Stable NNLO QCD predictions for quarkonium production and decay



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Pills of quarkonium physics

Quarkonia are bound states of a heavy quark-antiquark pair $\mathbb{Q} = Q\overline{Q}'$

Heavy mass ⟶ relative velocity is non relativistic $v^2 \ll c^2$

Bodwin, Braaten, Lepage, *PRD 51* (1994)

Non-relativistic QCD (**NRQCD**) exploits v as an additional expansion parameter

Example of a Fock-state decomposition:

$$|J/\psi\rangle = \underbrace{|{}^3S_1^{[1]}\rangle}_{v^0} + \underbrace{|{}^3P_J^{[8]}\, g\rangle}_{v^4} + \underbrace{|{}^1S_0^{[8]}\, g\rangle}_{v^3} + \underbrace{|{}^3S_1^{[8]}\, gg\rangle}_{v^4} + \ldots$$

Spectroscopic notation $n = {}^{2S+1}L_J^{[c]}$

S = spin L = orbital angular mom. J = total angular mom. c = colour (1 or 8)

Quarkonium observables

The NRQCD factorization of an observable $\mathscr{A}$

$$\mathscr{A} = \sum_n \underbrace{H[n](\mu_R, \mu_\Lambda)}_{\text{SDC}} \times \underbrace{\langle \mathcal{O}[n] \rangle (\mu_s, \mu_\Lambda)}_{\text{LDME}}$$

- $H[n]$ is the short-distance coefficient (SDC) and is purely hard
- $\langle \mathcal{O}[n] \rangle$ is the long-distance matrix element (LDME)

- μ_R renormalization scale
- μ_Λ NRQCD factorization scale
- μ_s soft scale

Only the SDC × LDME combination is physical

→ Perturbative expansions are scheme dependent

Working enviorement

In this talk only:

- charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) mesons
- S-wave states ($L = 0$)
- color-singlet states ($c = 1$) example: $|J/\psi\rangle = |^3S_1^{[1]}\rangle + \ldots$
- leading contribution in the v expansion (no relativistic corrections) → $M_{\mathbb{Q}} \approx 2m_Q$
- spin singlet and spin triplet ground states ($n = 1$)

$$V_Q = J/\psi \vee \Upsilon \qquad \eta_Q = \eta_c \vee \eta_b$$

State of the problem

The standard approach adopts the $\overline{\text{MS}}$ scheme

$$\langle \mathcal{O}_{\text{bare}} \rangle = \langle \mathcal{O}_{\overline{\text{MS}}} \rangle Z_{\overline{\text{MS}}} \quad \longrightarrow \quad \text{the counterterm removes the } 1/\epsilon \text{ pole}$$

A clean example of an NNLO observable: $\eta_c \to \gamma\gamma$ **decay**

Feng, Jia, Sang, *PRL 115* (2015)

Abreu, Becchetti, Duhr, Ozcelik, *JHEP* 02 (2023)

$$\mu_\Lambda = m_c$$

$$\frac{\Gamma_{\text{NNLO}}(\eta_c \to \gamma\gamma)}{\Gamma_{\text{LO}}(\eta_c \to \gamma\gamma)} = 1 - 1.07\alpha_s - 11.16\alpha_s^2$$

- NNLO K factor is large $\longrightarrow$ bad convergence
- NLO and NNLO coefficients are negative $\longrightarrow$ negative predictions

Why caring?

It is a broaden theoretical problem

→ Unphysical predictions were also observed for $J/\psi \to e^+e^-$, $J/\psi \to 3\gamma$, $e^+e^- \to J/\psi$, …

Observable	Data	LO	NLO	$\text{NNLO}_{\overline{\text{MS}}}$
$\Gamma(\eta_c \to \gamma\gamma)$ [keV]	6.4 ± 0.6	9.3	6.8	-0.5 🔗
$\Gamma(J/\psi \to e^+e^-)$ [keV]	5.5 ± 0.1	10.0	5.7	-0.5 🔗
$\Gamma(J/\psi \to \gamma\gamma\gamma)$ [eV]	1.1 ± 0.2	5.3	-0.1	-2.3 🔗
$\sigma(e^+e^- \to J/\psi\, J/\psi)$ [fb]	< 9.1	7.5	2.2	-3.6 🔗

Central scales

$\mu_R = 2m_Q$ (decay)
$\mu_R = 5.29\,\text{GeV}$ (production)

Data

Decay from PDG2026 🔗
Upper limit from Belle 🔗

And it directly affects pheno studies Bodwin, Chung, Kang, Lee, Yu, *PRD 77* (2008) 🔗 Braaten, Lee, *PRD 67* (2003) 🔗

→ Leptonic decay widths are used to extract LDME values

$$\Gamma(\mathcal{Q} \to \ell^+\ell^-) \propto \langle \mathcal{O}_{\mathcal{Q}} \rangle$$

A hint from QED

In NRQED we can compute things perturbatively

Large cancellations are observed between the SDC and the LDME

$$\frac{\Gamma_{\text{NNLO}}}{\Gamma_{\text{LO}}} = 1 + c_1\,\alpha_s + \Big(\underbrace{c_2^{\text{SDC}}}_{\text{large, }<0} + \underbrace{c_2^{\text{soft}}}_{\text{large, }>0}\Big)\,\alpha_s^2 \quad \longrightarrow \quad c_2 = \mathcal{O}(1)$$

NNLO correction is small

In the $\overline{\text{MS}}$ scheme the finite piece from the LDME is lost ($c_2^{\text{LDME}} = 0$)

Kiyo, Pineda, Signer, *Nucl.Phys.B 841* (2010) 🔗

In the weak-coupling approach such a cancellation is also observed

⟶ Valid for top and (barely) bottom production: how to improve?

Pinning the source of the problem

The LDME is related to the wavefunction at the origin $\langle \mathcal{O} \rangle \propto |\psi_{S,n}(0)|^2$

NNLO non-Coulombic perturbative soft contribution to the wavefunction is known

Kniehl, et al., *PRL* (2003) 🔗 Penin, et al., *NPB* (2004) 🔗 Beneke, et al., *NPB* (2005) 🔗 Kiyo, et al., *NPB* (2010) 🔗

$$\delta\psi^{\text{Soft}}_{S,n}(0) = \psi^{\text{LO}}_n(0)\left\{\frac{\alpha_s^2(\mu_s)}{2}\left[C_F^2\left(\frac{L_n}{2S+1} - \frac{15}{8n^2} + \frac{4}{(2S+1)n} + \frac{5}{3} + \frac{7}{3(2S+1)} - \frac{2\Sigma_1(n)}{2s+1}\right) + C_A C_F\left(\frac{L_n}{2} + \frac{2}{n} + \frac{5}{4} - \Sigma_1(n)\right)\right]\right\}$$

$L_n = \log(\mu_\Lambda^2 a_n^2)$ ⟶ $\boldsymbol{L_\Lambda} = \log(\mu_\Lambda^2/m_c^2)$ + $\widetilde{L}_n = \log(m_c^2 a_n^2)$ a_n Bohr radius

$\psi^{\text{LO}}(0)$ includes the resummed Coulomb contribution

Presence of a short-distance (**SD**) and a long-distance (**LD**) constribution

- **SD**: energy-level independent and UV divergent ⟶ isolated by taking $n \to \infty$
- **LD**: included as part of the normalisation (Coulomb + NP)

The SIM scheme

Also the SDC presents L_Λ logs

→ variation on μ_Λ are sometimes used to suppress the NNLO coefficient

(somehow mimicking the cancellation from the soft expansion)

Instead: A variation on the $\overline{\text{MS}}$ scheme can be introduced to improve convergence

Soft-improved matching (SIM) scheme:

$$\langle \mathcal{O}_{\text{bare}} \rangle = \langle \mathcal{O}_{\text{SIM}} \rangle Z_{\text{SIM}} \qquad Z_{\text{SIM}} = Z_{\overline{\text{MS}}} + \alpha_s^2(\mu_R)\, \delta Z_{\text{SIM}}$$

$$\delta Z_{\text{SIM}} = \left[C_F^2 \left(\frac{L_\Lambda}{2S+1} + \frac{5}{3} + \frac{7}{3(2S+1)} \right) + C_F C_A \left(\frac{L_\Lambda}{2} + \frac{5}{4} \right) \right]$$

SIM features

- Deviation from the $\overline{\text{MS}}$ scheme only from NNLO (**LO** & **NLO unchanged**)
- L_Λ exactly cancels between SDC and LDME
- Adds a **positive finite** contribution to the NNLO coefficient
- Leaves the hard part unchanged

Reorganisation of the perturbative expansion

$$\mathscr{A}^{\overline{\text{MS}}} = H \times \langle \mathcal{O}_{\overline{\text{MS}}} \rangle Z_{\overline{\text{MS}}} \longrightarrow \mathscr{A}^{\text{SIM}} = H \times \langle \mathcal{O}_{\text{SIM}} \rangle Z_{\text{SIM}}$$

The NNLO SIM coefficient

The NNLO contribution from the LDME counterterm within SIM scheme

Spin singlet ⟶ $$Z_{\rm SIM}^{S=0} = Z_{\overline{\rm MS}} + \left[\left(C_F^2 + \frac{C_F C_A}{2}\right)\log\frac{\mu_\Lambda^2}{m_Q^2} + 12.11\right]\alpha_s^2(\mu_{\rm R})$$

Spin triplet ⟶ $$Z_{\rm SIM}^{S=1} = Z_{\overline{\rm MS}} + \left[\left(\frac{C_F^2}{3} + \frac{C_F C_A}{2}\right)\log\frac{\mu_\Lambda^2}{m_Q^2} + 9.35\right]\alpha_s^2(\mu_{\rm R})$$

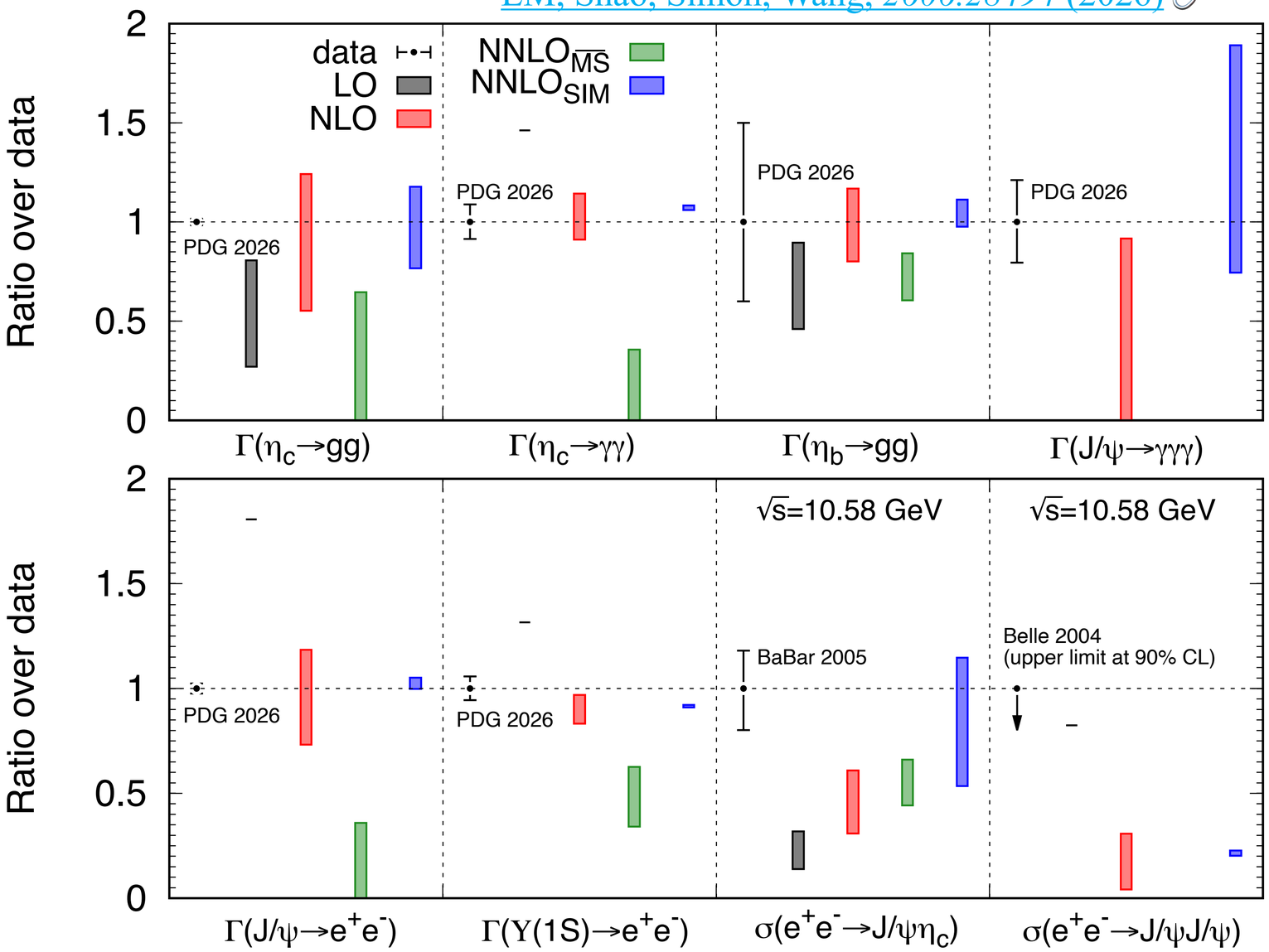
LM, Shao, Simon, Wang, *2606.28494* (2026) 🔗

$\eta_c \to \gamma\gamma$ revised: $$\frac{\Gamma_{\rm NNLO}(\eta_c \to \gamma\gamma)}{\Gamma_{\rm LO}(\eta_c \to \gamma\gamma)} = 1 - 1.07\alpha_s \left[\begin{pmatrix} -11.16_{n_f=3} \\ -10.40_{n_f=4} \\ -11.19_{n_f=5} \end{pmatrix} + \begin{pmatrix} 0_{\overline{\rm MS}} \\ 12.11_{\rm SIM} \end{pmatrix}\right]\alpha_s^2$$

SDC ↓ LDME ↓

An eight-observable comparison

LM, Shao, Simon, Wang, *2606.28494* (2026)



Scales

$\mu_R = 2m_Q$ for decay

$\mu_R = \sqrt{s}/2$ for production

$\mu_\Lambda = m_Q$

Charmonium LDMEs
tuned from data

$\langle \mathcal{O}_{\psi} \rangle = 1.36\ \text{GeV}^3$ from $\Gamma(J/\psi \to e^+e^-)$

$\langle \mathcal{O}_{\eta_c} \rangle = 0.318\ \text{GeV}^3$ from $\Gamma(\eta_c \to \gamma\gamma)$

Bottomonium LDMEs
from BT potential

Eichten, Quigg, *PRD 52* (1995)

$\langle \mathcal{O}_{\Upsilon} \rangle = 9.28\ \text{GeV}^3$

$\langle \mathcal{O}_{\eta_b} \rangle = 3.093\ \text{GeV}^3$

Data

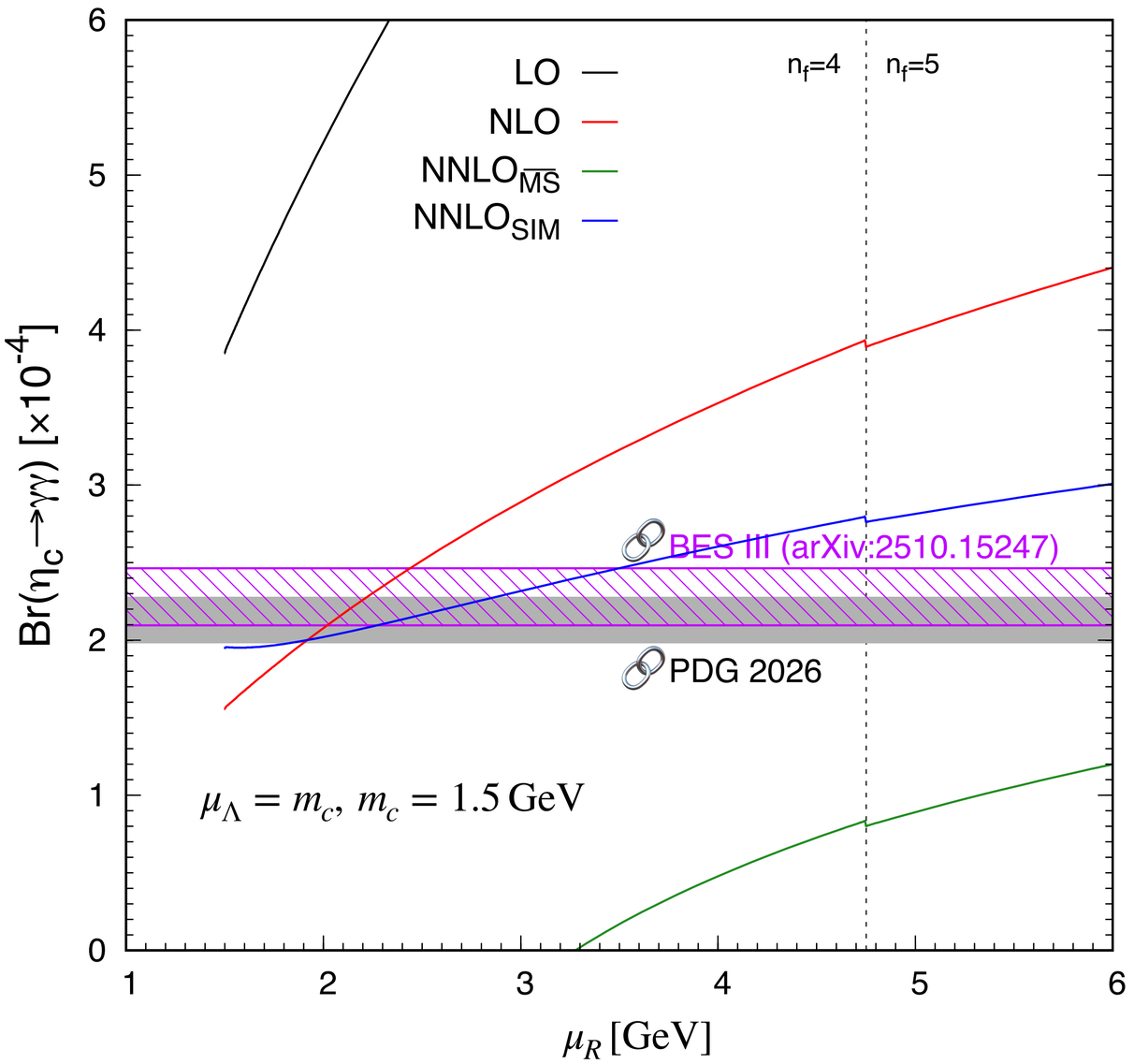
PDG2026

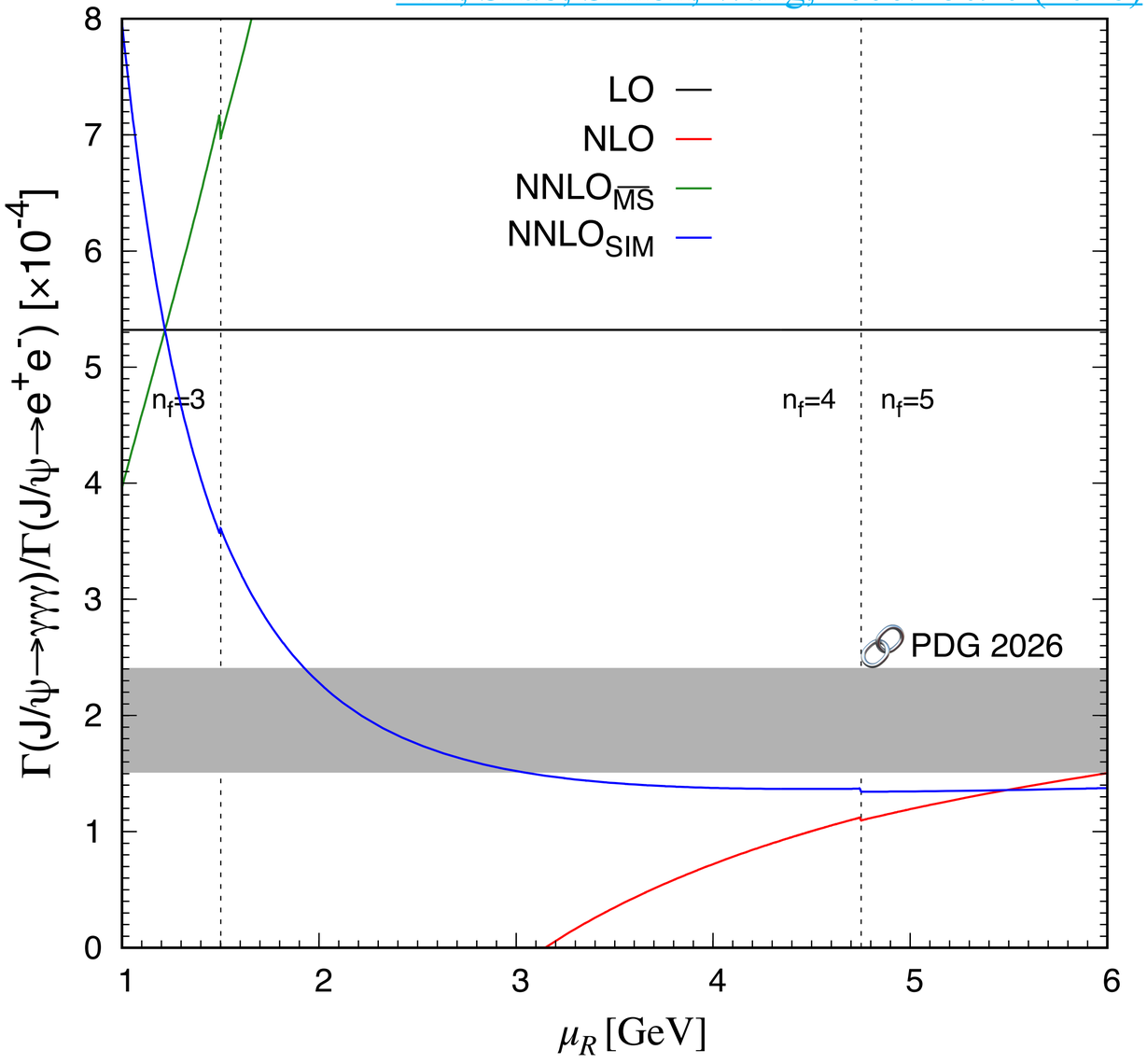
Belle2004

BaBar2005

Normalisation-independent ratios

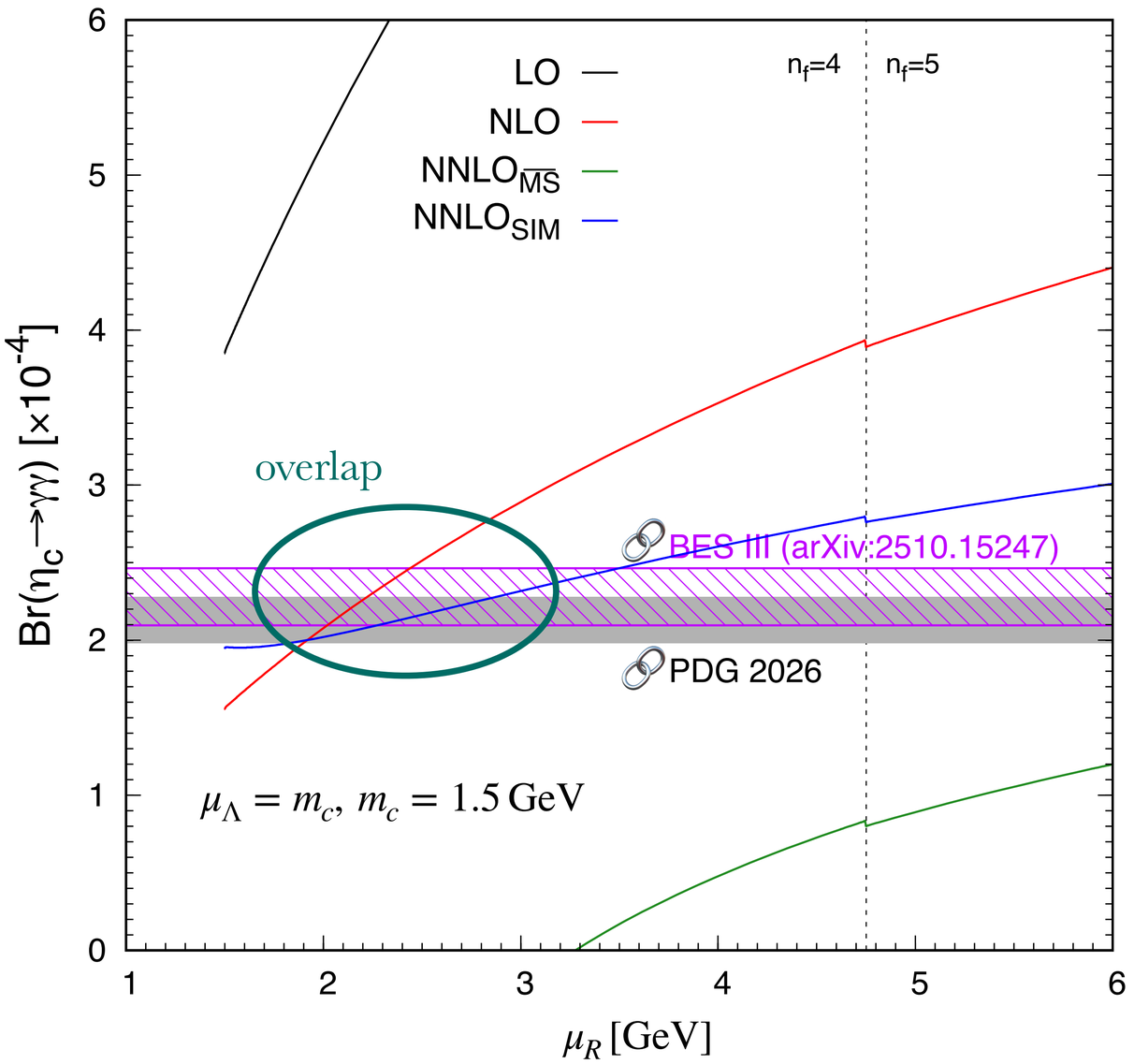
LM, Shao, Simon, Wang, *2606.28494* (2026)

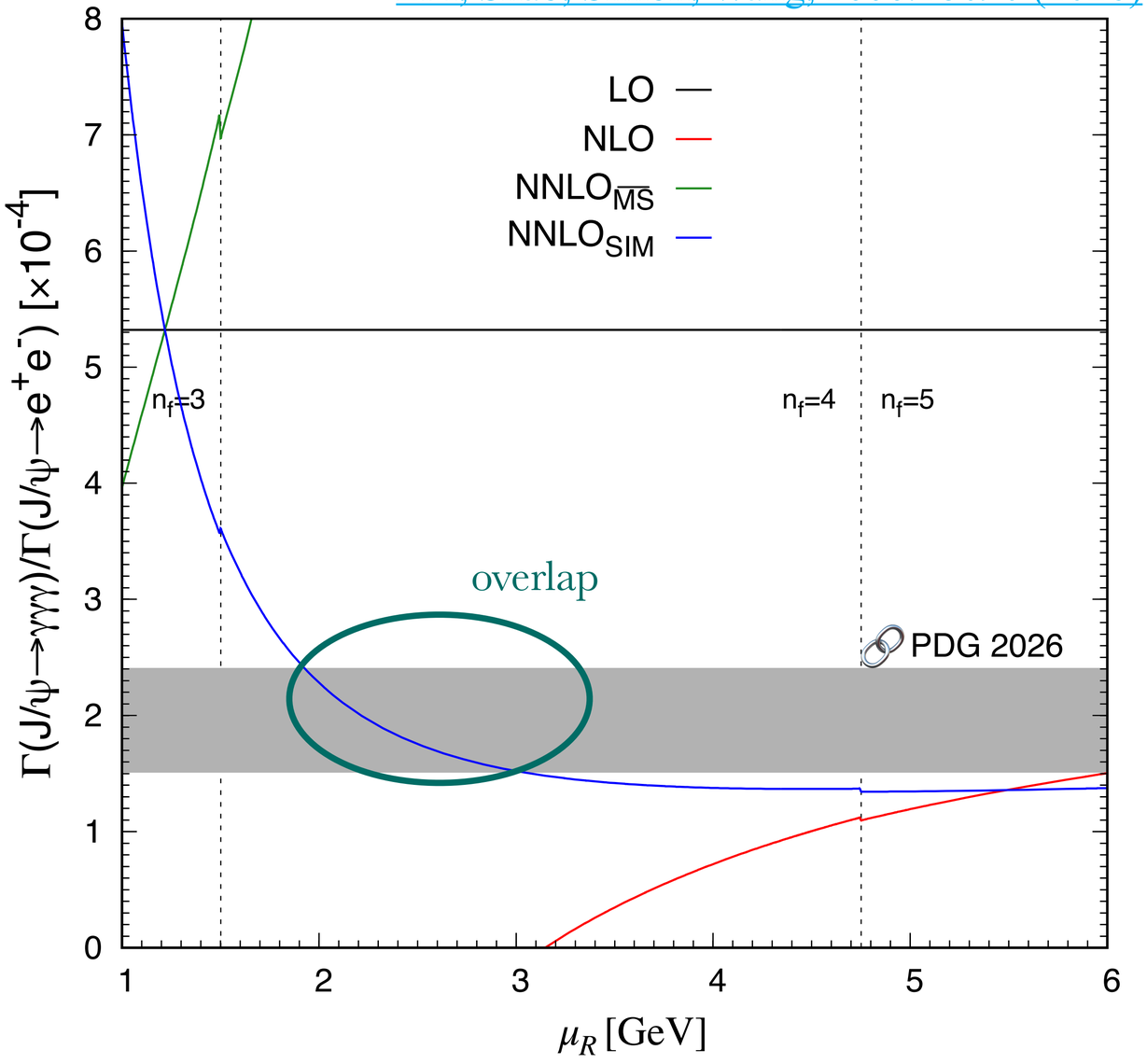




Normalisation-independent ratios

LM, Shao, Simon, Wang, *2606.28494* (2026)





End of the story?

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Our scheme does not resolve all problems

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- **Relativistic corrections** should also be taken into account

  ⟶ Do they present instabilities as well?

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- **Multi-scale processes** might be polluted by additional logarithms

  ⟶ $\log(s/m_c^2)$ enhancement in $\sigma(e^+e^- \to J/\psi\eta_c)$

  ⟶ $-\log(m_H^2/m_Q^2)$ enhancement in $\Gamma(H \to V_Q\gamma)$

End of the story?



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  ⟶ $\log(s/m_c^2)$ enhancement in $\sigma(e^+e^- \to J/\psi\eta_c)$

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- What about **colour-octet** states?

  ⟶ Colour octet potential repulsive and no bound state to expand around

Summary and final remarks



I presented a new matching scheme: SIM

- Minimal variation from $\overline{\text{MS}}$ that leads to (more) stable NNLO results
- Applicable to both **exclusive** and **inclusive** processes
- Goes beyond the weak-coupling approach
  (LDME soft expansion applied to charmonia and bottomonia)

It differs from other remedies proposed in the literature

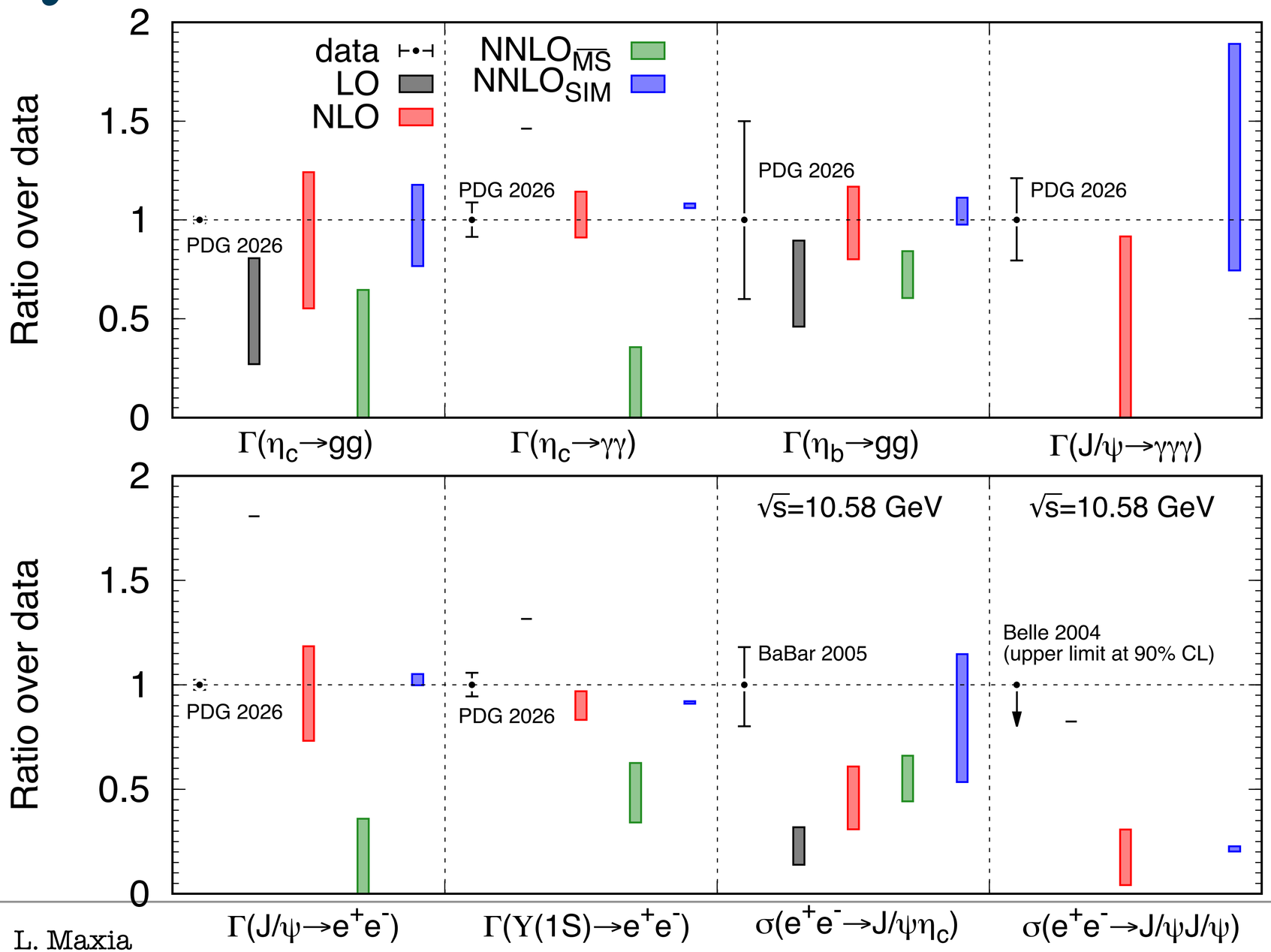
- Amplitude-level truncation
  (only for exclusive)
- Direct expansion of branching ratios

→ Discrepancies are formally of higher order in α_s

(size is not clear for a poorly converging series)

Thank you for the attention!





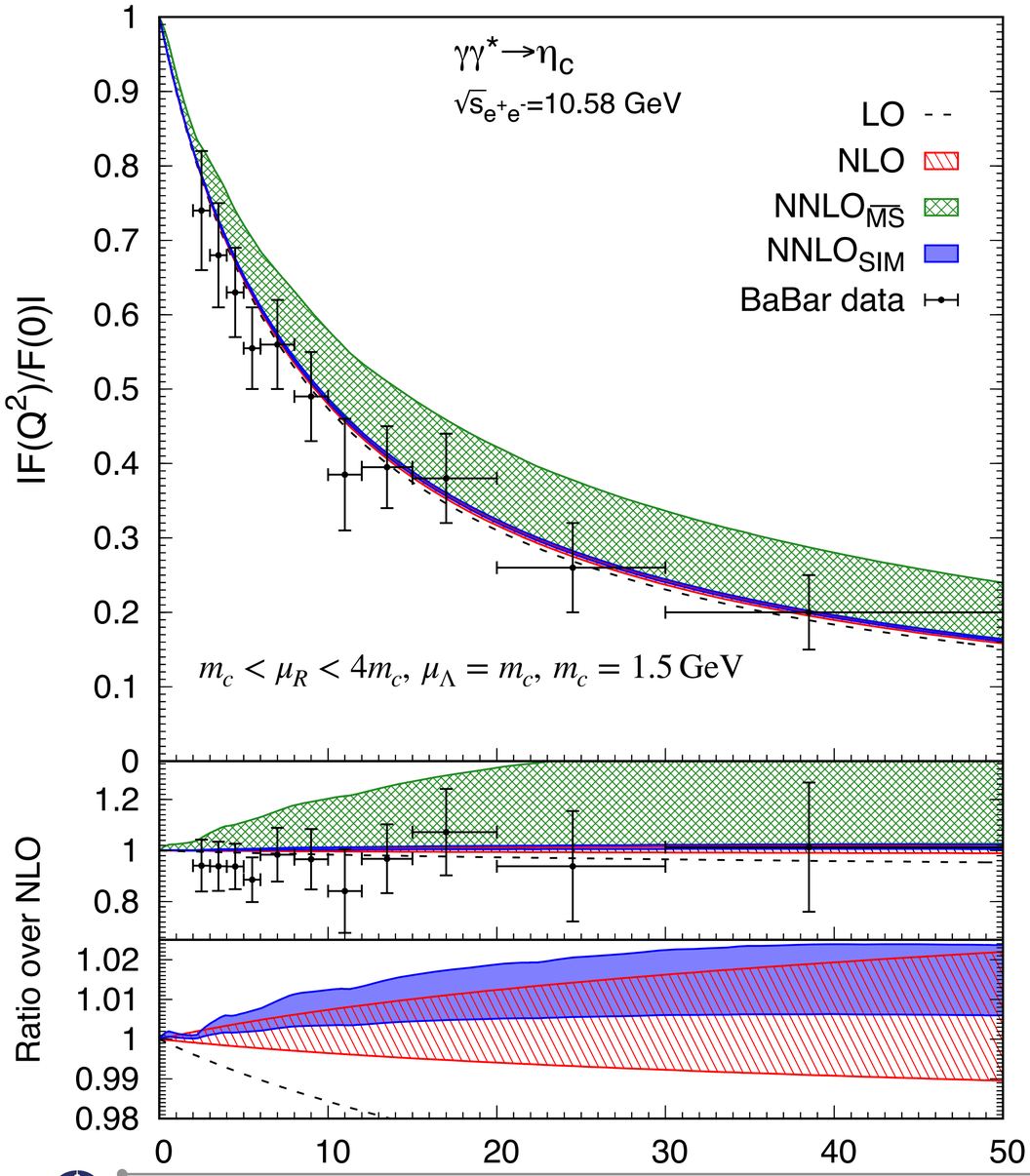


Stable NNLO QCD predictions for quarkonium production and decay

Back-up slides

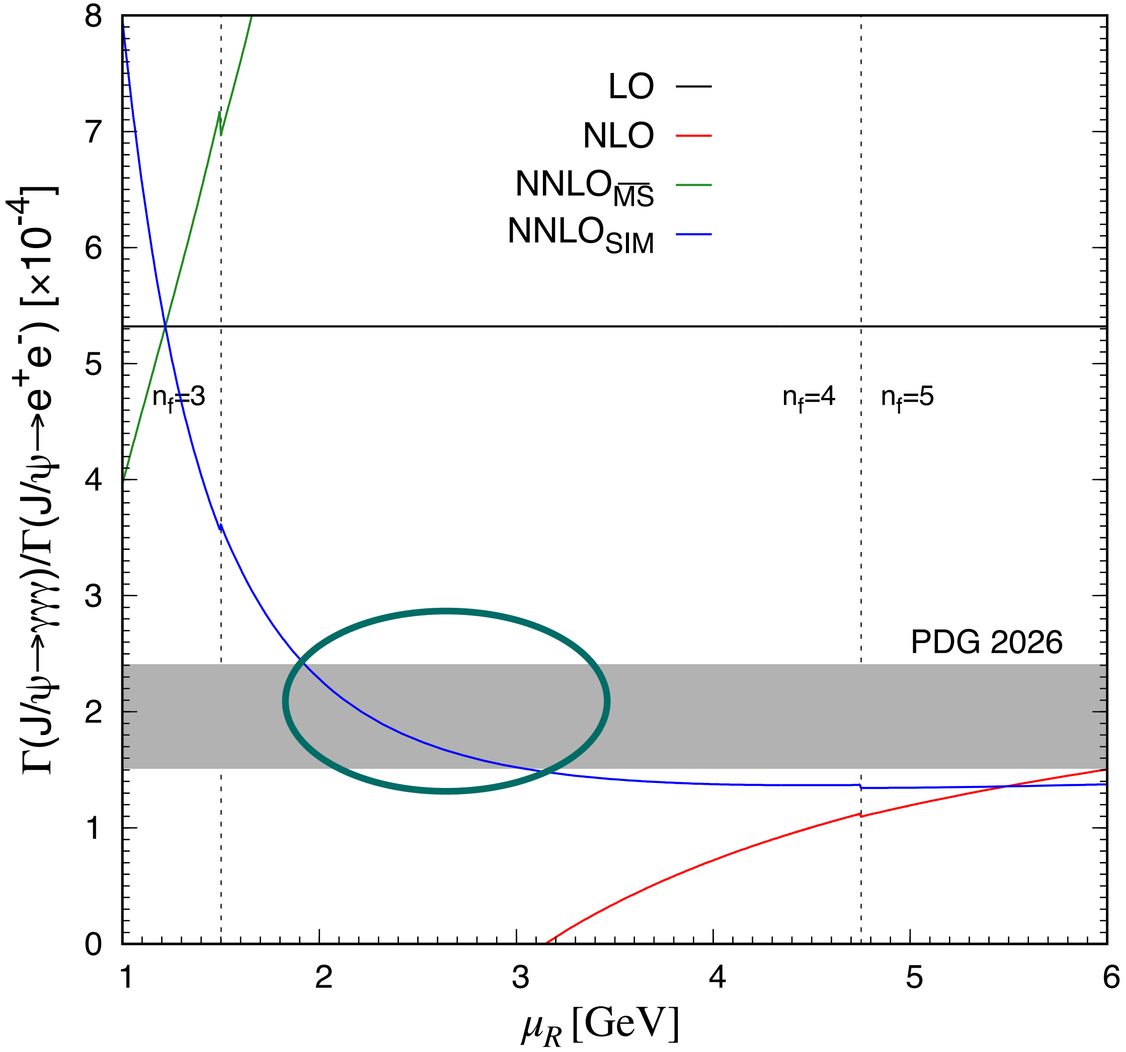
L. Maxia
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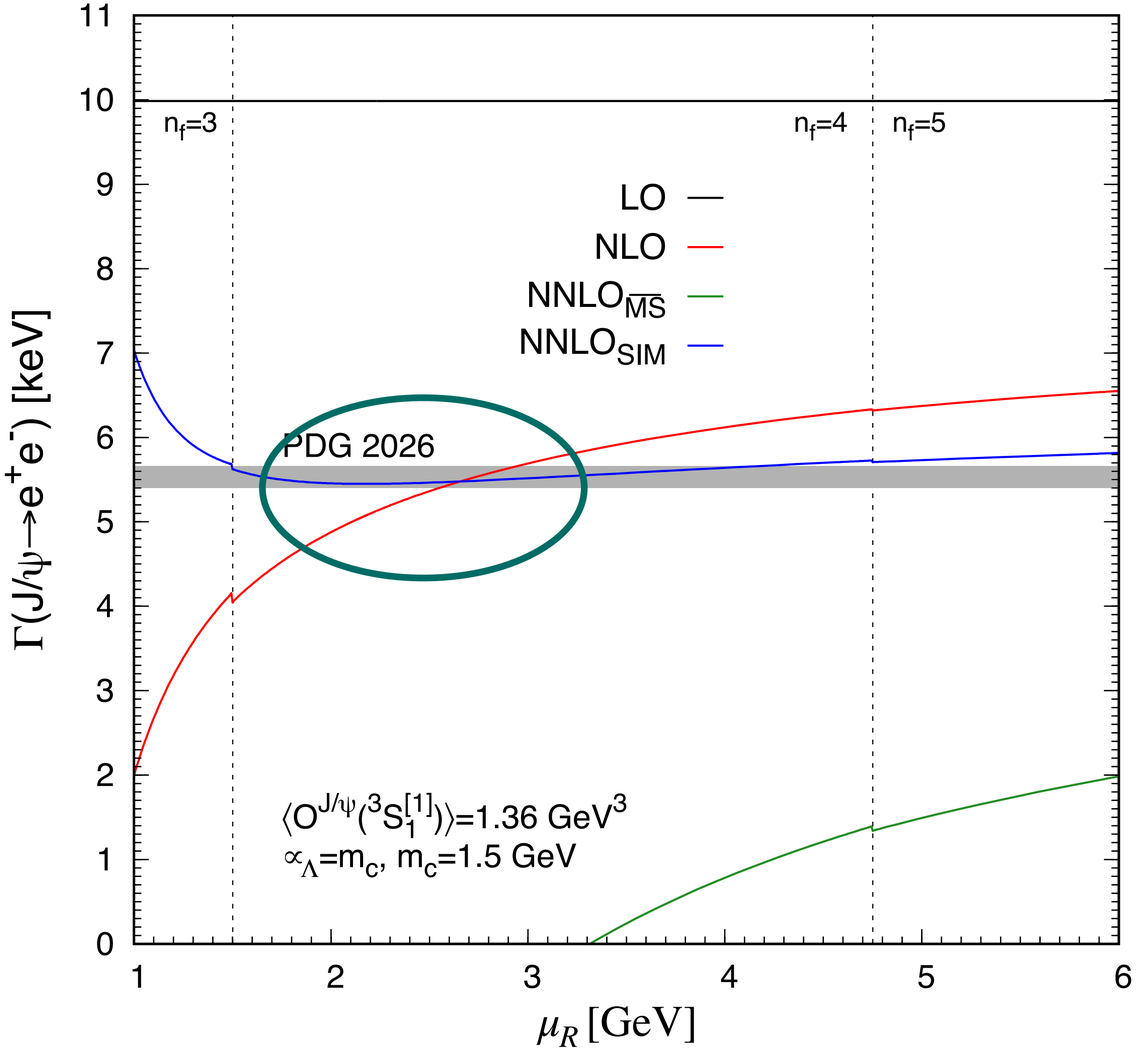
Form Factor stability



- LDME normalisation independent
- Amplitude-level observable
- NNLO$_{SIM}$ prediction stays centered close to BaBar
- NNLO$_{SIM}$ scale uncertainty reduced by 1.5 respect NLO

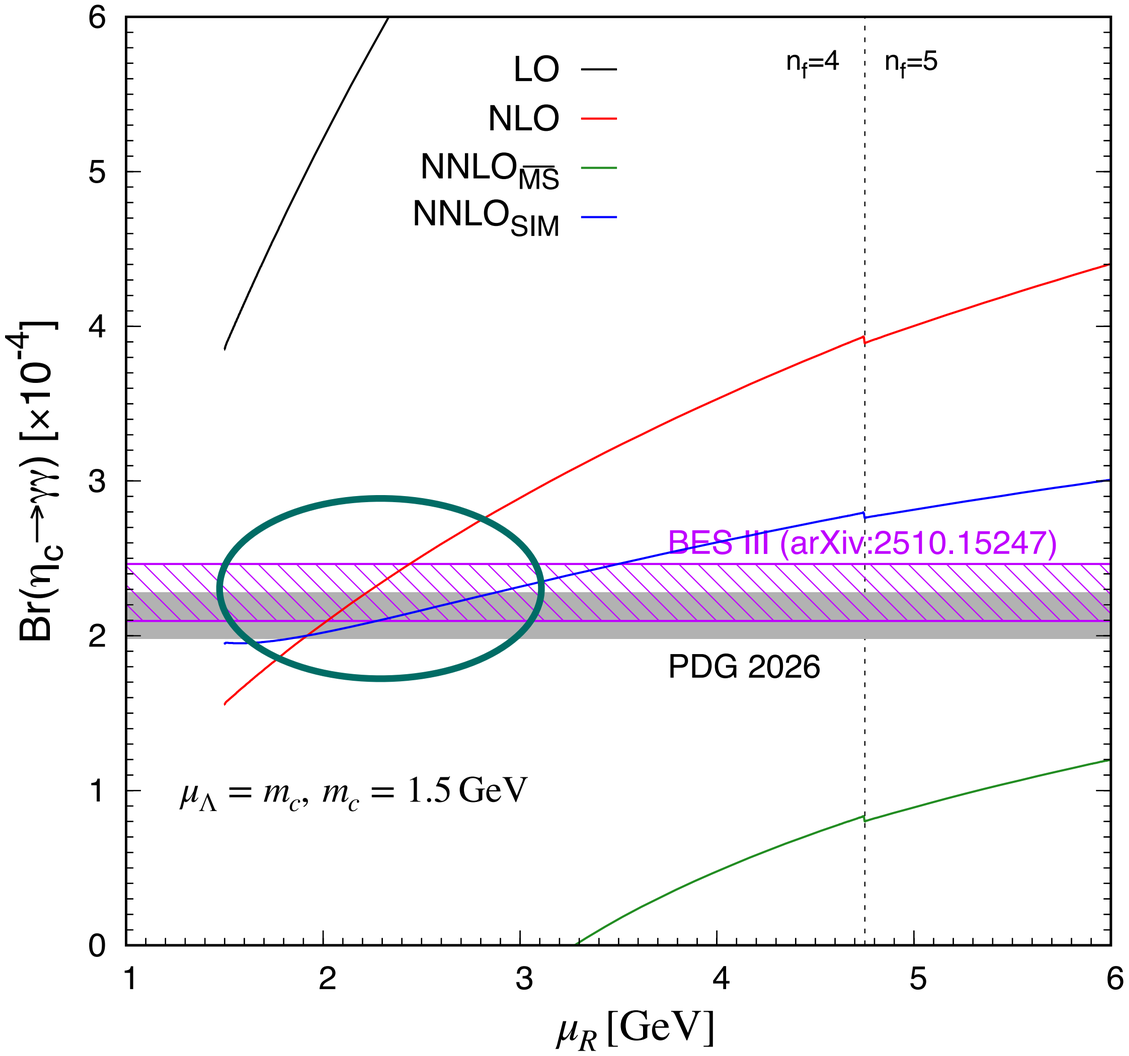
LDME tuning strategy - J/ψ

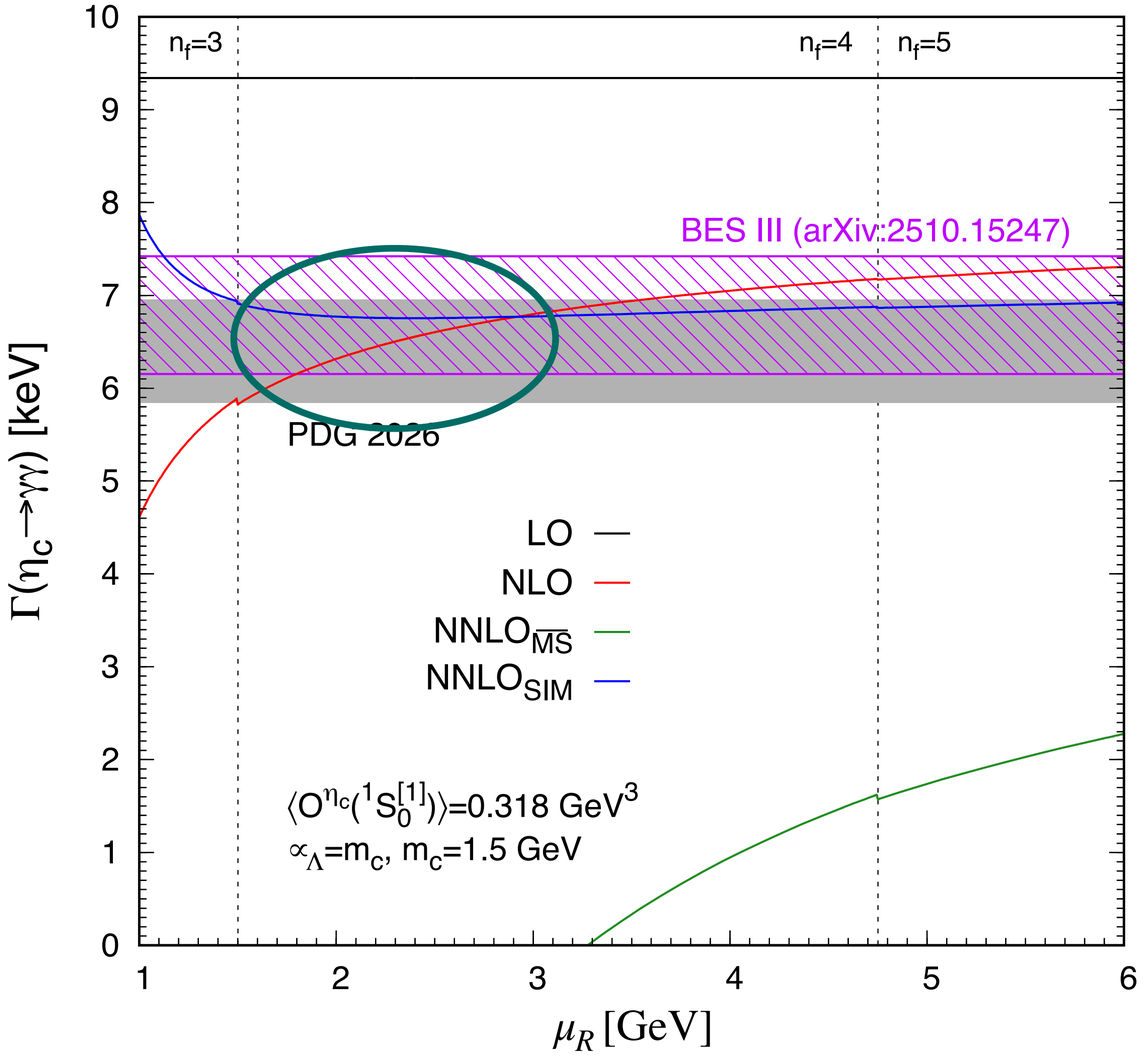




$J/\psi \to e^+e^-$ presents a weak scale dependence for $\mu_R \sim 2$ - 3 GeV while data is precise

LDME tuning strategy - η_c





$\eta_c \to \gamma\gamma$ presents a weak scale dependence for $\mu_R \sim 2$ - 3 GeV (tuned for BESIII)