCD: HTL: *Spira '16; Gerlach, Herren, Steinhauser '18; Chen, Li, Shao, Wang '19; Ajjath, Shao '22*

Small-t expansion

Davies, Mishima, Schönwald, Steinhauser '23

- Consider hierarchy of scales: $s, m_t^2 \gg |t|, m_H^2$
 - Form factors written in terms of Feynman integrals: $I(m_H, m_t, s, t, \epsilon)$
 - Taylor expand in $m_H^2 \rightarrow 0$: $I(m_H, \dots) = I(0, \dots) + m_H^2 I'(0, \dots) + \dots$
 - IBP reduction to master integrals: $J(0, m_t, s, t, \epsilon)$
 - Expand master integrals around $t \rightarrow 0$:

$$J(0, m_t, s, t, \epsilon) = \sum_{i,j} c_{ij}(s, m_t^2) \epsilon^i t^j$$



High-energy expansion

Davies, Mishima, Steinhauser, Wellman '18, '19

- Consider hierarchy of scales: $s, |t| \gg m_t^2 \gg m_H^2$
 - Form factors written in terms of Feynman integrals: $I(m_H, m_t, s, t, \epsilon)$
 - Taylor expand in $m_H^2 \rightarrow 0$: $I(m_H, \dots) = I(0, \dots) + m_H^2 I'(0, \dots) + \dots$
 - IBP reduction to master integrals: $J(0, m_t, s, t, \epsilon)$
 - Expand master integrals around $m_t^2 \rightarrow 0$:

$$J(0, m_t, s, t, \epsilon) = \sum_{i,j,k} c_{ijk}(s, t) \epsilon^i (m_t)^j \log(m_t)^k$$

- Improve convergence with Padé approximation

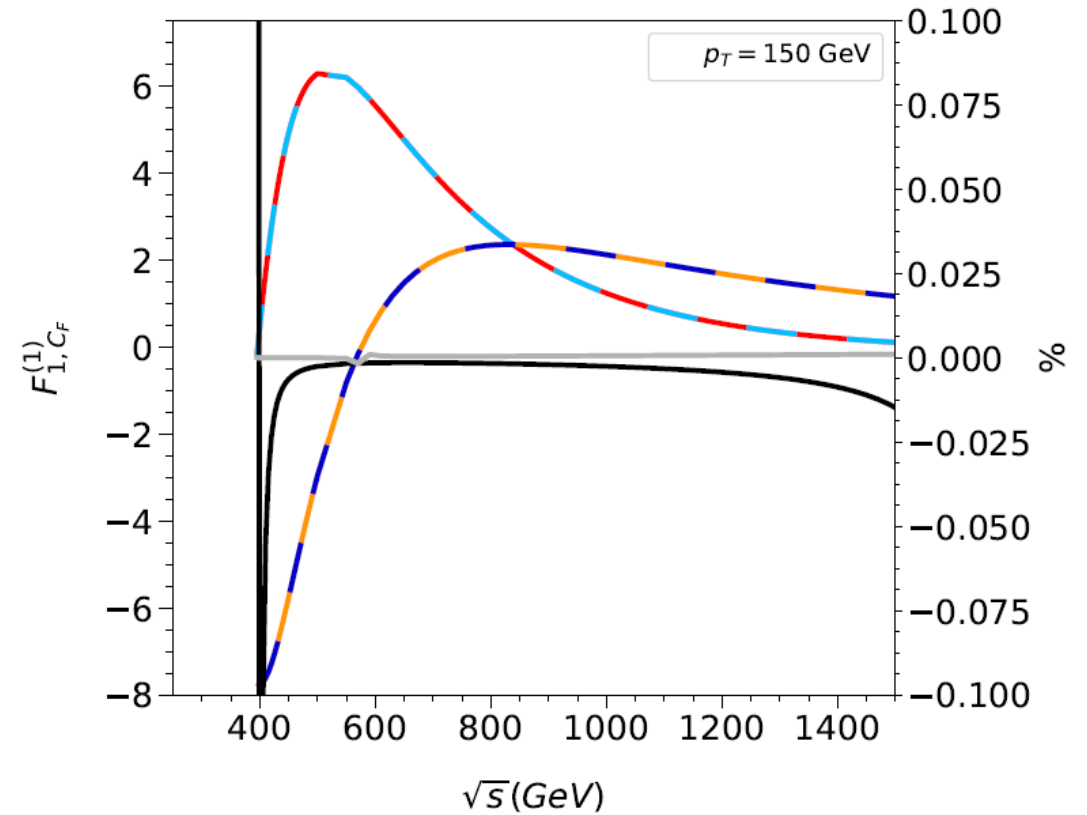
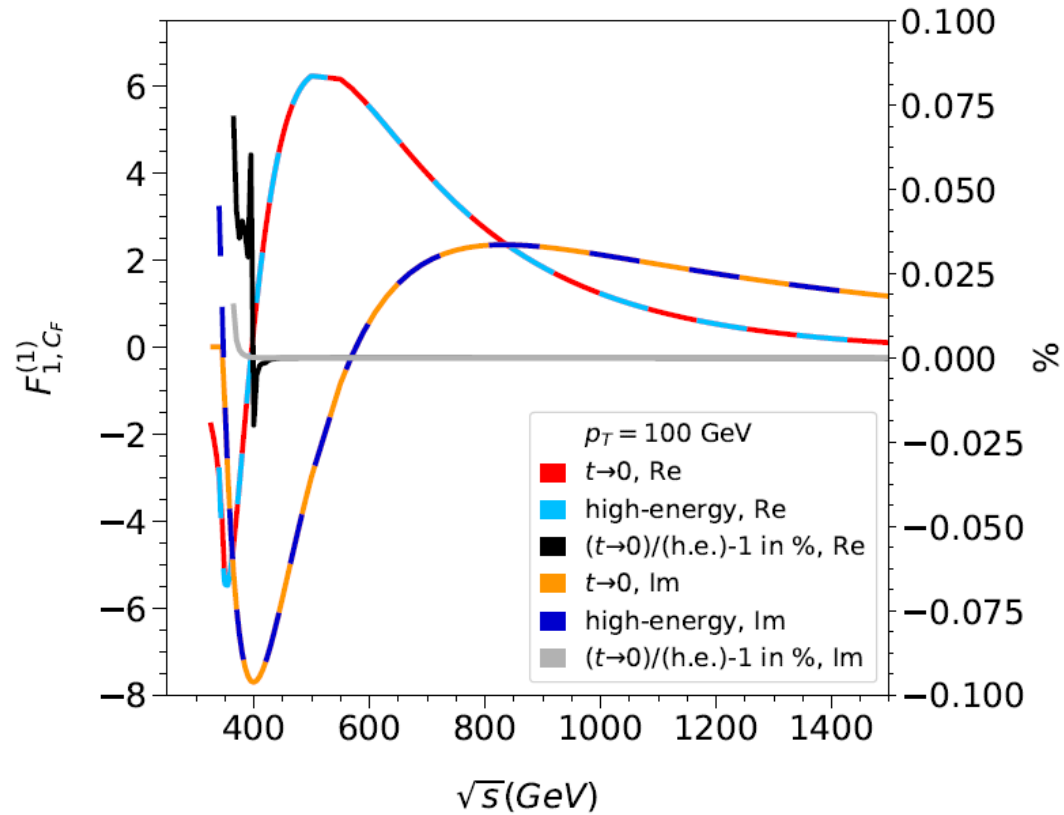
Davies, Mishima, Steinhauser, Wellman '20

$$\sum_{k=0}^N c_k (m_t^2)^k = \frac{a_0 + a_1 (m_t^2) + \dots + a_r (m_t^2)^r}{1 + b_1 (m_t^2) + \dots + b_s (m_t^2)^s} = [r/s](m_t^2), \quad r+s = N$$

- Region of convergence increased from $p_T \gtrsim 500 \text{ GeV}$ to $p_T \gtrsim 150 \text{ GeV}$

Combination of small- t and high-energy expansions

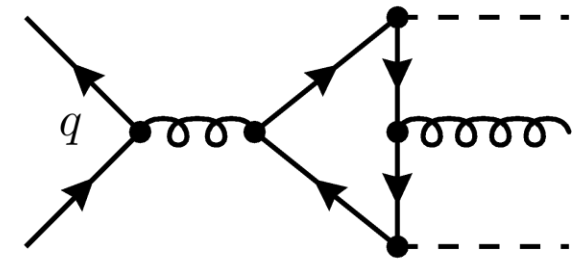
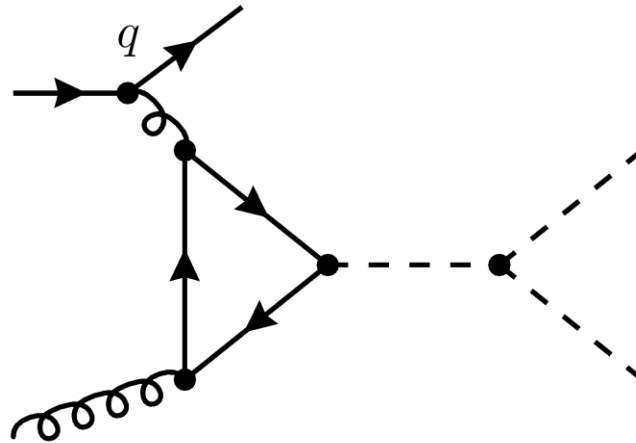
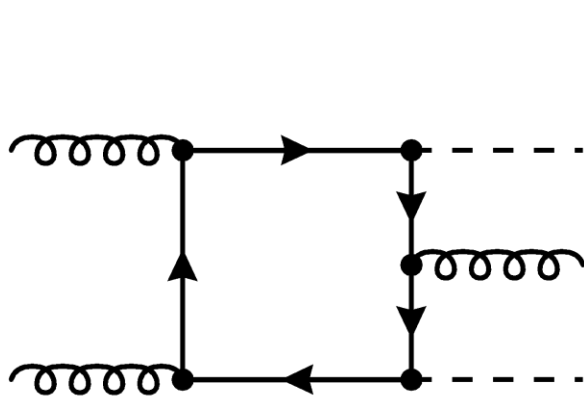
Davies, Mishima, Schönwald, Steinhauser '23



- Coverage of whole space with small- t and high-energy expansion with Padé

Real corrections

- New partonic processes: $gg \rightarrow gHH$, $qg \rightarrow qHH$, $q\bar{q} \rightarrow gHH$
- IR singularities handled with [Catani-Seymour](#) subtraction scheme *Catani, Seymour '97*
- Calculation of one-loop matrix elements with [Recola](#) (*Actis, Denner, Hofer, Lang, Scharf, Uccirati '17*) and [Collier](#) (*Denner, Dittmaier, Hofer '17*)
 - Implemented alternative reduction to scalar integrals for exceptional phase-space points with [CutTools](#) (*Ossola, Papadopoulos, Pittau '09*) and [OneLoop](#) (*van Hameren '11*)



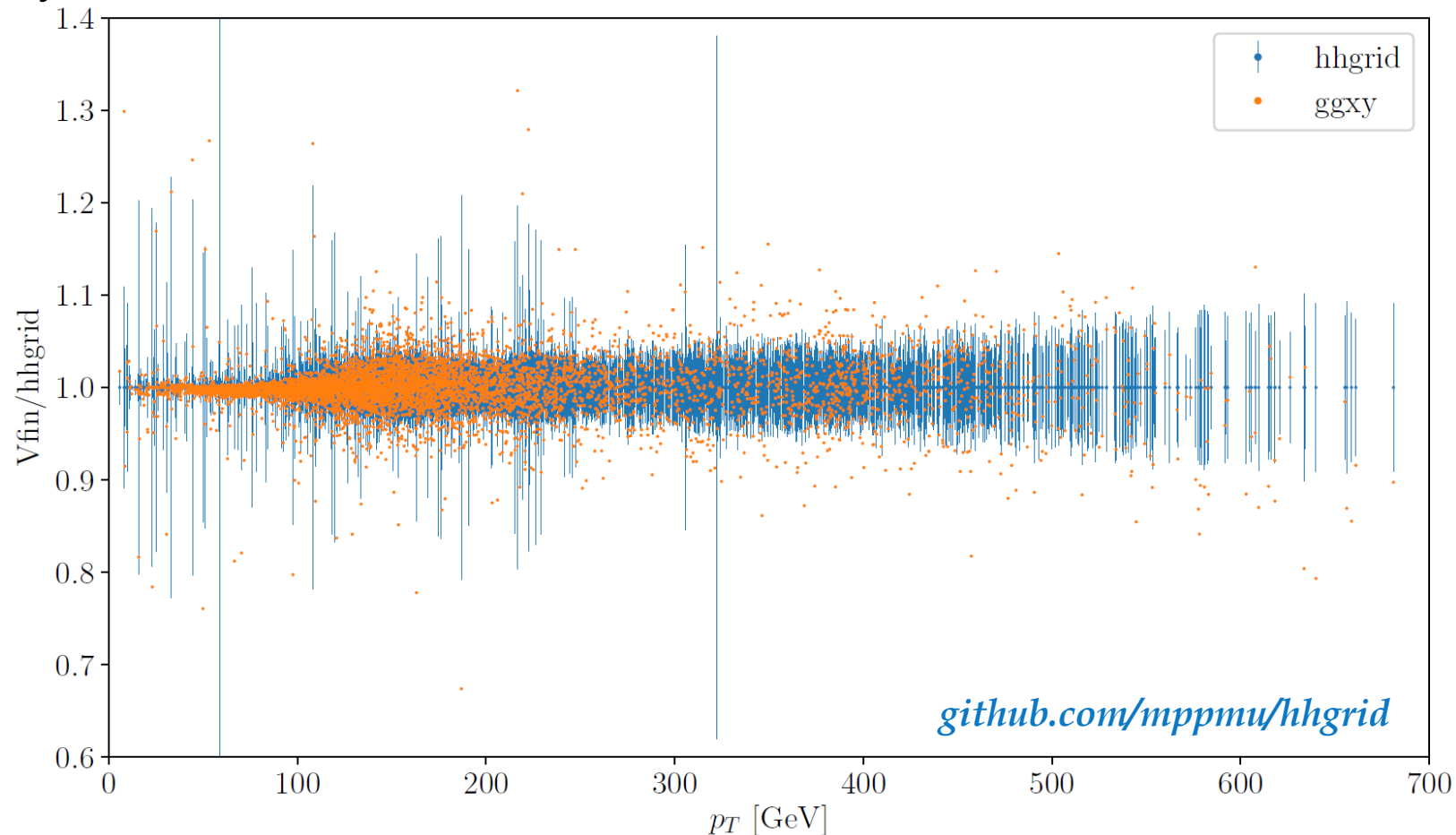
- Public C++ library for the computation of LO and NLO amplitudes and cross sections for [gluon initiated top-quark loop](#) induced processes
- In first version contains all necessary functions for $gg \rightarrow HH$ at [NLO QCD](#)
- Analytical expansions allow to modify m_t , m_H and top-quark mass renormalization scheme ([OS](#), $\overline{\text{MS}}$)
- Supports variation of $\kappa_\lambda = \lambda_3/\lambda_3^{\text{SM}}$

Installation

- External dependencies: [Boost](#), [CMake](#), [Eigen](#)
- Specify path to [LHAPDF](#) installation in build script for hadronic cross sections
- Following libraries are directly installed with and linked to [ggxy](#):
 - [Collier](#) (*Denner, Dittmaier, Hofer '17*)
 - [CutTools](#) (*Ossola, Papadopoulos, Pittau '09*)
 - [OneLOop](#) (*van Hameren '11*)
 - [Recola](#) (*Actis, Denner, Hofer, Lang, Scharf, Uccirati '17*)
 - [avhlib/Kaleu](#) (*van Hameren '10*)
 - [CRunDec](#) (*Herren, Steinhauser '18*)
 - [lievaluate](#) (*Frellesvig, Tommasini, Wever '16*)

Example: check-hhgrid.cpp

- Comparison of 2-loop finite reminder $\mathcal{V}_{\text{fin}}$ with `hhgrid`
 - Interpolation grid of 6320 phase-space points, combination of numerical evaluation with `pySecDec` and (lower order) high-energy expansion
- `ggxy` reproduces results within given uncertainties in less than 10 seconds on a single core
 - Sufficiently fast for a direct use in MC tools and more flexible



Example: nlo-gghh.cpp

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16

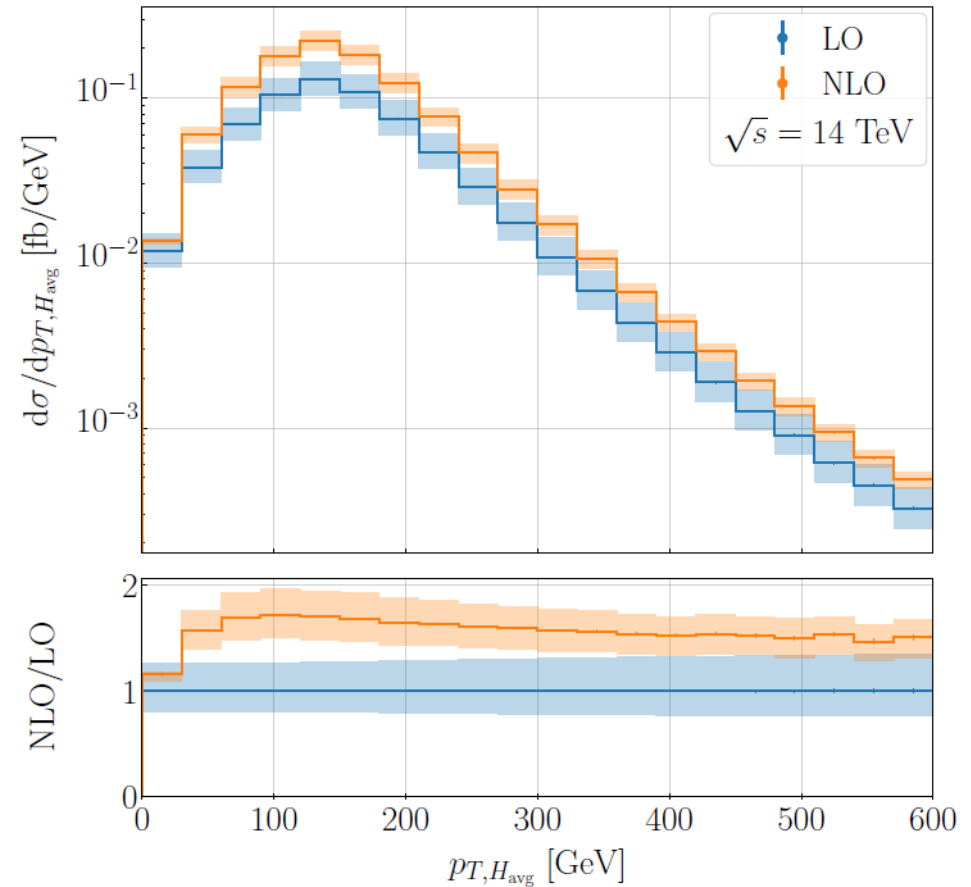
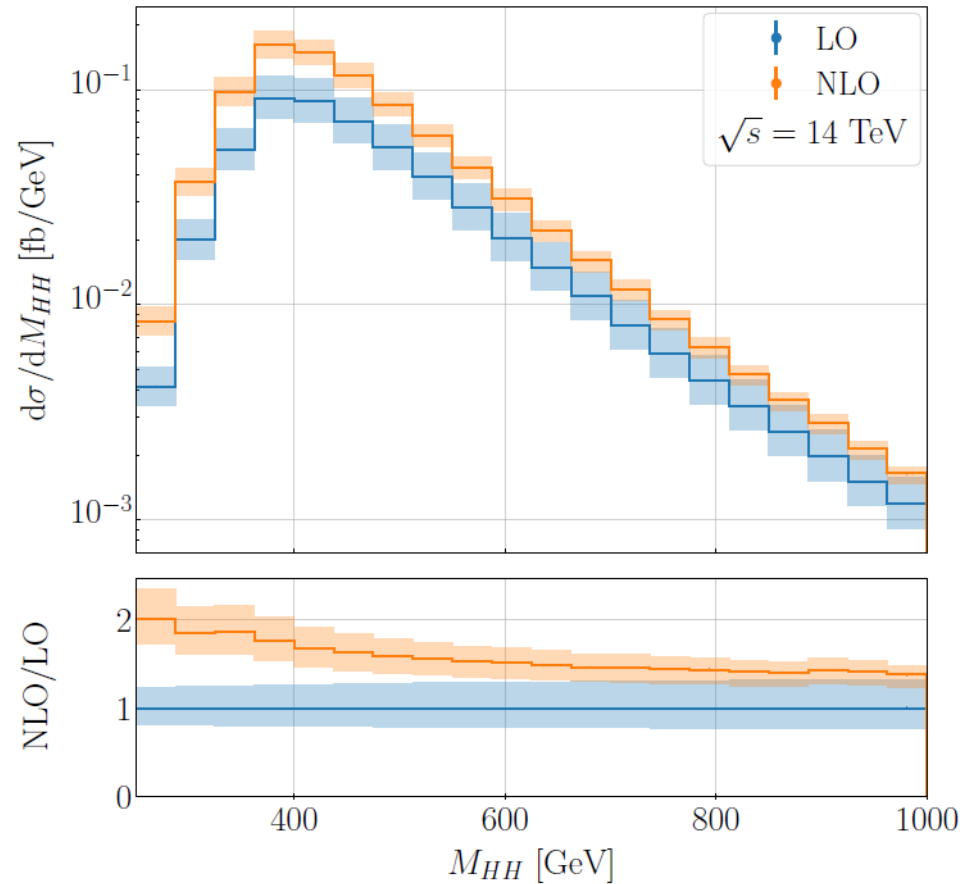
$\sqrt{s}$		ggxy	Ref.
14 TeV	σ^{LO} [fb]	$19.848(4)^{+27.6\%}_{-20.5\%}$	$19.85^{+27.6\%}_{-20.5\%}$
	σ^{NLO} [fb]	$32.92(2)^{+13.6\%}_{-12.6\%}$	$32.91^{+13.6\%}_{-12.6\%}$
100 TeV	σ^{LO} [fb]	$731.2(2)^{+20.9\%}_{-15.9\%}$	$731.3^{+20.9\%}_{-15.9\%}$
	σ^{NLO} [fb]	$1150(1)^{+10.8\%}_{-10.0\%}$	$1149^{+10.8\%}_{-10.0\%}$

Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '20

$\sqrt{s}$		ggxy	Ref.
13 TeV	σ^{NLO} [fb]	27.72(2)	27.73(7)
14 TeV	σ^{NLO} [fb]	32.79(2)	32.81(7)

- Reproduces LO/NLO hadronic cross sections from literature together with scale uncertainties from 7-point scale variation
- Fast runtime: 30 minutes on a single core for a precision of 0.1% – 0.2%

Example: nlo-gghh.cpp



- Differential cross sections with scale uncertainties also produced by example program
- Runtime depends highly on requested precision
 - Naive parallelization by running multiple seeds in parallel and combine results afterwards

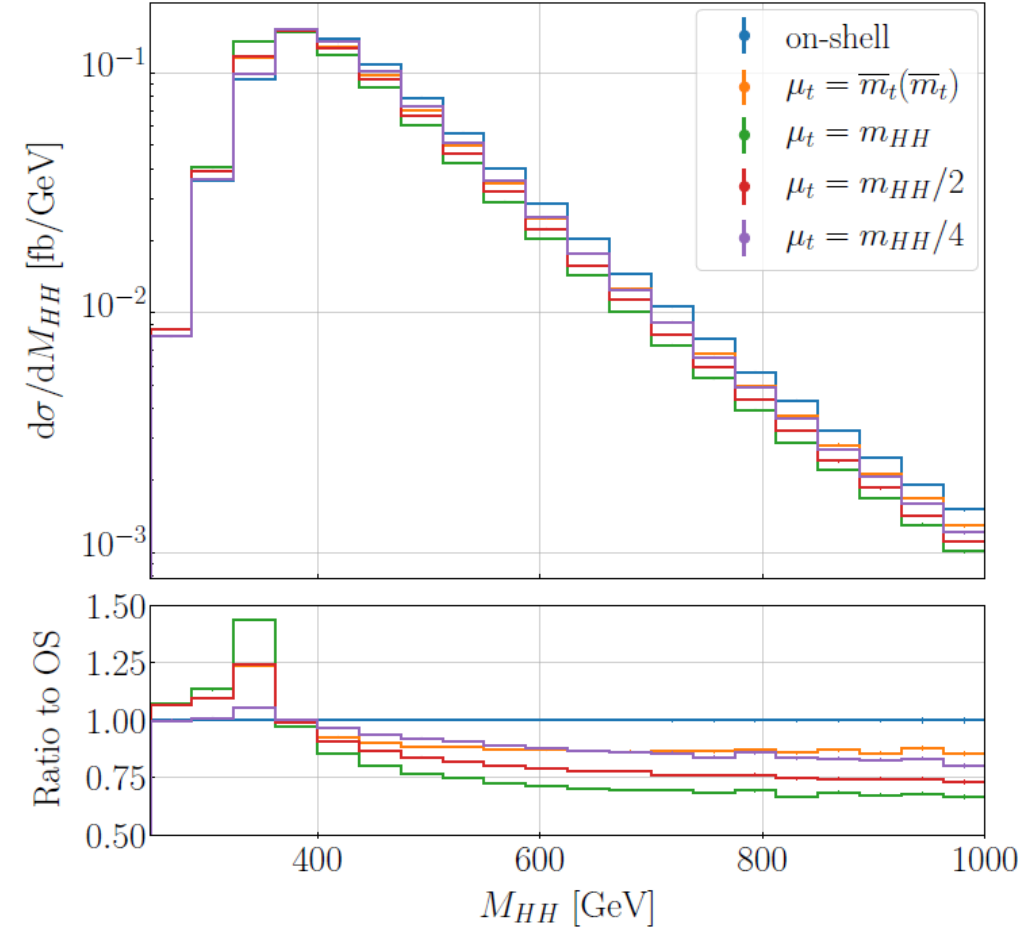
Example: nlo-gghh.cpp

- Straightforward to modify κ_λ and/or to use $\overline{\text{MS}}$ top-quark mass renormalization scheme

Bagnaschi, Degrassi, Gröber '23

κ_λ	Top-mass scheme	$\sigma_{\text{ggxy}}^{\text{NLO}}$ [fb]	$\sigma_{\text{Ref.}}^{\text{NLO}}$ [fb]
-0.6	on-shell	$100.34(6)^{+15.7\%}_{-13.6\%}$	$100.77^{+15.8\%}_{-13.7\%}$
0.0	on-shell	$68.08(4)^{+15.1\%}_{-13.4\%}$	$68.38^{+15.1\%}_{-13.4\%}$
1.0	on-shell	$30.83(2)^{+13.8\%}_{-12.7\%}$	$30.93^{+13.7\%}_{-12.7\%}$
1.0	$\overline{\text{MS}}, \mu_t = \overline{m}_t(\overline{m}_t)$	$29.78(2)^{+14.3\%}_{-13.0\%}$	$29.78^{+14.3\%}_{-13.0\%}$
1.0	$\overline{\text{MS}}, \mu_t = M_{HH}/2$	$28.79(2)^{+15.3\%}_{-13.5\%}$	$28.90^{+15.2\%}_{-13.5\%}$
2.4	on-shell	$13.369(9)^{+14.7\%}_{-13.2\%}$	$13.41^{+14.8\%}_{-13.1\%}$
6.6	on-shell	$203.4(1)^{+19.0\%}_{-15.1\%}$	$203.91^{+19.0\%}_{-15.2\%}$

- Overall good agreement
- Large dependence on input scheme of top-quark mass
Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher '20



Conclusion

- Presented new public C++ library `ggxy`:
 - All relevant functions implemented for $gg \rightarrow HH$ at NLO QCD
 - 2-loop amplitudes calculated with analytical expansions
 - Input parameters (m_t , m_H , κ_λ) or top-quark mass renormalization scheme (`OS`, `$\overline{MS}$`) can be freely chosen
- Several cross-checks with calculations in the literature
- Can be used directly to compute hadronic cross section or can be interfaced to other MC tools
- Fast runtime: 30 minutes for per mille precision

Outlook

- Interface to POWHEG (*Alioli, Nason, Oleari, Re '10*)
- Straightforward to include other processes:
 - $gg \rightarrow ZH$
 - $gg \rightarrow ZZ$ ($gg \rightarrow Z\gamma$ and $gg \rightarrow \gamma\gamma$)