Theoretical Advances in Parton Showers

Simon Plätzer (for B1d Czakon, Gieseke & Plätzer)

Particle Physics, University of Vienna

at the
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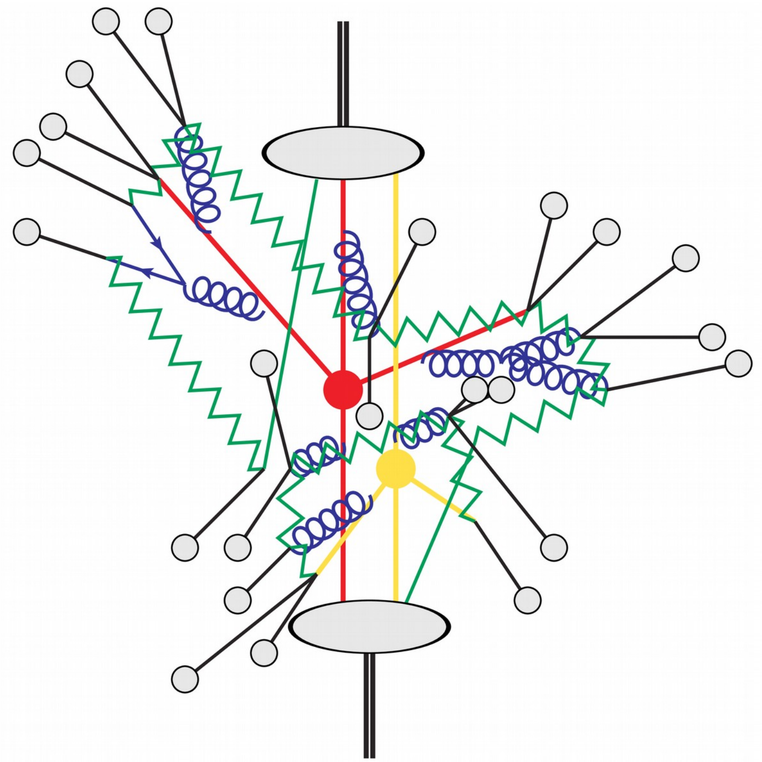
QCD Description of Collider Events

Indispensable input
for experiments & phenomenology.

Realistic, fully detailed description
spanning orders of magnitude in
relevant energy scales.

Factorization dictates work flow.

Hard partonic scattering
Jet evolution
Multiple interactions
Hadronization



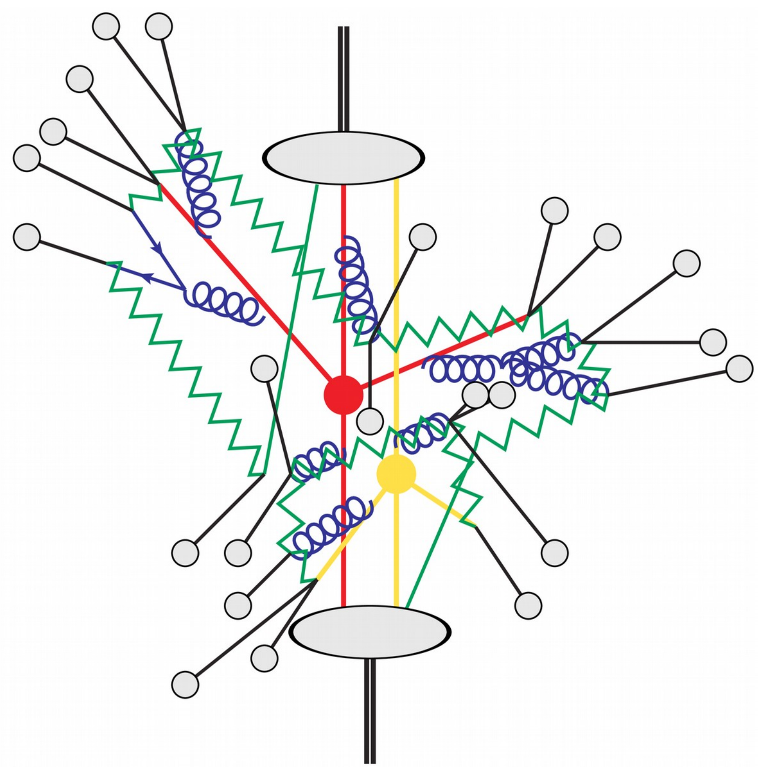
$$d\sigma \sim d\sigma_{\mathrm{hard}}(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Event Generator Simulations

Precision collider phenomenology demands reliable simulations with the highest level of theoretical control.

Open questions in **existing** shower algorithms and **future development**:

- Formal accuracy?
- Relation to analytic resummation?
- Relation to effective field theories?
- New formulation, methods and algorithms?



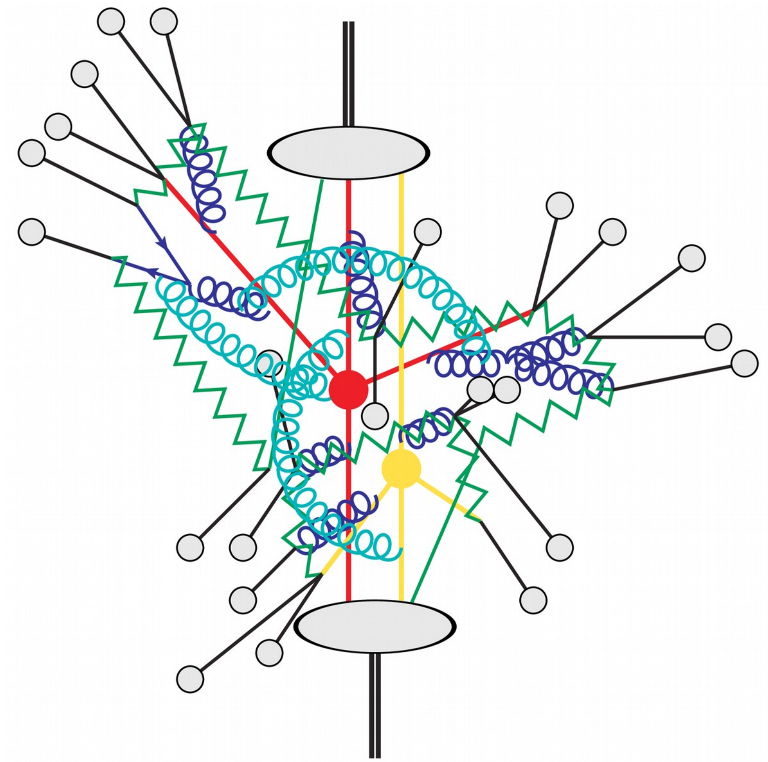
$$\mathrm{d}\sigma \sim \mathrm{d}\sigma_{\mathrm{hard}}(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Bottlenecks

Parton shower algorithms
Lack a systematic expansion,
obstruct NNLO for the hard process.

Hadronization models
Lack constraints from perturbative
evolution: Hiding perturbative effects?

Rethink foundations of parton showers:
Systematic picture including virtual
corrections and quantum mechanical
interference.



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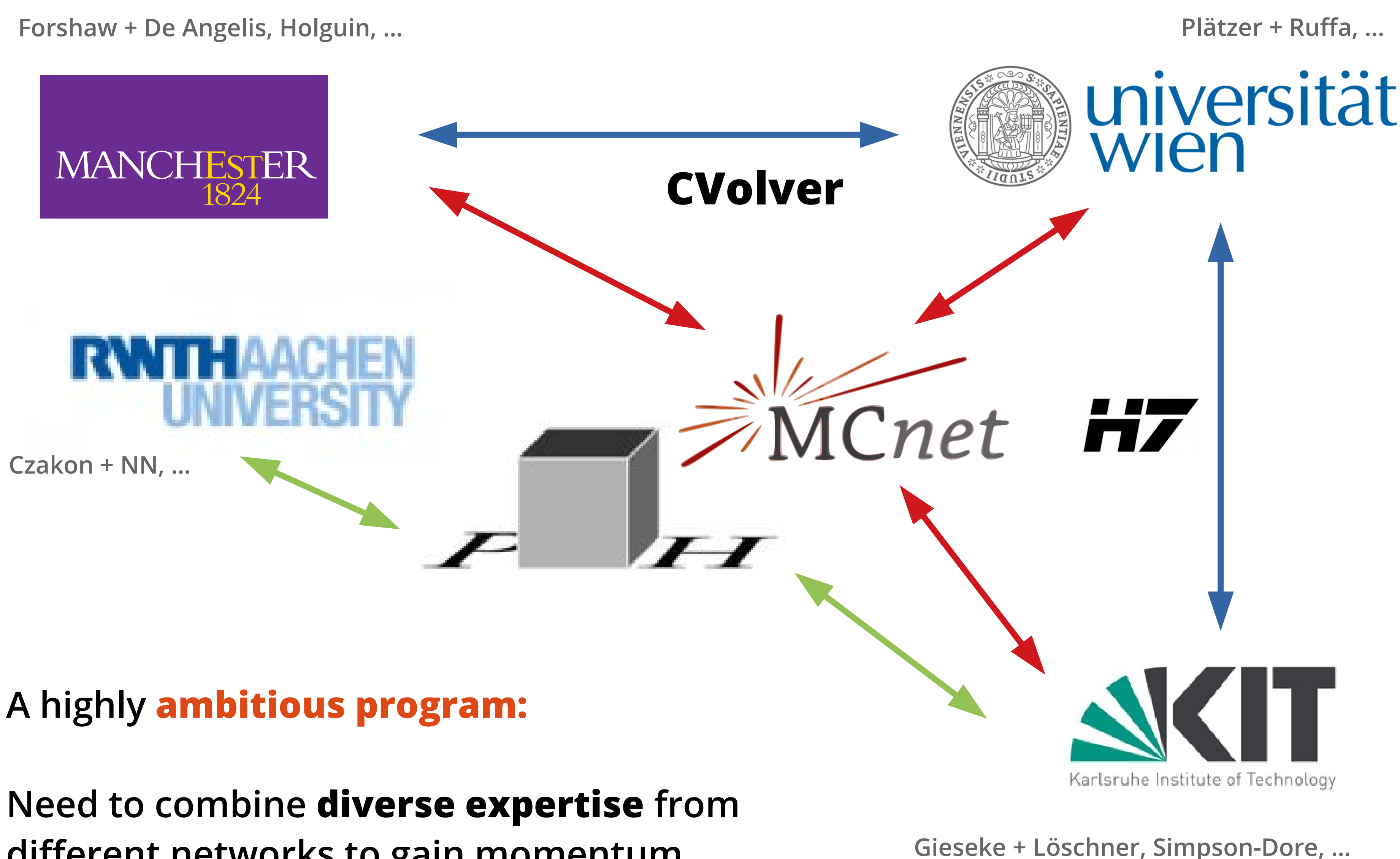
Tackling the bottlenecks



A highly **ambitious program:**

Need to combine **diverse expertise** from different networks to gain momentum.

Main goals

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Seek a framework to systematically address and improve **a new kind of parton shower algorithm**, not relying on ad-hoc constructions, treating colour and spin exactly as far as possible.

Gain most detailed analytic control over the algorithm to make **decisive statements about logarithmic accuracy** for a certain class of observables.

Extend the algorithm **beyond the customary leading colour limit**, include spin correlations and prepare to include higher orders in the shower algorithm itself.

Determine the relevant ingredients to have **resummation beyond (N)LL** in reach.

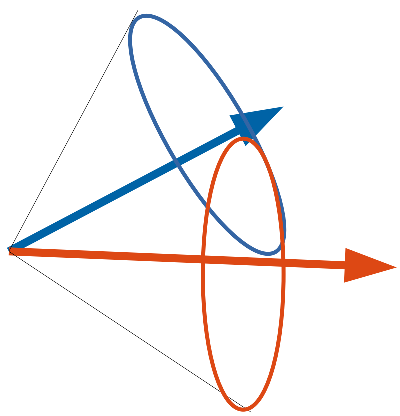
Facilitate the **matching to NNLO** fixed-order perturbative calculations.

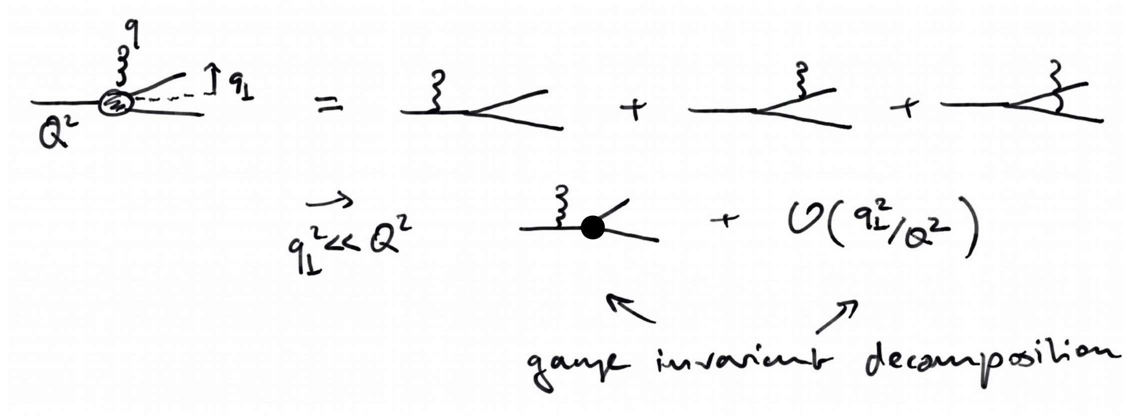
Contribute to the development of the **CVolver** package which targets new evolution algorithms, and include into the multi-purpose event generator **Herwig 7**.

Resummation and Parton Showers

Coherent branching algorithms essential to direct QCD resummation of global event shapes, and to designing parton shower algorithms.

[Catani, Marchesini, Webber]
[Gieseke, Stephens, Webber]

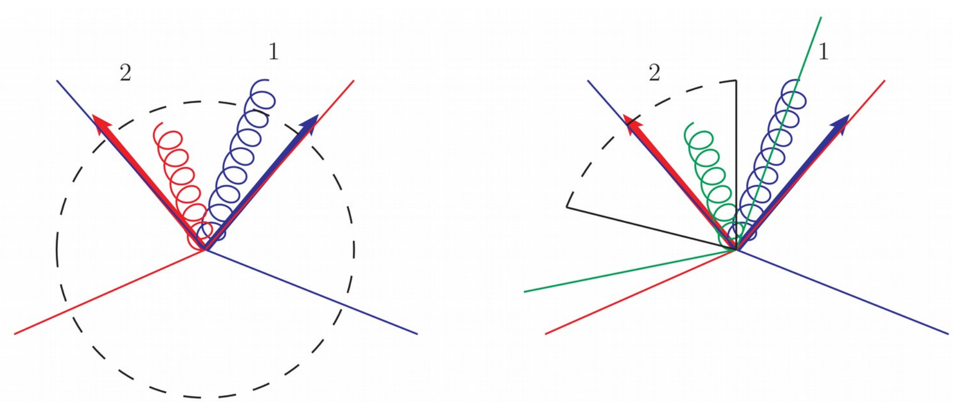




Large-angle soft effects included on average by a clever choice of ordering variable.

Non-global logarithms require dipole-type soft gluon evolution to take into account change in colour structure after each emission.

Systematic inclusion of collinear effects needs to be addressed.



[Dasgupta, Salam; Banfi, Marchesini, Smye]
[Angeles, DeAngelis, Forshaw, Plätzer, Seymour]

Theoretical Framework

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[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[see also Nagy, Soper]

Unified framework requires evolution at the amplitude level as most general basis:

$$\sigma = \sum_n \int \mathrm{Tr}\left[\mathbf{A}_n(\mu)\right] u(p_1, \ldots, p_n)\, \mathrm{d}\phi_n$$

$$\mathbf{A}_n(\mu) = |\mathcal{M}_n(\mu)\rangle\langle\mathcal{M}_n(\mu)|$$

Evolved `density operator`

Observable

Phase space

General expression of (partonic) cross section including all multiplicities and virtual corrections and colour mixing in all orders perturbation theory.

$$|\mathcal{M}_n(\mu)\rangle = \mathbf{Z}^{-1}(\mu, \epsilon)|\tilde{\mathcal{M}}_n(\epsilon)\rangle$$

[Sterman, Becher & Neubert, ...]

Theoretical Framework

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Parton shower picture encoded in recursive definition including emission and virtual evolution operators

$$\mathbf{A}_n(E) = \mathbf{V}(E, E_n)\mathbf{D}_n\mathbf{A}_{n-1}(E_n)\mathbf{D}_n^\dagger\mathbf{V}^\dagger(E, E_n)\theta(E - E_n)$$

Corresponds to tower of evolution equations for each partonic multiplicity.

Questions to be addressed include:

- Origin and generalizations of the evolution towards full parton showers, constraints for certain classes of observables
  $\leftrightarrow$ **systematic expansion**
- Role and impact of the ordering variable, relation to virtual corrections
  [Angeles, Forshaw, Seymour]
- Anomalous dimensions at higher orders
- Kinematic mappings and factorization properties of amplitudes

Anomalous Dimensions and Emission Kernels

Soft evolution at leading order established, even beyond for NGL [Weigert, Caron-Huot]

$$\mathbf{A}_n(E) = \mathbf{V}(E, E_n)\mathbf{D}_n\mathbf{A}_{n-1}(E_n)\mathbf{D}_n^\dagger\mathbf{V}^\dagger(E, E_n)\theta(E - E_n)$$

$$\mathbf{V}_n(E, Q) = \mathrm{P}\exp\left(-\int_E^Q \frac{\mathrm{d}q}{q}\mathbf{\Gamma}_n(q)\right) \qquad \mathbf{D}_n = \sum_{i=1}^{n-1}\frac{p_i\cdot\epsilon^*(p_n)}{p_i\cdot p_n}\mathbf{T}_i$$

$$\mathbf{\Gamma}_n = \frac{\alpha_s}{\pi}\sum_{i<j}\int \mathrm{d}\Omega\frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j}(-\mathbf{T}_i\cdot\mathbf{T}_j)$$

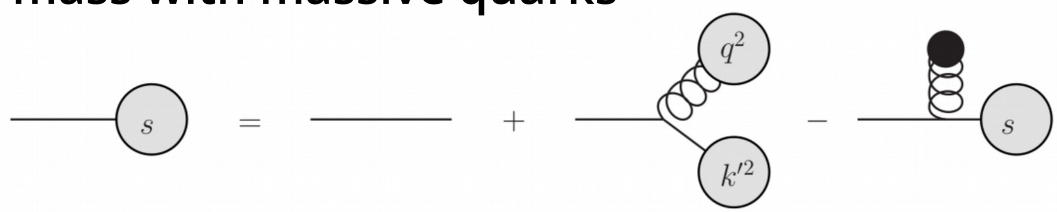
Next steps to be undertaken are:

- Systematic inclusion of soft- and hard-collinear effects
  $\leftrightarrow$ relate algorithm to coherent branching, access spin correlations
- Implement momentum conservation and clarify role of kinematic mappings
- Push to next-to-leading order in the soft sector, and beyond
- Smooth coverage of phase space and algorithmic construction desirable
- Make contact with NNLO subtraction

Analytic results

Analytic results are vital to judge on the accuracy of the algorithms: Link the general algorithm to analytic results focusing on both **direct QCD** as well as **effective field theory** setups. Not at all impossible!

Coherent branching calculation of jet mass with massive quarks



$$
\begin{aligned}
J(s,Q^2) \;=\; \delta(s) + \int_0^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_0^1 \mathrm{d}z\, P_{qq}\Big[\alpha_s\big(z(1-z)\tilde{q}\big), z\Big] \\
\times \Bigg[\int_0^\infty \mathrm{d}k'^2 \int_0^\infty \mathrm{d}q^2 \delta\Big(s - \frac{k'^2}{z} - \frac{q^2}{1-z} - z(1-z)\tilde{q}^2\Big) J(k'^2, z^2\tilde{q}^2) J_g(q^2, (1-z)^2\tilde{q}^2) \\
- J(s, \tilde{q}^2)\Bigg],
\end{aligned}
$$

[Hoang, Plätzer, Samitz – JHEP 10 (2018) 200]

Link soft gluon evolution algorithm to effective theory for jets

$$
E\frac{\partial \mathbf{G}_n(E)}{\partial E} = -\boldsymbol{\Gamma}\mathbf{G}_n(E) - \mathbf{G}_n(E)\boldsymbol{\Gamma}^\dagger + \mathbf{D}_n\mathbf{G}_{n-1}(E)\mathbf{D}_n^\dagger \; u(E, \hat{p}_n)
$$

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Derive how (non-)global resummations are recovered from colour-space evolution.

Further work

Fully **integrate new evolution inside the Herwig 7** event generator and perform extensive comparison against collider data.

- Role of colour reconnection models needs to be clarified.
- Availability of fixed-order calculations at amplitude level required.

Perform true **matching to NNLO calculations.**

- Link shower kinematics and kernels to subtraction schemes
- Devise matching subtractions from expanding new evolution to second order

Preliminary and ongoing work

Loads of preliminary studies and 'tools' underway which ensure that the **project gets moving very quickly:**

- Density operator evolution in colour matrix element corrections in Herwig

  [Plätzer, Sjödahl – JHEP 1207 (2012) 042]
  [Plätzer, Sjödahl, Thoren – JHEP 11 (2018) 009]

- Soft gluon evolution algorithm and systematic expansion around large-N

  [Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
  [De Angelis, Forshaw, Plätzer – soon]

- Spin correlation algorithm in Herwig [Webster, Richardson – arXiv:1807.01955]
- MC algorithms for non-probabilistic evolution [Plätzer, Sjödahl – EPJ Plus 127 (2012) 26]

- Kinematics and factorization of double real emissions [Gieseke, Plätzer, Simpson – in progress]
- First steps towards collinear contributions [Forshaw, Holguin, Plätzer – soon]

- Touching ground with colour reconnection models

  [Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]

- NNLO cross sections available within STRIPPER framework

  [Czakon – PLB 693 (2010) 259]
  [Czakon, Heymes – NPB 890 (2014) 152]

Thank you!

A General Algorithm

Evolution equations for factorizing observables $u(p_1, \cdots p_n) = u(E_1, \hat{p}_1) \cdots u(E_n, \hat{p}_n)$ integrating over the intermediate scales:

$$E \frac{\partial \mathbf{G}_n(E)}{\partial E} = -\mathbf{\Gamma} \mathbf{G}_n(E) - \mathbf{G}_n(E) \mathbf{\Gamma}^\dagger + \mathbf{D}_n \mathbf{G}_{n-1}(E) \mathbf{D}_n^\dagger \; u(E, \hat{p}_n)$$

Recursive observables: $u_n(p_1, \cdots, p_n) = u(p_n, \{p_1, \cdots, p_{n-1}\})\, u_{n-1}(p_1, \cdots, p_{n-1})$

Genuine non-global case: $u(k, \{q\}) = \Theta_{\text{out}}(k) + \Theta_{\text{in}}(k) u_{\text{in}}(k, \{q\})$

Can identify global and non-global contributions simply by splitting anomalous dimension into \`out' and \`in' contributions.

Reproduce all available literature.

[Dasgupta, Salam – Phys.Lett. B512 (2001) 323]
[Forshaw, Kyrieleis, Seymour – JHEP 0608 (2006) 059]
[Weigert – Nucl.Phys. B685 (2004) 321]
[Caron-Huot – JHEP 1803 (2018) 036]
[Becher, Neubert, Rothen, Shao – JHEP 1611 (2016) 019]

Includes a re-derivation of the BMS equation at leading-N, being able to calculate subleading-N corrections to it.

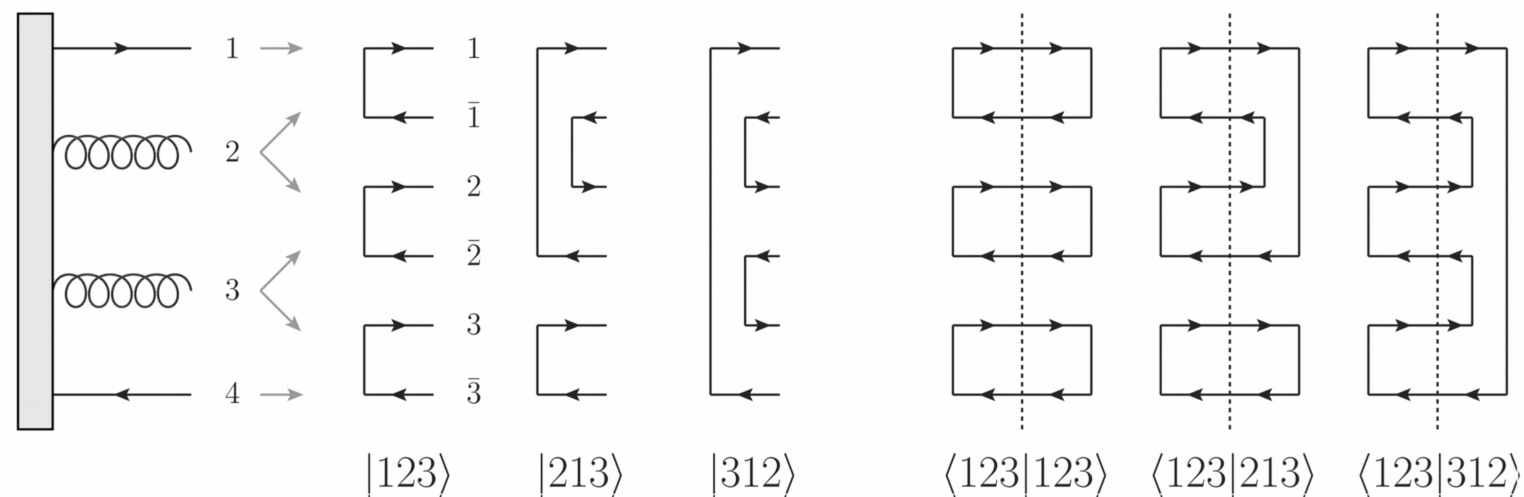
[Banfi, Marchesini, Smye – JHEP 0208 (2002) 006]

Colour Flows

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Express amplitudes in combinations of fundamental/anti-fundamental indices:

$$|\mathcal{M}\rangle = \sum_\sigma \mathcal{M}_\sigma |\sigma\rangle \qquad |\sigma\rangle = \left| \begin{array}{ccc} 1 & \cdots & n \\ \sigma(1) & \cdots & \sigma(n) \end{array} \right\rangle = \delta^{\alpha_1}_{\bar{\alpha}_{\sigma(1)}} \cdots \delta^{\alpha_n}_{\bar{\alpha}_{\sigma(n)}}$$



Non-orthogonal basis:

$$1 = \sum_\sigma |\sigma\rangle[\sigma| \qquad [\sigma|\tau\rangle = \langle\sigma|\tau] = \delta_{\tau\sigma} \qquad \mathrm{Tr}[\mathbf{A}] = \sum_{\tau,\sigma} [\tau|\mathbf{A}|\sigma]\langle\sigma|\tau\rangle$$

Also overcomplete … but computationally very handy: It’s all about permutations.

Resumming in Colour Space

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Evolution operator in colour flow basis:

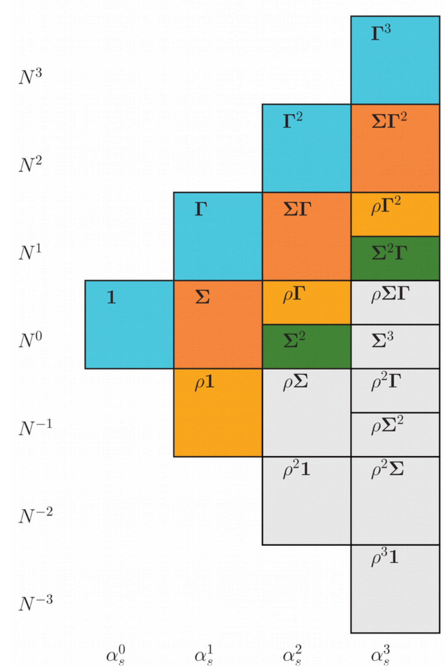
$$[\tau|e^{\mathbf{\Gamma}}|\sigma\rangle = \sum_{k=0}^{\infty}\frac{(-1)^l}{N^l}\sum_{k=0}^{l}\frac{(-\rho)^k}{k!}\sum_{\sigma_0,\ldots,\sigma_{l-k}}\delta_{\tau\sigma_0}\delta_{\sigma_{l-k}\sigma}\prod_{\alpha=0}^{l-k-1}\Sigma_{\sigma_\alpha\sigma_{\alpha+1}}R(\{\sigma_0,\ldots,\sigma_{l-k}\})$$

$$= \delta_{\tau\sigma}\left(e^{-N\Gamma_\sigma} + e^{-N\Gamma_\sigma}\frac{\rho}{N}\right) - \frac{1}{N}\frac{e^{-N\Gamma_\tau} - e^{-N\Gamma_\sigma}}{\Gamma_\tau - \Gamma_\sigma}\Sigma_{\tau\sigma} + \text{NNLC}$$

[Plätzer – EPJ C 74 (2014) 2907]

Sum terms enhanced by $\alpha_s N$ to all orders, insert perturbations in 1/N.

Take into account real emission contributions and the final suppression by the scalar product matrix element.



virtuals	reals
$(0\text{ flips}) \times 1 \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[\ldots]\mathbf{t}\vert_{2\text{ flips}} \times 1$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[\ldots]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{s}[\ldots]\mathbf{s}\vert_{0\text{ flips}} \times N^{-2}$
$(1\text{ flip}) \times \alpha_s \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r}$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[\ldots]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$
$(0\text{ flips}) \times \alpha_s N^{-1} \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r}$
$(0\text{ flips}) \times \alpha_s^2 \times (\alpha_s N)^n$ $(2\text{ flips}) \times \alpha_s^2 \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r}$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[\ldots]\mathbf{t}\vert_{2\text{ flips}}$

Basic algorithm

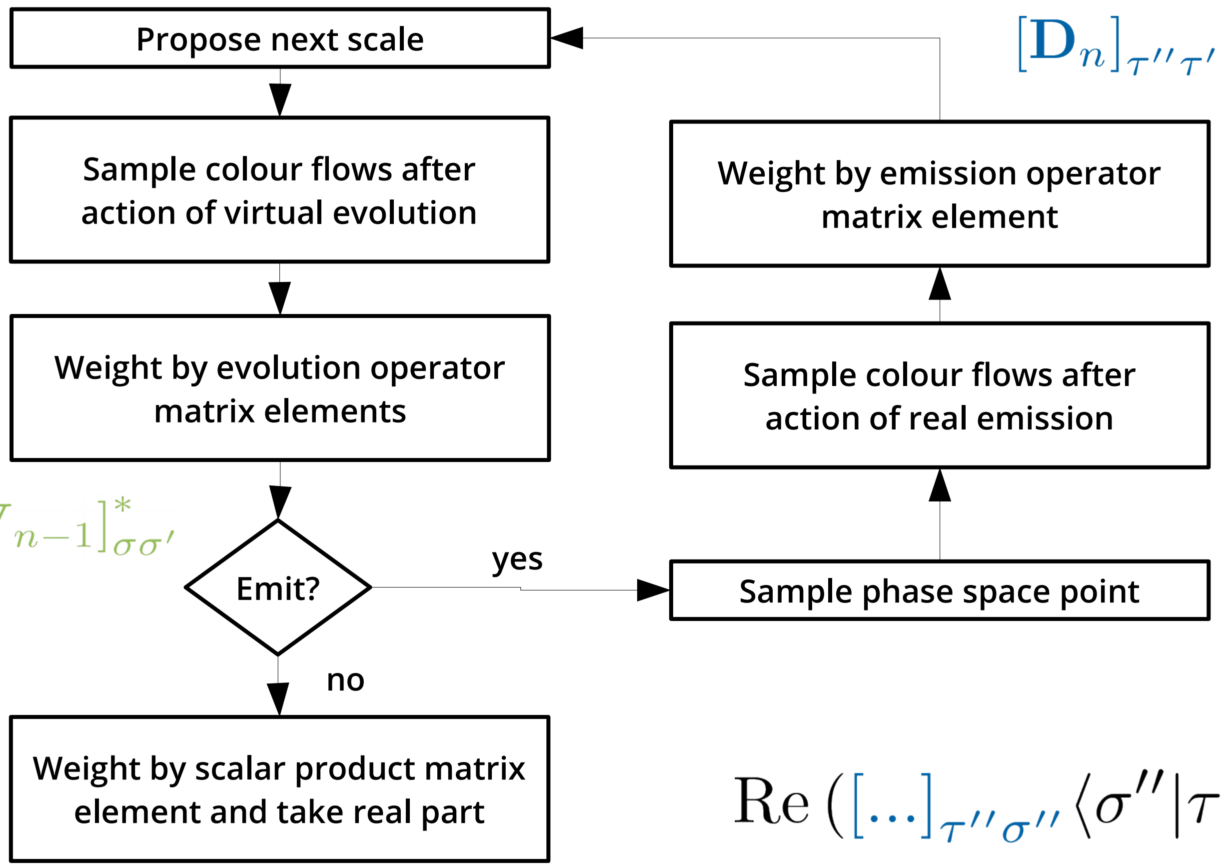
[De Angelis, Forshaw, Plätzer – arXiv:1902.xxxxx, first presented at PSR ‘18 and others]

$$\mathbf{A}_n(E) = \mathbf{V}(E,E_n)\mathbf{D}_n\mathbf{A}_{n-1}(E_n)\mathbf{D}_n^\dagger\mathbf{V}^\dagger(E,E_n)\theta(E-E_n)$$

$$[\mathbf{M}_{n-1}]_{\tau\sigma}$$

$$\tau\sigma \to \tau'\sigma'$$

$$[\mathbf{V}_{n-1}]_{\tau'\tau}[\ldots]_{\tau\sigma}[\mathbf{V}_{n-1}]^*_{\sigma\sigma'}$$



$$[\mathbf{D}_n]_{\tau''\tau'}[\ldots]_{\tau'\sigma'}[\mathbf{D}_n]^*_{\sigma'\sigma''}$$

$$\sigma',\tau' \to \sigma'',\tau''$$

$$\mathrm{Re}\left([\ldots]_{\tau''\sigma''}\langle\sigma''|\tau''\rangle\right)$$

Exploring Colours with CVolver

[De Angelis, Forshaw, Plätzer – arXiv:1902.xxxxx, first presented at PSR ‘18 and others]

A framework to solve multi-differential evolution equations in colour space.
Concise, simple, and light-weight code structure.

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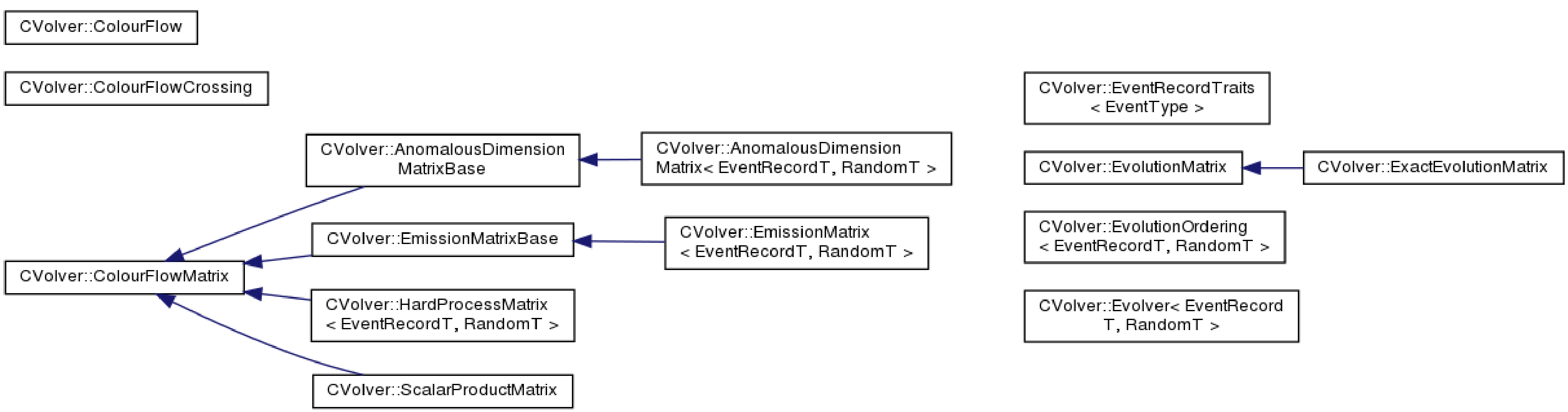

Dedicated Monte Carlo algorithms to sample colour structures.
Plugin approach can accommodate anything from (N)GLs to full parton showers.

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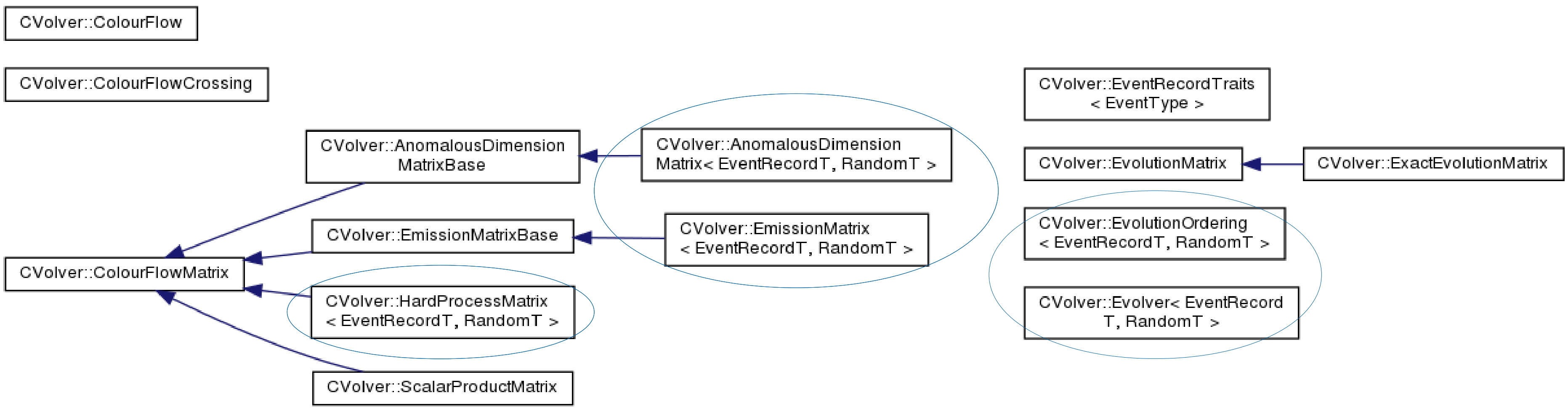

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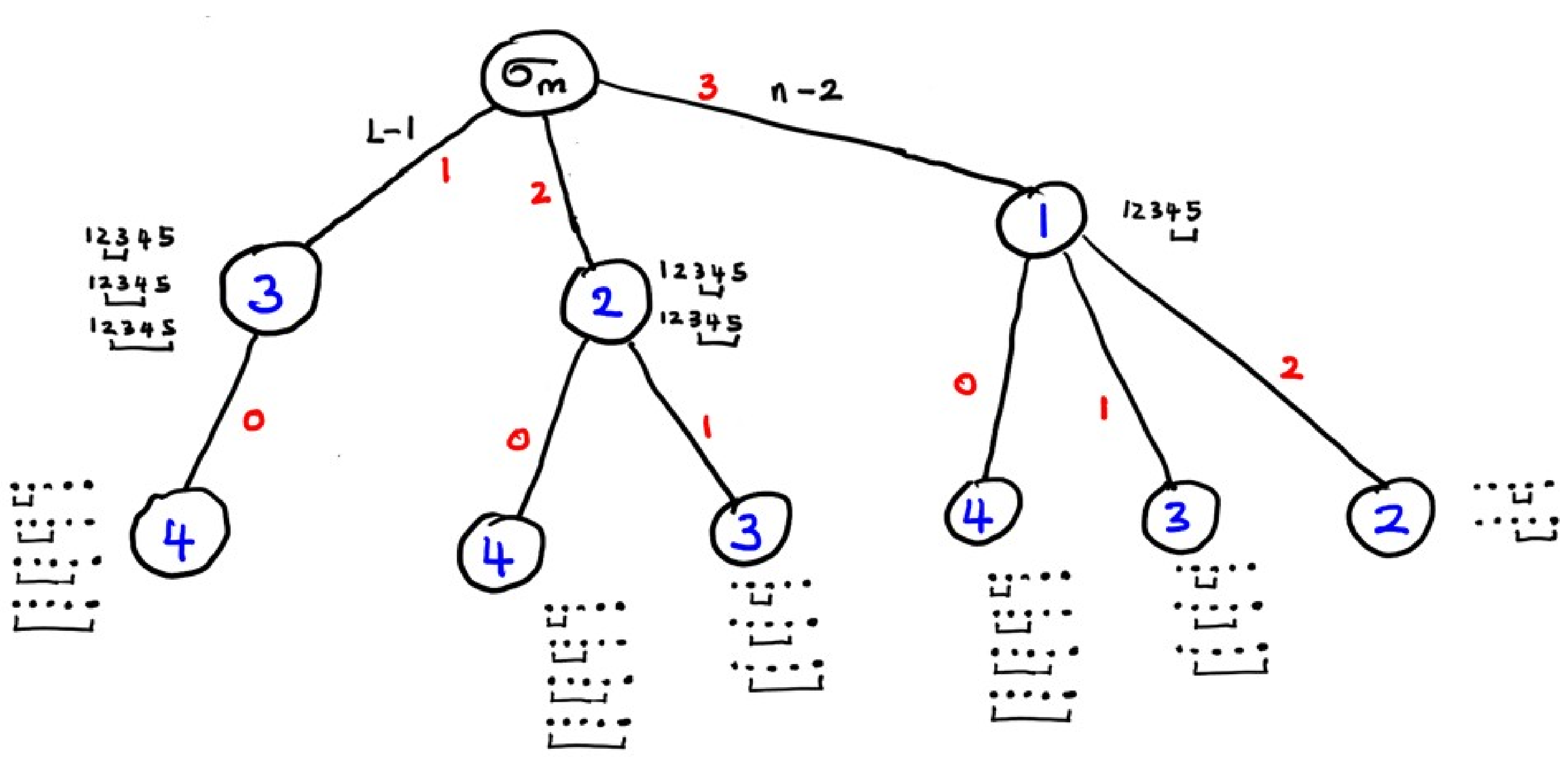
Importance Sampling in Colour Space

[De Angelis, Forshaw, Plätzer – arXiv:1902.xxxxx, first presented at PSR '18 and others]

Importance sampling in colour space rules: $\#(\tau, \tau') \sim 1/N^{\#(\tau,\tau')}$

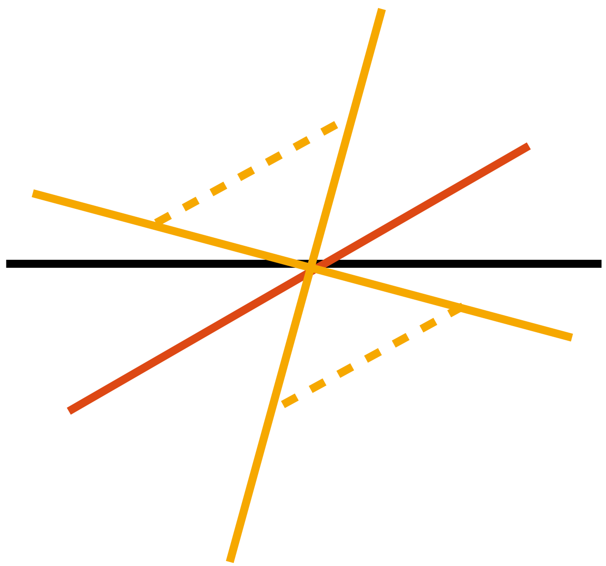
Enumerate and address permutations with fixed cycle length:

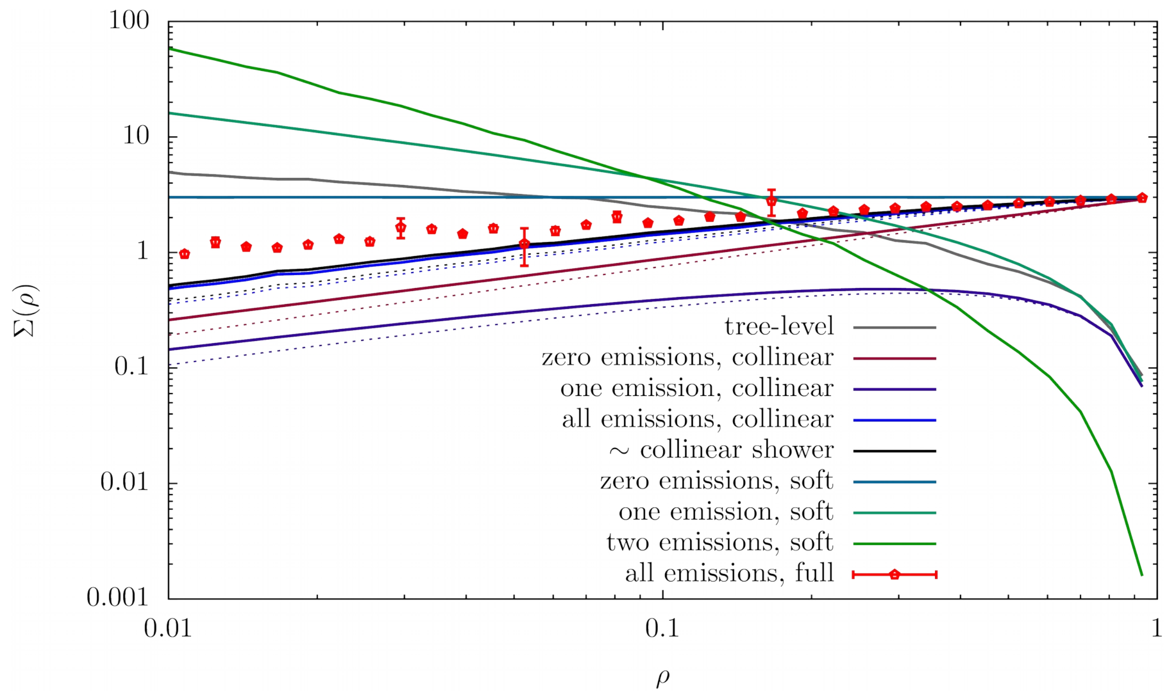


Numerical Results from CVolver

[De Angelis, Forshaw, Plätzer – arXiv:1902.xxxxx, first presented at PSR ‘18 and others]

Code is differential for a large class of (non-global) observables.
Example: Cone-dijet veto cross section.





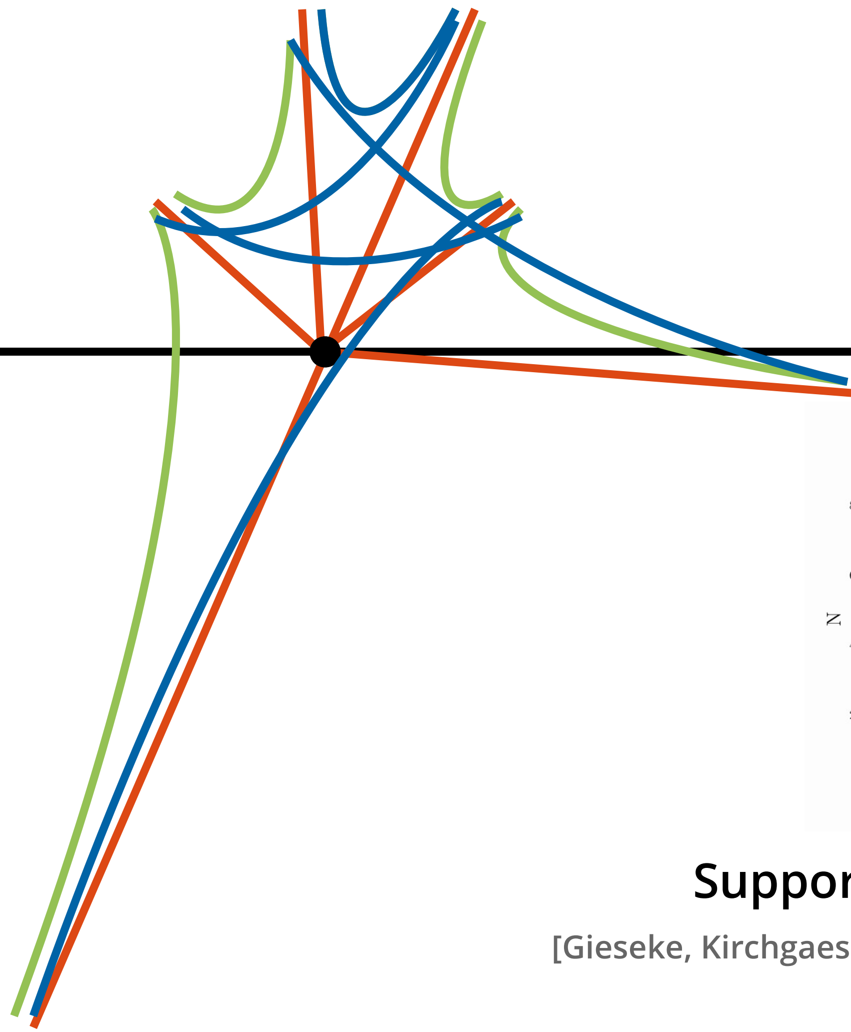
1/N breakdowns possible, scales up to several 10s of emissions for d=2.

A Fresh Look at Colour Reconnection Models

[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]

The cluster model is based on a single colour flow after the shower has stopped.

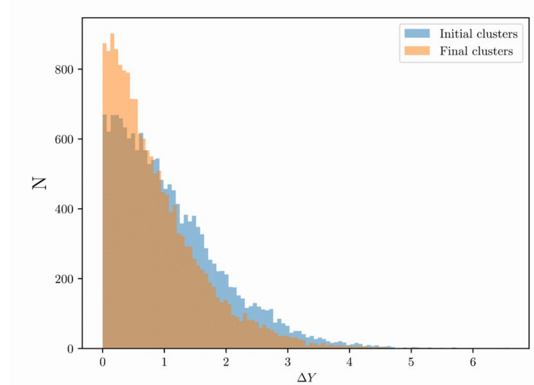
Essentially no evolution is considered, just decays.



View as an evolving amplitude, driven by a single initial colour flow:

$$|\mathcal{M}\rangle = e^{\mathbf{\Gamma}}|\text{clusters}\rangle$$

$$P_{\text{reco}} \sim |\langle\text{clusters'}|\mathcal{M}\rangle|^2$$



Supports geometric models

[Gieseke, Kirchgaesser, Plätzer – EPJ C 78 (2018) 99]