

Λ_b baryon LCDAs in the short distance expansion

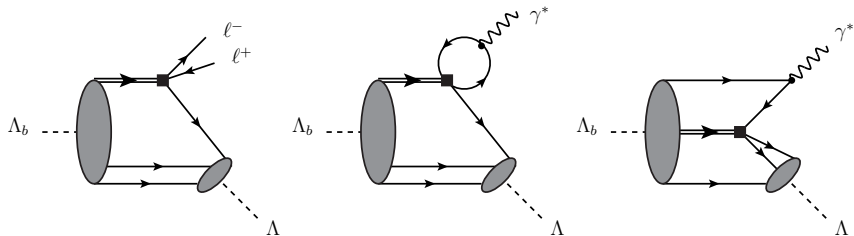
Daniel Vladimirov
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In collaboration with Thorsten Feldmann (2505.02570)



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- What are Light-Cone Distribution Amplitudes (LCDAs) ?
- Short-distance expansion
- Results
- Modelling the LCDAs
- Conclusion & Outlook



(Fig. taken from arXiv:2312.14146)

- ▶ LCDAs appear in the context of QCD factorization or light-cone sum rules, based on an expansion in Λ_{QCD}/m_b
- ▶ LCDAs provide the relevant information about the hadronic bound state: momentum distribution of light quarks (coupled to static heavy quark in HQET)
- ▶ Universal quantities, but not directly accessible in experiment

The “leading-twist” LCDA is defined as a *non-local* hadronic matrix element:

$$\epsilon^{abc} \langle 0 | \left(u^a(\tau_1 n) C \gamma_5 \not{n} d^b(\tau_2 n) \right) h_v^c(0) | \Lambda_b(v, s) \rangle = f_{\Lambda_b}^{(2)} \tilde{\phi}_2(\tau_1, \tau_2) u_{\Lambda_b}(v, s)$$

[Ball, Braun, Gardi, 0804.2424]

- ▶ Colour-neutral baryon current $\rightarrow \epsilon^{abc}$
- ▶ Charge-conjugation matrix C (Lorentz-invariant definition of diquark current)
- ▶ Normalization constant $f_{\Lambda_b}^{(2)}$
- ▶ On-shell spinor $u_{\Lambda_b}(v, s)$ for Λ_b moving with velocity v^μ in HQET ($v^2 = 1$)

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- ▶ Light-cone direction: n^μ with $n^2 = 0$ and $v \cdot n = 1$
- ▶ Implicit: Gauge-links (“Wilson lines”), make the definition gauge-invariant

$$u(\tau_1 n) \rightarrow [0, \tau_1 n] u(\tau_1 n), \quad d(\tau_2 n) \rightarrow [0, \tau_2 n] d(\tau_2 n)$$

- ▶ Momentum distribution from Fourier transform $(\tau_1, \tau_2) \rightarrow (\omega_1, \omega_2)$
- ▶ Dirac structure between light quarks depends on the process

Hadronic matrix elements cannot be computed perturbatively

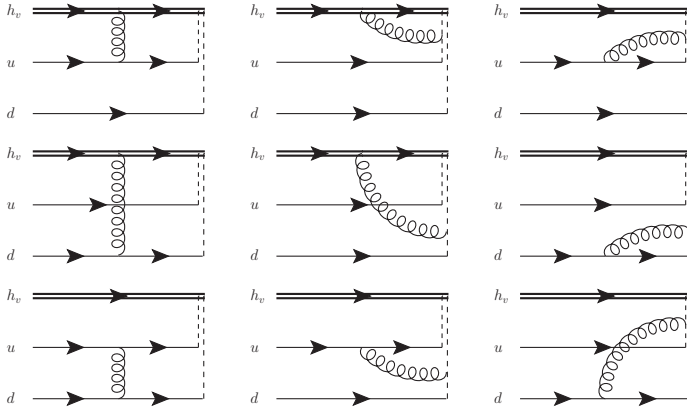
⇒ OPE allows us to express the non-local operator in terms of local structures:

$$\begin{aligned} & \epsilon^{abc} \left(u^a(\tau_1 n) C \gamma_5 \not{n} d^b(\tau_2 n) \right) h_v^c(0) \\ &= \sum_{i=0}^{\infty} \sum_{k=1}^{K_i} c_k^{(i+9/2)}(\tau_1, \tau_2) \mathcal{O}_k^{(i+9/2)}(0) \end{aligned}$$

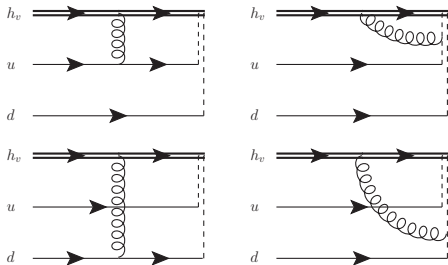
(for mesonic case, see [Kawamura, Tanaka, 0810.5628], [Feldmann, Lügghausen, Seitz, 2306.14686])

- ▶ Here: Truncate the expansion including order τ_1, τ_2 and α_s
- ▶ Determine coefficients c_k from matching calculation with on-shell *partonic* states

All possible diagrams at order α_s :



Only diagrams where gluons couple to heavy quark contribute to the matching:



Reason: Light quark dynamics is reproduced in local matrix elements

- ▶ The definition of the LCDA requires hadronic matrix elements
- ▶ We can use equations of motion, together with Lorentz and isospin symmetry,

$$iv \cdot \partial \langle 0 | \mathcal{O}_1^{(9/2)} | \Lambda_b(v, s) \rangle = \bar{\Lambda} \langle 0 | \mathcal{O}_1^{(9/2)} | \Lambda_b(v, s) \rangle ,$$

to express the local matrix elements in terms of $\bar{\Lambda} = M_{\Lambda_b} - m_b$ and $f_{\Lambda_b}^{(2)}$

- ▶ Example:

$$\epsilon^{abc} \langle 0 | \left((u^a C \gamma_5)_\alpha d_\beta^b \right) h_{v,\gamma}^c | \Lambda_b(v, s) \rangle = \frac{1}{4} \left[M^{(9/2)}(v) \right]_{\beta\alpha} u_\gamma(v, s)$$

$$M^{(9/2)}(v) = f_{\Lambda_b}^{(1)} + f_{\Lambda_b}^{(2)} \not{v}$$

- ▶ Putting everything together results in short-distance expansion for the LCDA:

$$\begin{aligned}\tilde{\phi}_2(\tau_1, \tau_2) = & \left(1 - i(\tau_1 + \tau_2) \frac{2\bar{\Lambda}}{3}\right) \left(1 - \frac{\alpha_s C_F}{4\pi} \left(\mathcal{L}_1^2 + \mathcal{L}_2^2 + \mathcal{L}_1 + \mathcal{L}_2 + \frac{5\pi^2}{12}\right)\right) \\ & + i\bar{\Lambda} \frac{\alpha_s C_F}{4\pi} \left(\tau_1 \left(\frac{2\mathcal{L}_1}{3} - \frac{3}{4}\right) + \tau_2 \left(\frac{2\mathcal{L}_2}{3} - \frac{3}{4}\right)\right) + \mathcal{O}(\tau_i^2),\end{aligned}$$

- ▶ Notice the logarithmic dependence on the light-cone separations, entering via

$$\mathcal{L}_j = \ln(i\tau_j \mu e^{\gamma_E})$$

These give rise to a non-trivial “radiative tail” in momentum space (!)

- ▶ Reason: Heavy quark can radiate an infinite amount of energy to the light quark

(Similar expressions for sub-leading LCDAs ϕ_3^s , ϕ_3^σ and ϕ_4 can be found in the paper)

Idea:

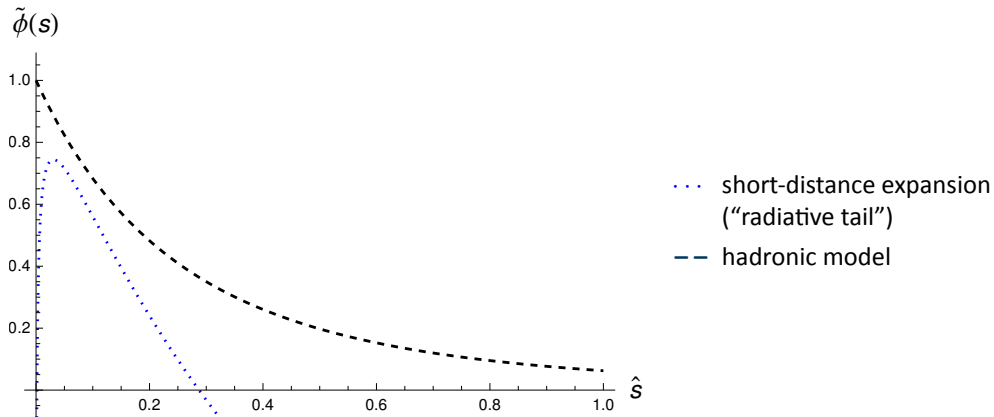
- ▶ "radiative tail" to describe behaviour for small $|\tau_i| \sim \mathcal{O}(1/\mu)$
- ▶ hadronic model to describe behaviour for large $|\tau_i| \sim \mathcal{O}(1/\Lambda_{\text{QCD}})$

$$\tilde{\phi}_2(\tau_1, \tau_2) = \frac{1}{(1 + i\omega_0\tau_1)^2(1 + i\omega_0\tau_2)^2}$$

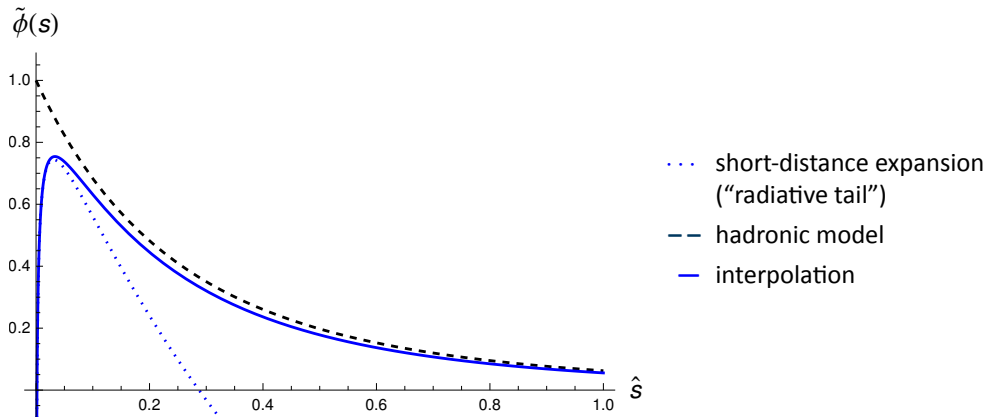
here, $\omega_0 \sim \text{few } 100 \text{ MeV}$ is to be fixed in terms of $\bar{\Lambda} = M_{\Lambda_b} - m_b$

- ▶ interpolate between the two regions, using the 1-loop RGE
(see paper for details)

Result of the interpolation at equal light-cone times $\hat{s} = i\tau_1\omega_0 = i\tau_2\omega_0$



Result of the interpolation at equal light-cone times $\hat{s} = i\tau_1\omega_0 = i\tau_2\omega_0$



- ▶ We calculated the 3-particle Λ_b baryon LCDAs in the short-distance expansion
- ▶ Our result for the “radiative tail” provides perturbative constraints, on an otherwise poorly known hadronic quantity
- ▶ Allows for QCD-improved modelling of Λ_b LCDAs
- ▶ Possible applications:
 - ⇒ $|V_{ub}|$ extraction from $\Lambda_b \rightarrow p\ell\nu$
 - ⇒ New physics studies in rare decays $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$

- ▶ Light-like vectors:

$$n^2 = \bar{n}^2 = 0 \quad \text{and} \quad n \cdot \bar{n} = 2$$

- ▶ Heavy-quark velocity:

$$v^\mu = \frac{n^\mu + \bar{n}^\mu}{2}$$

- ▶ HQET field:

$$\not{n} h_v = h_v, \quad (i v \cdot D) h_v = 0 \quad (\text{on-shell})$$

► Chiral-odd:

$$\epsilon^{abc} \langle 0 | \left(u^a(\tau_1 n) C \gamma_5 \not{n} d^b(\tau_2 n) \right) h_v^c(0) | \Lambda_b(v, s) \rangle = f_{\Lambda_b}^{(2)} \tilde{\phi}_2(\tau_1, \tau_2) u_{\Lambda_b}(v, s)$$

$$\epsilon^{abc} \langle 0 | \left(u^a(\tau_1 n) C \gamma_5 \not{n} d^b(\tau_2 n) \right) h_v^c(0) | \Lambda_b(v, s) \rangle = f_{\Lambda_b}^{(2)} \tilde{\phi}_4(\tau_1, \tau_2) u_{\Lambda_b}(v, s)$$

► Chiral-even:

$$\epsilon^{abc} \langle 0 | \left(u^a(\tau_1 n) C \gamma_5 d^b(\tau_2 n) \right) h_v^c(0) | \Lambda_b(v, s) \rangle = f_{\Lambda_b}^{(1)} \tilde{\phi}_3^s(\tau_1, \tau_2) u_{\Lambda_b}(v, s)$$

$$\epsilon^{abc} \langle 0 | \left(u^a(\tau_1 n) C \gamma_5 i \sigma_{\mu\nu} \bar{n}^\mu n^\nu d^b(\tau_2 n) \right) h_v^c(0) | \Lambda_b(v, s) \rangle = 2 f_{\Lambda_b}^{(1)} \tilde{\phi}_3^\sigma(\tau_1, \tau_2) u_{\Lambda_b}(v, s)$$

$$\epsilon^{abc} \langle 0 | \left(u^a(0) C \gamma_5 \gamma^\mu d^b(0) \right) h_v^c(0) | \Lambda_b(v, s) \rangle = f_{\Lambda_b}^{(2)} v^\mu u_{\Lambda_b}(v, s)$$

$$\epsilon^{abc} \langle 0 | \left(u^a(0) C \gamma_5 d^b(0) \right) h_v^c(0) | \Lambda_b(v, s) \rangle = f_{\Lambda_b}^{(1)} u_{\Lambda_b}(v, s)$$

$$\phi_i(\omega_1, \omega_2) = \int_{-\infty-i\epsilon}^{\infty-i\epsilon} \frac{d\tau_1}{2\pi} \int_{-\infty-i\epsilon}^{\infty-i\epsilon} \frac{d\tau_2}{2\pi} e^{i\omega_1\tau_1 + i\omega_2\tau_2} \tilde{\phi}_i(\tau_1, \tau_2)$$

$$[x, y] \equiv \mathcal{P} \exp -ig_s n^\mu \int_x^y d\lambda \mathcal{A}_\mu^A(n\lambda) T^A$$

$$\begin{aligned}\mathcal{O}_\Gamma(\tau_1, \tau_2) &\equiv \epsilon^{abc} \left(u^a(\tau_1 n) C \gamma_5 \Gamma d^b(\tau_2 n) \right) h_v^c(0) \\ &= \sum_{i=0}^{\infty} \sum_P \sum_{k=1}^{K_i} c_{P,k}^{(i+9/2)}(\tau_1, \tau_2) \mathcal{O}_{P\Gamma,k}^{(i+9/2)}(0)\end{aligned}$$

$$\begin{aligned}P\Gamma : \Gamma_{++} &= \frac{\not{n}\not{\bar{n}}}{4} \Gamma \frac{\not{\bar{n}}\not{n}}{4}, & \Gamma_{+-} &= \frac{\not{n}\not{\bar{n}}}{4} \Gamma \frac{\not{n}\not{\bar{n}}}{4} \\ \Gamma_{-+} &= \frac{\not{\bar{n}}\not{n}}{4} \Gamma \frac{\not{\bar{n}}\not{n}}{4}, & \Gamma_{--} &= \frac{\not{\bar{n}}\not{n}}{4} \Gamma \frac{\not{n}\not{\bar{n}}}{4}\end{aligned}$$

$$\begin{aligned}c_P^{(9/2)}(\tau_1, \tau_2) &= 1 + \mathcal{O}(\alpha_s) \\c_{P,1}^{(11/2)}(\tau_1, \tau_2) &= c_{P,2}^{(11/2)}(\tau_2, \tau_1) = -i\tau_1 + \mathcal{O}(\alpha_s) \\c_{P,3}^{(11/2)}(\tau_1, \tau_2) &= c_{P,4}^{(11/2)}(\tau_2, \tau_1) = \mathcal{O}(\alpha_s)\end{aligned}$$

$$\tilde{I}_{\text{subtr.}}(\tau_1, \tau_2) = \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \left(e^{-i\omega_1\tau_1 - i\omega_2\tau_2} - 1 + i\omega_1\tau_1 + i\omega_2\tau_2 - \dots \right) I(\omega_1, \omega_2).$$

$$\epsilon^{abc} \langle 0 | \left((u^a C \gamma_5)_\alpha d_\beta^b \right) h_{v,\gamma}^c | \Lambda_b(v, s) \rangle = \frac{1}{4} \left[M^{(9/2)}(v) \right]_{\beta\alpha} u_\gamma(v, s)$$

$$M^{(9/2)}(v) = f_{\Lambda_b}^{(1)} + f_{\Lambda_b}^{(2)} \not{v}$$

$$\epsilon^{abc} \langle 0 | \left((u^a C \gamma_5)_\alpha (i \overleftarrow{D}_\mu)_\beta^b \right) h_{v,\gamma}^c | \Lambda_b(v, s) \rangle = \frac{1}{4} \left[M_\mu^{(11/2)}(v) \right]_{\beta\alpha} u_\gamma(v, s)$$

$$M_\mu^{(11/2)}(v) = f_{\Lambda_b}^{(2)} (C \gamma_\mu + D \not{v} v_\mu) + f_{\Lambda_b}^{(1)} (E \not{v} \gamma_\mu + F \gamma_\mu \not{v})$$

$$C = -\frac{\bar{\Lambda}}{6} \quad D = \frac{2\bar{\Lambda}}{3} \quad E = \frac{\bar{\Lambda}}{6} \quad F = \frac{\bar{\Lambda}}{3}$$

$$\begin{aligned}\tilde{\phi}_2(\tau_1, \tau_2) = & \left(1 - i(\tau_1 + \tau_2) \frac{2\bar{\Lambda}}{3}\right) \left(1 - \frac{\alpha_s C_F}{4\pi} \left(L_1^2 + L_2^2 + L_1 + L_2 + \frac{5\pi^2}{12}\right)\right) \\ & + i\bar{\Lambda} \frac{\alpha_s C_F}{4\pi} \left(\tau_1 \left(\frac{2L_1}{3} - \frac{3}{4}\right) + \tau_2 \left(\frac{2L_2}{3} - \frac{3}{4}\right)\right) + \mathcal{O}(\tau_i^2)\end{aligned}$$

$$\begin{aligned}\tilde{\phi}_4(\tau_1, \tau_2) = & \left(1 - i(\tau_1 + \tau_2) \frac{\bar{\Lambda}}{3}\right) \left(1 - \frac{\alpha_s C_F}{4\pi} \left(L_1^2 + L_2^2 - L_1 - L_2 + \frac{5\pi^2}{12}\right)\right) \\ & + i\bar{\Lambda} \frac{\alpha_s C_F}{4\pi} \left(\tau_1 \left(\frac{2L_1}{3} - \frac{3}{4}\right) + \tau_2 \left(\frac{2L_2}{3} - \frac{3}{4}\right)\right) + \mathcal{O}(\tau_i^2)\end{aligned}$$

$$\begin{aligned}\tilde{\phi}_3^s(\tau_1, \tau_2) = & \left(1 - i(\tau_1 + \tau_2)\frac{\bar{\Lambda}}{2}\right) \left(1 - \frac{\alpha_s C_F}{4\pi} \left(L_1^2 + L_2^2 + \frac{5\pi^2}{12}\right)\right) \\ & + i\bar{\Lambda} \frac{\alpha_s C_F}{4\pi} \left(\tau_1 \left(\frac{5}{6}L_1 - \frac{1}{6}L_2 - \frac{3}{4}\right) + \tau_2 \left(\frac{5}{6}L_2 - \frac{1}{6}L_1 - \frac{3}{4}\right)\right) + \mathcal{O}(\tau_i^2)\end{aligned}$$

$$\begin{aligned}\tilde{\phi}_3^\sigma(\tau_1, \tau_2) = & \left(-i(\tau_1 - \tau_2)\frac{\bar{\Lambda}}{6}\right) \left(1 - \frac{\alpha_s C_F}{4\pi} \left(L_1^2 + L_2^2 + \frac{5\pi^2}{12}\right)\right) \\ & + i\bar{\Lambda} \frac{\alpha_s C_F}{4\pi} \left((\tau_1 + \tau_2)\frac{1}{2}(L_1 - L_2)\right) - \frac{\alpha_s C_F}{4\pi} (L_1 - L_2) + \mathcal{O}(\tau_i^2)\end{aligned}$$

$$\begin{aligned} \frac{d\tilde{\phi}(s; \mu)}{d \ln \mu} = & -\frac{\alpha_s C_F}{4\pi} \left(\Gamma_c^{(1)} L + \gamma^{(1)} \right) \tilde{\phi}(s; \mu) \\ & + \frac{\alpha_s C_F}{4\pi} \Gamma_c^{(1)} \int_0^1 dz \frac{z}{1-z} \left(\tilde{\phi}_2(-izs, -is; \mu) - \tilde{\phi}_2(-is, -is; \mu) \right) \end{aligned}$$

with $\Gamma_c^{(1)} = 4$, $\gamma^{(1)} = 2$, and $L = \ln(s\mu e^{\gamma_E})$

$$\begin{aligned} \tilde{\phi}(s, \mu) = & \left(1 - \frac{\alpha_s C_F}{4\pi} \left(\left(4L - 2 \ln \frac{\mu}{\mu_0(s)} \right) \ln \frac{\mu}{\mu_0(s)} + 2 \ln \frac{\mu}{\mu_0(s)} + \text{const}_1(s) \right) \right) \tilde{f}(s) \\ & + \frac{\alpha_s C_F}{4\pi} \left(4 \ln \frac{\mu}{\mu_0(s)} + 4 \text{const}_2(s) \right) \int_0^1 dz \frac{z}{1-z} \left(\tilde{f}_2(zs, s) - \tilde{f}_2(s, s) \right) \end{aligned}$$

$$\hat{\mu}_0(s) = \frac{1 + \hat{\mu}_F s}{s}, \quad \text{const}_1 = \frac{5\pi^2}{12}, \quad \text{const}_2 = -\frac{9}{8} \frac{1}{1 + \hat{\mu}_F s}$$

$$\alpha_s = 0.3 \text{ GeV}, \bar{\Lambda} = 0.6 \text{ GeV}, \mu e^{\gamma_E} = \mu_f e^{\gamma_E} = 2 \text{ GeV}, \omega_0 \simeq 0.23 \text{ GeV}$$