

Quark-mass effects in Higgs-boson production in gluon fusion

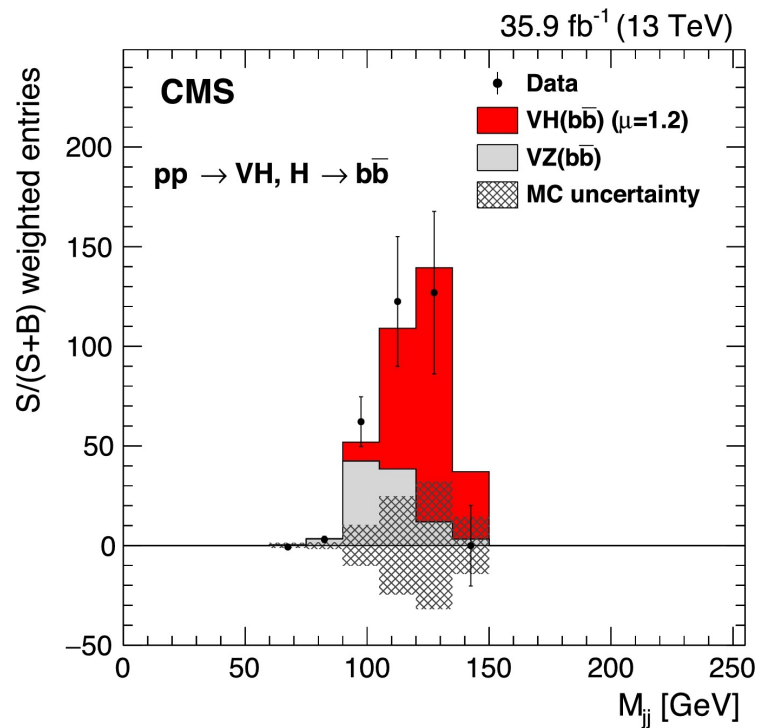
Project A1a

M. Czakon, R. Harlander

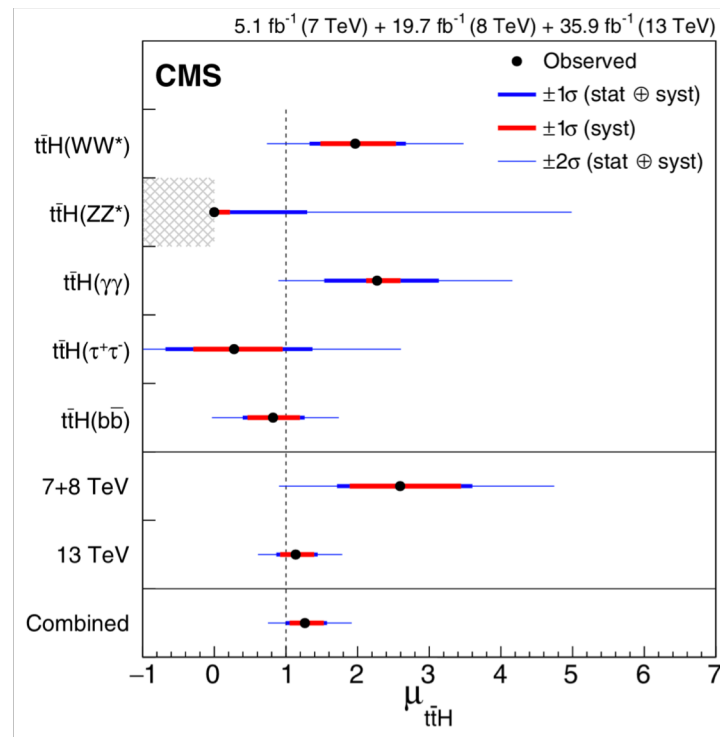
Plan:

- Status
- Some open problems in Higgs-boson production description:
 - Total inclusive cross section
 - Differential cross section
 - Kinematical limits
 - Resummation and bottom Yukawa coupling
- Semi-numerical methods
- Conclusions

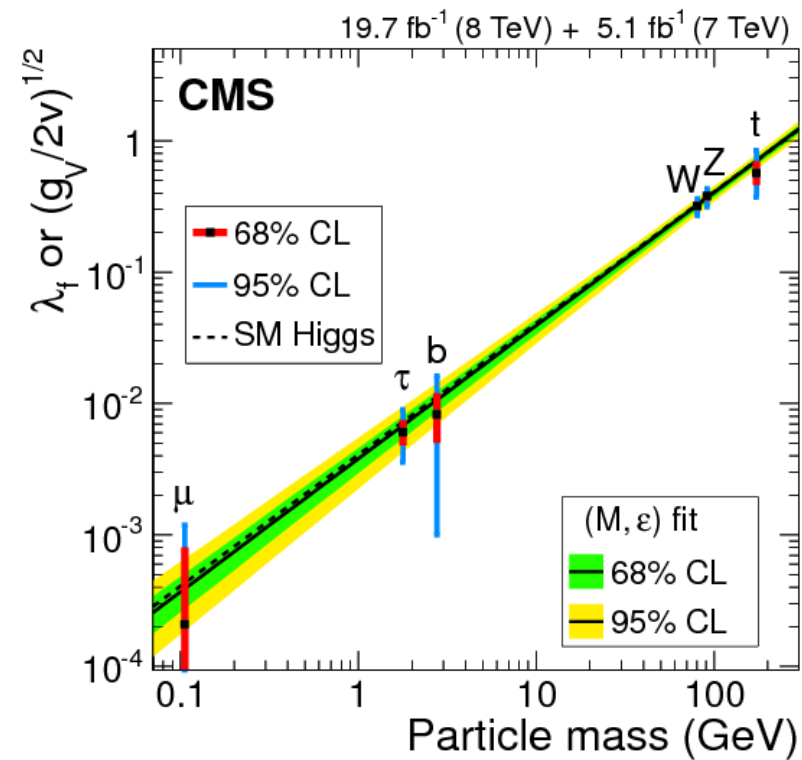
Status experiment



PLB 780 (2018) 501



PRL 120 (2018) 231801



Status theory

Handbook of LHC Higgs cross sections:

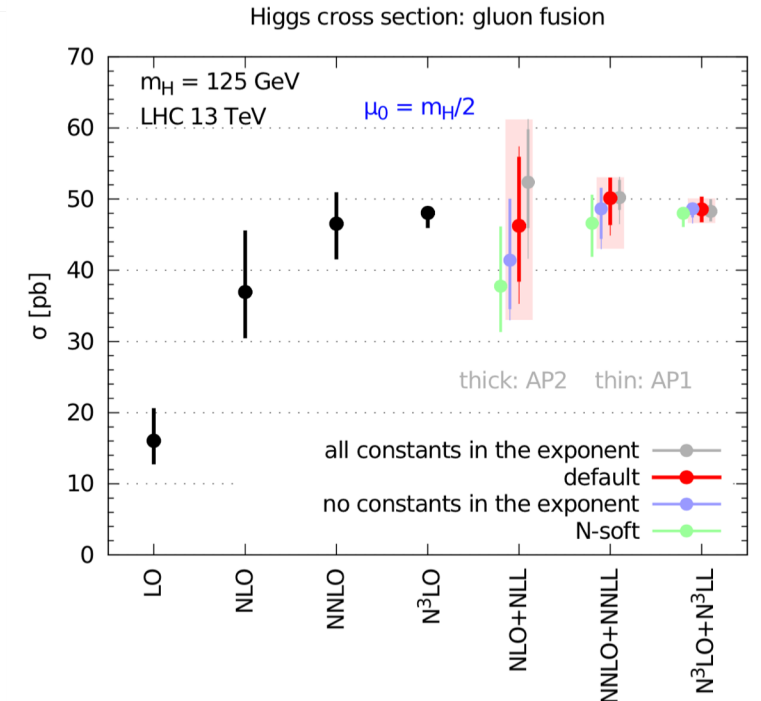
4. Deciphering the nature of the Higgs sector

Report of the LHC Higgs Cross Section Working Group `16

LHC @13 TeV

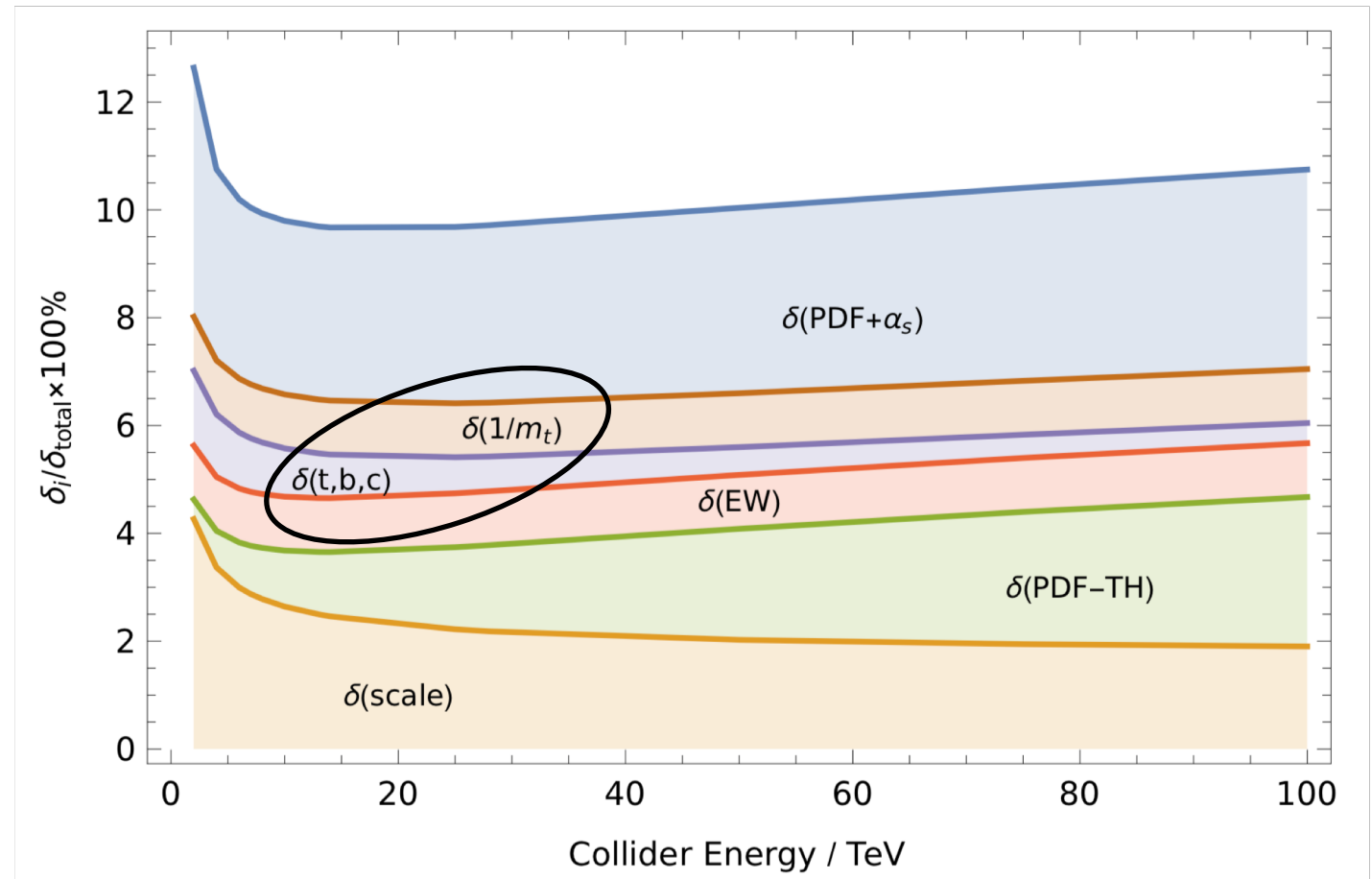
$$\sigma = 48.58 \text{ pb}^{+2.22 \text{ pb} (+4.56\%)}_{-3.27 \text{ pb} (-6.72\%)} (\text{theory}) \pm 1.56 \text{ pb} (3.20\%) (\text{PDF}+\alpha_s).$$

48.58 pb =	16.00 pb	(+32.9%)	(LO, rEFT)
	+ 20.84 pb	(+42.9%)	(NLO, rEFT)
	− 2.05 pb	(−4.2%)	((t, b, c), exact NLO)
	+ 9.56 pb	(+19.7%)	(NNLO, rEFT)
	+ 0.34 pb	(+0.7%)	(NNLO, 1/m _t)
	+ 2.40 pb	(+4.9%)	(EW, QCD-EW)
	+ 1.49 pb	(+3.1%)	(N ³ LO, rEFT)



Status theory

Higgs Physics
at the HL-LHC and HE-LHC
Report from Working Group 2
on the Physics of the HL-LHC,
and Perspectives at the HE-LHC '19



- Missing higher-order effects of QCD corrections beyond N3LO ($\delta(\text{scale})$).
- Missing higher-order effects of electroweak and mixed QCD-electroweak corrections at and beyond $\mathcal{O}(\alpha_s \alpha)$ ($\delta(\text{EW})$).
- Effects due to finite quark masses neglected in QCD corrections beyond NLO ($\delta(t,b,c)$ and $\delta(1/m_t)$)
- Mismatch in the perturbative order of the parton distribution functions (PDF) evaluated at NNLO and the perturbative QCD cross sections evaluated at N3LO ($\delta(\text{PDF-TH})$).

Top quark mass & inclusive cross section $gg \rightarrow H + X$

Exploiting the large mass expansion

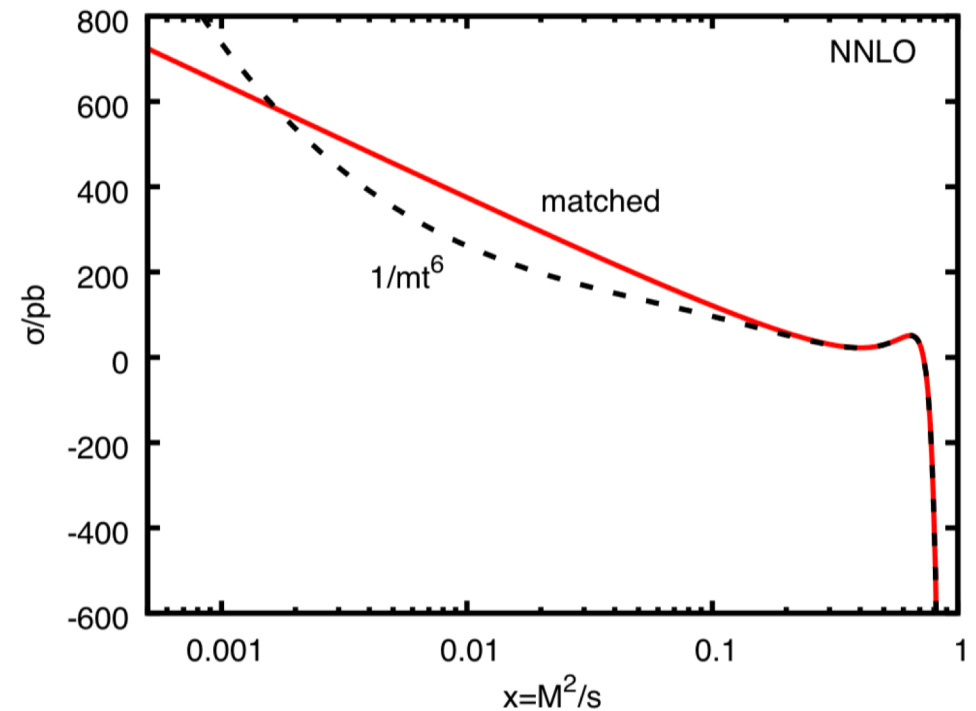
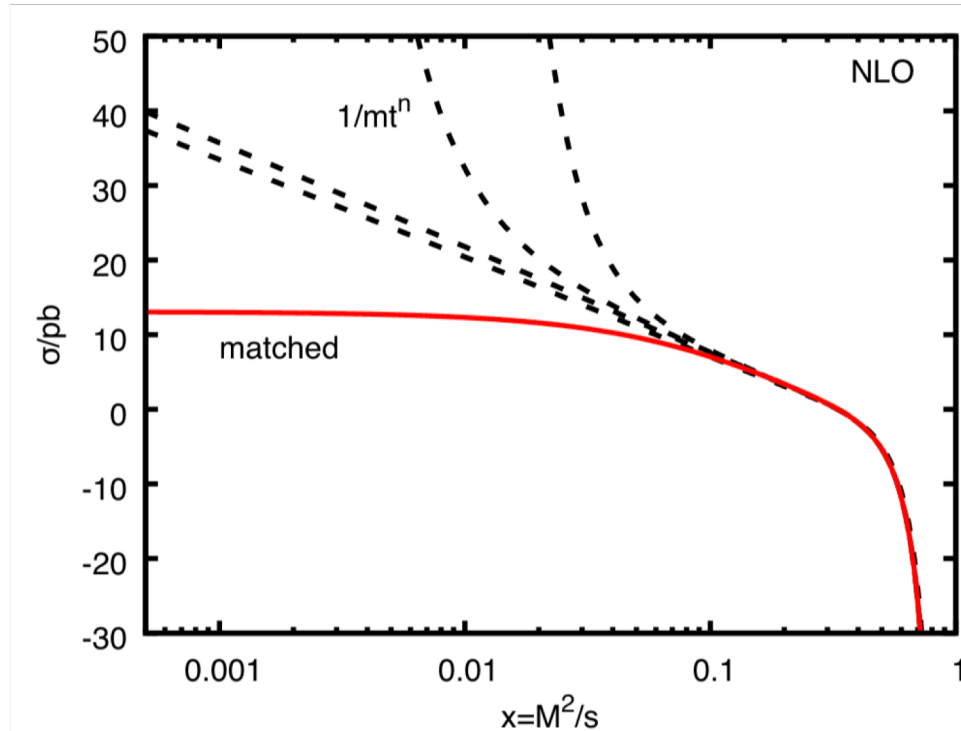
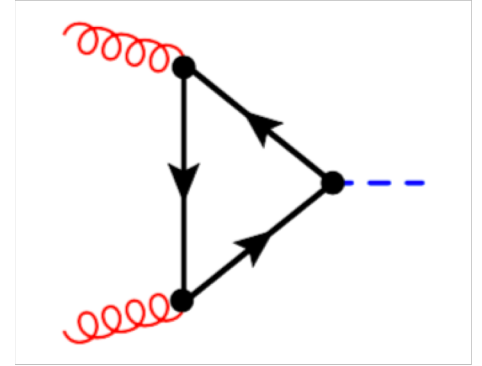
RH, Ozeren, '09

RH, Neumann, Ozeren, M. Wiesemann, '12

RH, Mantler, S. Marzani, K. Ozeren, '10

Marzani, Ball, Del Duca, Forte, Vicini '08

Pak, Rogal, Steinhauser '10



Top quark mass & inclusive cross section $gg \rightarrow H + X$

...and the high-energy limit

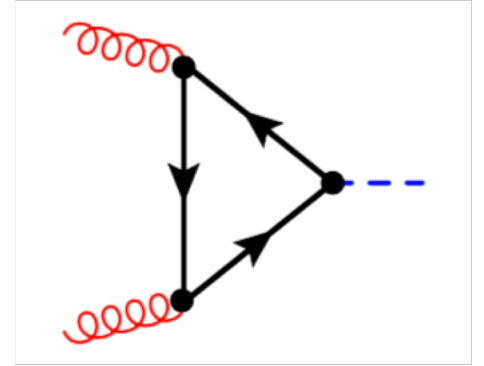
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$$\Delta_{gg}(N, \tau, \mu_F) = h(0, \tau, \gamma_s, \gamma_s) R^2(\gamma_s) \left(\frac{M_H^2}{\mu_F^2} \right)^{2\gamma_s}$$

$$R = 1 + \mathcal{O} \left(\left(\frac{\alpha_s}{N} \right)^3 \right)$$

$$h(N, \tau, M_1, M_2) = M_1 M_2 \int_0^1 d\zeta \zeta^{N-1} \int_0^\infty d\xi_1 \xi_1^{M_1-1} \int_0^\infty d\xi_2 \xi_2^{M_2-1} \int_0^{2\pi} \frac{d\varphi}{2\pi} M_H^2 \sigma^{\text{off}}(\zeta, \tau, \xi_1, \xi_2, \varphi),$$

$$\xi_i = \frac{|\mathbf{k}_i|^2}{M_H^2}, \quad \zeta = \frac{M_H^2}{2(k_1 \cdot k_2 - \mathbf{k}_1 \cdot \mathbf{k}_2)}$$

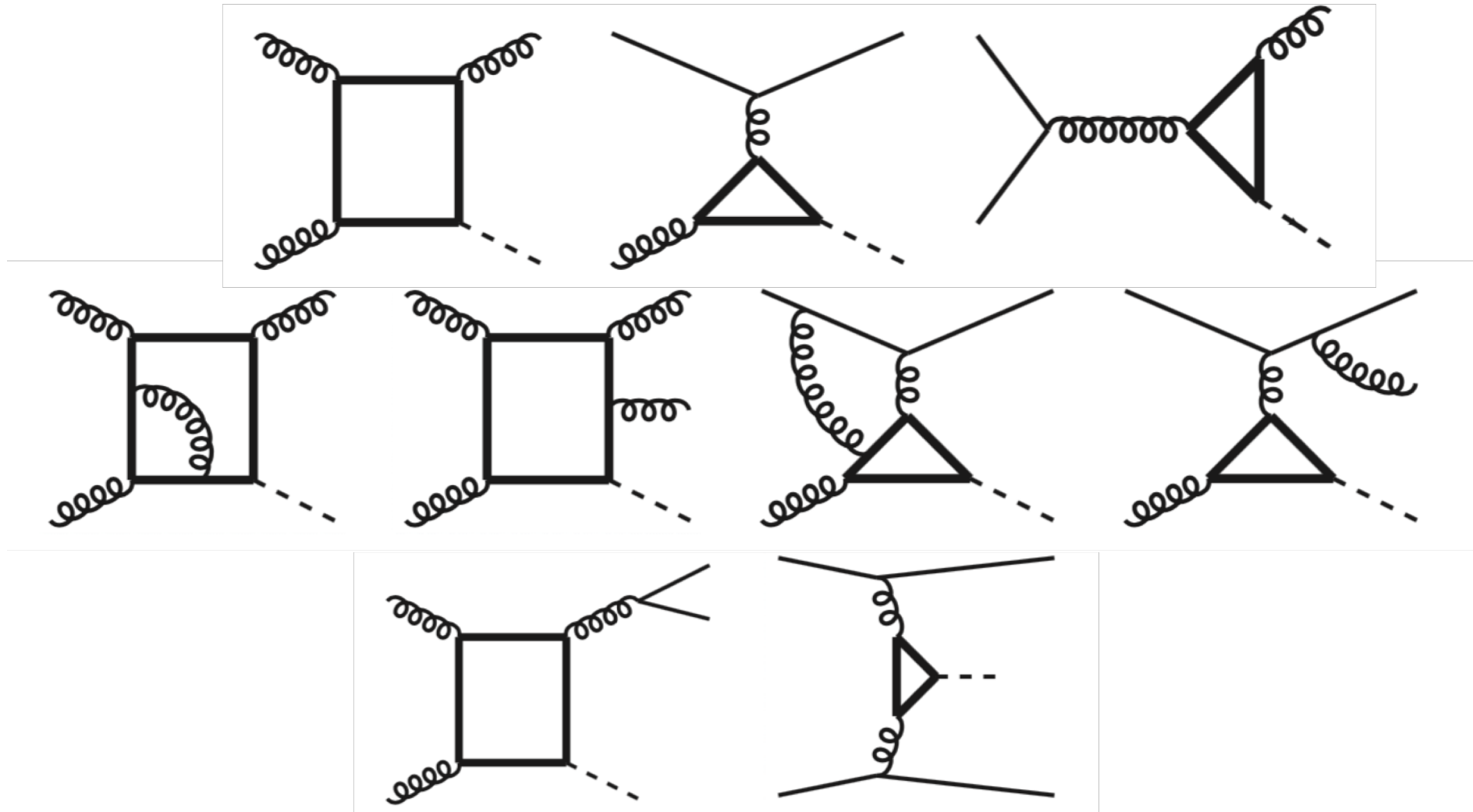
$$M_1 = M_2 = \gamma_s \left(\frac{\alpha_s}{N} \right)$$

$$\gamma_s \left(\frac{\alpha_s}{N} \right) = \sum_{k=1}^{\infty} c_k \left(\frac{C_A \alpha_s}{\pi N} \right)^k, \quad c_k = 1, 0, 0, 2\zeta(3), \dots$$

- Extend beyond leading high-energy behavior ?
- How about threshold behavior ?

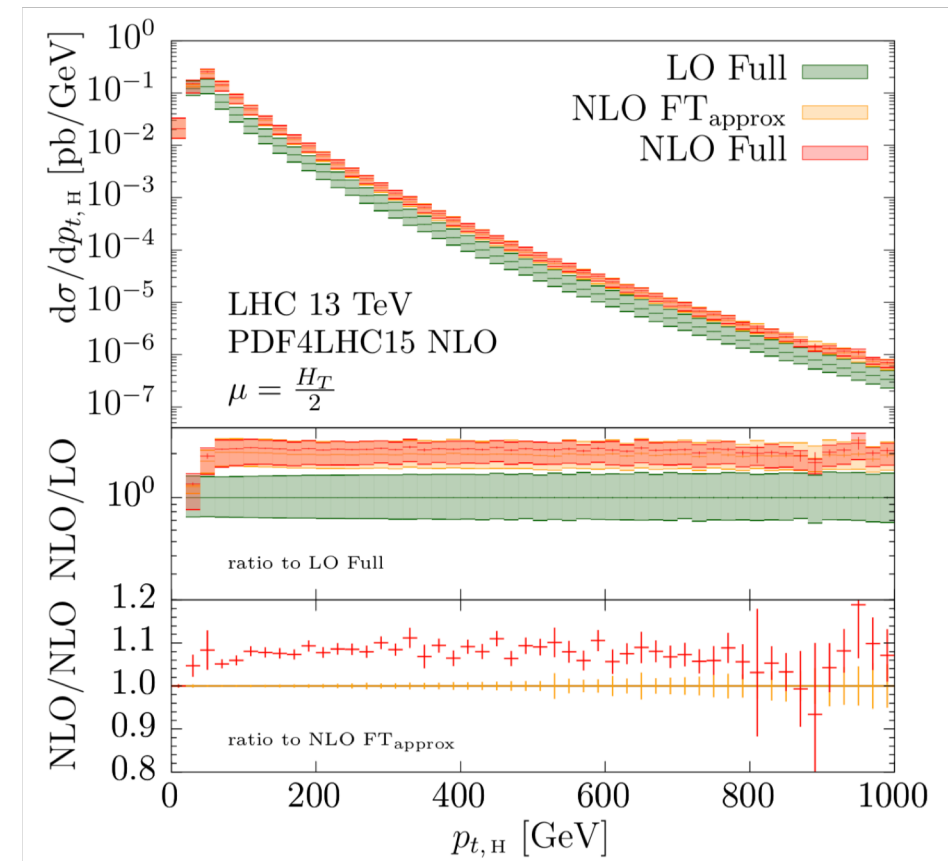
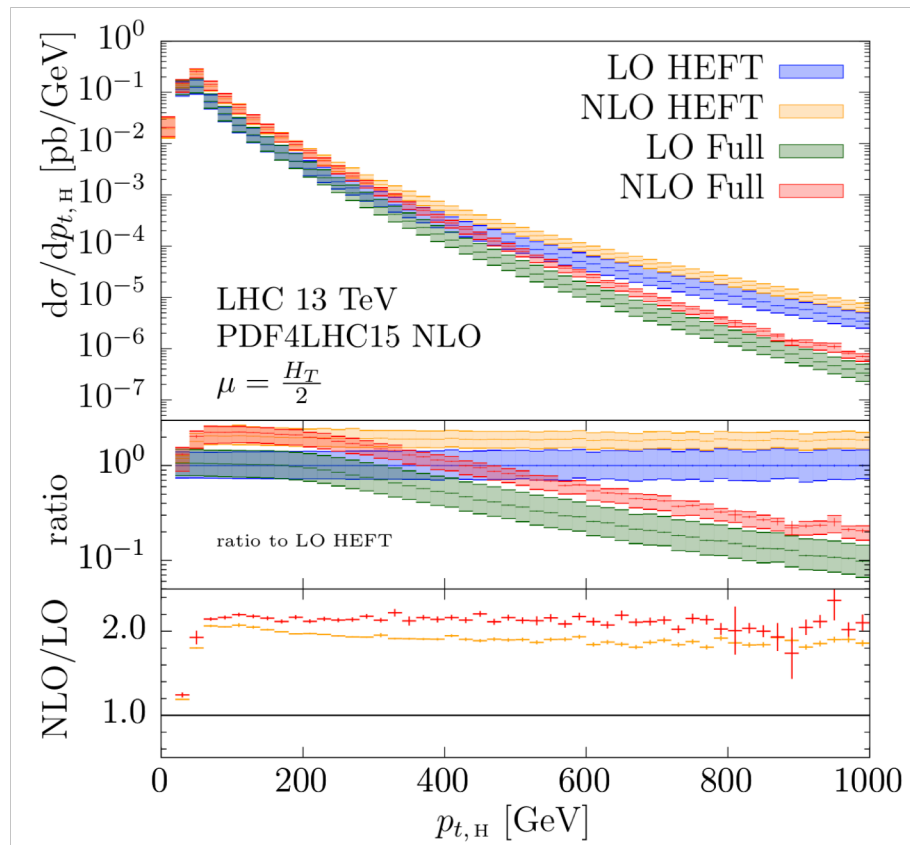
Higgs + jet studies with mass effects @ NLO

- Complicated two-loop amplitudes



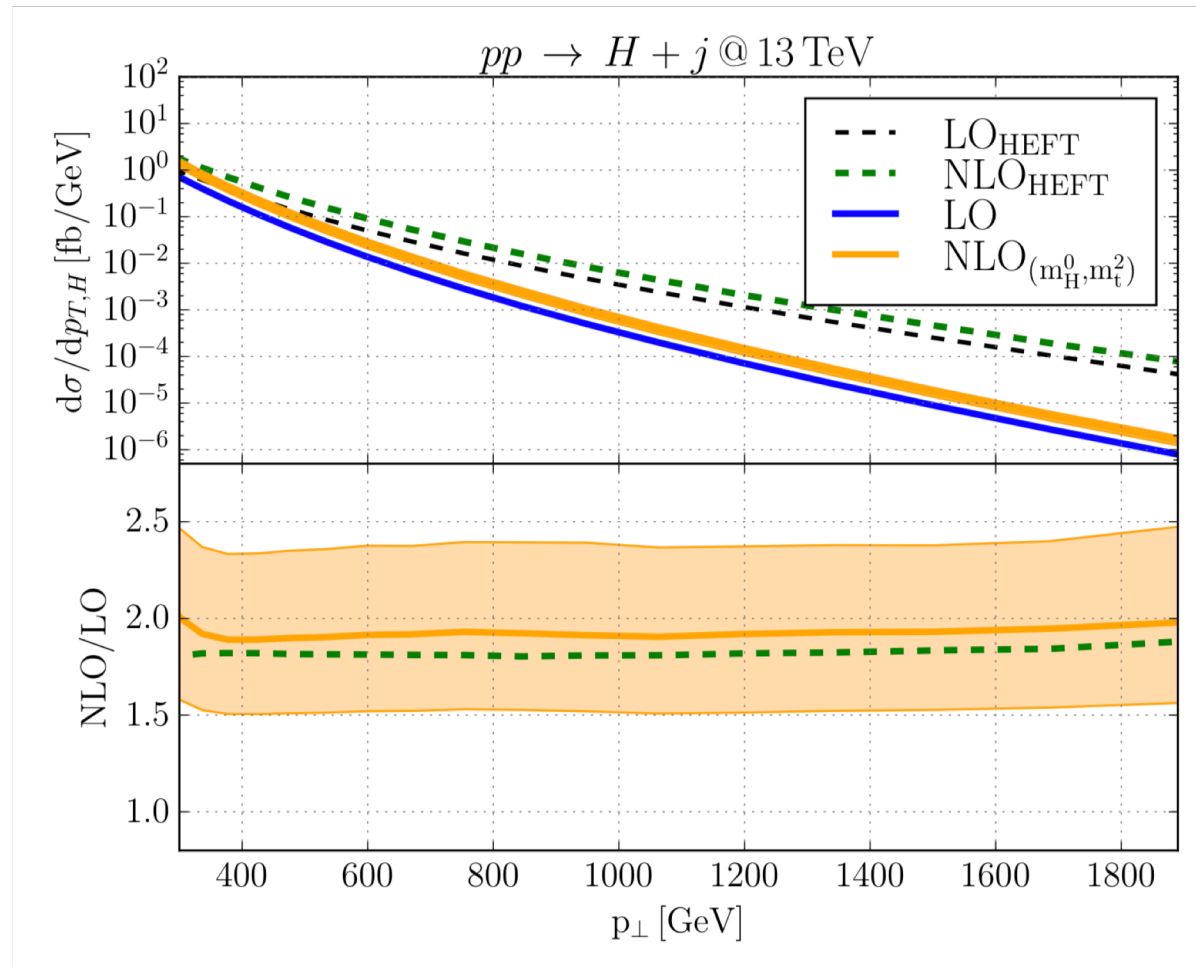
Higgs + jet studies with mass effects @ NLO

- Exact numerical result using sector decomposition for two-loop amplitudes [Jones, Kerner, Luisioni '18](#)



Higgs + jet studies with mass effects @ NLO

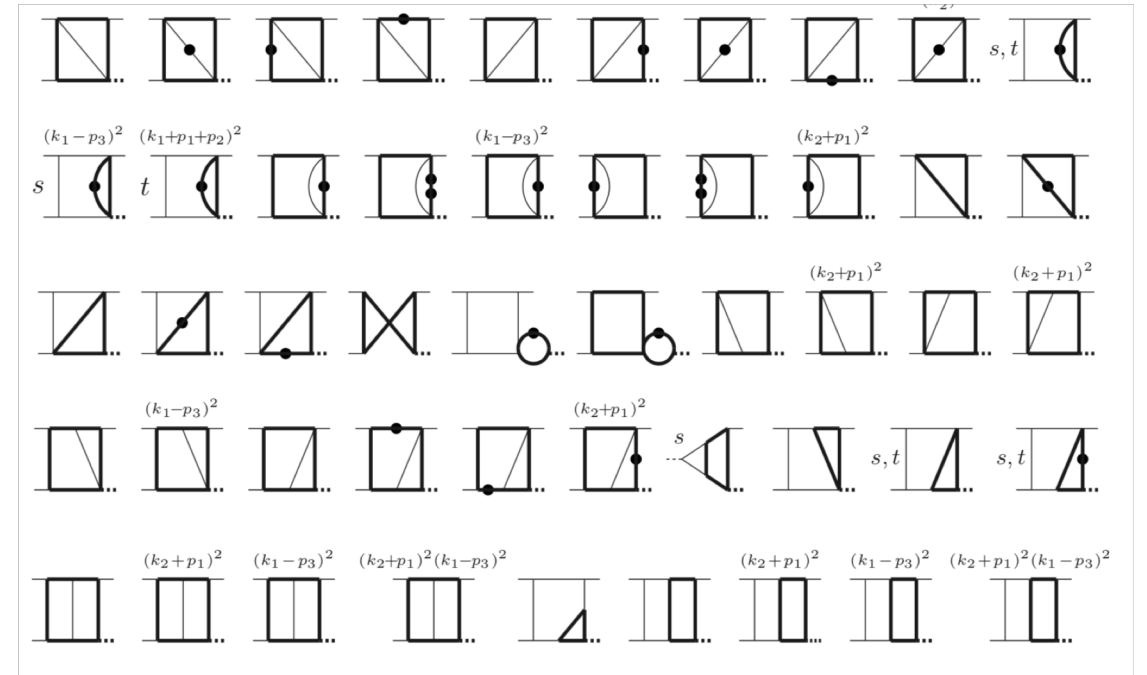
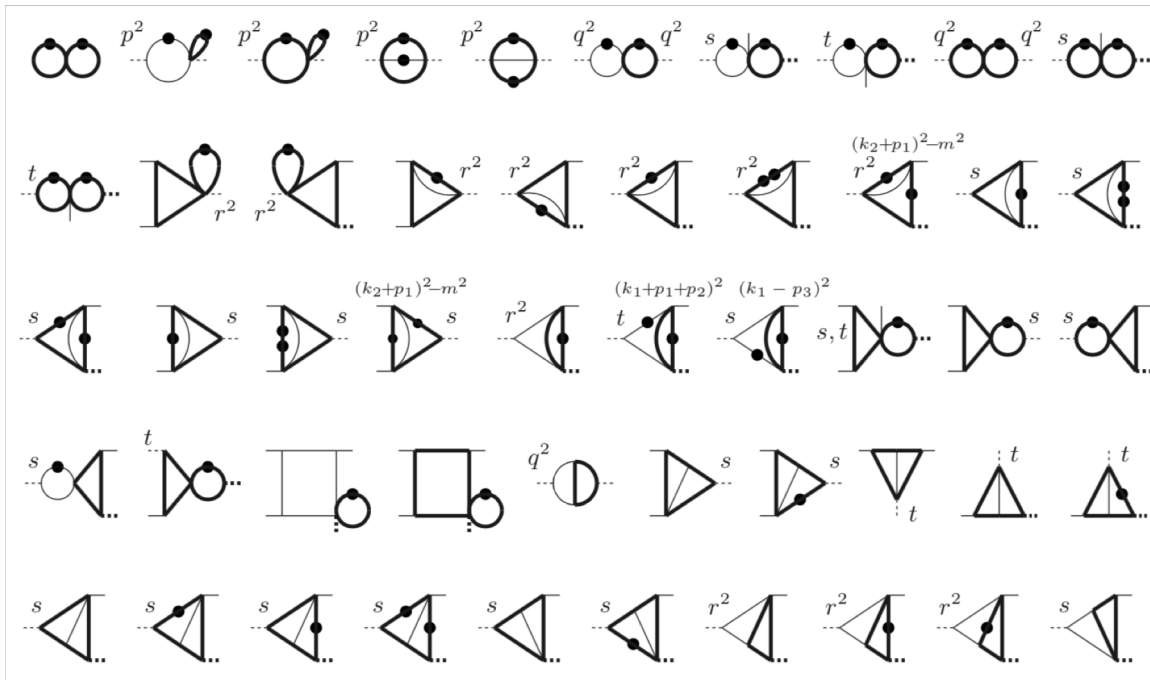
- Results using full two-loop amplitudes for nearly massless top quarks and exact real radiation [Lindert, Kudashkin, Melnikov, Wever '18](#)



Higgs + jet studies with mass effects @ NLO

- Exact two-loop amplitudes in the planar approximation

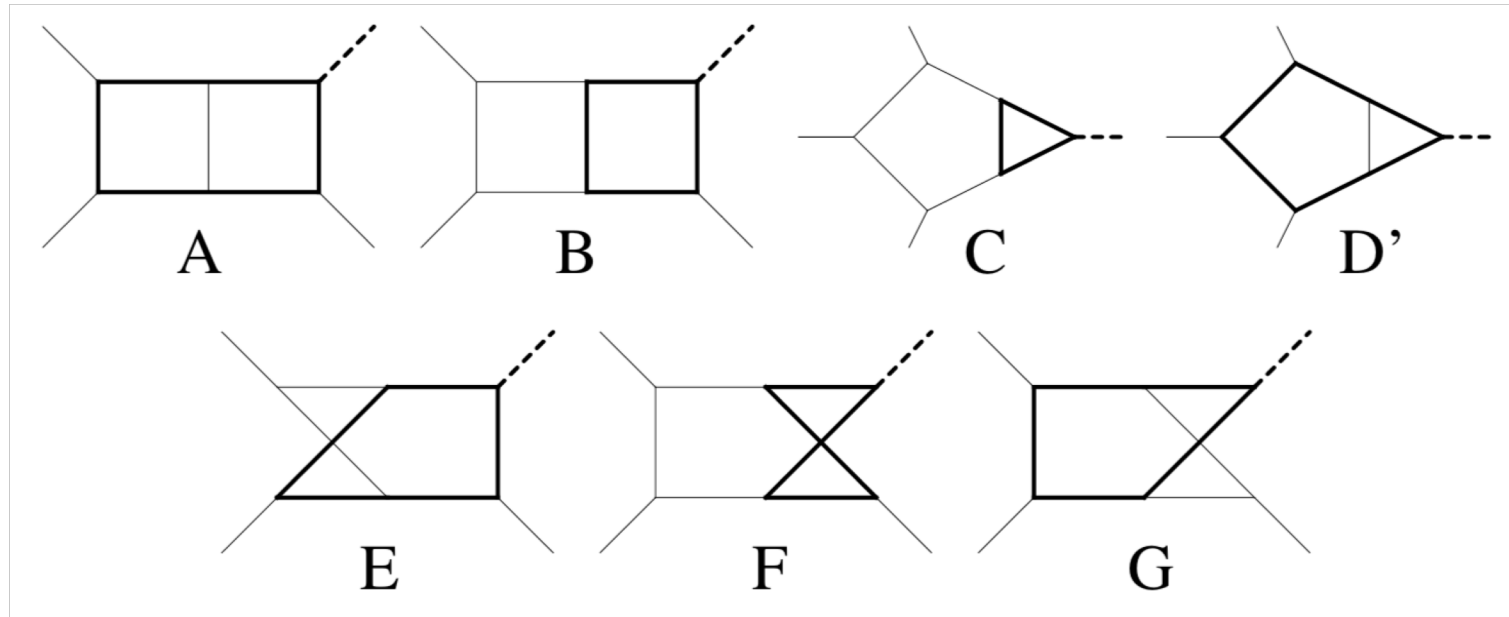
Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov '16



Higgs + jet studies with mass effects @ NLO

- Exact two-loop amplitudes and some non-planars

Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov '18



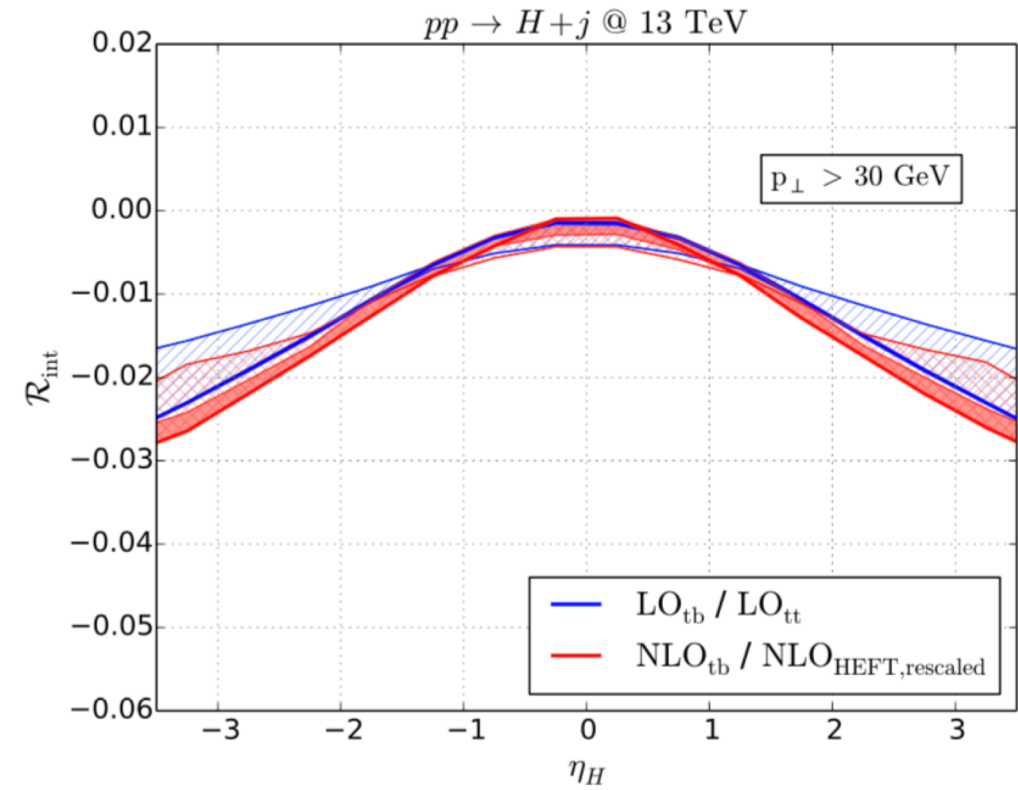
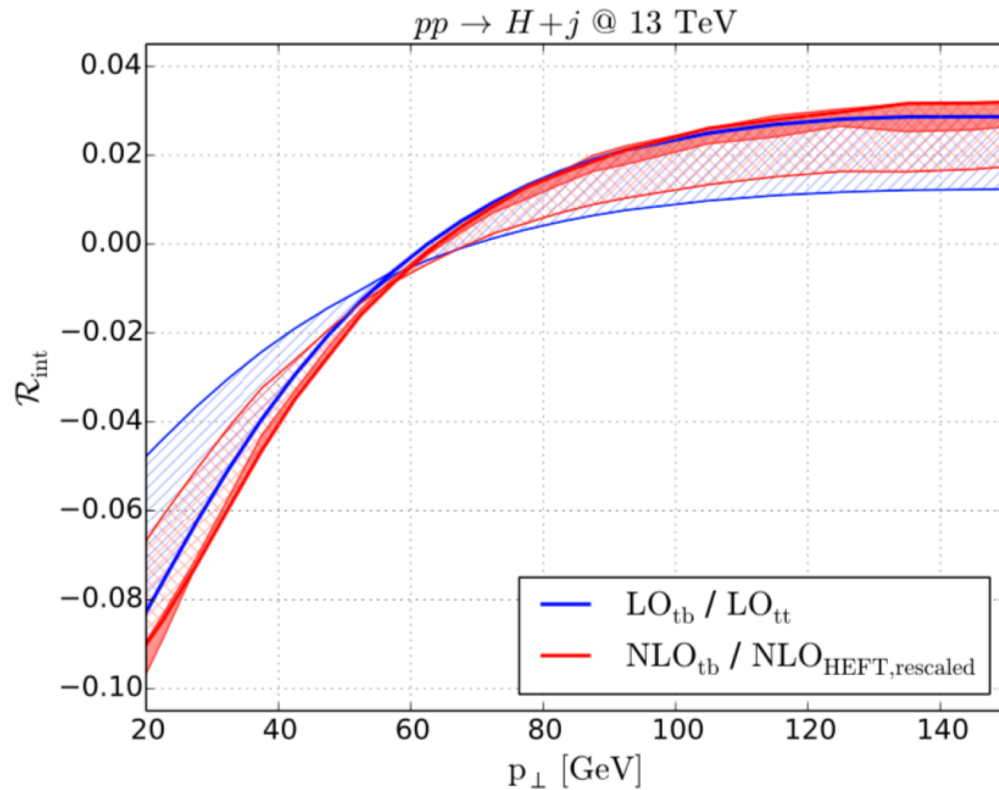
- Elliptic polylogs

$$A_{66}|_{\text{max cut}} \propto \int \frac{dz}{s \sqrt{P_{4;1}(z)}}$$

$$P_{4;1} \equiv ((m_H^2 + z)^2 - 4m_H^2 m_t^2) (4m_t^2 t u / s + (t + z)^2)$$

Top-bottom-loop interference for Higgs + jet

Lindert, Melnikov, Tancredi, Wever '17



- Integrate to obtain the effect on the inclusive cross section ?

Cross sections with mass effects: summary

- 3-loop 3-points amplitudes and 2-loop 4-point amplitudes allow to evaluate the missing mass effects at NNLO by means of a direct Monte Carlo integration using an NNLO subtraction scheme, e.g. STRIPPER [MC '10](#), [MC and Heymes '14](#)
- Do we have to wait for the complete analytic results ?

Semi-numerical methods

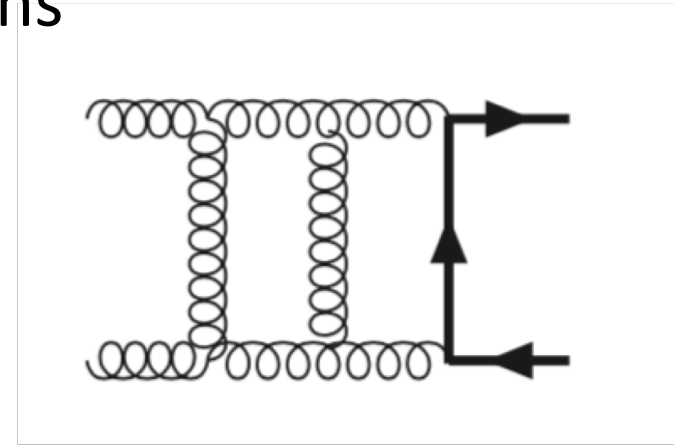
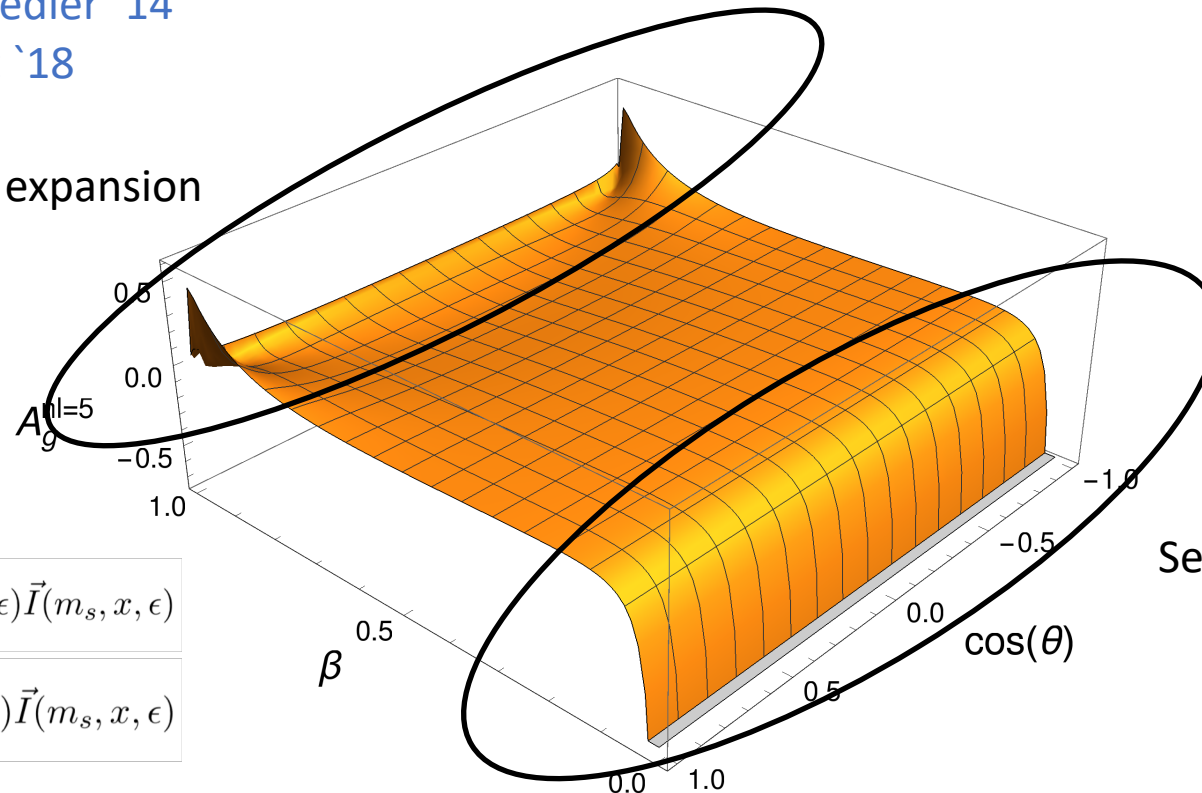
- Numerical solution of differential equations
- Example: top-quark pair production in hadron collisions

MC '08

Bärnreuther, MC, Fiedler '14

Chen, MC, Poncelet '18

Analytic high-energy expansion



Semi-numerical threshold expansion

$$m_s \frac{\partial}{\partial m_s} \vec{I}(m_s, x, \epsilon) = A^{(m_s)}(m_s, x, \epsilon) \vec{I}(m_s, x, \epsilon)$$

$$x \frac{\partial}{\partial x} \vec{I}(m_s, x, \epsilon) = A^{(x)}(m_s, x, \epsilon) \vec{I}(m_s, x, \epsilon)$$

Semi-numerical methods

- Numerical solution of differential equations
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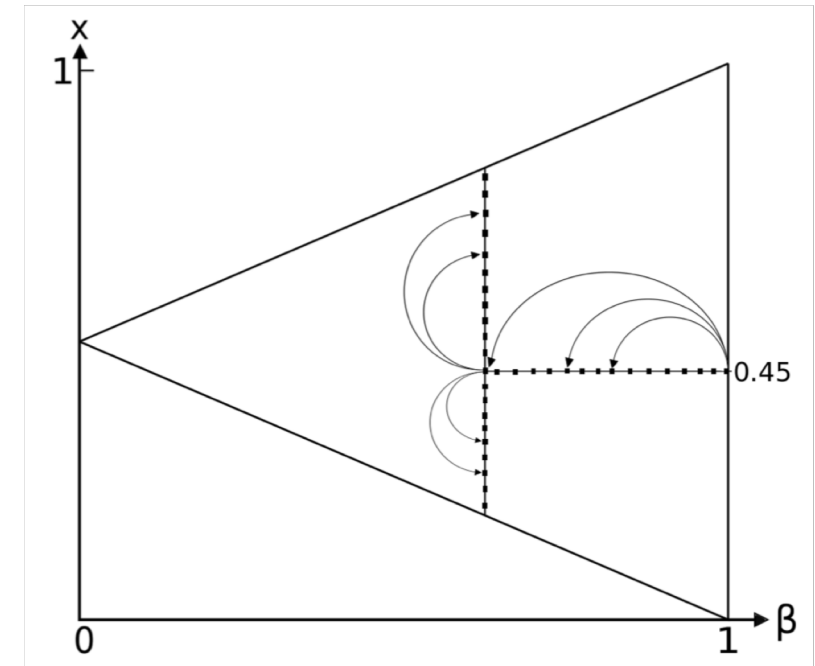
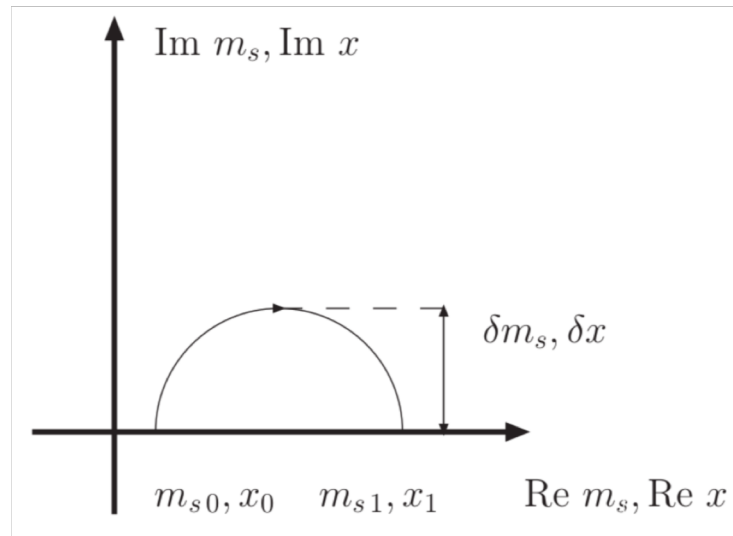
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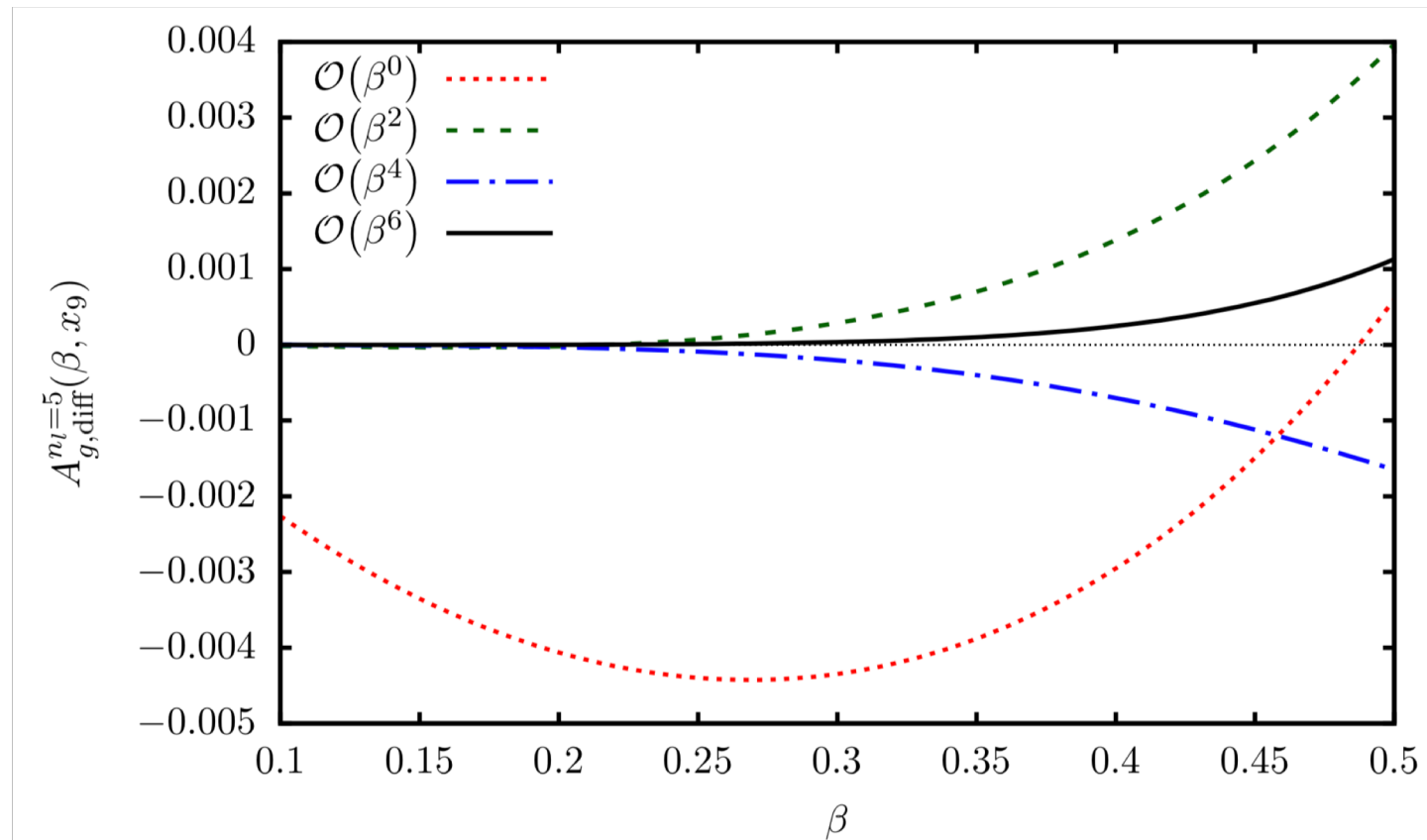
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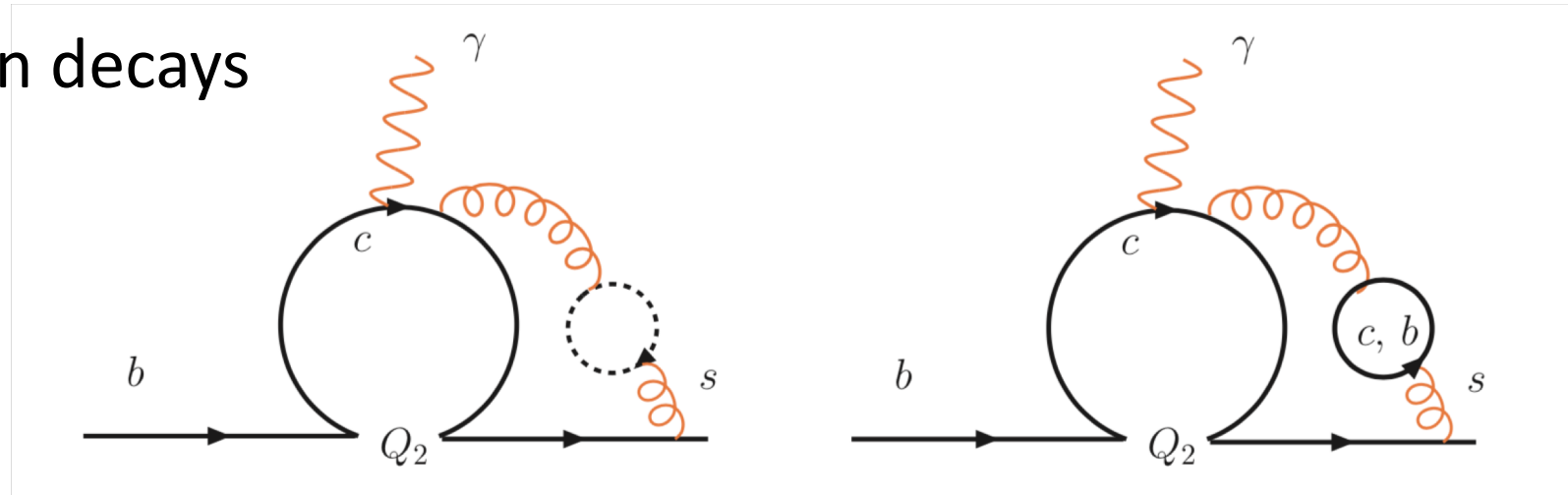


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Semi-numerical methods

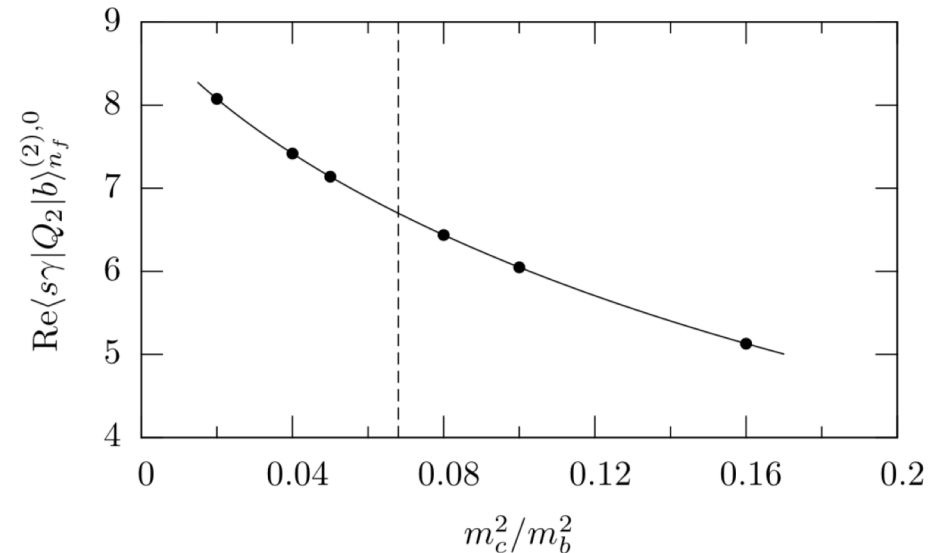
- Numerical solution of differential equations
- Example: Rare B-meson decays



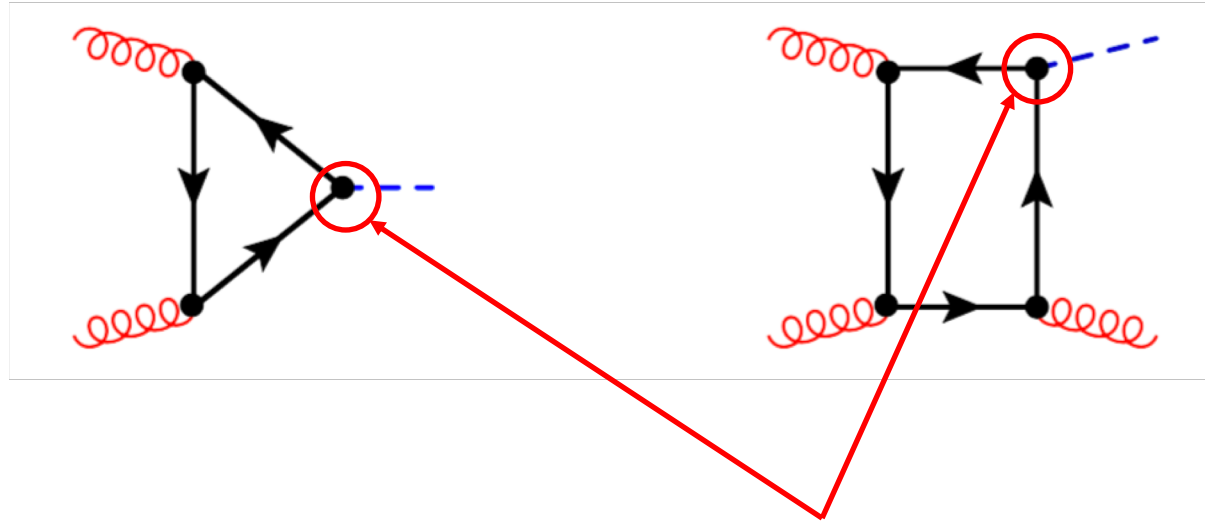
Boughezal, MC, Schutzmeier '07

MC, Fiedler, Huber, Misiak, Schutzmeier, Steinhauser '15

Misiak et al '15



One last question...



- How to renormalize the bottom Yukawa coupling ?
- Hints from subleading Sudakov resummation ? [Melnikov, Penin '16](#)

Conclusions

- In principle, one can obtain both total and differential cross sections using purely numerical methods
- However, there is potential for physical insight by analytic methods in high-energy, threshold and other regimes
- Progress expected through synergy between the two approaches

Conclusions

Links to other projects:

- A1b - Higgs boson physics with higher order QCD corrections within the Higgs Effective Theory
- A3b - Precision predictions for Higgs boson properties as a probe for New Physics
- B1a - Production of colour-singlet final states through N3LO QCD
- B1b - Precision top-quark physics at the LHC

Methodologically related to:

- | | |
|----------------------------------|----------|
| • Subtraction methods | A1a, B1a |
| • Massive virtual amplitudes | A3b, B1b |
| • Evaluation of master integrals | C1a, C1b |

Relevant input to Higgs-boson physics analyses:

- | | |
|--|----------|
| • Bottom-mass effects in BSM theories | A3a, A3b |
| • Mass effects in Higgs-boson production | A1b |

