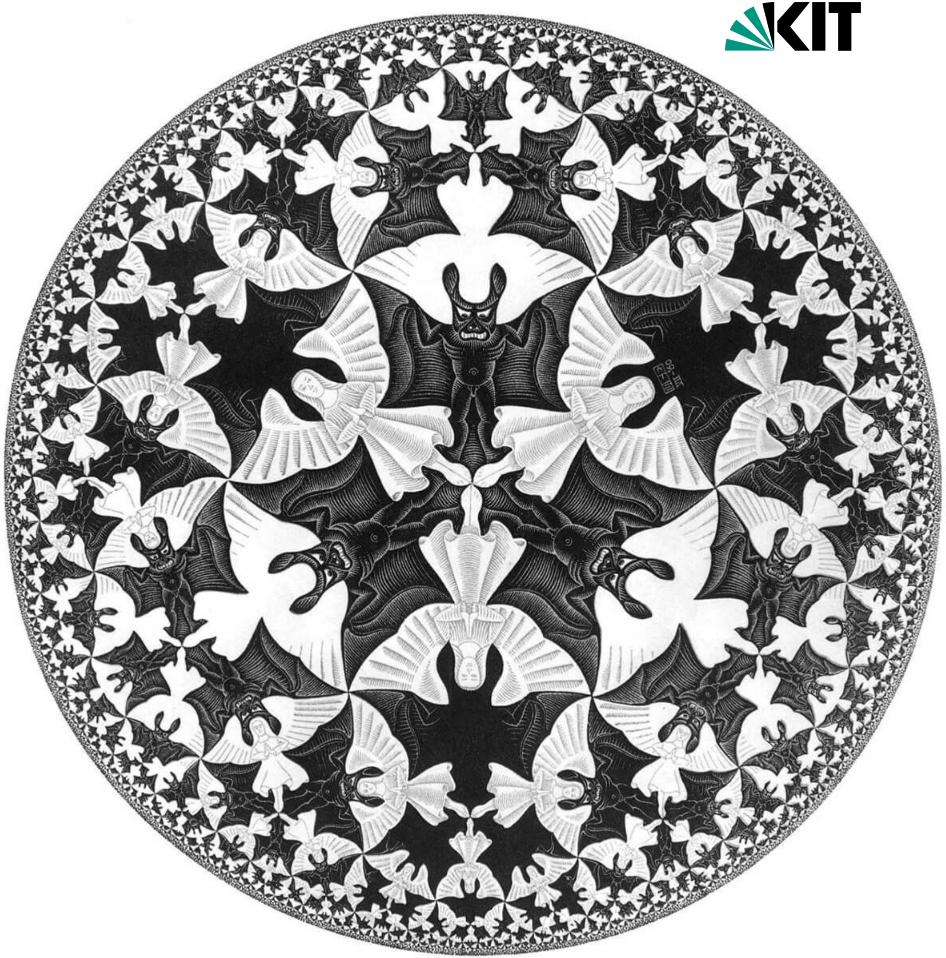
Classically conformal dark sectors



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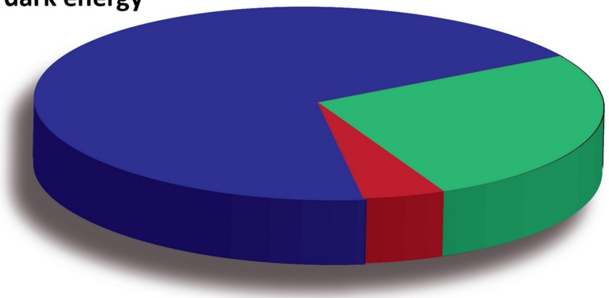
Heidelberg, 22 July 2025

Motivation for dark sectors

Given the complexity of visible matter, it seems preposterous to require dark matter to be simple



- Intriguing possibility: Dark matter particles do not appear in isolation, but together with other (unstable) particles
- The interactions between these particles are (much) stronger than their couplings to the SM

Example:

- Extended gauge groups:

  ➞ DM charged under new (Abelien or non-Abelian) gauge group, while SM particles are uncharged

Spontaneous mass generation

- The mass of visible matter is generated spontaneously
  - Electron mass generated through Higgs field (electroweak symmetry breaking)
  - Proton mass generated through strong interactions (chiral symmetry breaking)

- What if something similar happens for the dominant form of mass (i.e. dark matter)?

Example:

- Dark Higgs mechanism:
  ➞ DM particle obtains its mass from the vacuum expectation value of a (SM singlet) scalar field

- In a conventional Higgs mechanism, the symmetry breaking scale is set “by hand”

- **This talk:** Consider the case where the scale arises from loops via *dimensional transmutation*

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A simple dark sector model

- Consider U(1)’ gauge extension of the Standard Model and minimal particle content:
  - U(1)’ gauge boson (called *dark photon*)
  - Complex scalar field Φ with $Q_\Phi = 1$
  - Two left-handed fermions $\chi_{1,2}$ with $Q_1 = ½$ and $Q_2 = -½$

- Charges chosen such that the model is anomaly-free and there exist Majorana-like Yukawa interactions:

$$\mathcal{L} = |D_\mu \Phi|^2 - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \bar{\chi}_1 i \not{D} \chi_1 + \bar{\chi}_2 i \not{D} \chi_2 - \left(\frac{y_1}{2} \Phi \bar{\chi}_1^c \chi_1 + \frac{y_2}{2} \Phi^* \bar{\chi}_2^c \chi_2 + \text{h.c.} \right) - V(\Phi).$$

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Conformal symmetry

- Assume that the dark sector has a conformal symmetry at the classical level:
  - ➙ Invariance under rescaling

$$x^\mu \to \alpha x^\mu, \; \Phi(x) \to \alpha^{-1}\Phi(\alpha x), \; A'_\mu(x) \to \alpha^{-1}A'_\mu(\alpha x) \text{ and } \chi(x) \to \alpha^{-3/2}\chi(\alpha x)$$

  - ➙ No dimensionful couplings, no mass terms!
  - ➙ Scalar potential given by

$$V(\Phi) = \lambda(\Phi^*\Phi)^2$$

- Although the Lagrangian has no explicit scale, the β function for λ does not vanish!
  - ➙ Scale dependence λ(Λ) of the potential at the quantum level
  - ➙ Spontaneous symmetry breaking

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Effective potential

- At tree level: Potential minimised for $\Phi = 0$
- Assume nevertheless that scalar field takes homogeneous and static background value Φ_b which breaks both U(1)' and conformal symmetry
- Expand around background: $\Phi = (\phi_\mathrm{b}+\phi+\mathrm{i}\varphi)/\sqrt{2}$
- Radiative corrections give Coleman-Weinberg potential:

$$V_\mathrm{CW}(\phi_\mathrm{b}) = \sum_a \eta_a g_a \frac{m_a^4(\phi_\mathrm{b})}{64\pi^2}\left[\log\frac{m_a^2(\phi_\mathrm{b})}{\bar{\mu}^2} - C_a\right]$$

with $\eta = 1$ ($\eta = -1$) for bosons (fermions), $C = 3/2$ ($C = 5/6$) for scalars/fermions (vectors) and

$$m_\phi^2(\phi_\mathrm{b}) = 3\lambda\phi_\mathrm{b}^2\,, \qquad m_\varphi^2(\phi_\mathrm{b}) = \lambda\phi_\mathrm{b}^2\,, \qquad m_{A'}^2(\phi_\mathrm{b}) = g^2\phi_\mathrm{b}^2\,, \qquad m_\chi(\phi_\mathrm{b}) = \frac{y\phi_\mathrm{b}}{\sqrt{2}}$$

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Coleman-Weinberg mechanism

- Adding tree-level potential and CW potential with renormalisation condition $m_\phi(\phi_\mathrm{b} = 0) = 0$ gives the one-loop effective potential at scale Λ

$$V_\mathrm{eff}(\phi_\mathrm{b}, T=0) = \frac{\lambda}{4}\phi_\mathrm{b}^4 + \sum_a \frac{\eta_a g_a}{64\pi^2} m_a^4(\phi_\mathrm{b}) \left[\log\left(\frac{\phi_\mathrm{b}^2}{\Lambda^2}\right) - \frac{25}{6}\right]$$

  Remember: Running coupling λ = λ(Λ)

- V_eff has maximum at $\Phi_\mathrm{b} = 0$ and minimum at $\Phi_\mathrm{b} = v$ with

$$\lambda = \sum_a \frac{g_a \eta_a}{48\pi^2} \frac{m_a^4(v)}{v^4} \left[11 - 3\log\left(\frac{v^2}{\Lambda^2}\right)\right]$$

  The appropriate scale to evaluate this expression is Λ = v, which leads to

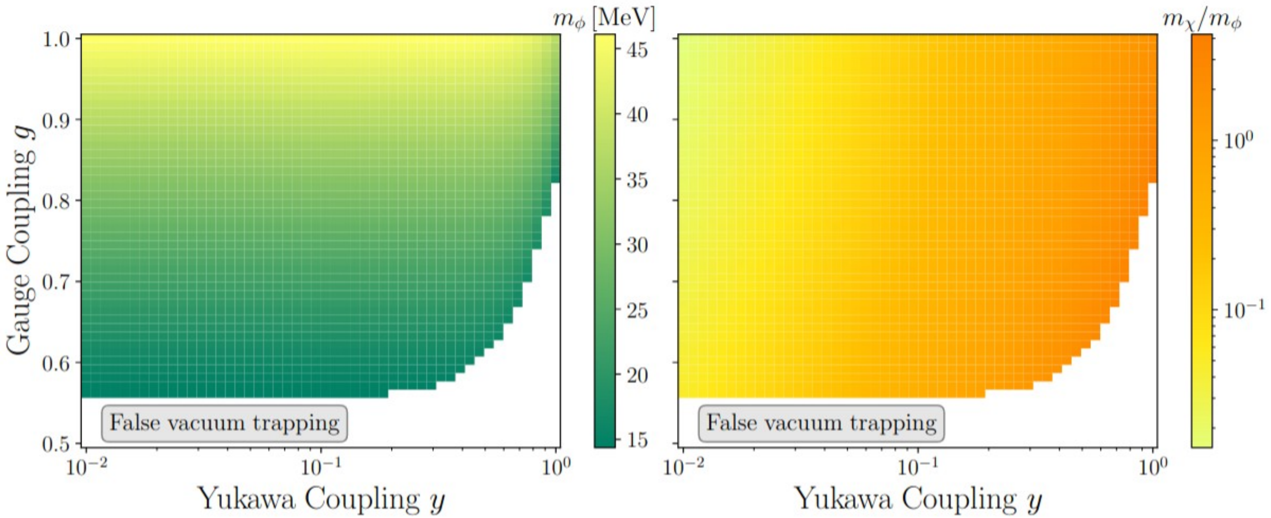
$$\lambda = 11 \sum_a \frac{g_a \eta_a}{48\pi^2} \frac{m_a^4(v)}{v^4} = \frac{11}{48\pi^2}\left(10\lambda^2 + 3g^4 - y^4\right)$$

Dimensional transmutation

- The classically scale-invariant potential V(Φ) depends on the renormalisation scale at the quantum level, leading to the generation of a dimensionful scale v
  - ➞ Has been studied in the past in the context of the hierarchy problem

- Trade λ(v) for v as one of the fundamental parameters of the model (together with g and y)

  Example: v = 100 MeV



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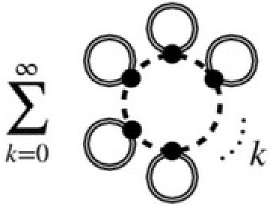
Finite-temperature effective potential

- So far: Calculation in vacuum (T=0)
- In plasma: Temperature gives additional dimensionful scale
  - → Particles obtain plasma mass (like Debye mass of plasmons) proportional to T
  - → Corrections to effective potential:

$$V_T(\phi_\mathrm{b}) = \frac{T^4}{2\pi^2}\sum_a \eta_a g_a J_{\mathrm{b/f}}\left(\frac{m_a^2(\phi_\mathrm{b})}{T^2}\right)$$

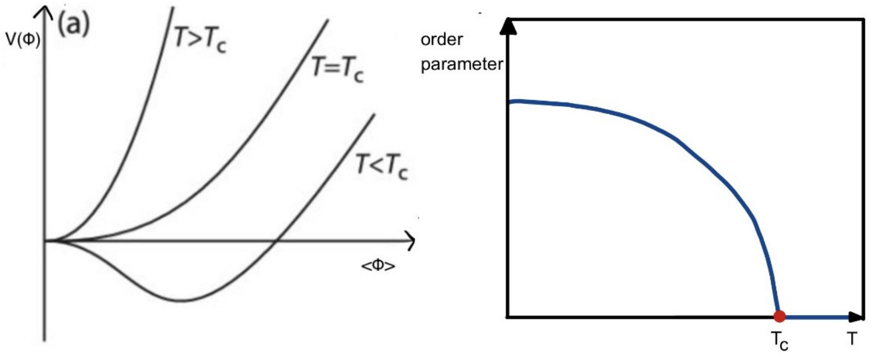
- Note: Infrared divergence for bosonic modes cancelled by "daisy resummation"



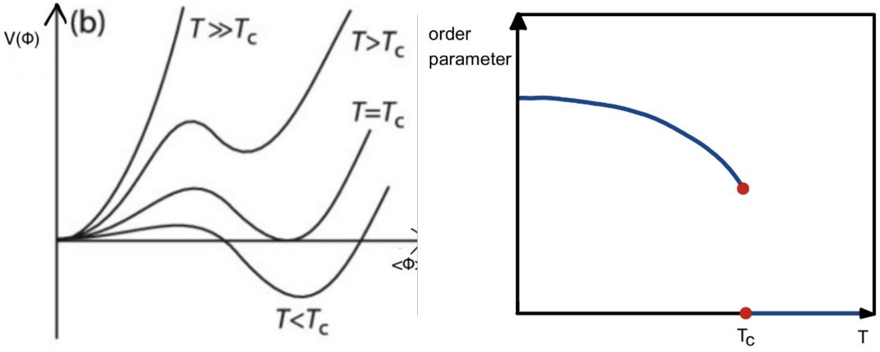
$$V_\mathrm{daisy}(\phi_\mathrm{b}) = -\frac{T}{12\pi}\sum_{a=\phi,\varphi,A'_\mathrm{L}} g_a\left[(m_a^2(\phi_\mathrm{b})+\Pi_a(T))^{3/2}-(m_a^2(\phi_\mathrm{b}))^{3/2}\right]$$

Phase transition

- Key result: Scalar field obtains mass term $\Pi_\phi = \left(\frac{\lambda}{3} + \frac{y^2}{12} + \frac{g^2}{4}\right) T^2$
  - ➞ At sufficiently high temperatures, $\Phi_b = 0$ becomes global minimum
  - ➞ U(1)’ symmetry is restored
- There exists a critical temperature T_c when the global minimum switches to $\Phi_b \neq 0$
- Two possibilities:



Second-order phase transition



First-order phase transition

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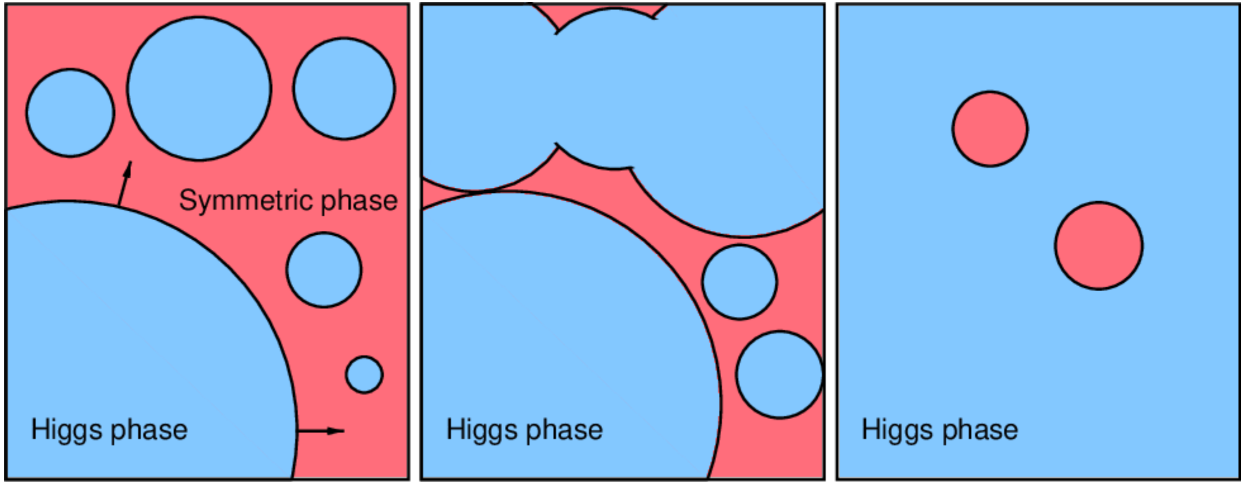
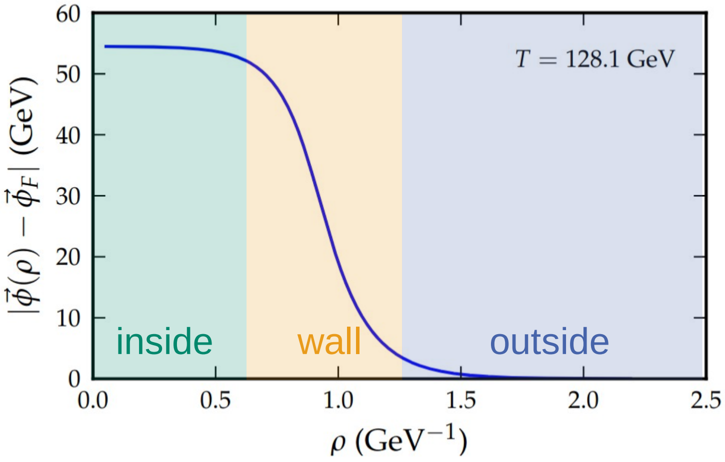
First-order phase transition

- Potential barrier between true minimum at $\Phi_b \neq 0$ and false minimum at $\Phi_b = 0$
  - → False minimum is metastable
  - → Field remains at $\Phi_b = 0$ even for $T < T_c$ (supercooling)
  - → Transition to the true vacuum via bubble nucleation



A closer look at bubble nucleation

- A bubble is a localised transition from $\Phi_b \neq 0$ (inside) to $\Phi_b = 0$ (outside)



- Continuity of field implies large potential energy for the bubble wall (or large contribution to total energy from steep gradients for thin walls)
  - → Need to minimize surface area (→ spherical symmetry)
  - → Need sufficiently small ratio surface/volume (→ minimal radius)

Bounce action

- The optimal field configuration minimises the O(3)-symmetric Euclidean bounce action

$$S_3(T) = \int d^3x \left[\frac{(\nabla \bar{\phi}_b)^2}{2} + V_{\text{eff}}(\bar{\phi}_b, T) \right]$$

- For this minimal action, we can calculate the corresponding bubble nucleation rate (number of bubbles per time and volume)

$$\Gamma(T) = A(T) \exp\left[-\frac{S_3(T)}{T} \right]$$ with $A(T) \sim T^4$

- For constant temperature, $\Gamma(T)^{-1}$ quantifies the time it takes the phase transition to complete

- But in an expanding universe, the temperature decreases with time

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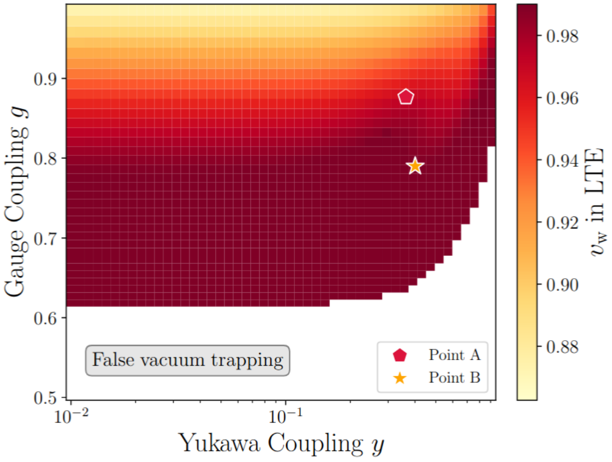
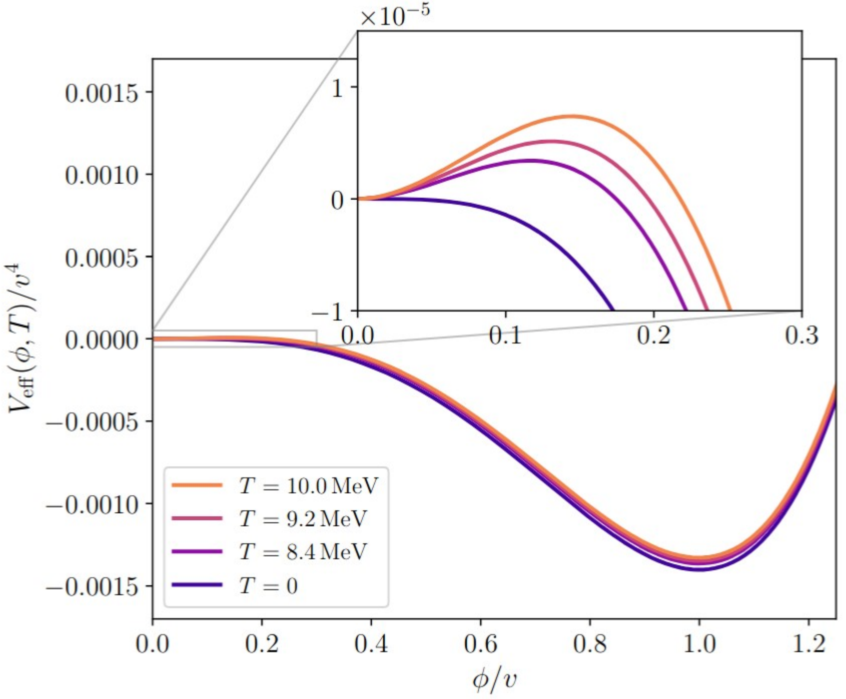
Bubble nucleation temperature

- As the temperature decreases, potential barrier becomes smaller
  - → Bounce action decreases rapidly
  - → Sudden enhancement of bubble nucleation

- Bubbles start expanding, with bubble walls often reaching relativistic velocities v_w





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Completion of the phase transition

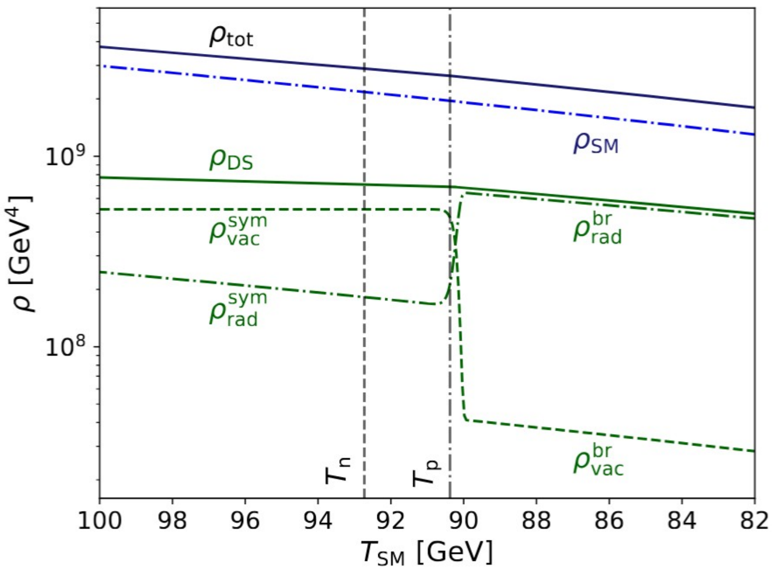
- The expanding bubbles eventually collide, creating a continuous path of $\Phi_b \neq 0$ (percolation)

- Latent heat (~ difference in potential energy ΔV_{eff} between metastable and true minimum) is released into the plasma

- **Consequence 1:** Reheating of thermal bath
  - ➞ Non-standard cosmological evolution
  - ➞ Interesting for dark matter production

see talk by Henda

- **Consequence 2:** Generation of plasma sound waves
  - ➞ Cosmological source of gravitational waves



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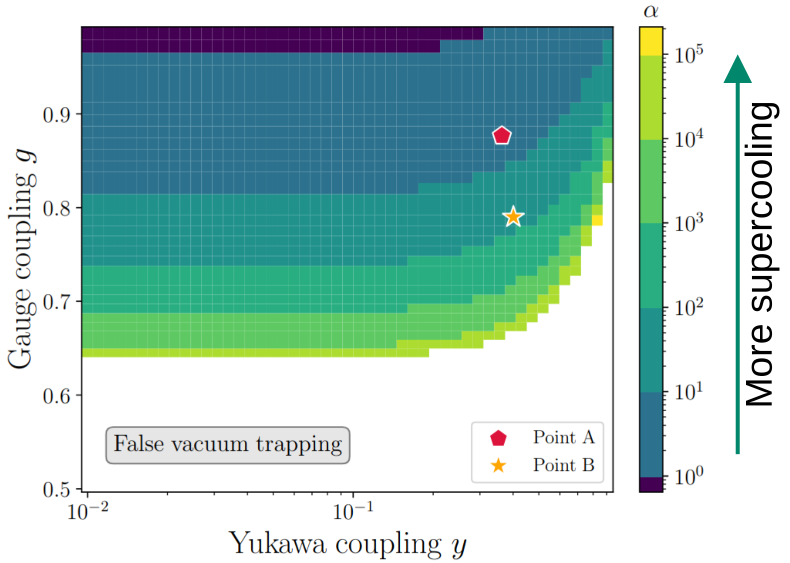
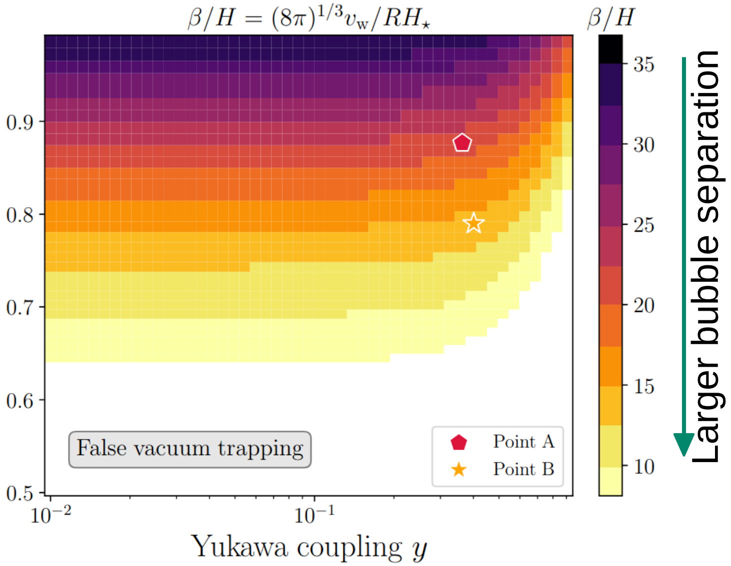
Why study a classically conformal model?

- Bounce action changes very slowly with temperature
- Huge supercooling $\alpha \sim \Delta V_{eff} / E_{tot}$
- Large separation R between bubbles

} → Larger gravitational wave signal





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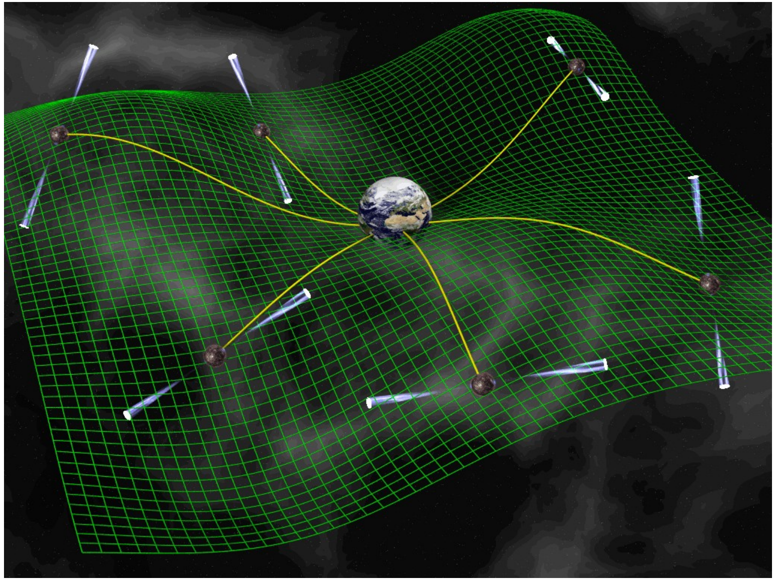
Pulsar timing arrays

- **Pulsars:** Astrophysical objects emitting extremely regular signals
- **Pulsar timing:** Observe a pulsar over several years to search for periodic shift of arrival times
  - → Indication for an oscillating distortion of space-time (i.e. gravitational waves)
- **Pulsar timing array:** Observe many such pulsars simultaneously to correlate arrival times from different directions
  - → Eliminate systematics and confirm gravitational wave origin of signal



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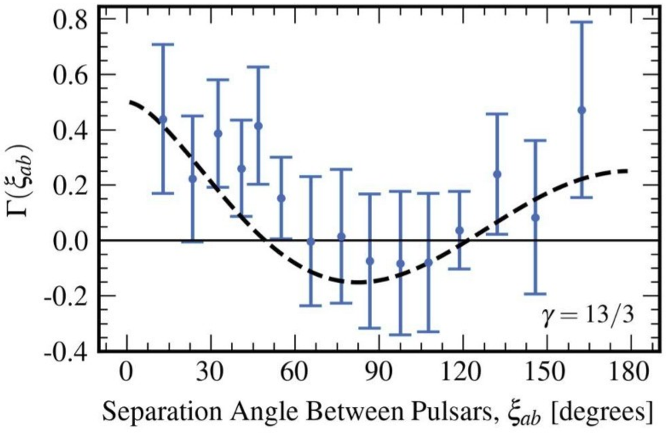
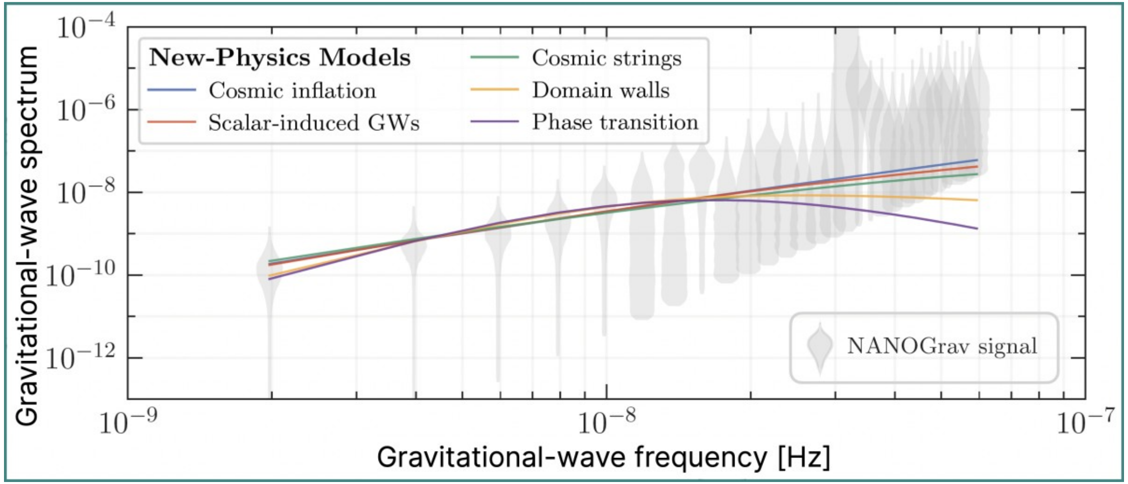
Evidence for nano-Hertz gravitational waves

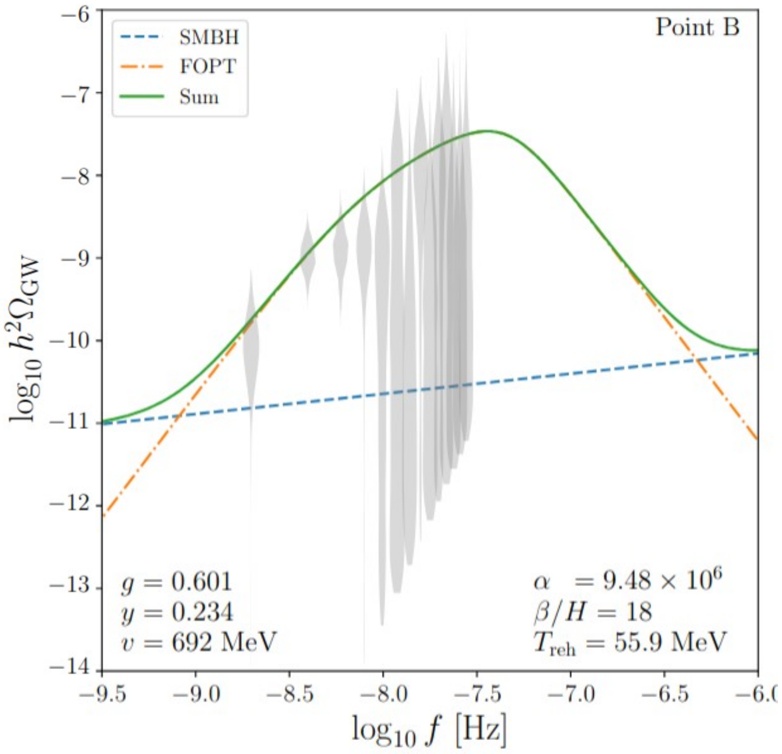
- **Result:** Clear indication for oscillating arrival times with frequencies in the nHz range
- Correlations consistent with expectation and preferred over uncorrelated signal at 3–4σ





- **Simplest explanation:** Mergers of supermassive black hole binaries
- But: signal larger than expected and different shape!

First-order phase transition



- Spontaneous symmetry breaking can happen through a first-order phase transition
- Strong supercooling possible for classically conformal dark sector
- Potential explanation of gravitatational wave signal observed by pulsar timing arrays
- Peak frequency around 10 nHz requires dark Higgs vev of order 100 MeV

Balan, FK et al., arXiv:2502.19478

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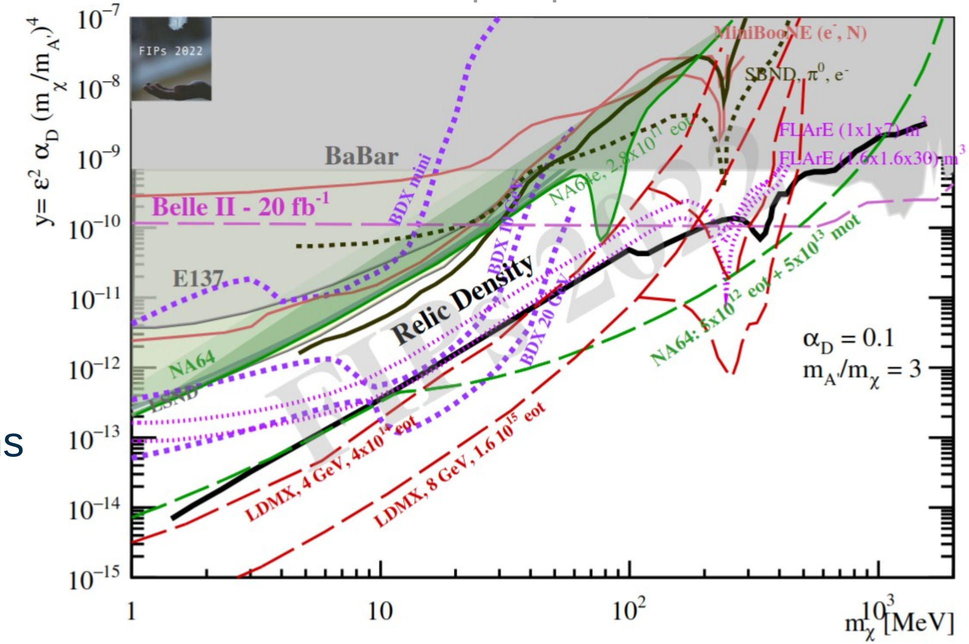
Challenge 1: Coupling dark and visible sector

- Calculation assumes equilibrium of the dark sector with Standard Model thermal bath
- Need some coupling between the two sectors
- Mixed term in the scalar potential $|H|^2 \, |\Phi|^2$ spoils conformal symmetry after EWSB
- Simplest solution: Kinetic mixing between U(1)' gauge boson and hypercharge:

$$\mathcal{L} \supset \frac{\kappa}{2} F_{\mu\nu} F'^{\mu\nu}$$

→ Search for invisible decays of dark photons produced in collisions of SM particles (Leading constraints: BaBar & NA64)

FIPs 2022 Workshop Report, arXiv:2305.01715



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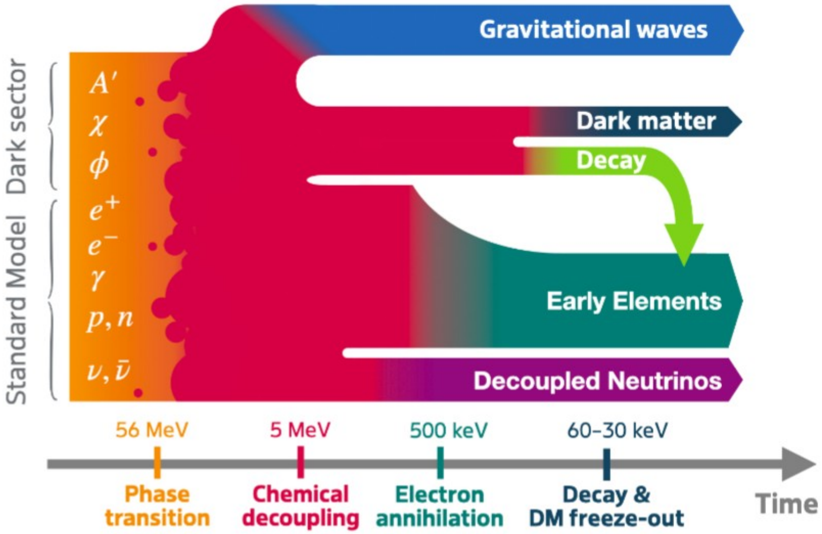
Challenge 2: Depleting the dark sector

- After the phase transition, the dark sector dominates the energy density of the universe
- To recover standard cosmology, need to transfer this energy to SM particles
- Dark photon interactions are too weak to fully deplete the dark sector
- Non-negligible abundance of decoupled dark Higgs bosons remains after BBN
- Decay into e^+e^- via dark-photon loops:

$$\tau_\phi \approx 2500\,\mathrm{s}\left(\frac{\kappa}{10^{-4}}\right)^4\left(\frac{g}{0.75}\right)^2\frac{m_\phi}{30\,\mathrm{MeV}}\left(\frac{m_{A'}}{100\,\mathrm{MeV}}\right)^{-2}$$



Balan, FK et al., arXiv:2502.19478

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Challenge 3: Dark matter relic density

- After symmetry breaking, the chiral fermions $\chi_{1,2}$ combine into a Dirac fermion χ
- This fermion is stable and could be a viable DM candidate
- Cosmological abundance set by freeze-out mechanism: $\Omega_{\rm DM} \simeq 0.1 \, \frac{10^{-8}\,\mathrm{GeV}^{-2}}{\langle \sigma_{\rm ann} v \rangle}$
- For direct annihilation of DM into dark Higgs bosons: $\langle \sigma_{\rm ann} v \rangle \sim \frac{y^4}{m_{\rm DM}^2} \sim \frac{y^2}{v_\phi^2}$
- First-order phase transition requires dark sector couplings of order unity: y ~ 1
  - → Observed DM relic density implies $v_\Phi \sim$ TeV
  - → Perfect for LISA, but not for pulsar timing arrays
  - → Need to find a way to suppress annihilation cross section

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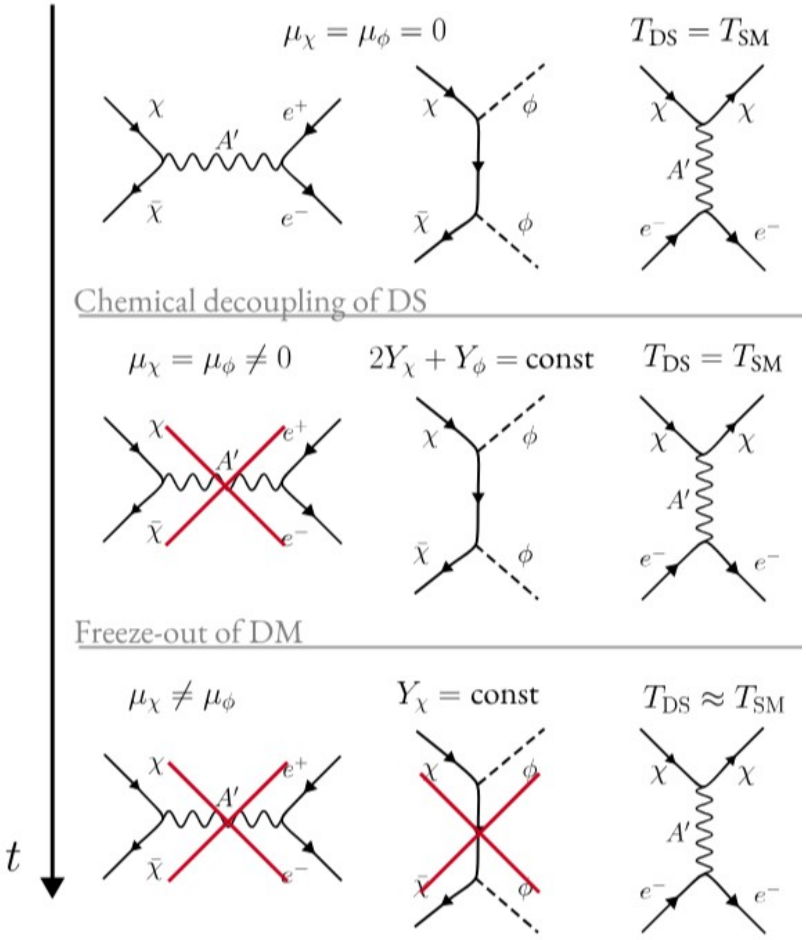
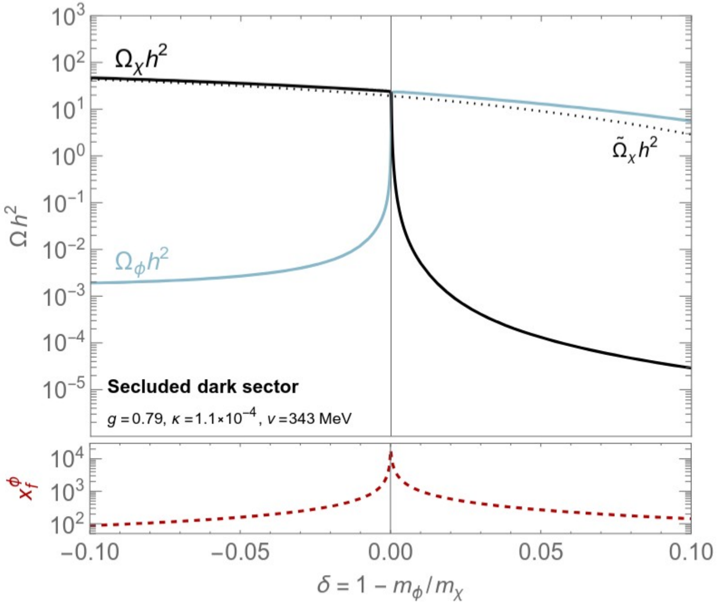
Detailed evolution



- Reproducing DM relic density requires m_χ and m_Φ to be very close



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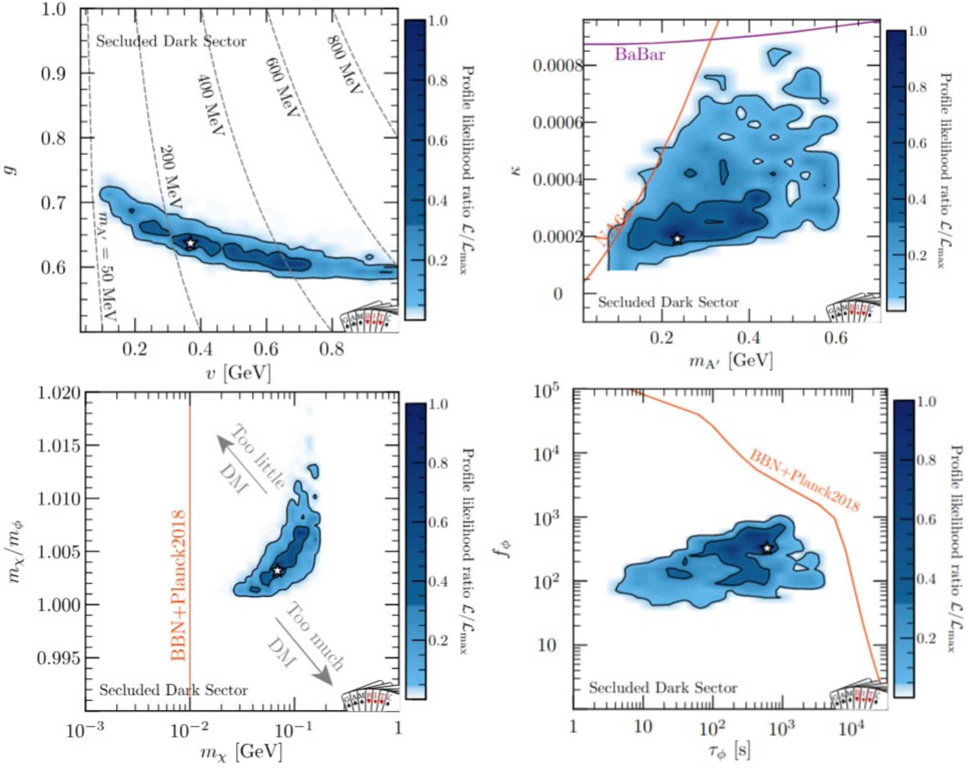
Results



- Possible to fit PTA signal and reproduce observed DM relic abundance while satisfying bounds from dark photon searches
- Abundance and lifetime of dark Higgs bosons small enough to comply with BBN constraints
- Well-defined allowed parameter regions

  → Predictions for future searches

Balan, FK et al., arXiv:2502.19478

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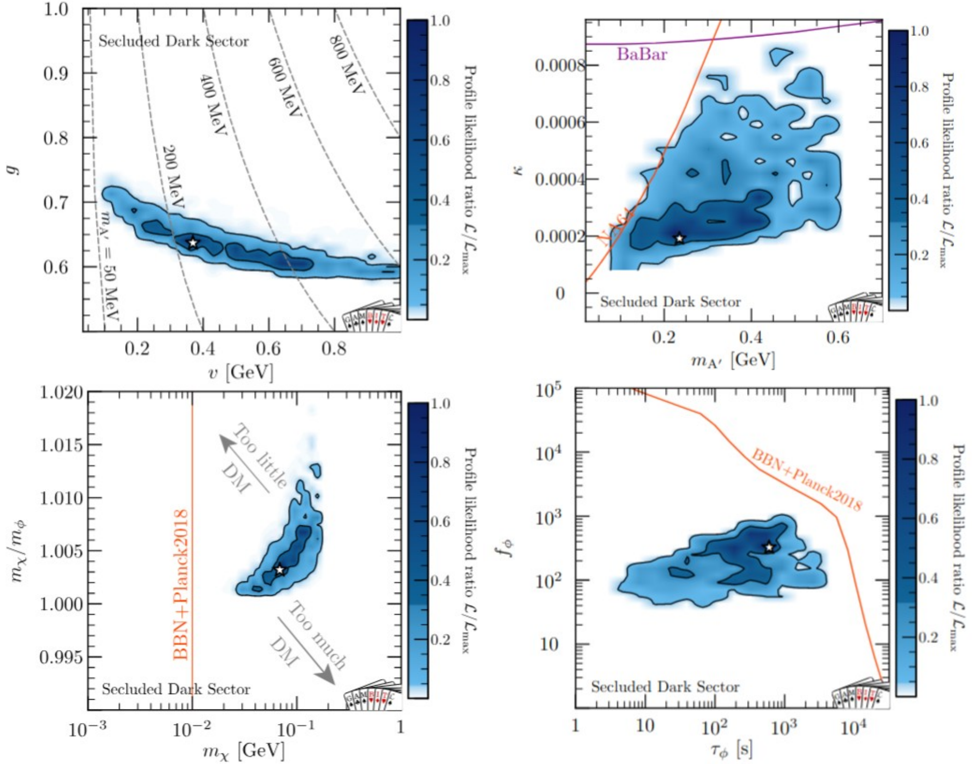
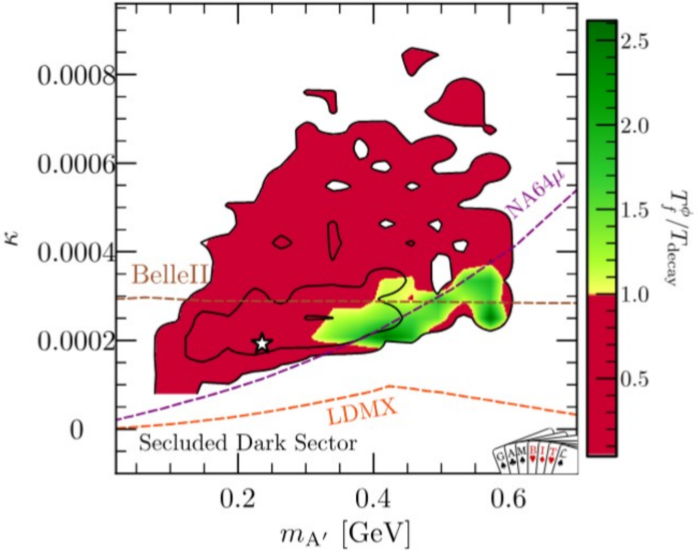
Results





Model can be tested with future searches for invisibly decaying dark photons

Balan, FK et al., arXiv:2502.19478

Conclusions

- The mass scale of dark matter may be generated via the Coleman-Weinberg mechanism
  - ➞ Dimensional transmutation from a classically conformal (i.e. scale-invariant) dark sector
  - ➞ Spontaneous breaking of a U(1)' gauge symmetry via a dark Higgs mechanism

- Thermal corrections restore the gauge symmetry at high temperatures
  - ➞ Expansion of the universe triggers a phase transition
  - ➞ Potential barrier allows for supercooling and bubble nucleation

- A strong first-order phase transition generates gravitational waves
  - ➞ For a sub-GeV dark sector, these gravitational waves are in the nano-Hertz range
  - ➞ Potential expanation of signal seen by pulsar timing arrays

- Viable model: Conformal dark sector coupled to the Standard Model via kinetic mixing
  - ➞ Provides a viable dark matter candidate
  - ➞ Satisfies cosmological and laboratory constraints on dark Higgs bosons and dark photons