Karlsruher Institut für Technologie

Particle Physics Phenomenology after the Higgs Discovery

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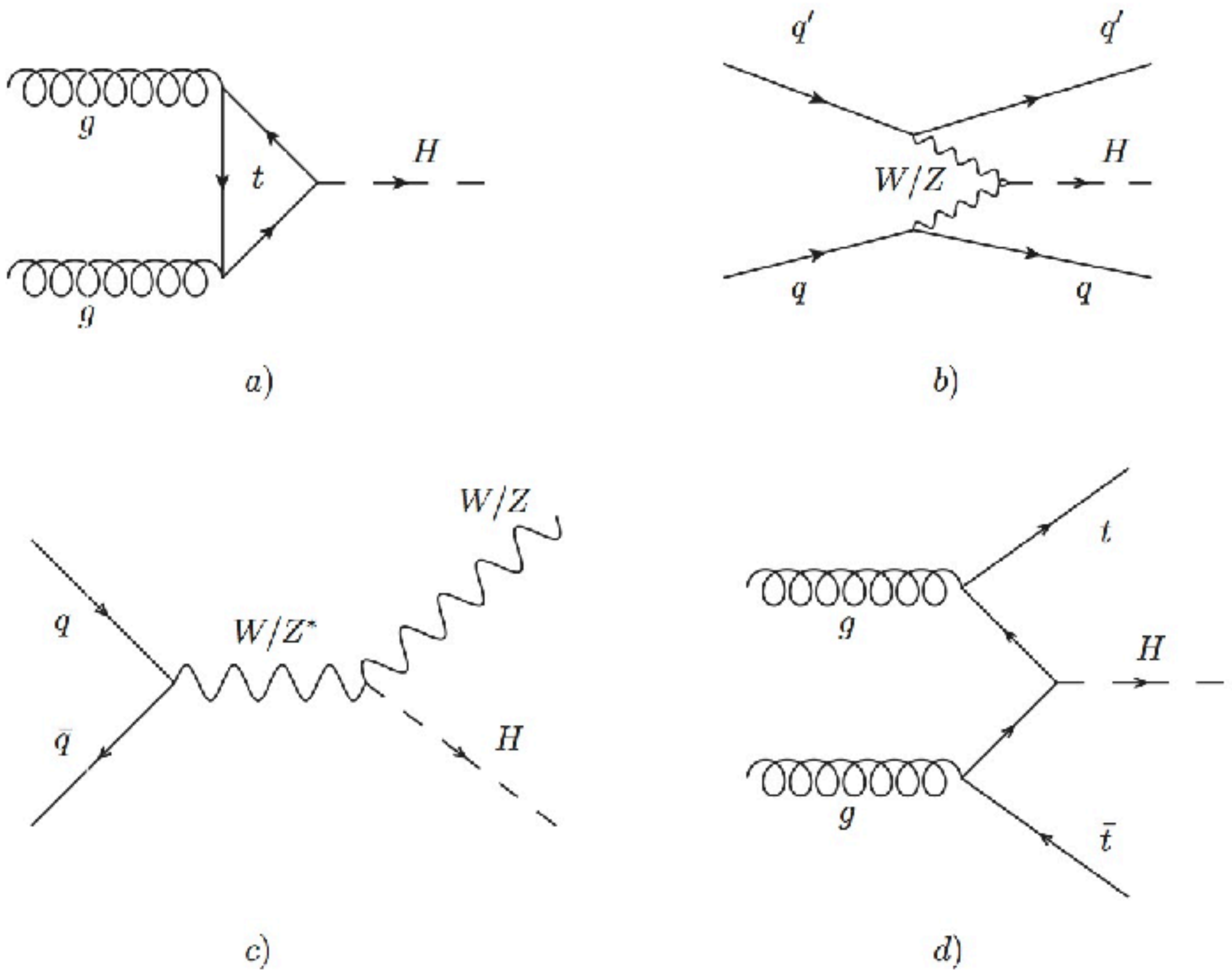
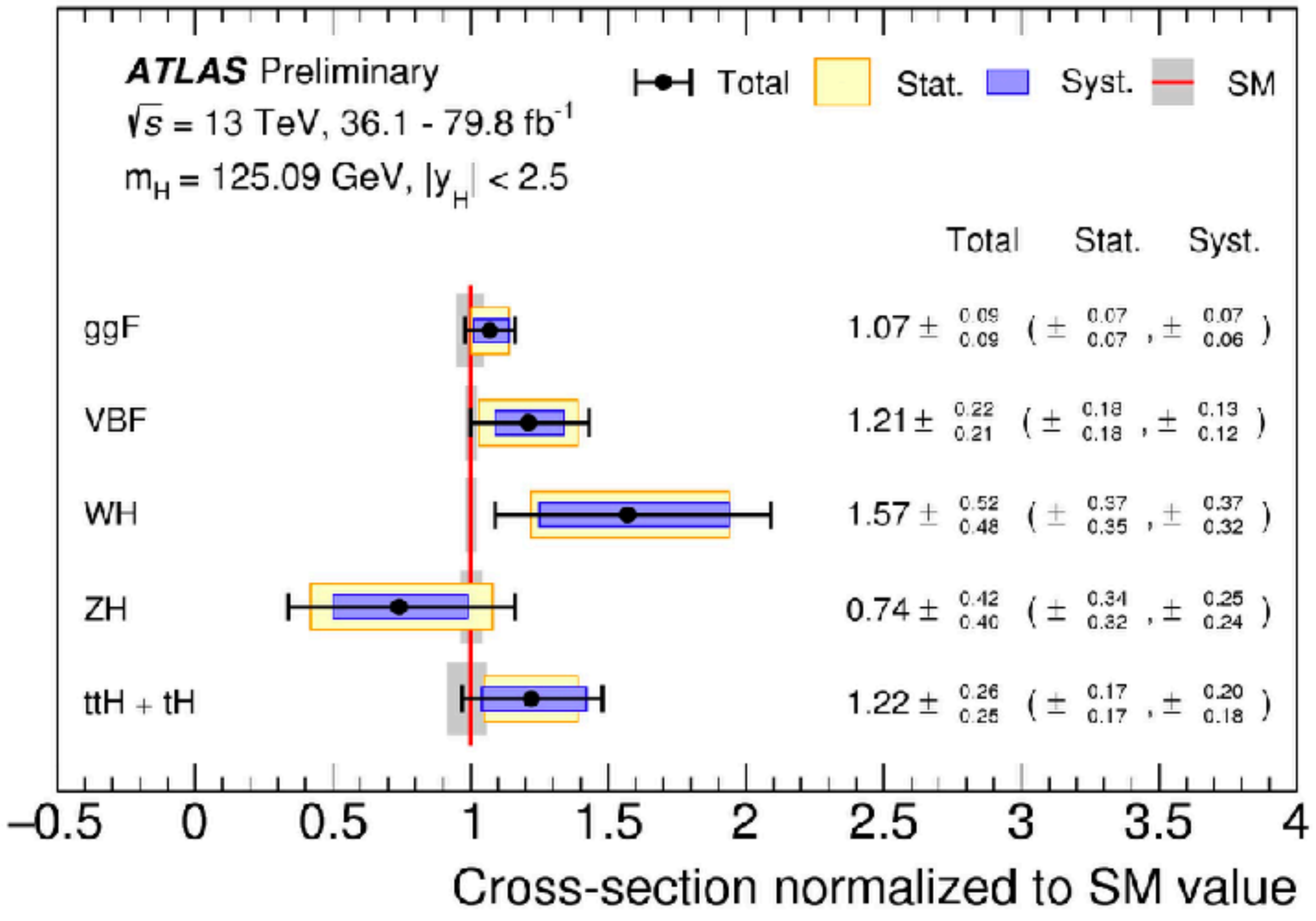
PROJECT A1C: HIGHER ORDER QCD CORRECTIONS TO HIGGS BOSON PRODUCTION IN WEAK BOSON FUSION

Kick-off meeting of the CRC 257, March 2019

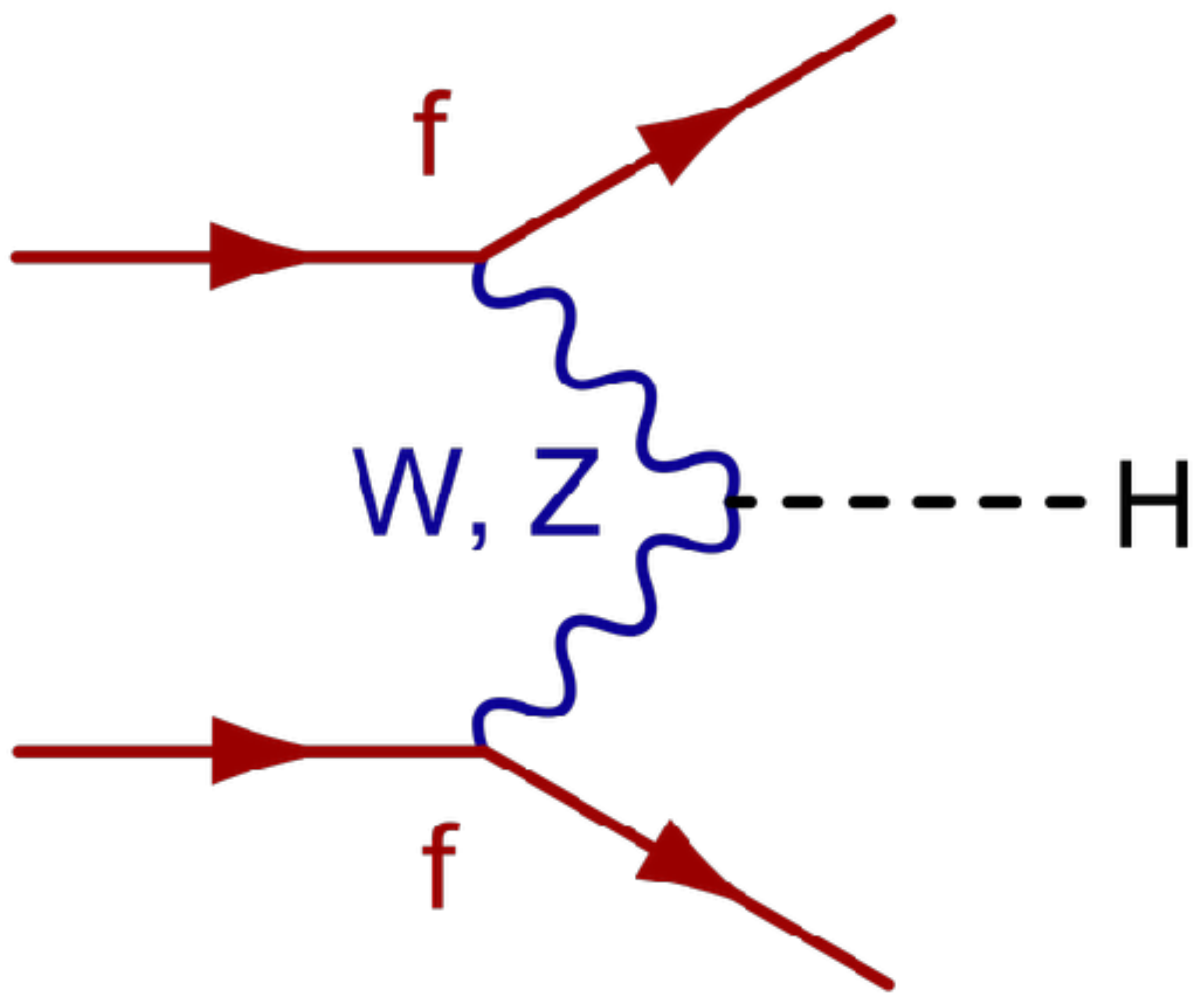
Higgs boson discovery structurally completed the Standard Model and launched an era of the detailed exploration of this new particle.

Higgs boson properties are in good agreement with the SM expectations. Nevertheless, there is still room for O(10-20) percent deviations even in the largest cross sections.





Higgs production in weak boson fusion is interesting as it has a relatively large cross section and because it is proportional to tree-level HVV couplings, which are fixed by gauge symmetries, it is sensitive to HVV anomalous couplings and since it provides one of the best signatures to study CP properties of the Higgs boson.

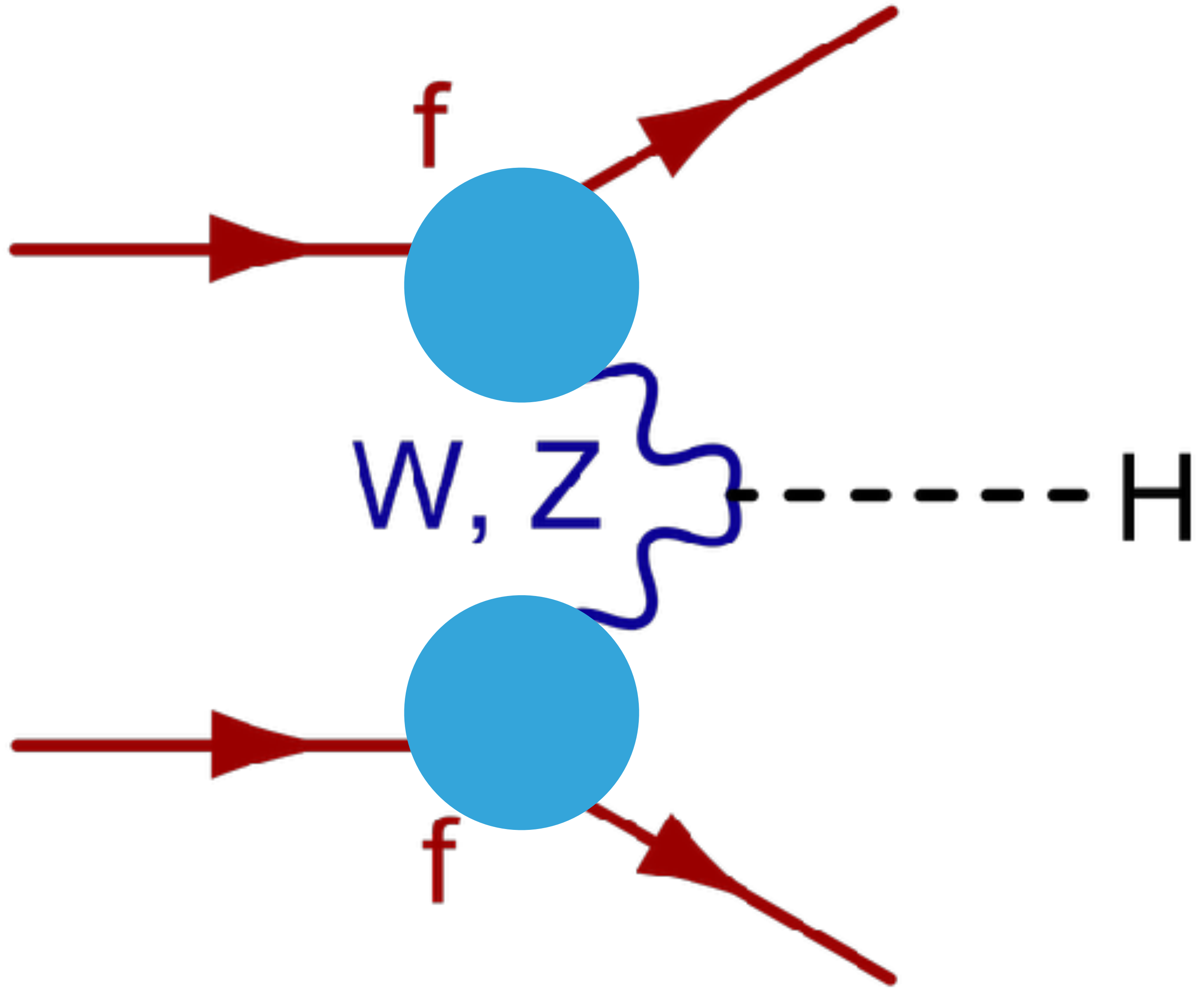


We will focus on the QCD corrections to Higgs boson production in weak boson fusion; those corrections involve two distinct types of contributions:

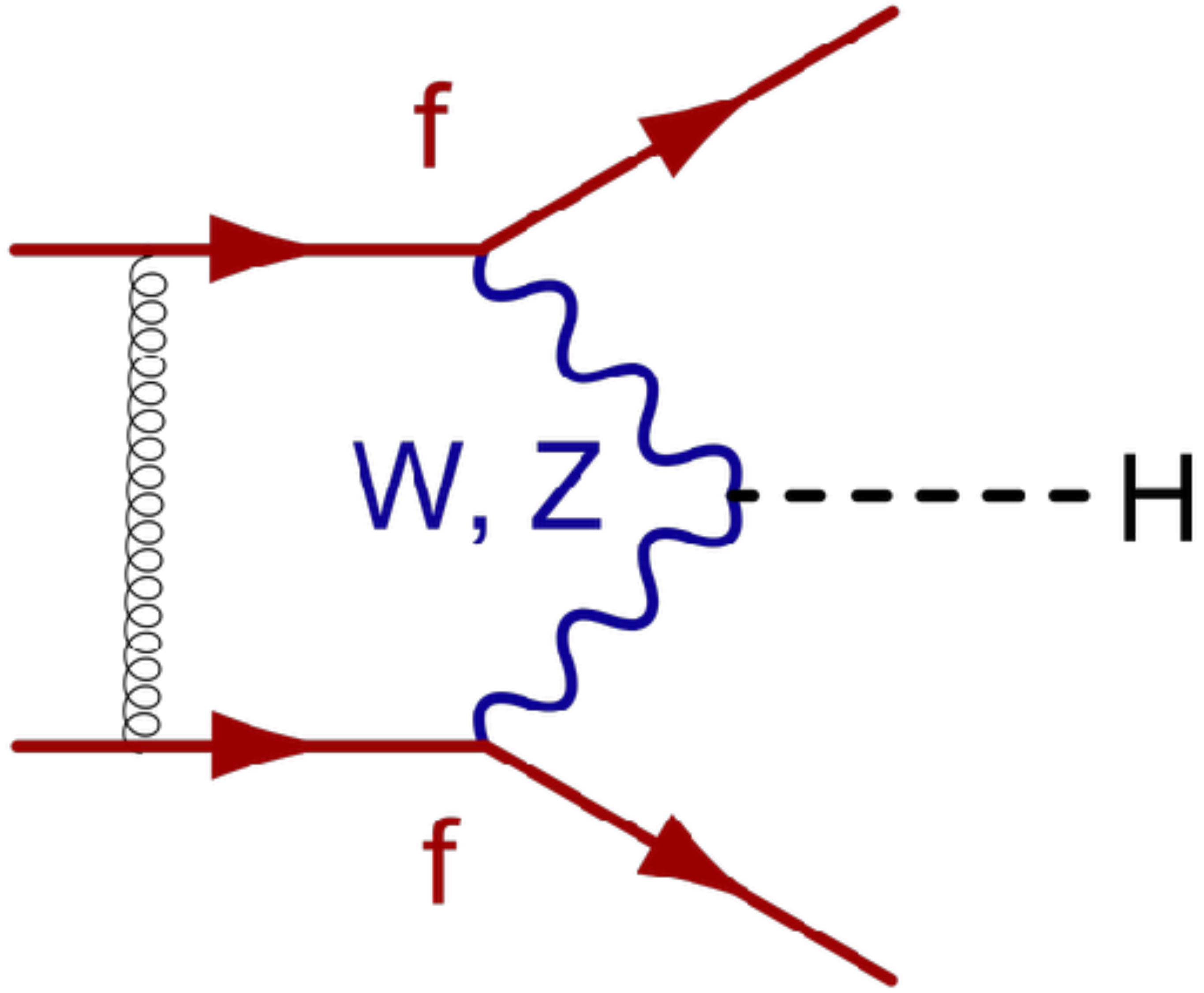
a) corrections to the same fermion line (DIS-like);

b) interactions between two fermion lines.

There are other contributions to WBF that do not fit into this template: the interference of the two fermion lines (e.g. uu -> Huu) or interference of WBF and other contributions (gg->H with two forward jets or V*H production followed by the decay V* -> 2 jets). These contributions have been computed at leading and sometimes at next-to-leading order in the strong coupling constant and were found to be small (per mille).

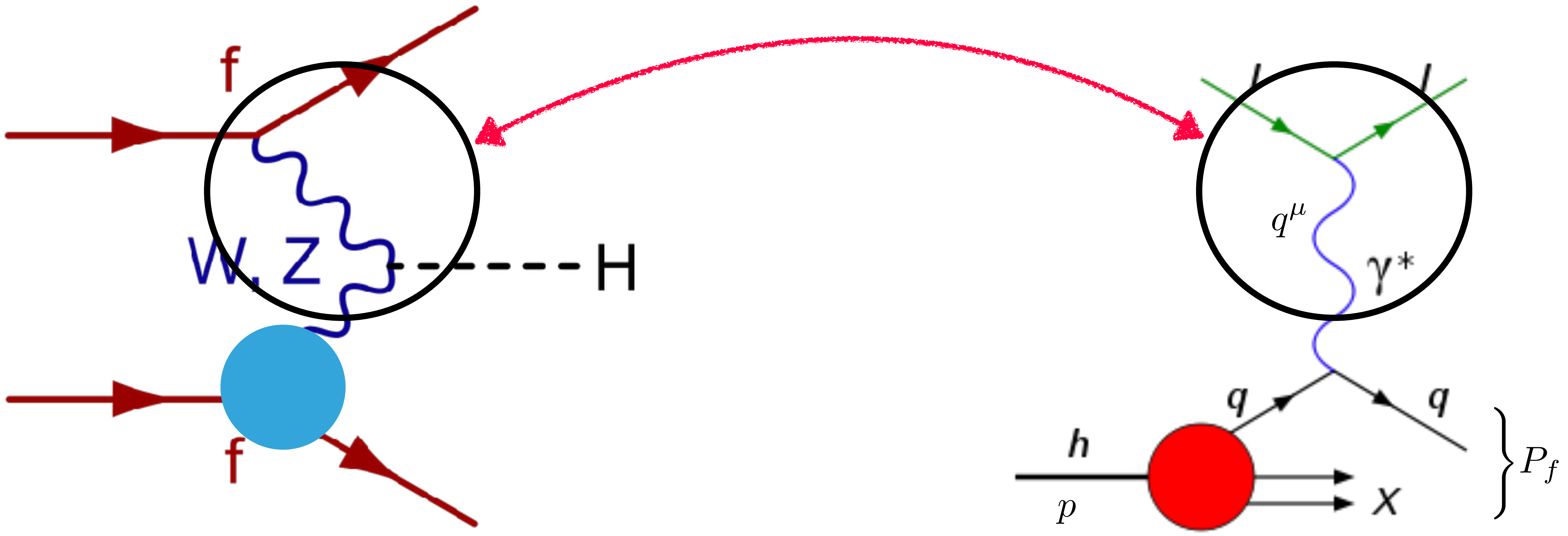


DIS-like effects



Cross-talk between two fermion lines

The DIS-like contribution is called so for a good reason: apart from clear differences in the upper parts of these diagrams, their lower parts can be related to each other and to inclusive DIS structure functions.

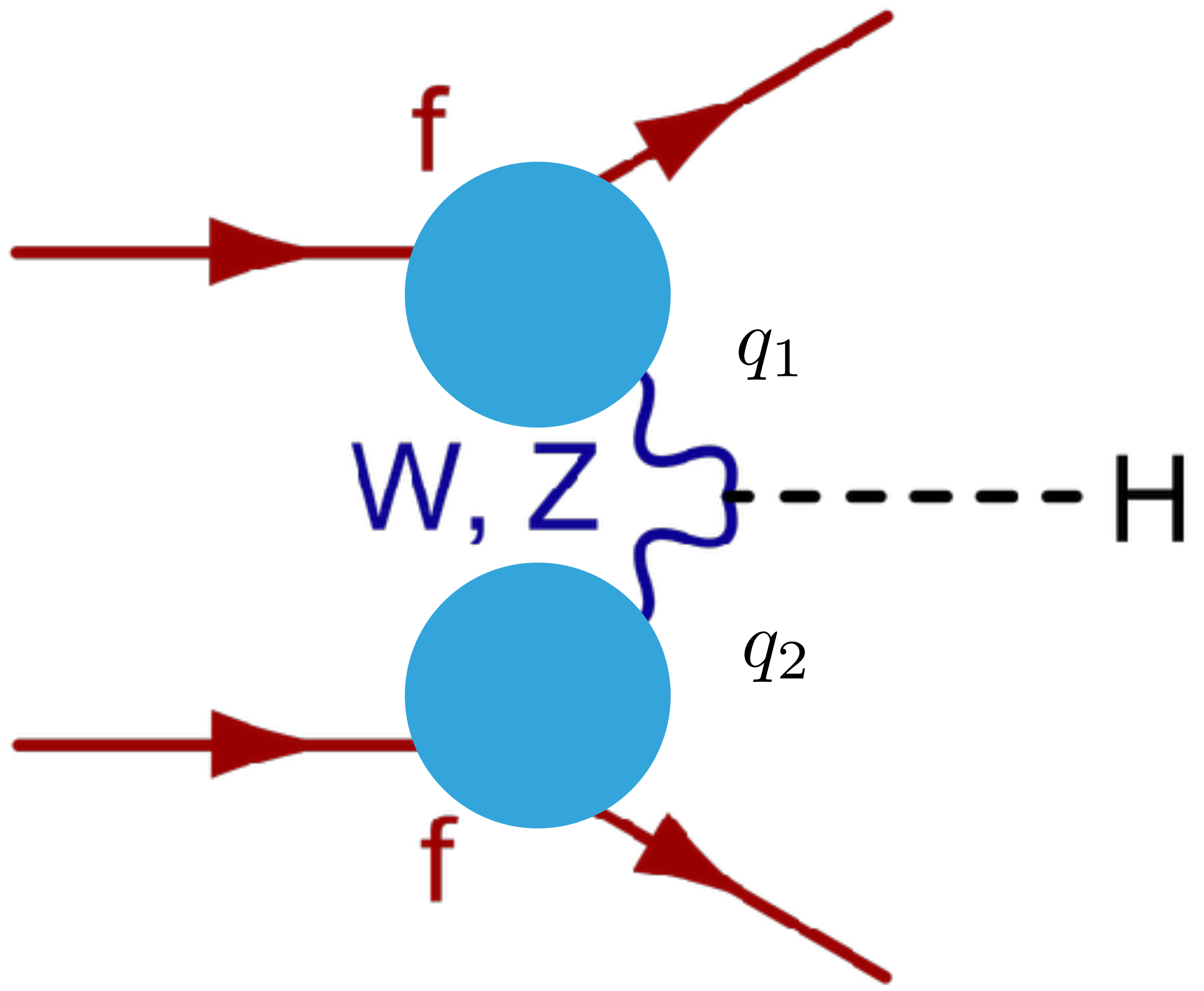


$$W_{\mu\nu} = \frac{1}{2}\sum_{\lambda}\sum_{X_f}\int [\mathrm{d}P_f](2\pi)^d\delta^{(d)}(p - P_f - q)\,\langle p,\lambda|J^\mu|f\rangle\,\langle f|J^\nu|p,\lambda\rangle$$

$$W^{\mu\nu} = W_1(q^2, p{\cdot}q)\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2(q^2, p{\cdot}q)\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)$$

In case of WBF, where exchanges of electroweak bosons contribute, also structure functions of the axial-vector current are needed. However, all DIS coefficient functions are currently known through N3LO and can be used to describe the WBF.

$$W^{\mu\nu} = W_1(q^2, p\cdot q)\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2(q^2, p\cdot q)\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) - i\epsilon^{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{2p\cdot q}W_3(q^2, p\cdot q)$$



van Neerven, Zijlstra; Moch, Vermaseren, Vogt

$$d\sigma_{\rm VBF} \sim \frac{(2\pi)^4\delta^{(4)}(q_1+q_2-p_H)d^4q_1 d^4q_2}{(q_1^2-m_V^2)(q_2^2-m_V^2)} W^{\mu\nu}(q_1^2, q_1\cdot p_1)W^{\mu'\nu'}(q_2^2, q_2\cdot p_2)\mathcal{M}^{\mu\mu',\nu\nu'}_{VV\to H}\frac{d^3p_H}{2E_H(2\pi)^3}$$

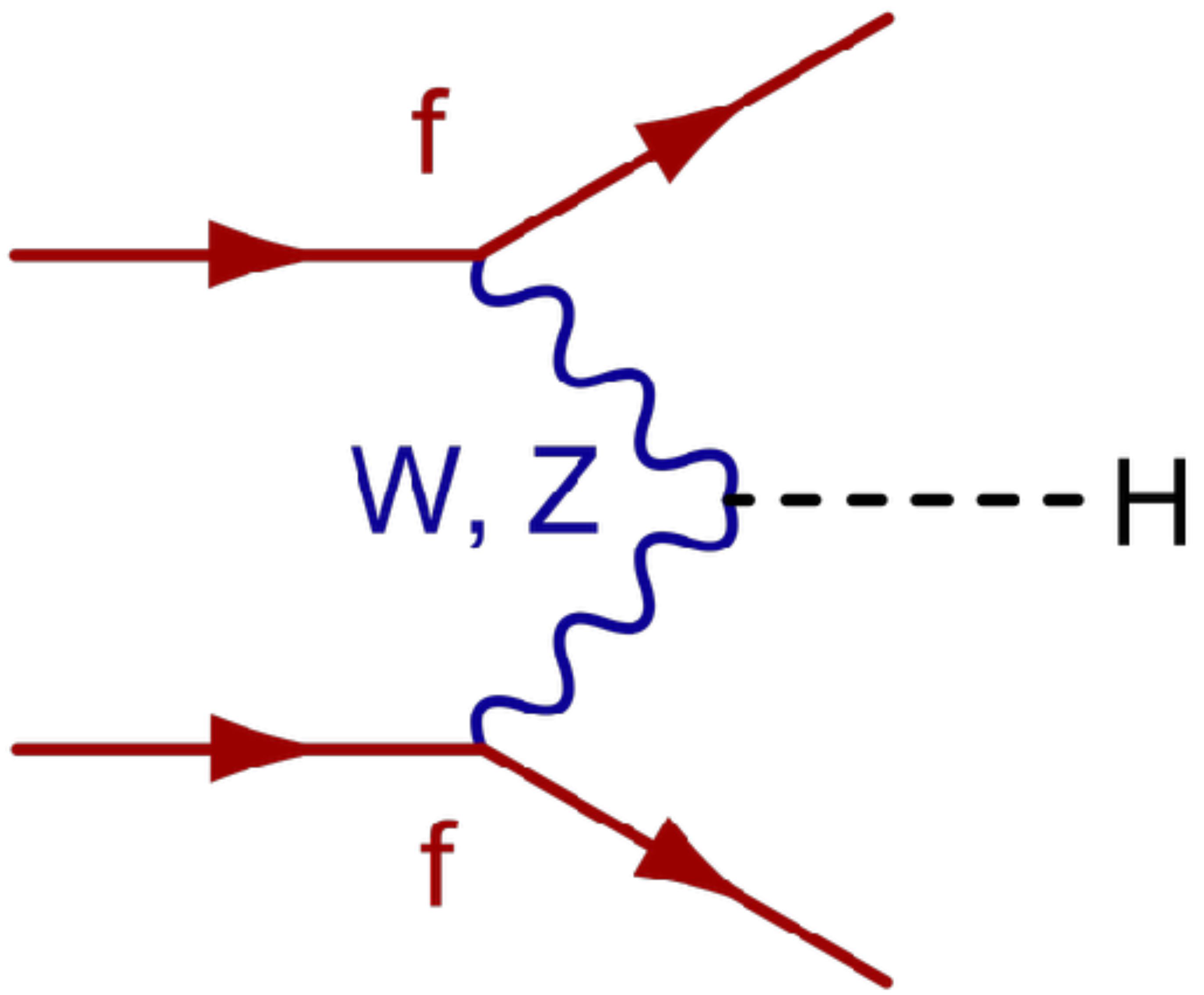
If the DIS structure functions are known in the perturbative expansion in QCD up to a certain order, the VBF cross section can be immediately computed through the same perturbative order as well. In practice, this has been done through N3LO QCD, since this is the order through which DIS structure functions are known.

Boltoni, Maltoni, Moch, Zaro
Kalberg, Dryer

Within the structure functions approach, quite moderate QCD corrections to WBF were found (O(3%)@NLO, O(1%)@NNLO). However it is not clear if these numbers are really relevant for physics of weak boson fusion.

The structure function approach involves integration over final state partons and does not allow us to impose constraints on QCD radiation. This is not ideal since VBF cuts are quite severe (the VBF cross section is only about 20 percent of the cross section without the VBF cuts) and involve forward tagging jets.

For this reason, it is important to perform a fully differential computation (even within the DIS-approximation!) that accounts for the VBF cuts on the tagging jets.



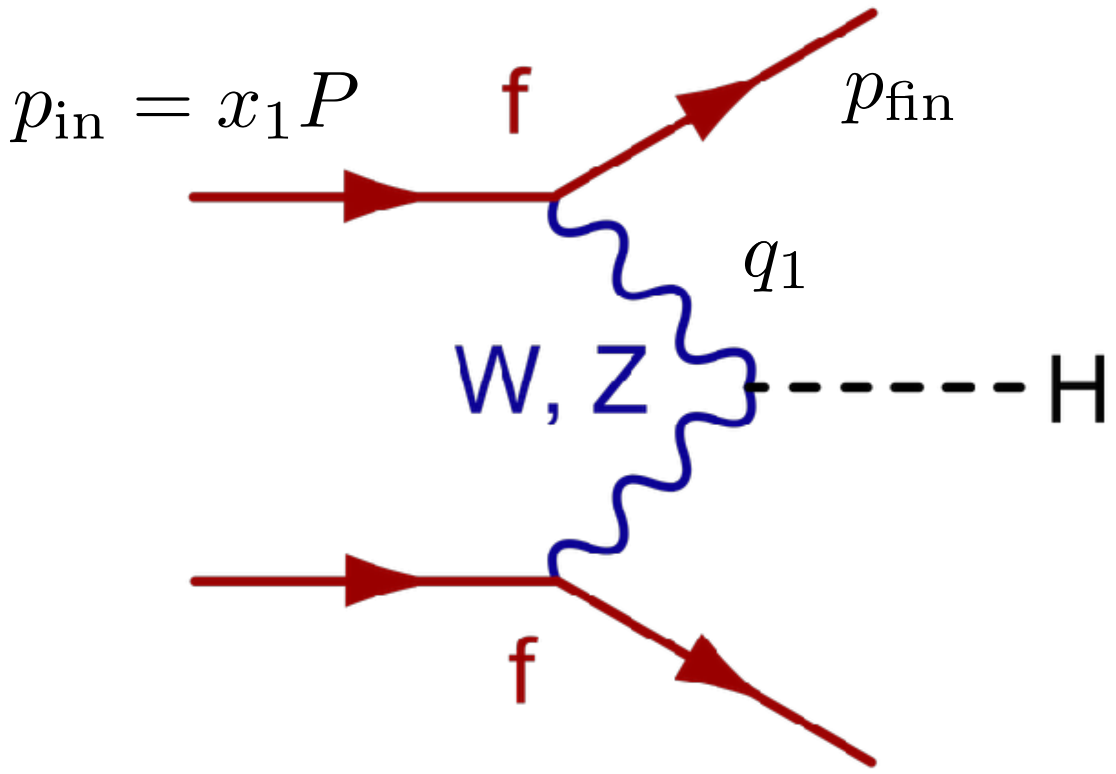
Typical VBF cuts

$$p_\perp^{j_{1,2}} > 25 \text{ GeV}, \quad |y_{j_{1,2}}| < 4.5,$$
$$\Delta y_{j_1,j_2} = 4.5, \quad m_{j_1,j_2} > 600 \text{ GeV},$$
$$y_{j_1} y_{j_2} < 0, \quad \Delta R > 0.4$$

A fully differential NNLO QCD computation within the DIS-approximation can be performed without much ado: we know enough about "generic" NNLO subtractions by now.

Cruz-Martinez, Glover, Gehrmann, Huss

However, historically, the NNLO QCD corrections to WBF in the DIS-approximation where computed using the so-called "projection-to-Born" method. The method is based on the observation that, for WBF in the factorization approximation, the Born kinematics follows uniquely from values of vector boson momenta since both incoming and outgoing quarks are on their mass shells.



$$q_1: \quad (x_1 P - q_1)^2 = 0 \quad \Rightarrow \quad x_1 = \frac{q_1^2}{2 q_1 P}$$

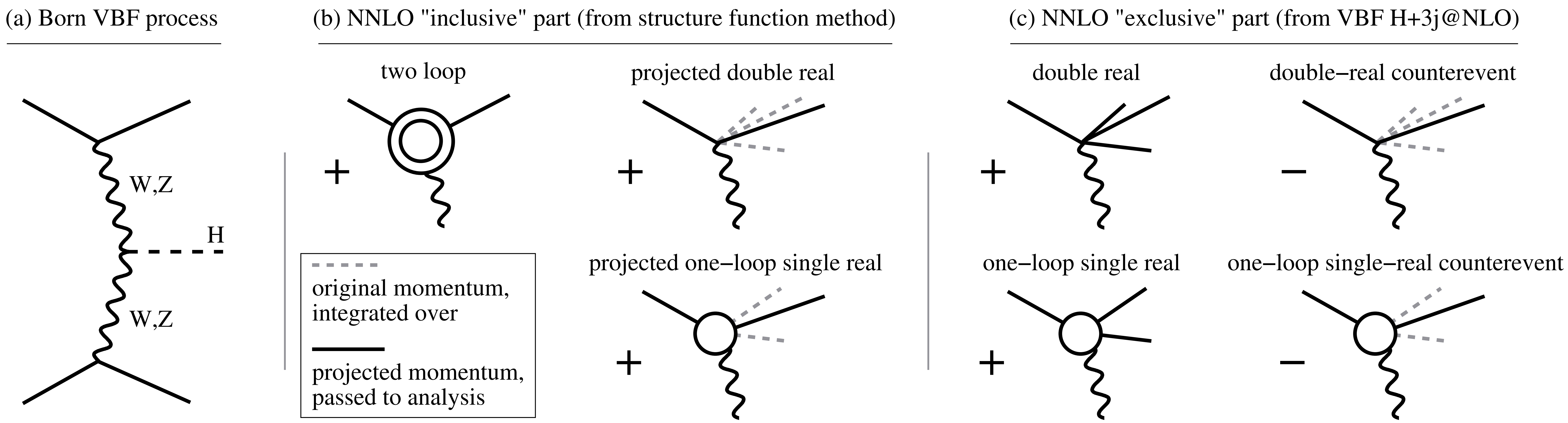
$$p_{\rm in} = x_1 P \quad p_{\rm fin} = x_1 P - q_1$$

A NNLO QCD computation involves many different ingredients, such as leading order contribution for VBF+2 jets, a next-to-leading order contribution for VBF + 1 jet and, of course, the NNLO contribution to VBF + 0 jets. The first two contributions do not require a genuine NNLO computation whereas the last one does. However, the VBF+0 jets must have a LO kinematics so that, at each q_1, there is one unknown ``constant". This constant, however, can be determined if we know one quantity through NNLO, at fixed q_1.

Cacciari, Dreyer, Karlberg, Salam, Zanderighi

The following picture and formula illustrate the projection-to-Born method. It is important that this method can be easily extended to one order higher in perturbative QCD provided, of course, that NNLO QCD corrections to VBF+jet production are known.

$$q_1 : \quad (x_1 P - q_1)^2 = 0 \quad \Rightarrow x_1 = \frac{q_1^2}{2 q_1 \cdot P} \quad \Rightarrow \quad p_{\rm in} = x_1 P, \quad p_{\rm fin} = q_1 - x_1 P$$



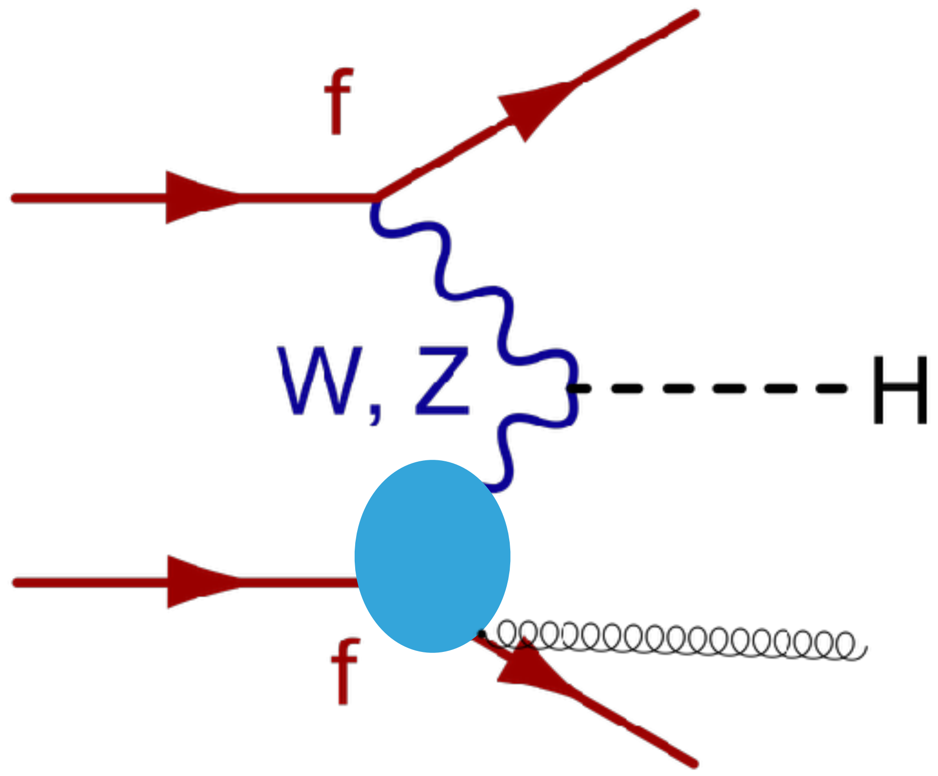
$$d\sigma^{\rm NNLO}_{\rm VBF}|_{q_1} = w(\{p\}) \frac{d\Phi_{\rm VBF+1j}}{\Phi_{\rm VBF+1j}} - w(\{p\}) \frac{d\Phi_{\rm B}}{\Phi_{\rm B}} + C(q_1) \frac{d\Phi_{\rm B}}{\Phi_{\rm B}}$$

$$\int d\sigma^{\rm NNLO}_{\rm VBF}|_{q_1} = W(q_1^2, P q_1) \quad \Leftrightarrow \quad C(q_1)$$

Cacciari, Dreyer, Karlberg, Salam, Zanderighi

The importance of the projection-to-Born method derives from the fact that it can be easily generalised to N3LO. To this end, we need to compute the NNLO QCD corrections to VBF+1 jet and supplement it with the projection-to-Born prescription. The N3LO QCD results for VBF in the structure function approximation are already available and can be used directly to compute the unknown “constant”.

Kalberg, Dryer



A calculation of NNLO QCD corrections to VBF + 1 jet will be performed using the so-called nested soft-collinear subtraction scheme which should contain an analytic integration of the subtraction counter-terms constructed from universal soft- and collinear limits of matrix elements.

We have integrated all the double-unresolved subtraction terms that may appear in any NNLO computation, including the double-eikonal function and the triple-collinear splitting functions for both initial and final state.

We have analytic formulas for integrated counter-terms for processes with initial-initial (color singlet production) and final-final (color singlet decay) and we are working on the initial-final (DIS-like) contributions. We believe that — once these building blocks are available — we will be able to put them together in a relatively straightforward fashion.

$$\sigma = \int [\mathrm{d}\sigma_R - \mathrm{d}\sigma_{\mathrm{CT}}] + \int \mathrm{d}\sigma_{\mathrm{CT}}$$

M. Delto, R. Röntsch, F. Caola, H. Frellesvig

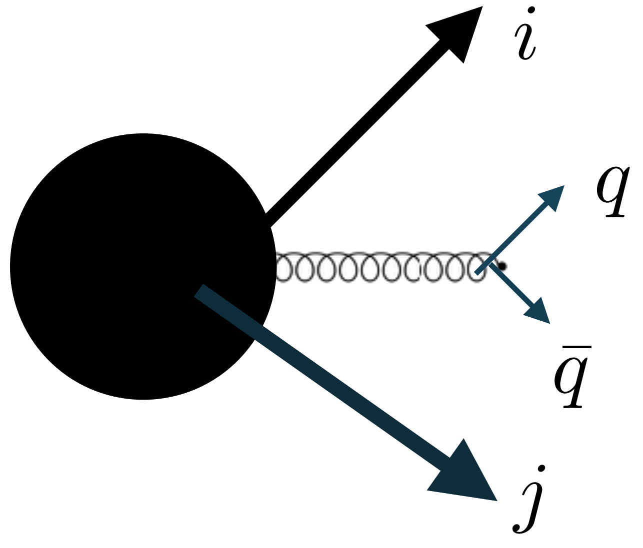
A few examples that illustrate the simplicity of the integrated double-soft and triple-collinear subtraction terms

M. Delto, F. Caola, H. Frellesvig

$$
\begin{aligned}
\mathbb{S}_{ij}^{(q\bar{q})} = (2E_{\max})^{-4\epsilon}\left[\frac{1}{8\pi^2}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\right]^2 \Bigg\{ & -\frac{1}{3\epsilon^3} + \frac{1}{\epsilon^2}\left[\frac{2}{3}\ln(s^2) - \frac{4}{3}\ln 2 \right.\\
& \left. + \frac{13}{18}\right] + \frac{1}{\epsilon}\left[-\frac{4}{3}\mathrm{Li}_2(c^2) - \frac{2}{3}\ln^2(s^2) + \ln(s^2)\left(\frac{8}{3}\ln 2 - \frac{13}{9}\right) + \frac{\pi^2}{9} \right.\\
& \left. + \frac{4}{3}\ln^2 2 + \frac{35}{9}\ln 2 - \frac{125}{54}\right] - \frac{8}{3}\mathrm{Ci}_3(2\delta) - \frac{2}{3\tan(\delta)}\mathrm{Si}_2(2\delta) - \frac{4}{3}\mathrm{Li}_3(c^2) \\
& - \frac{8}{3}\mathrm{Li}_3(s^2) + \mathrm{Li}_2(c^2)\left[\frac{29}{9} - \frac{8}{3}\ln 2\right] + \frac{4}{9}\ln^3(s^2) + \ln^2(s^2)\left[-\frac{4}{3}\ln(c^2) \right.\\
& \left. - \frac{8}{3}\ln 2 + \frac{13}{9}\right] + \ln(s^2)\left[-\frac{8}{3}\ln^2 2 - \frac{70}{9}\ln 2 + \frac{2}{9}\pi^2 + \frac{107}{27}\right] + 9\zeta_3 \\
& + \frac{2\pi^2}{3}\ln 2 - \frac{8}{9}\ln^3 2 - \frac{23}{108}\pi^2 - \frac{35}{9}\ln^2 2 - \frac{223}{27}\ln 2 + \frac{601}{162} + \mathcal{O}(\epsilon)\Bigg\}.
\end{aligned}
$$

$$s = \sin\delta, \quad c = \cos\delta,$$

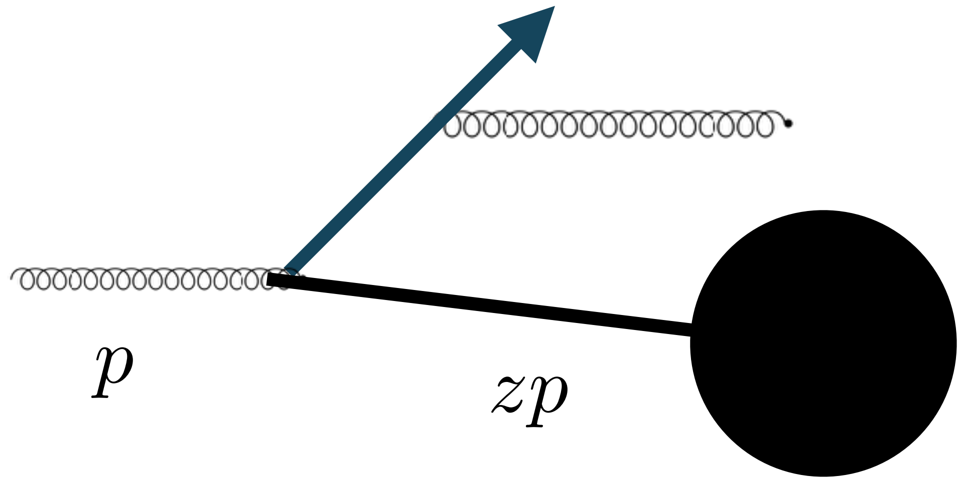
$$p_i p_j = 2E_i E_j \sin^2\delta$$



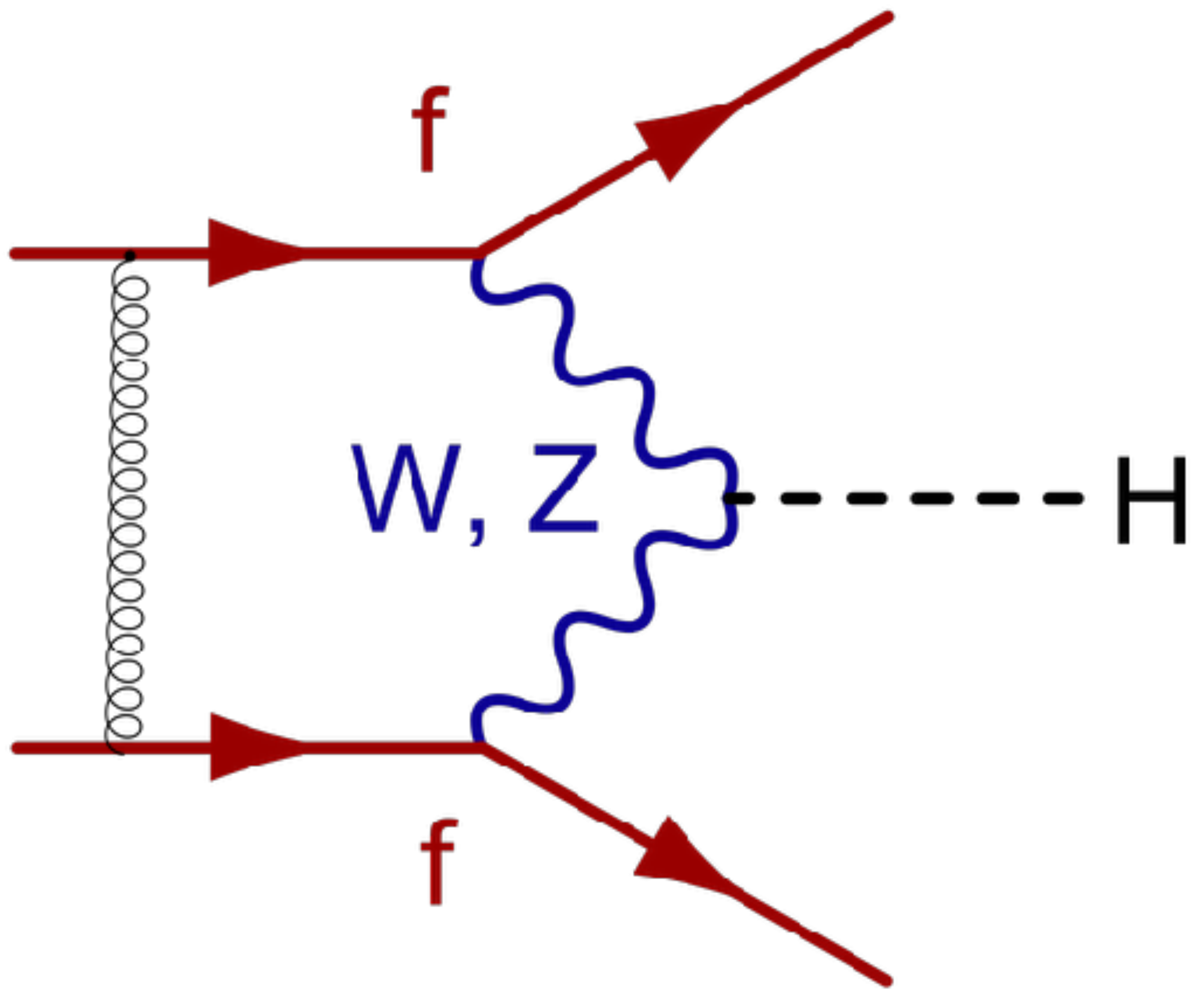
$$
\begin{aligned}
\tilde{R}_{\rm reg}^{\rm NA} = \frac{1}{\epsilon}\Bigg(& \frac{-6\pi^2 z^3 - 67z^3 + 3\pi^2 z^2 + 81z^2 - 3\pi^2 z - 27z + 13}{9z} \\
& + (2z^2 - 2z + 1)\ln(1-z)\ln(2) + (2z^2 - 2z + 1)\ln(1-z)\ln(z) \\
& - (2z^2 + 2z + 1)\ln(1+z)\ln(z) + (4z+1)\ln(z)\ln(2) + \frac{4 - 31z^3 + 24z^2 + 3z}{6z}\ln(2) \\
& + \frac{6z+1}{2}\ln^2(z) + \frac{12z+1}{2}\ln(z) - \left(2z^2 + 2z + 1\right)\mathrm{Li}_2(-z) + \left(2z^2 - 2z + 1\right)\mathrm{Li}_2(z)\Bigg) \\
& + \Big((8z^2 + 8z + 4)\left(\ln(1-z) + \ln(2)\right) + (2z^2 - 6z + 1)\ln(z)\Big)\mathrm{Li}_2(-z) \\
& + \Big((-8z^2 + 8z - 4)\ln(1-z) - 8(z-3)z\ln(2) - 4z\ln(z)\Big)\mathrm{Li}_2(z) \qquad (7.7) \\
& + \frac{44z^3 + 48z^2 + 15z + 8}{3z}\mathrm{Li}_2(-z) + \frac{-22z^3 + 96z^2 - 3z + 20}{3z}\mathrm{Li}_2(z) \\
& - \left(18z^2 - 2z + 9\right)\mathrm{Li}_3(1-z) + \left(10z^2 + 26z + 5\right)\mathrm{Li}_3(-z) \\
& + \left(4z^2 + 4z + 2\right)\left(3\mathrm{Li}_3\left(\frac{z}{1+z}\right) + \mathrm{Li}_3(1-z^2)\right) + \left(32z + 4\right)\mathrm{Li}_3(z).
\end{aligned}
$$

$$g \to q^* + qg$$

$$\tilde{R} = C_F^2 \tilde{R}_{\rm reg}^{A} + C_F C_A \tilde{R}_{\rm reg}^{\rm NA}$$



Pushing the computation of the DIS-like contributions to N3LO should be accompanied by the study of smaller effects that could have been neglected previously. An interesting contribution is that of non-factorizable corrections where different quark lines talk to each other.



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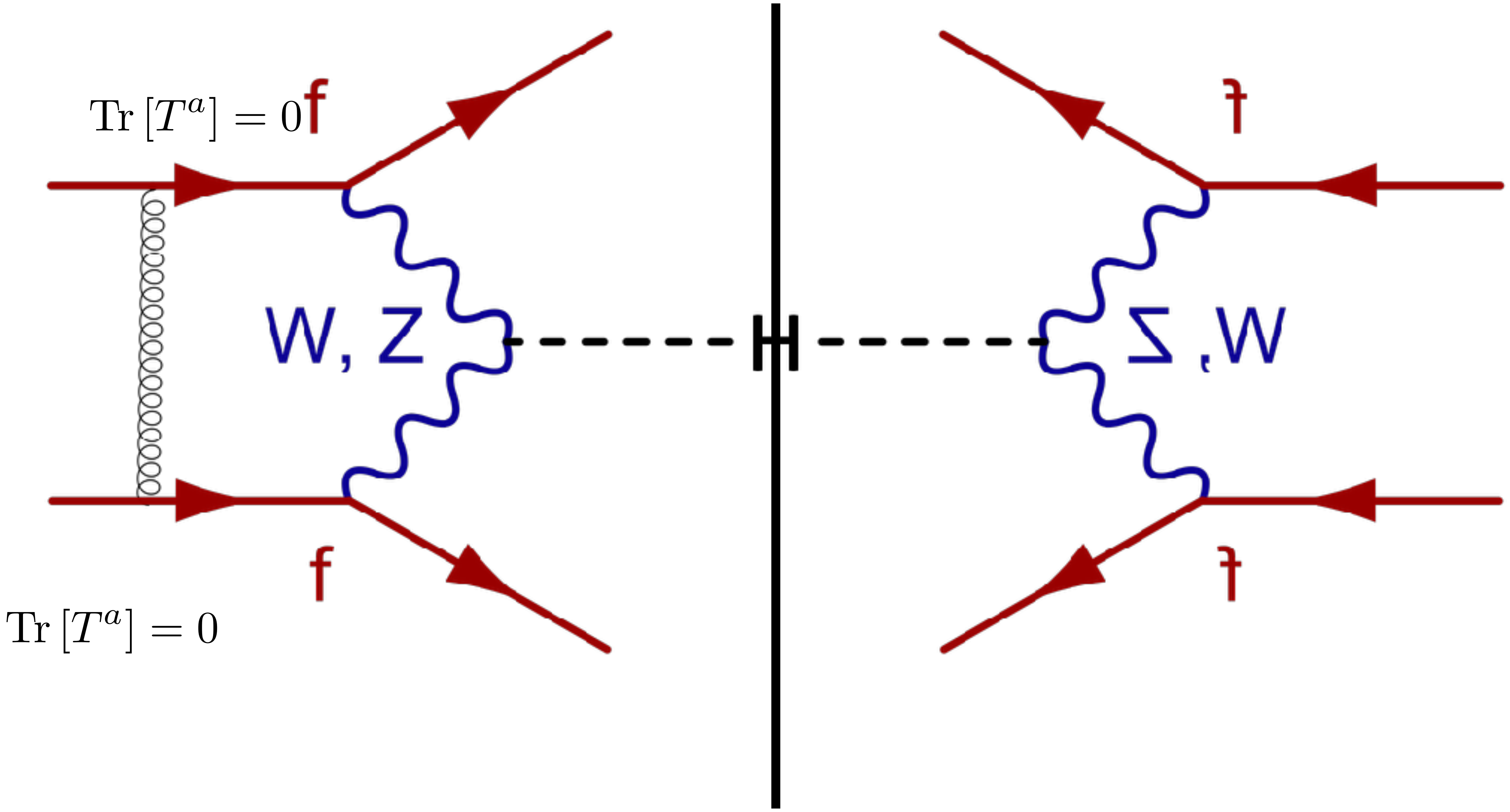
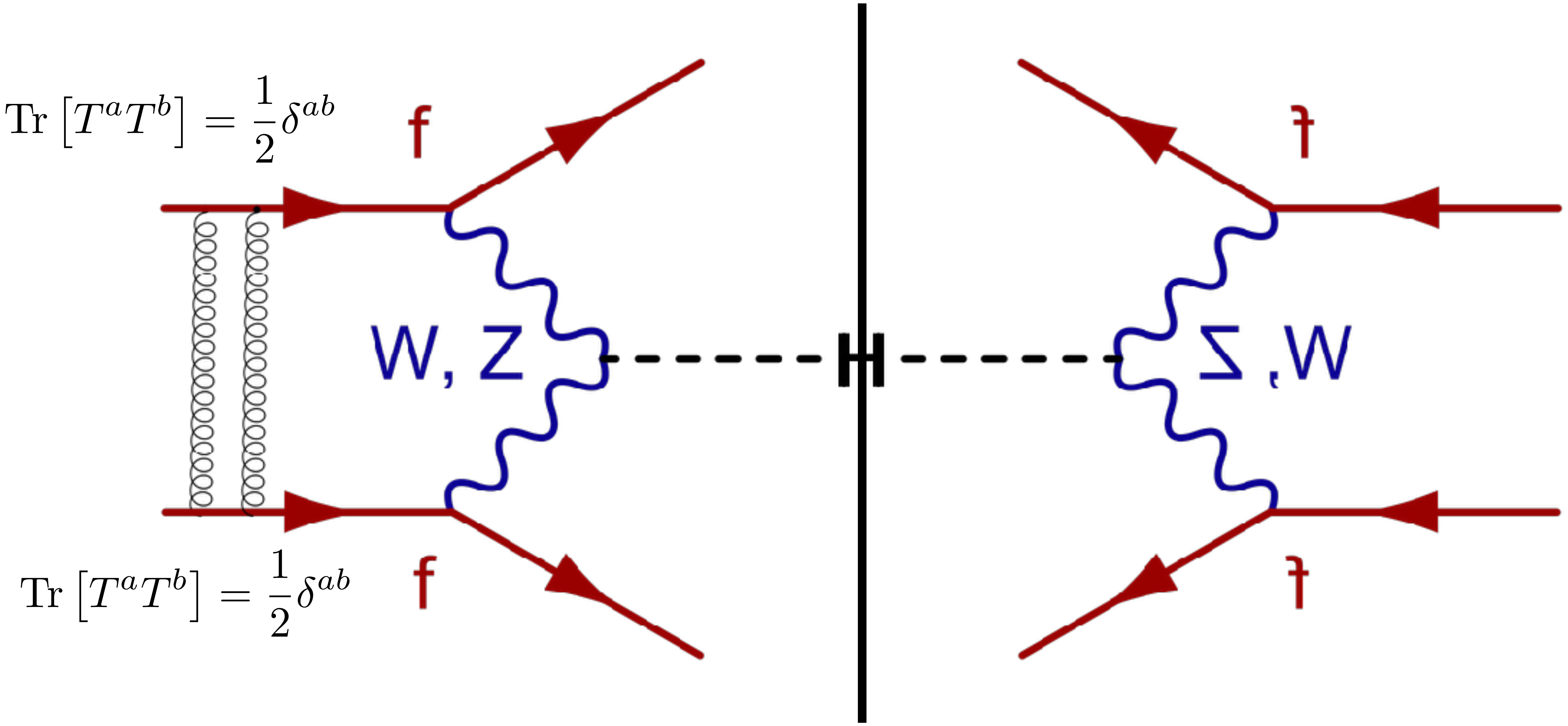


EFFECTS BEYOND THE DIS-LIKE CONTRIBUTIONS

Pushing the computation of the DIS-like contributions to N3LO should be accompanied by the study of smaller effects that have been neglected previously. An interesting contribution is that of non-factorizable corrections where different quark lines talk to each other. This contribution vanishes exactly at NLO QCD because of color conservation but it does appear at NNLO where it is color-suppressed.

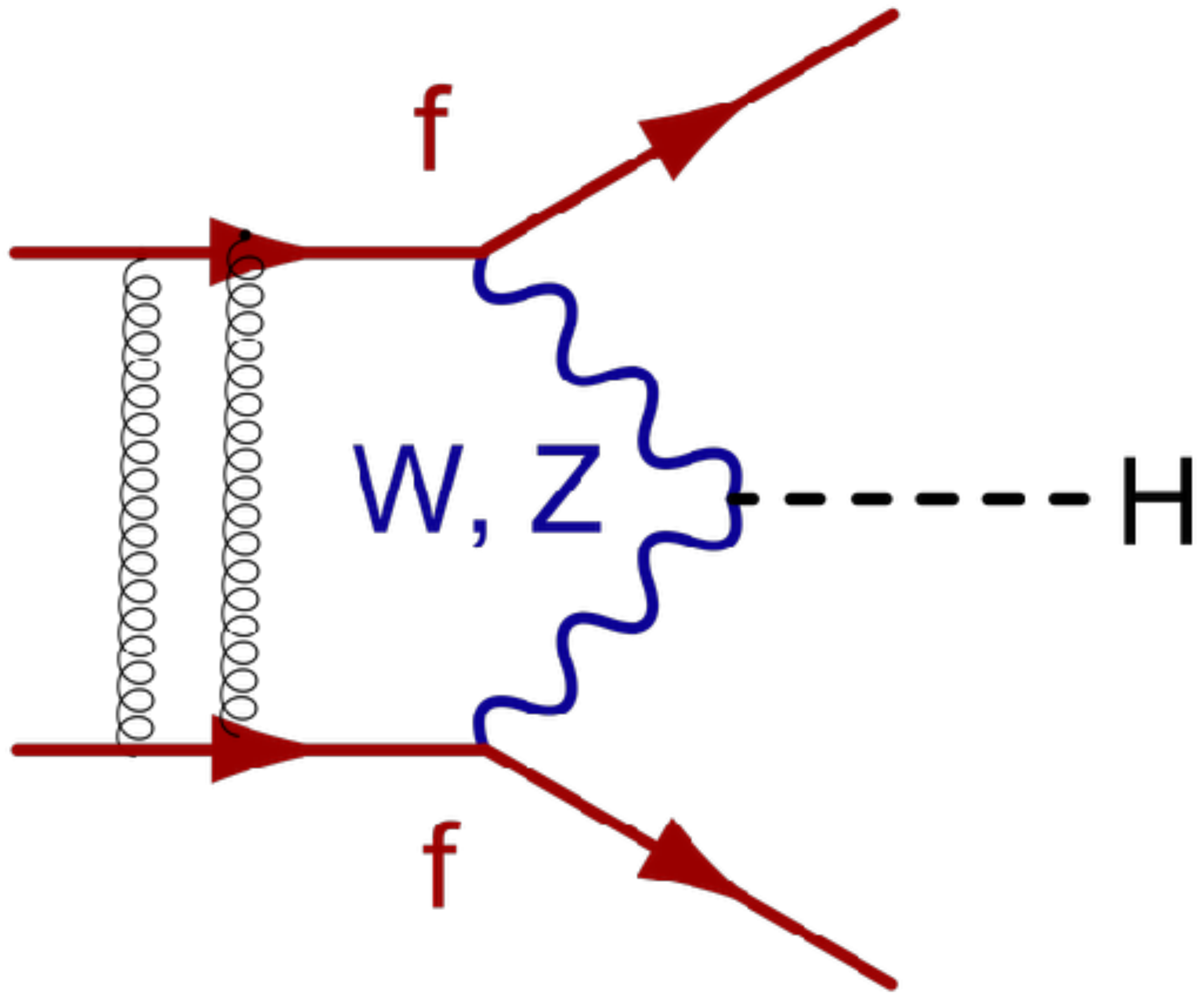


$$\frac{\sigma_{\mathrm{non-fact}}}{\sigma_{\mathrm{VBF}}} \sim \alpha_s^2 \qquad \frac{\sigma_{\mathrm{non-fact}}}{\Delta\sigma_{\mathrm{VBF}}^{\mathrm{NNLO}}} \sim \frac{1}{N_c^2}$$

These simple estimates suggest that the non-factorizable contributions are about 1%. However, at this point we do not know much beyond that.

EFFECTS BEYOND THE DIS-LIKE CONTRIBUTIONS

For example, we do not know how these non-factorizable contributions depend on kinematics. In particular, are there logarithms of vector boson masses or ratios of Mandelstam variables? Since we are in the forward limit, these parameters have a clear hierarchy and it is interesting to understand if there is an enhancement as the result.



$$m_H^2, m_V^2, t \ll s$$

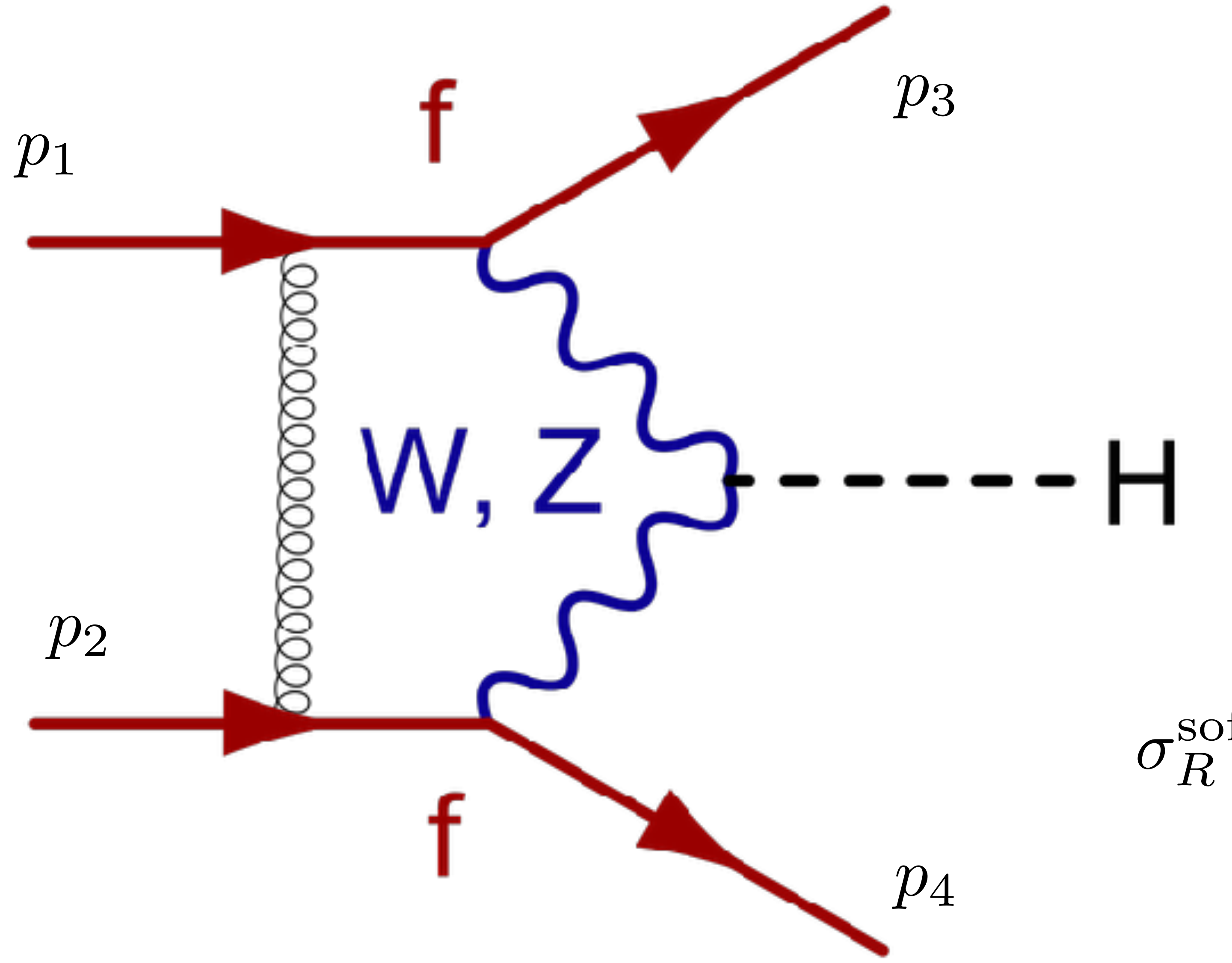
$$\sim \alpha_s^2 \log \frac{t}{s} \; , \; \alpha_s^2 \log \frac{t}{m_V^2} \quad ?$$

It is clear that an explicit computation of a two-loop 5-point function with two different masses is too complex a problem. Can one make use of the small parameters to calculate the scattering amplitude in the kinematic limit relevant for WBF?

What are the methodological issues that need to be addressed (for example, reductions to master integrals won't go through if used in a conventional way) ?

EFFECTS BEYOND THE DIS-LIKE CONTRIBUTIONS

Another possible way to better understand these contributions is to perform a real NNLO-like computation ignoring the finite part of the NNLO two-loop amplitude. An example at NLO QED (color conservation arises when the sum over electric charges of incoming "partons" is performed). Important observation is that only soft divergencies can appear in such contributions.



The real emission process

$$\mathcal{A} = \left[e_Q \left(\frac{p_3\epsilon}{p_3k} - \frac{p_1\epsilon}{p_1k} \right) + e_{\bar{Q}} \left(\frac{p_4\epsilon}{p_4k} - \frac{p_2\epsilon}{p_2k} \right) \right] \mathcal{A}_0$$

$$|\mathcal{A}|^2 \Rightarrow -2e_Qe_{\bar{Q}} \left[\frac{p_3p_4}{(p_3k)(p_4k)} + \frac{p_1p_2}{(p_1k)(p_2k)} - \frac{p_3p_1}{(p_3k)(p_1k)} - \frac{p_2p_4}{(p_2k)(p_4k)} \right] |\mathcal{A}_0|^2$$

$$\sigma_R^{\rm soft} = -2e_Qe_{\bar{Q}} \frac{\alpha_s}{\pi} \int [{\rm d}k] \theta(k_0 - E_{\rm max}) \left[\frac{p_3p_4}{(p_3k)(p_4k)} + \frac{p_1p_2}{(p_1k)(p_2k)} - \frac{p_3p_1}{(p_3k)(p_1k)} - \frac{p_2p_4}{(p_2k)(p_4k)} \right] \sigma_0$$

$$\sim \frac{\alpha_s}{\pi} \frac{{\rm Emax}^{-2\epsilon}}{\epsilon^2} \left[\rho_{12}^{-\epsilon} + \rho_{34}^{-\epsilon} - \rho_{13}^{-\epsilon} - \rho_{14}^{-\epsilon} + \mathcal{O}(\epsilon^2) \right] \sigma_0.$$

no collinear divergences left in this combination

The virtual amplitude follows from the Catani formula

$$\mathcal{A}_V = \mathcal{A}_0 + \frac{N_\epsilon \alpha}{\pi \epsilon^2} e_Q e_{\bar{Q}} \left[(-s_{12})^{-\epsilon} + (-s_{34})^{-\epsilon} - s_{13}^{-\epsilon} - s_{24}^{-\epsilon} \right] \mathcal{A}_0 + \mathcal{A}_{\rm fin}^{1l}$$

$$ {\rm d}\sigma_V + {\rm d}\sigma_R \sim \frac{\alpha}{\pi} e_Q e_{\bar{Q}} \pi^2 \sigma_0 + \ldots $$

Initial-initial and final-final state interactions lead to non-cancelling contributions proportional to pi².

The goal of the project A1c is to push the description of weak boson fusion to N3LO in perturbative QCD and do it in such a way that results remain fully differential in the kinematics of final state QCD radiation. We plan to do that using the combination of the projection-to-Born method and modern methods for NNLO QCD computations.

Simultaneously, we would like to explore and better understand non-factorizable corrections to WBF. These corrections start to contribute at two loops and are color-suppressed. The two-loop amplitude receives contributions from the five-point functions with two internal and one external massive lines. It is very likely that new ideas and significant methodological improvements will be needed to enable better understanding of these non-factorizable corrections.