The MINLO method for Fixed-Order NLO calculations

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in collaboration with
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Outline

- Introduction to the MINLO method
- Differences between MINLO and traditional NLO
- Comparison between MINLO and standard NLO for ttbar production in association with jets
- Long-term goal of the project and conclusions

The MINLO method in a nutshell

- The Multi-scale improved NLO (MINLO) method was first proposed in 2012 by Hamilton, Nason, Zanderighi '12
- Extraction of renormalization and factorization scales based on the *most likely branching history* via the CKKW procedure: Catani, Krauss, Webber, Kuhn '02
- Inclusion of *Sudakov form factors* of the form $\Delta_f(q_1^2, q_2^2)$ to resum large logarithms
- For a given process an inverse k_T clustering algorithm is applied to determine the scales that will enter the calculation $\rightarrow$ *nodal scales*
- These scales will enter the Sudakov form factors to account for the no-branching probability between two given scales
- Proper subtraction at NLO to avoid double counting and ensure the NLO accuracy of the calculation

The MINLO method at LO - MILO

- The following weight is applied in the Matrix element:

$$[\alpha_s(\mu_{core})]^m \times \prod_{i=1}^{n} \alpha_s(q_i) \times \prod_{i} \Delta_{f_i}^{\mathrm{ext}}(q_{res}^2; q_i^2) \times \prod_{ij} \Delta_{f_{ij}}^{\mathrm{int}}(q_{res}^2; q_i^2, q_j^2)$$

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user-defined scale assigned to the primary system

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Sudakov form factors to all external and internal lines

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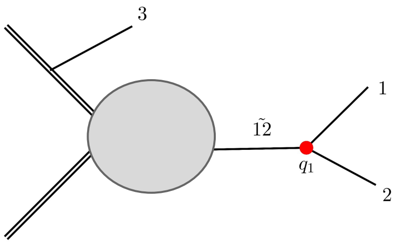
Example:



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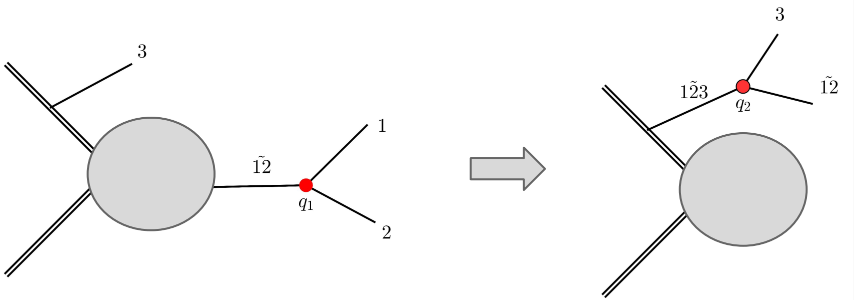

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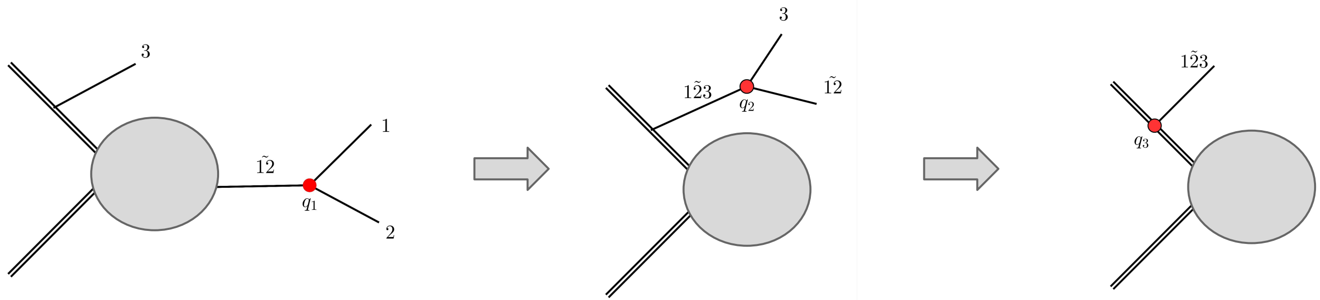

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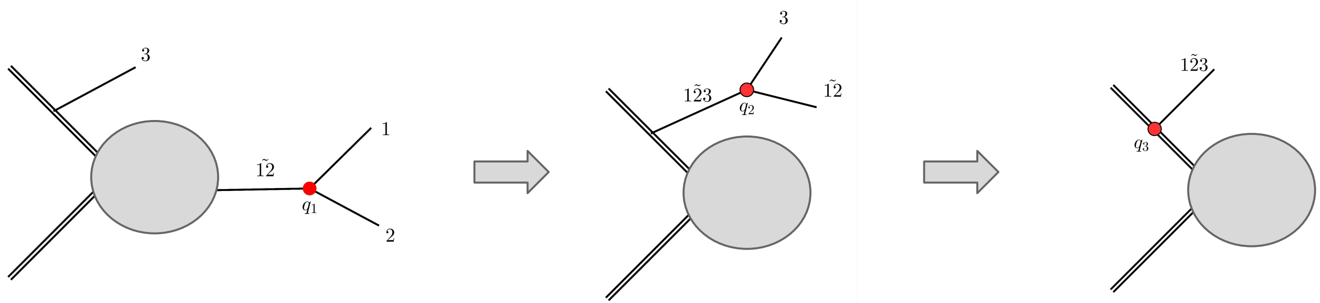
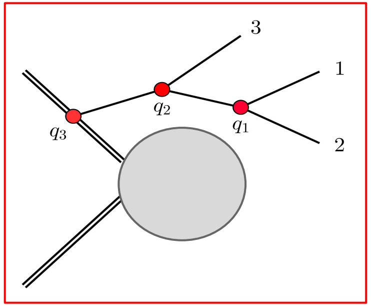
Example:





- We require *ordered clusterings*:

$$q_1 < ... < q_n \leq \mu_{core}$$

The MINLO method at NLO

$$\sigma^{\rm NLO} = \int d\Phi_B \left[\mathcal{B}(\Phi_B;\mu_F,\mu_R) + \mathcal{V}(\Phi_B;\mu_F,\mu_R) + \mathcal{I}^{(S)}(\Phi_B;\mu_F,\mu_R)\right] + \int d\Phi_R \left[\mathcal{R}(\Phi_R;\mu_F,\mu_R) - \mathcal{S}(\Phi_R;\mu_F,\mu_R)\right].$$

- Terms that live in the Born phase space are treated in the same manner as at LO
- The smallest nodal scale q_0 in $\mathcal{R}$ is excluded from the calculation to ensure proper cancellation of the IR poles
- The following replacement is applied to the Born contribution to avoid double counting of the $\mathcal{O}(\alpha_s)$ corrections that are included in the Virtual part

$$\mathcal{B} \to \mathcal{B}\times\left(1 - \sum_i \left[\Delta_{f_i}^{(1)}(q_{res}^2, q_i^2) - \Delta_{f_i}^{(1)}(q_{res}^2, q_1^2)\right] - \sum_{ij}\left[\Delta_{f_{ij}}^{(1)}(q_{res}^2, q_i^2) - \Delta_{f_{ij}}^{(1)}(q_{res}^2, q_j^2)\right]\right)$$

Scale variation in the MINLO method

- There are various ways for performing scale variation

Scale Variation	
$\mu_F = \xi_F q_{min}$	$\mu_F = \xi_F q_{min}$
$q_i \to \xi_R q_i, \quad \mu_{core} \to \xi_R \mu_{core}$ $\mu_R = \left((\xi_R \mu_{core})^m \times \prod_{i=1}^{n} (\xi_R q_i) \right)^{\frac{1}{m+n}}$	$\alpha_s(\mu_{eff}) = \left([\alpha_s(\mu_{core})]^m \times \prod_{i=1}^{n} \alpha_s(q_i) \right)^{\frac{1}{m+n}}$ $\mu_R = \xi_R \mu_{eff}$
$\alpha_s^{(n+m+1)} = \frac{1}{n+m} \left(\sum_{i=1}^{n} \alpha_s(\xi_R q_i) + m \alpha_s(\xi_R \mu_{core}) \right)$	$\alpha_s^{(n+m+1)} = \alpha_s(\mu_R)$
Hamilton, Nason, Zanderighi '12	Höche, Maierhoefer, Moretti, Pozzorini, Siegert '17

- The scales that enter the Sudakov form factors for the outgoing lines are not varied
- For incoming lines we make the following replacement

$$\Delta_{f_i}(q_{min}^2, q_i^2) \to \Delta_{f_i}(\xi_F^2 q_{min}^2, q_i^2)$$

ttbar+jets

These processes were used to cross-check our implementation with Sherpa…

Sherpa 3:
Bothmann, Flower, Gütschow, Höche, Hoppe, Isaacson, Knobbe, Krauss, Meinzinger, Napoletano, Price, Reichelt, Schönherr, Schumann, Siegert '24

Preliminary

Fixed-Order NLO vs MINLO for ttbar + 1 jet

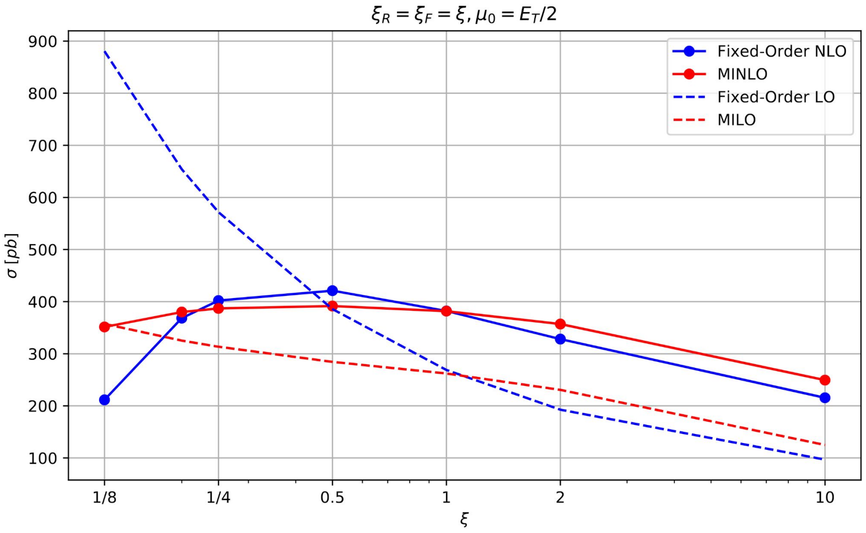
- The primary system is the stable ttbar system

$$E_T = \sum_{i=t,\bar{t},j} \sqrt{p_{T,i}^2 + m_i^2}$$

- Under 7-point scale variation:

$$\sigma_{t\bar{t}j}^{\mathrm{NLO}}(\mu_0 = E_T/2) = 382.1^{+10.2\%}_{-14.1\%}\ \mathrm{pb}$$
$$\sigma_{t\bar{t}j}^{\mathrm{MINLO}}(\mu_{core} = E_T/2) = 381.8^{+11.8\%}_{-13.2\%}\ \mathrm{pb}$$

-> Results agree within theoretical uncertainties
-> The sizes of the uncertainties are similar



- The MINLO results are less sensitive to scale variations both at LO and NLO in QCD

Preliminary

Fixed-Order NLO vs MINLO for ttbar + 2 jets

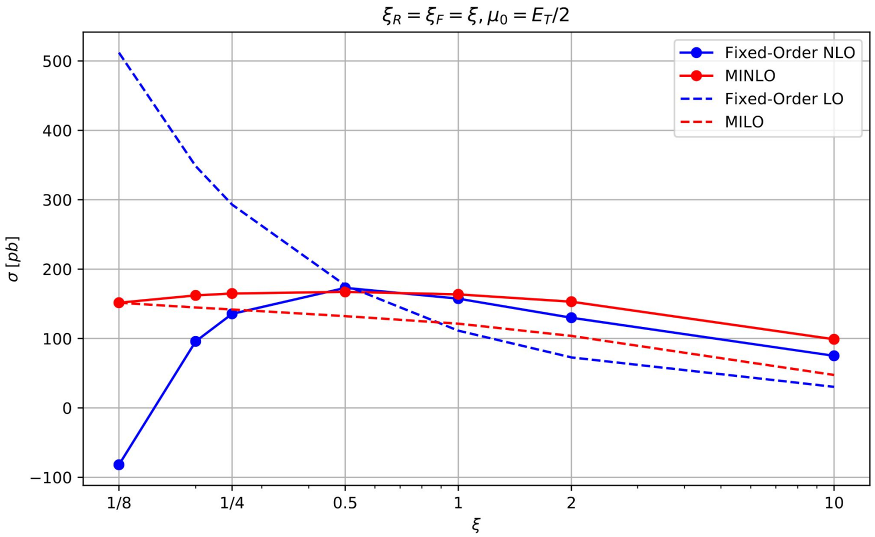
- The primary system is the stable ttbar system

$$E_T = \sum_{i=t,\bar{t},j} \sqrt{p_{T,i}^2 + m_i^2}$$

- Under 7-point scale variation:

$$\sigma^{\text{NLO}}_{t\bar{t}jj}(\mu_0 = E_T/2) = 157.4^{+10.9\%}_{-17.5\%}\ \text{pb}$$
$$\sigma^{\text{MINLO}}_{t\bar{t}jj}(\mu_{core} = E_T/2) = 163.6^{+9.1\%}_{-15.8\%}\ \text{pb}$$

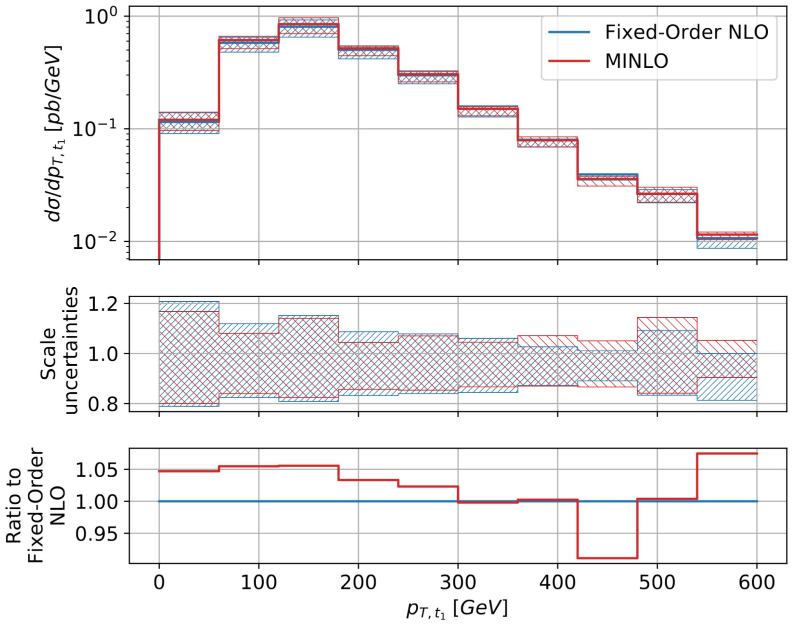
-> Results agree within theoretical uncertainties
-> Scale uncertainties are slightly smaller for MINLO

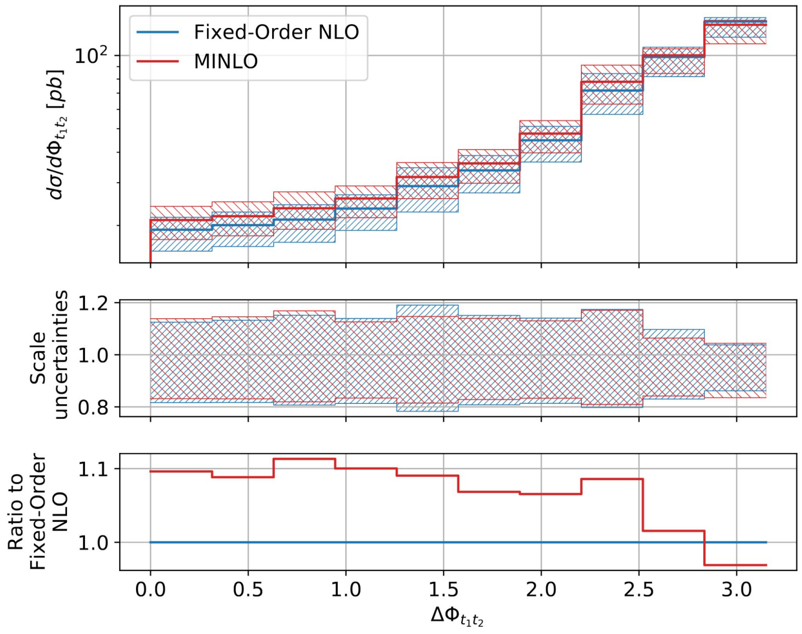


- With increasing jet multiplicity the MINLO method *further* improves the NLO predictions

Differential results for ttbar + 2jets

$$E_T = \sum_{i=t,\bar{t},j} \sqrt{p_{T,i}^2 + m_i^2}$$

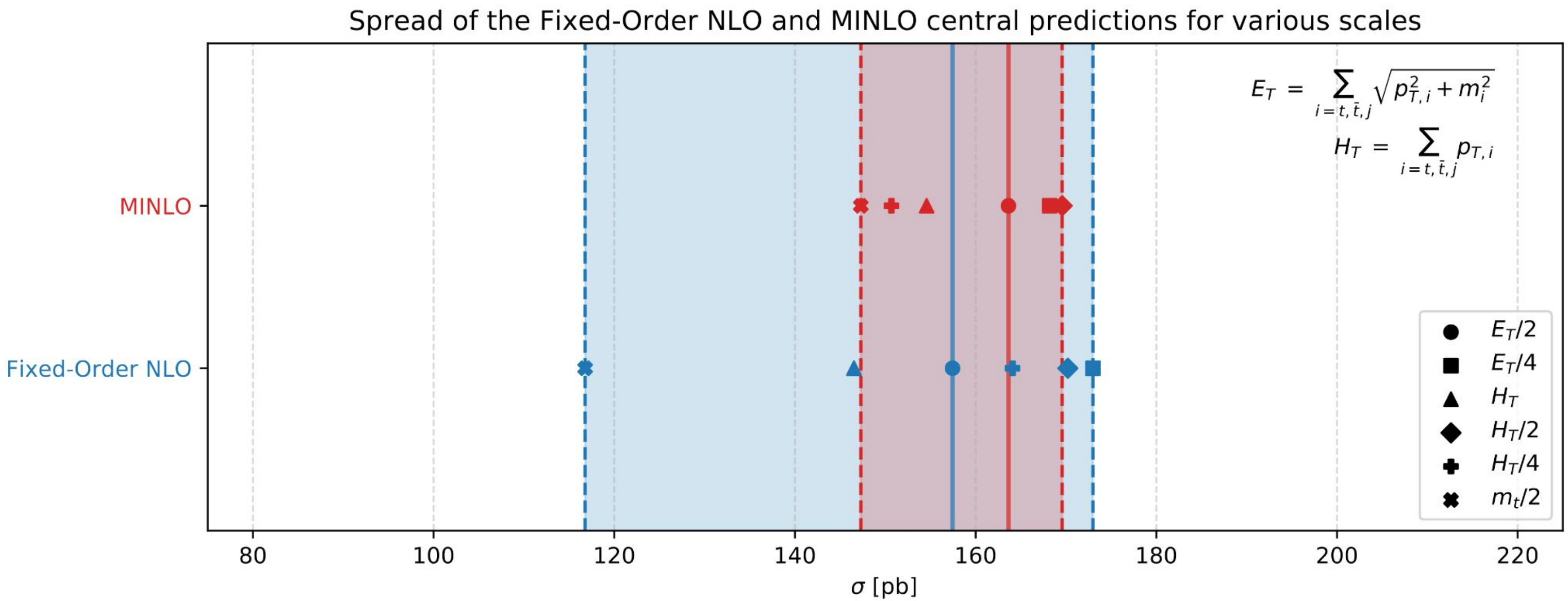




- Differences up to 10% which are covered by the scale uncertainties

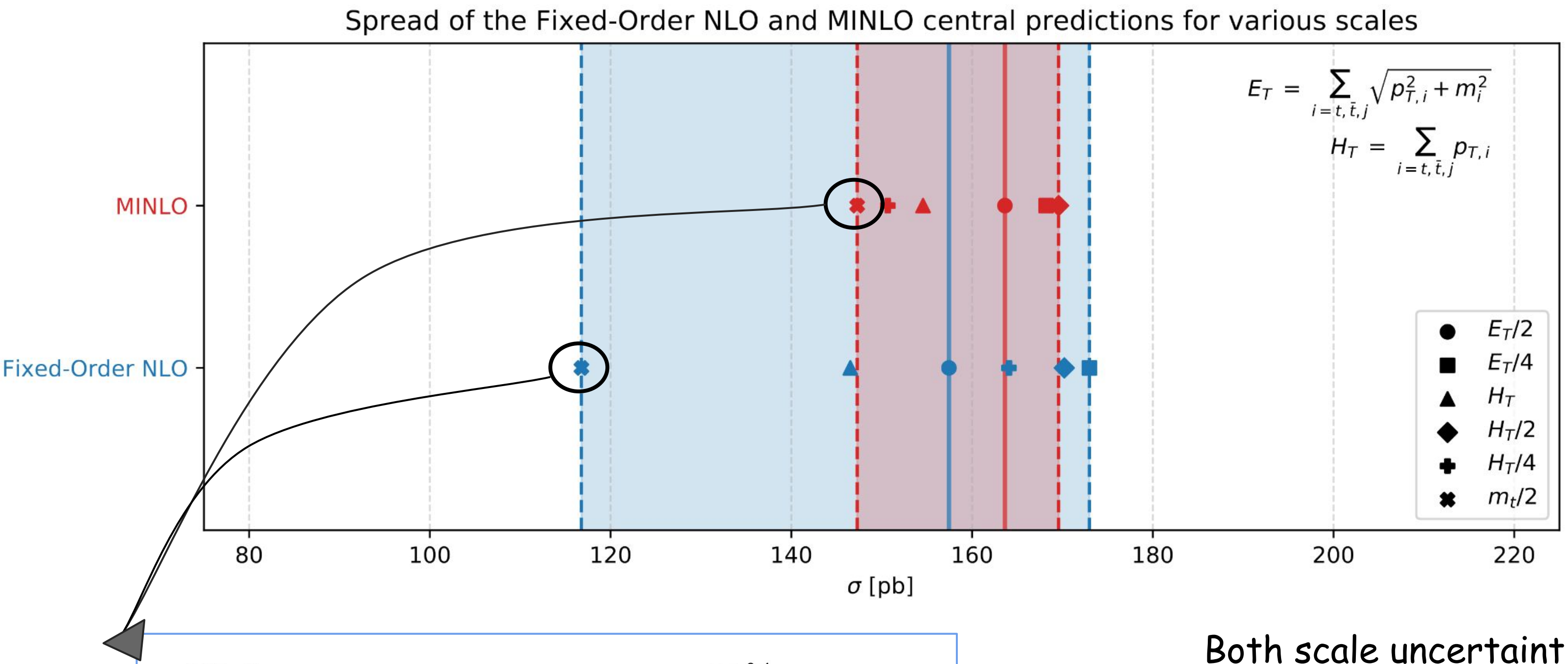
Preliminary

Fixed-Order NLO vs MINLO for ttbar + 2 jets



- The Fixed-Order NLO results are more spread around the default $E_T/2$ scale
- This is an indication that MINLO results are less sensitive to the scale that we use for performing the calculations

Fixed-Order NLO vs MINLO for ttbar + 2 jets



$$\sigma_{t\bar{t}jj}^{\text{NLO}}(\mu_0 = m_t/2) = 116.8^{+39\%}_{-203\%} \text{ pb}$$
$$\sigma_{t\bar{t}jj}^{\text{MINLO}}(\mu_{core} = m_t/2) = 147.3^{+9\%}_{-66\%} \text{ pb}$$

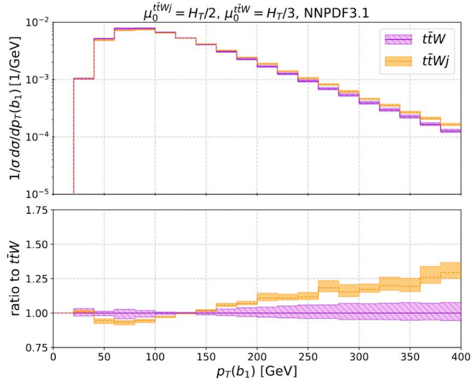
Both scale uncertainties are large but MINLO results are much more stable even if a “bad” scale is used

Long-term goal

- Full off-shell predictions for the pp -> ttW⁺ and pp -> ttW⁺j processes at NLO in QCD for the fully leptonic channel are available in the literature *Bevilacqua, Bi, Hartanto, Kraus, Worek '20* , *Denner, Pelliccioli '20* and *Bi, Kraus, Reinartz, Worek '23*

- The inclusion of the extra jet at NLO accuracy is quite significant for a correct modelling of the full off-shell pp -> ttW⁺ process at NLO in QCD

	$\sigma^{t\bar{t}W^+}_{H_T/3}$ [ab]	$\sigma^{t\bar{t}W^+j}_{H_T/2}$ [ab]
LO	$216.6^{+24\%}_{-18\%}$	$115.8^{+38\%}_{-26\%}$
NLO	$254.6^{+2.8\%}_{-5.9\%}$	$142.3^{+1.4\%}_{-8.1\%}$



- To improve the full off-shell predictions merging the ttW⁺ and ttW⁺j samples is needed

- Merging can be done using exclusive sums or MINLO + exclusive sums

Conclusions & Outlook

- Fixed-Order NLO and MINLO predictions are compatible within uncertainties especially if a “good” scale is used

- MINLO results are less sensitive to poorly chosen scales compared to Fixed-Order NLO

- Future goals:

  - Use MINLO also for merging the full off-shell ttW^+ and ttW^+j samples and comparison with the inclusive NLO full off-shell ttW^+ results ✔

  - Inclusion of the NLO ttW^+jj full off-shell sample in our merging procedure ...

Thanks for your attention!

BACKUP

Inverse k_T clustering algorithm

- For each final-final (FF) branching we consider:

$$y_{ij} = \frac{min(p_{T,i}^2, p_{T,j}^2)\Delta R_{ij}^2}{R^2}$$

- For each final-initial (FI) branching we consider:

$$y_{ij} = p_{T,i}^2$$

$$q = \sqrt{min\{y_{ij}\}}$$

⇩ nodal scale

- A branching is allowed only if is compatible in flavor:

$$\begin{array}{lllll} \text{FF:} & gg \to g, & gq \to q, & g\bar{q} \to \bar{q}, & q\bar{q} \to g \\ \text{FI:} & gg \to g, & gq \to q, & qg \to \bar{q}, & qq \to g \\ \text{FI:} & g\bar{q} \to \bar{q}, & \bar{q}g \to q, & \bar{q}\bar{q} \to g, & \end{array}$$

where q=u,d,s,c,b.

Sudakov form factors

- In line with Höche, Maierhoefer, Moretti, Pozzorini, Siegert '17 we employ the following Sudakov form factors:

$$\Delta_f(t_0,t_1) = \exp\left\{-\int_{t_0}^{t_1} dt \frac{\alpha_s(t)}{2\pi t} \sum_{b=q,g} \int_0^{1-\sqrt{t/t_1}} dz \left(zP_{ab}(z) + \delta_{ab}\frac{2C_f}{1-z}\Lambda_2(t)\right)\right\}$$

$$\Delta^{\text{ext}}_{f_j}(q^2_{res}; q^2_j) = \frac{\Delta_{f_j}(q^2_{res}, q^2_j)}{\Delta_{f_j}(q^2_{res}, q^2_1)}$$

$$\Delta^{\text{int}}_{f_{ij}}(q^2_{res}; q^2_i, q^2_j) = \frac{\Delta_{f_j}(q^2_{res}, q^2_i)}{\Delta_{f_j}(q^2_{res}, q^2_j)}$$

where:

$$\Lambda_2(t) = \frac{\alpha_s(t)}{2\pi}\left[\left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{10}{9}T_R N_f(t)\right]$$

$$q_{res} = \begin{cases} q_1, & \text{if the line is final} \\ \xi_F q_1, & \text{if the line is initial} \end{cases}$$

- This approach ensures that Sudakov form factors do not exceed unity allowing for their interpretation as no-branching probabilities

- These Sudakovs provide NLL resummation for soft and collinear logarithms that might appear in processes where there is a big separation of scales

Sudakov form factors

- We want to ensure that:

$$\mathcal{P}(q_i^2, q_k^2) = \mathcal{P}(q_i^2, q_j^2) \times \mathcal{P}(q_j^2, q_k^2)$$

where $\mathcal{P}(q_i^2, q_k^2)$ represents the probability of no emission between q_i and q_k

- This is guaranteed only if $\Delta_{f_{ij}}^{\text{int}}(q_{res}^2; q_i^2, q_j^2) = \frac{\Delta_{f_j}(q_{res}^2, q_i^2)}{\Delta_{f_j}(q_{res}^2, q_j^2)}$ because otherwise we would have:

$$\Delta_{f_{ij}}^{\text{int}}(q_i^2, q_j^2) \times \Delta_{f_{ij}}^{\text{int}}(q_j^2, q_k^2) = \exp\left\{-\int_{q_i^2}^{q_j^2} dt\, \Gamma(t, q_j^2)\right\} \times \exp\left\{-\int_{q_j^2}^{q_k^2} dt\, \Gamma(t, q_k^2)\right\}$$
$$\neq \exp\left\{-\int_{q_i^2}^{q_k^2} dt\, \Gamma(t, q_k^2)\right\} = \Delta_{f_{ij}}^{\text{int}}(q_i^2, q_k^2)$$

where:

$$\Gamma(t, q_i^2) = \frac{\alpha_s(t)}{2\pi t} \sum_{b=q,g} \int_0^{1-\sqrt{t/q_i^2}} dz \left(z P_{ab}(z) + \delta_{ab} \frac{2C_f}{1-z} \Lambda_2(t) \right)$$

NLO subtraction in the Born term

- The Sudakov form factors in the Born term include some higher-order effects

$$\mathcal{B} \times \left(\prod_k \frac{\Delta_{f_k}(q_{res}^2, q_k^2)}{\Delta_{f_k}(q_{res}^2, q_1^2)} \right) \times \left(\prod_{ij} \frac{\Delta_{f_{ij}}(q_{res}^2, q_i^2)}{\Delta_{f_{ij}}(q_{res}^2, q_j^2)} \right) =$$

$$\mathcal{B} \times \left(1 + \sum_i \left[\Delta_{f_k}^{(1)}(q_{res}^2, q_k^2) - \Delta_{f_k}^{(1)}(q_{res}^2, q_1^2) \right] + \mathcal{O}(\alpha_s^2) \right)$$

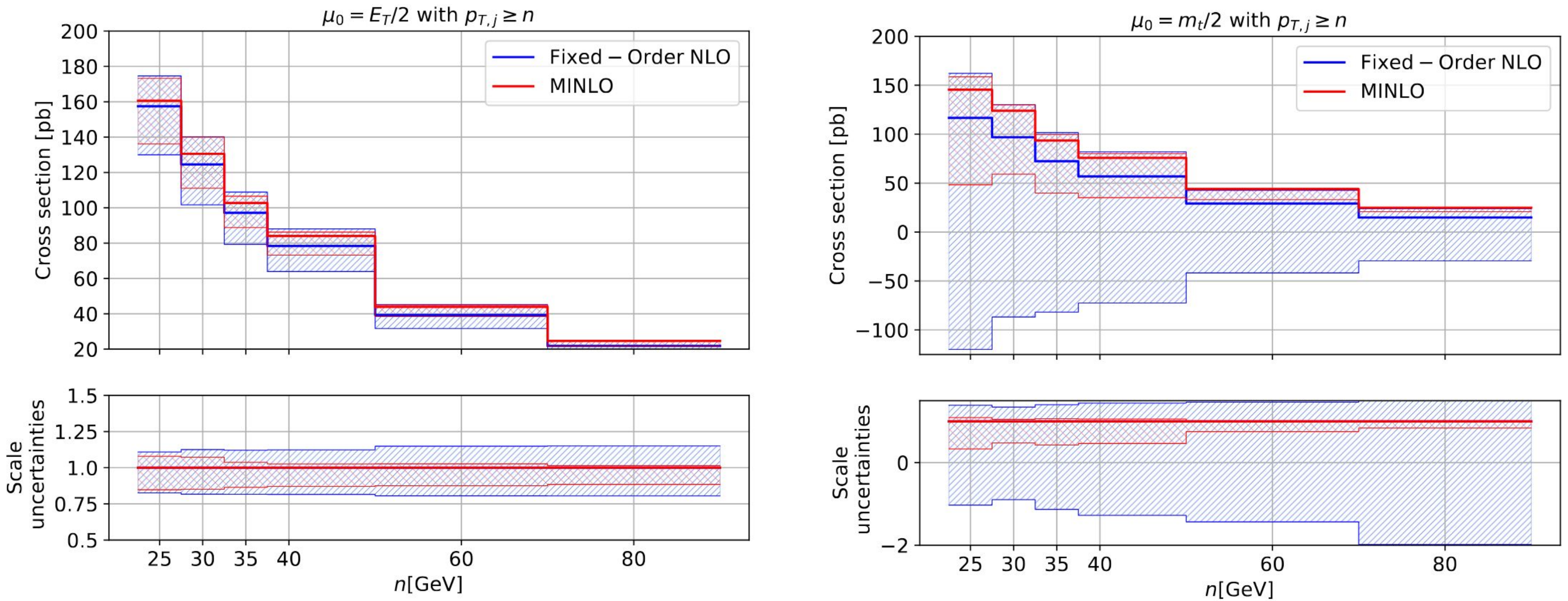
$$\times \left(1 + \sum_i \left[\Delta_{f_{ij}}^{(1)}(q_{res}^2, q_i^2) - \Delta_{f_{ij}}^{(1)}(q_{res}^2, q_j^2) \right] + \mathcal{O}(\alpha_s^2) \right) =$$

$$\mathcal{B} \times \left(1 + \boxed{\sum_i \left[\Delta_{f_k}^{(1)}(q_{res}^2, q_k^2) - \Delta_{f_k}^{(1)}(q_{res}^2, q_1^2) \right] + \sum_i \left[\Delta_{f_{ij}}^{(1)}(q_{res}^2, q_i^2) - \Delta_{f_{ij}}^{(1)}(q_{res}^2, q_j^2) \right]} + \mathcal{O}(\alpha_s^2) \right)$$

⇩

NLO part that needs to be subtracted since it is already included in the virtual contribution

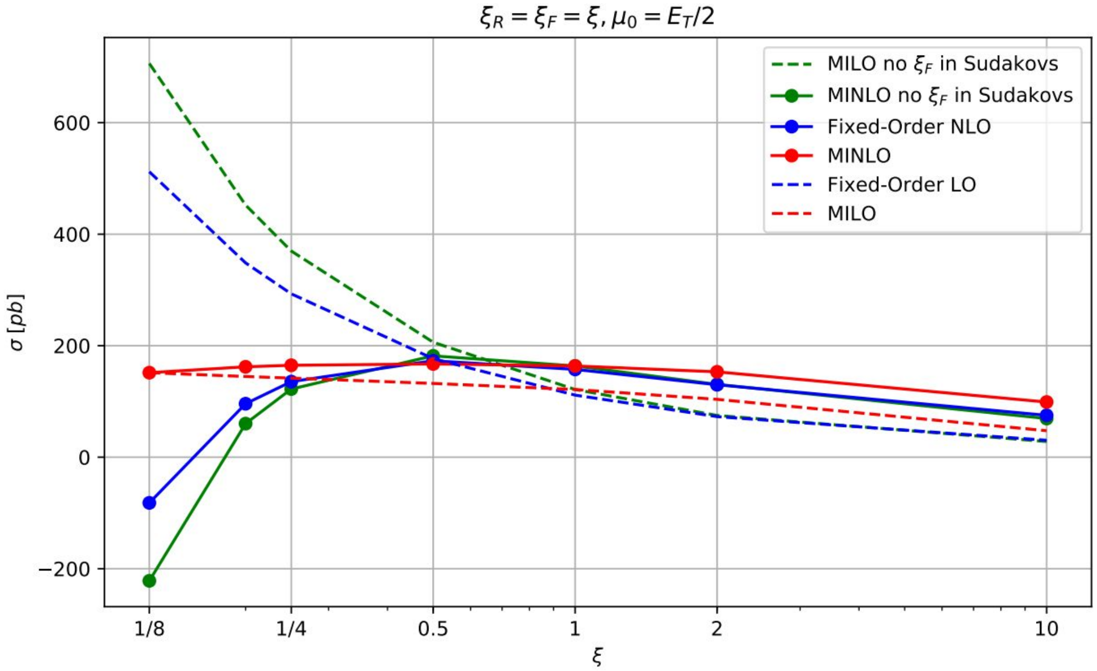
Fixed-Order NLO vs MINLO for ttbar + 2jets



- With increasing pTj threshold MINLO scale uncertainties become smaller than the ones from the Fixed-Order NLO

Preliminary

Fixed-Order NLO vs MINLO for ttbar + 2jets

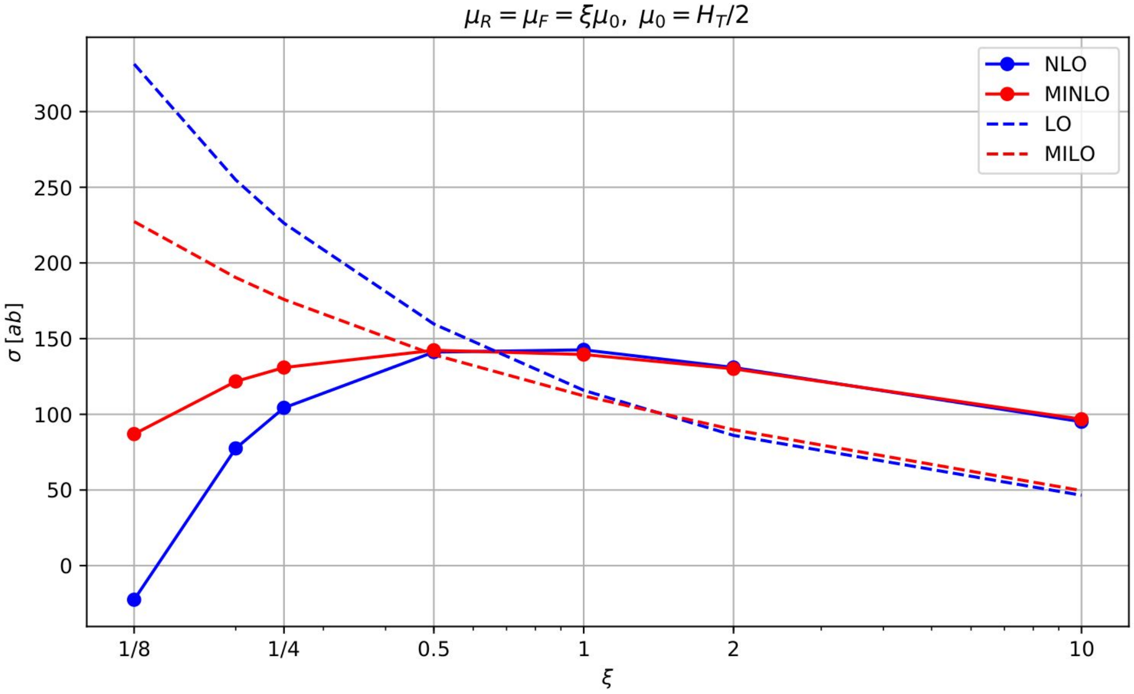


- The effects of not varying q_{min} in the Sudakov form factors for the incoming lines are large for small values of ξ

Preliminary

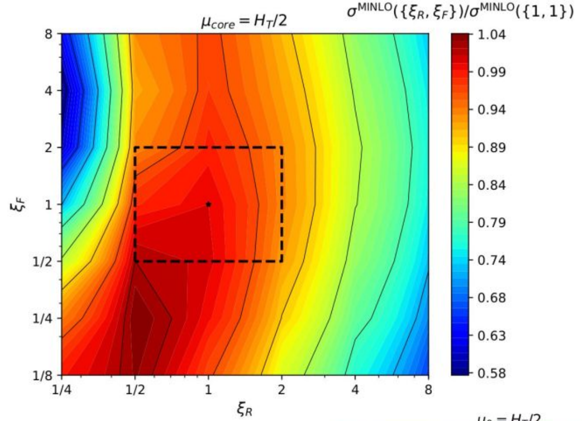
Fixed-Order NLO vs MINLO for the ttWj process

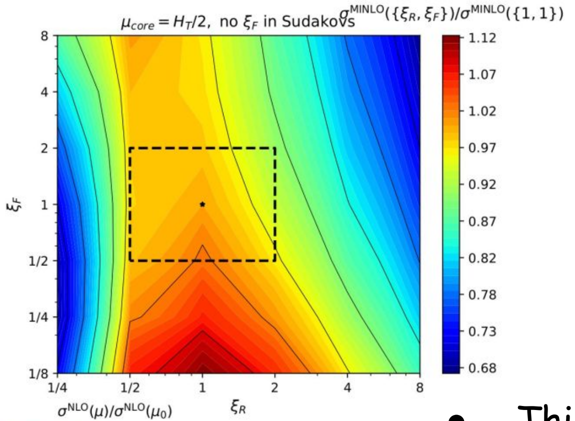
- Full off-shell predictions -> the core process consists of $e^+\nu_e\mu^-\bar{\nu_\mu}\tau^+\nu_\tau b\bar{b}$

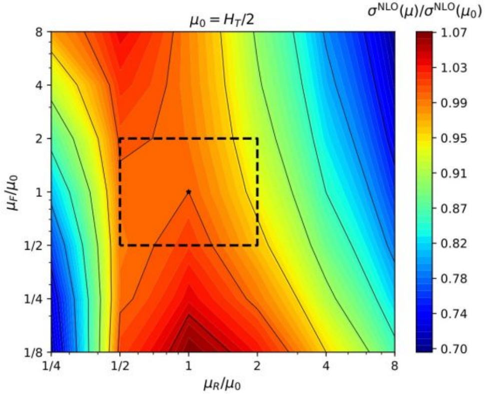


- Fixed-Order NLO and MINLO produce consistent results for ξ > 1/2
- For small ξ values MINLO behaves better compared to Fixed-Order NLO
- In general, both at LO and NLO, MI(N)LO is less sensitive to scale variations

Fixed-Order NLO vs MINLO for the ttWj process







- When no ξ_F is applied in the Sudakovs the scale uncertainties in MINLO behave similarly to the NLO ones

- This picture is inverted when ξ_F is applied in the Sudakovs

- Differences are more pronounced in the low ξ_R regime

Preliminary

Cross-checks with Sherpa

- The following processes have been cross-checked with the Sherpa code using the $E_T/2$ dynamical scale

$$\text{Sherpa}: \ \sigma^{\text{MILO}}_{\text{t}\bar{\text{t}}\text{j}} = 262.14(6)\ \text{pb}$$
$$\text{HEPlot}: \ \sigma^{\text{MILO}}_{\text{t}\bar{\text{t}}\text{j}} = 262.12(5)\ \text{pb}$$

$$\text{Sherpa}: \ \sigma^{\text{MINLO}}_{t\bar{t}j} = 381.26(67)\ \text{pb}$$
$$\text{HEPlot}: \ \sigma^{\text{MINLO}}_{t\bar{t}j} = 381.08(52)\ \text{pb}$$

+ 1jet

$$\text{Sherpa}: \sigma^{\text{MILO}}_{t\bar{t}jj} = 113.41(4)\ \text{pb}$$
$$\text{HEPlot}: \sigma^{\text{MILO}}_{t\bar{t}jj} = 113.40(2)\ \text{pb}$$

$$\text{Sherpa}: \sigma^{\text{MINLO}}_{t\bar{t}jj} = 162.0(2.2)\ \text{pb}$$
$$\text{HEPlot}: \sigma^{\text{MINLO}}_{t\bar{t}jj} = 160.1(1.3)\ \text{pb}$$

+ 2jets

$$\text{Sherpa}: \ \sigma^{\text{MILO}}_{t\bar{t}jjj} = 41.153(23)\ \text{pb}$$
$$\text{HEPlot}: \ \sigma^{\text{MILO}}_{t\bar{t}jjj} = 41.126(14)\ \text{pb}$$

+ 3jets

Sherpa 3:
Bothmann, Flower, Gütschow, Höche, Hoppe, Isaacson, Knobbe, Krauss, Meinzinger, Napoletano, Price, Reichelt, Schönherr, Schumann, Siegert '24

HEPlot:
Bevilacqua, "unpublished." '19