

Automated calculation of gluon jet function

Guido Bell, Kevin Brune, Goutam Das



Kevin Brune (JGU Mainz)

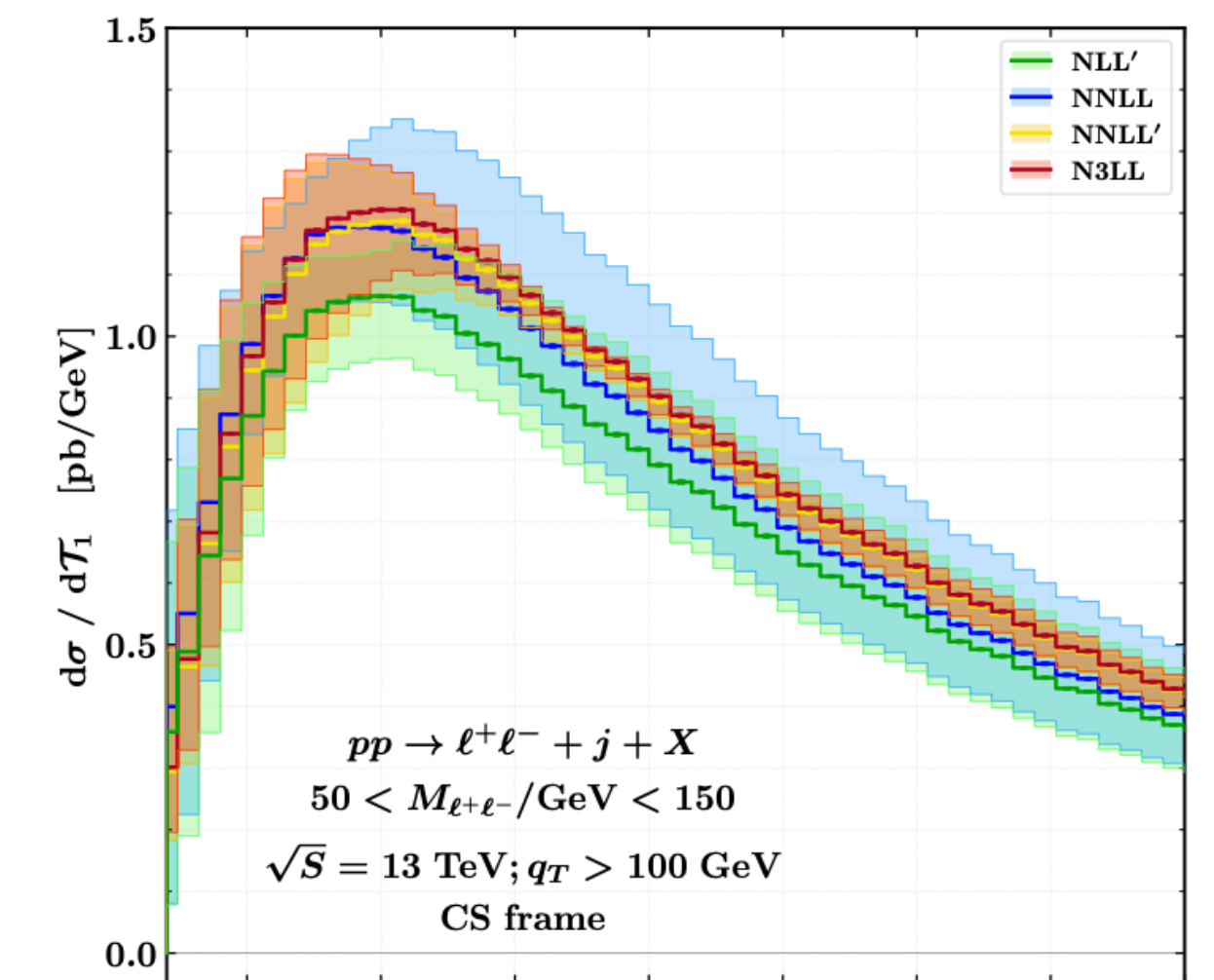
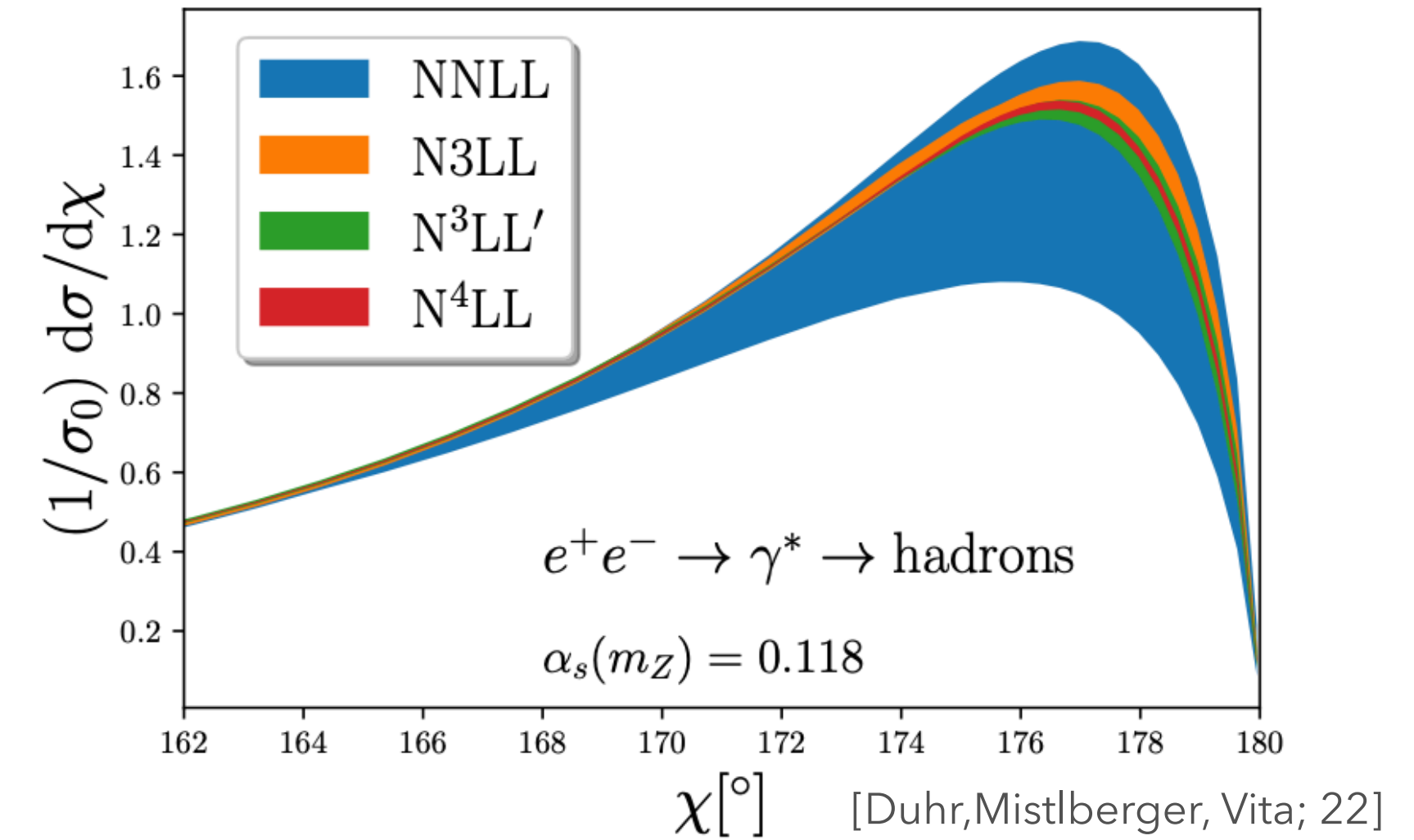
HP2 2026



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Motivation

- Resummation is required for collider observables
 - For some observables very precise
 - N^3LL and above
- For specific observable-types automation available
 - Event shapes $\rightarrow$ ARES [Banfi et al.; 17]
 - Transverse-momentum resummation $\rightarrow$ RadISH [Bizon et al.; 22]

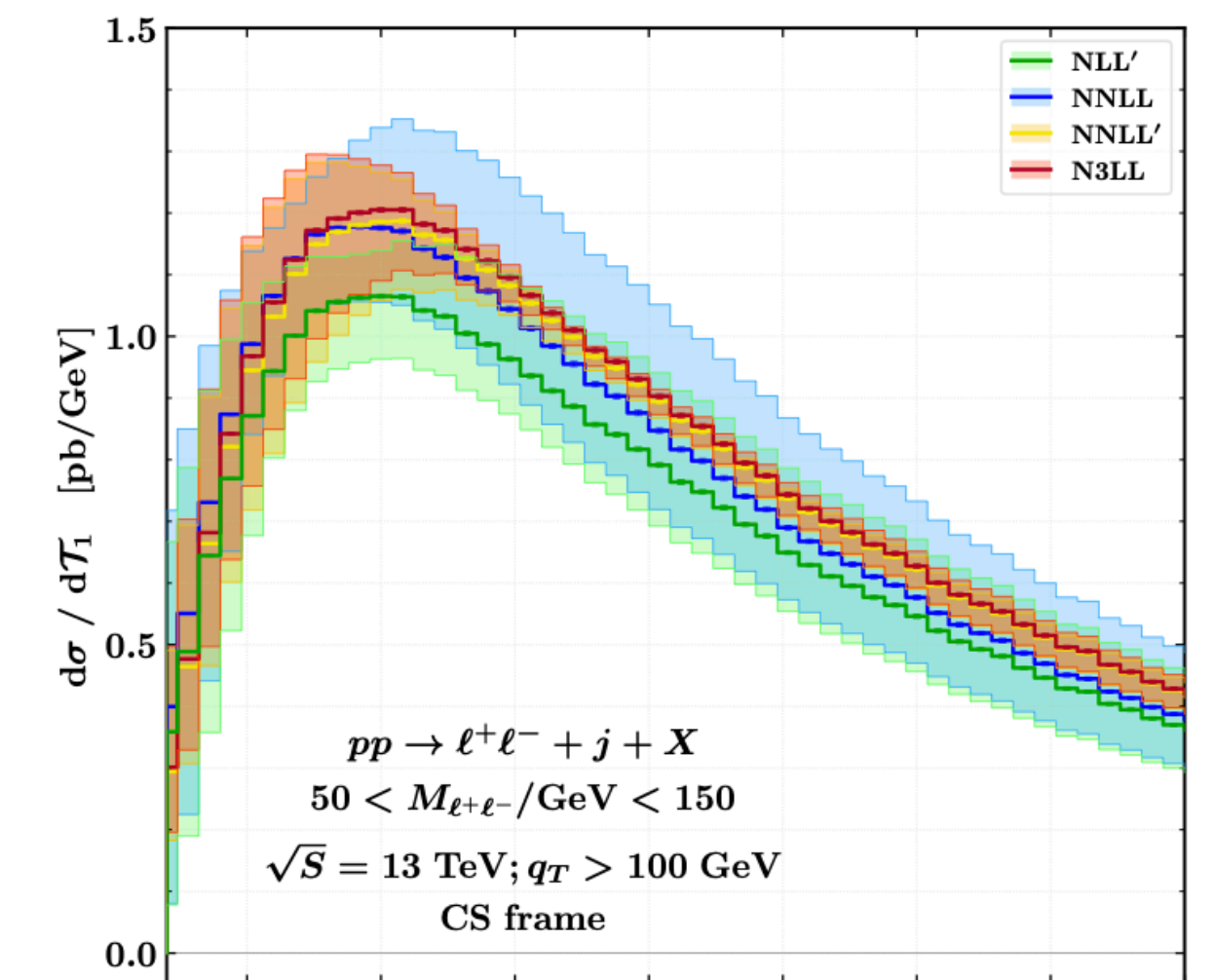
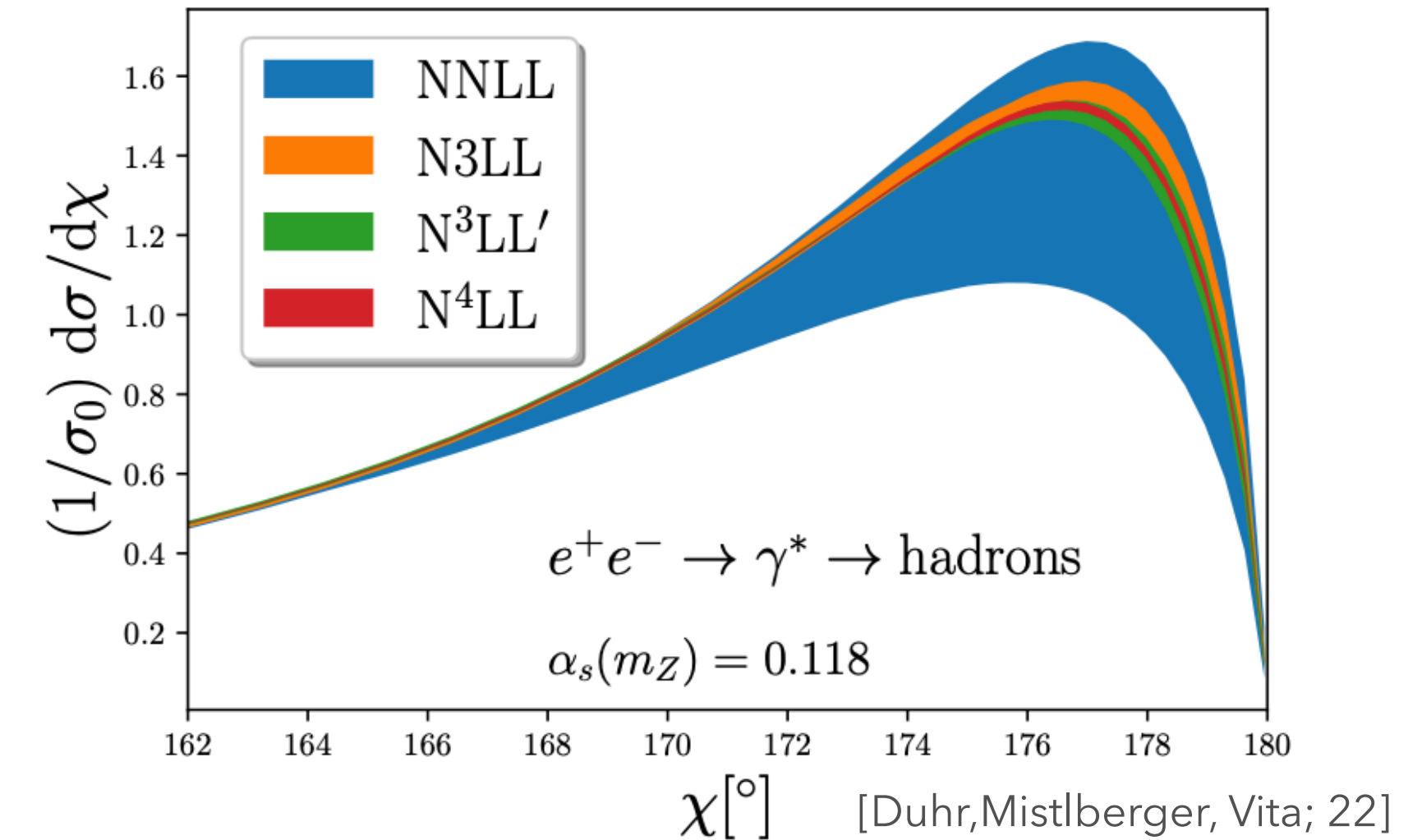


[Alioli et al.; 23]

Motivation

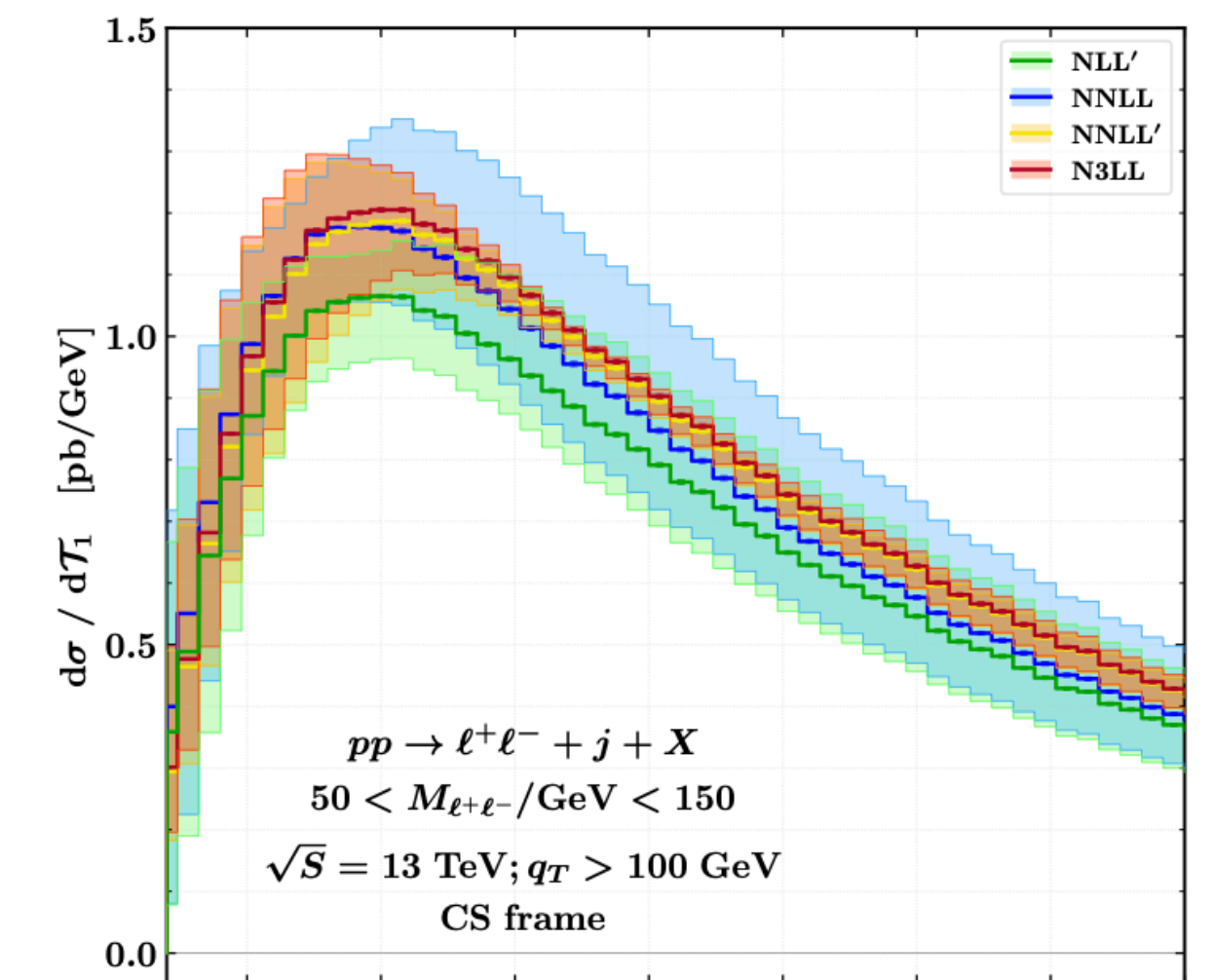
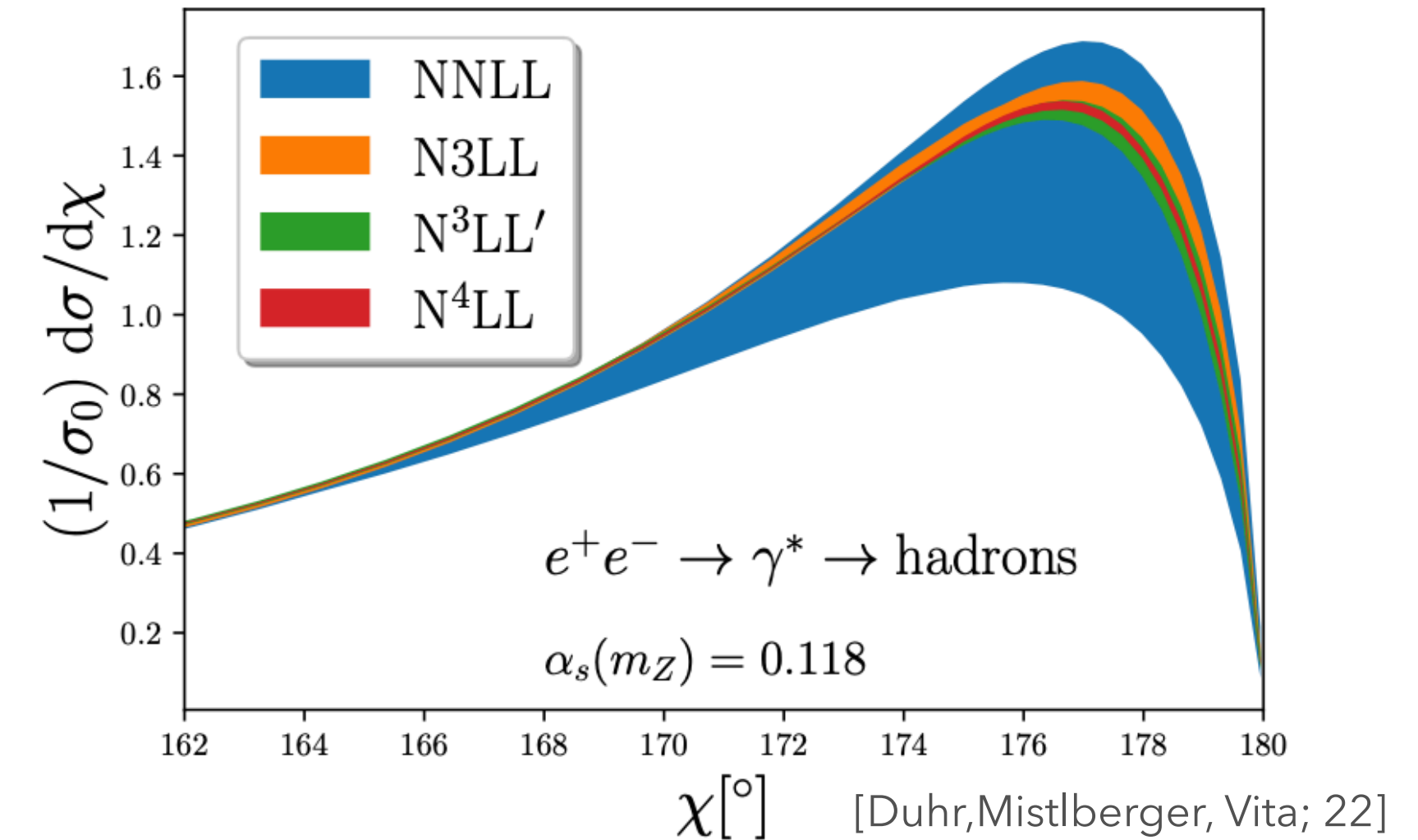
- Resummation is required for collider observables
 - For some observables very precise
 - N^3LL and above
- For generic observables only NLL accuracy

→ CAESAR [Banfi et al.; 04]



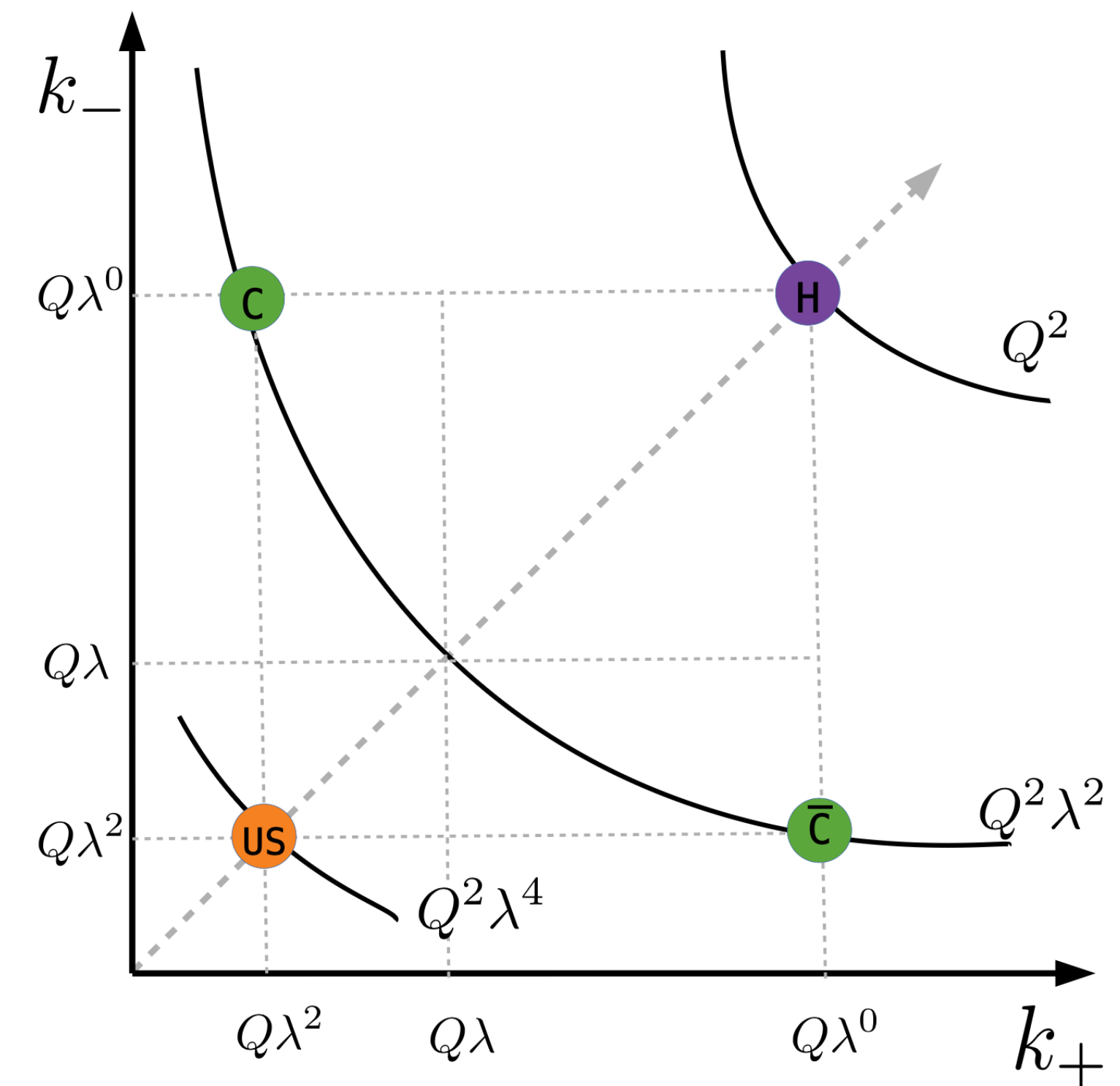
Motivation

- Resummation is required for collider observables
 - For some observables very precise
 - N^3LL and above
- For generic observables only NLL accuracy
 - CAESAR [Banfi et al.; 04]
- Push this to N^2LL' via the systematic framework of SCET
 - Inputs for slicing computations



Soft-Collinear Effective Theory (SCET)

- Effective theory
 - Hard modes are integrated out
 - Soft and collinear modes
 - Leading power $\rightarrow$ Soft and collinear modes decouple
- Typical scaling : $k^\mu \sim (k_-, k_+, k_\perp)$
 - Hard region: $k_H^\mu \sim (1, 1, 1) Q$
 - Collinear region: $k_C^\mu \sim (1, \lambda^2, \lambda) Q$
 - Ultrasoft region: $k_{US}^\mu \sim (\lambda^2, \lambda^2, \lambda^2) Q$



SCET-II

- Different scaling compared to SCET-I version

- Hard region: $k_H^\mu \sim (1,1,1) Q$

- Collinear region: $k_C^\mu \sim (1,\lambda^2,\lambda) Q$

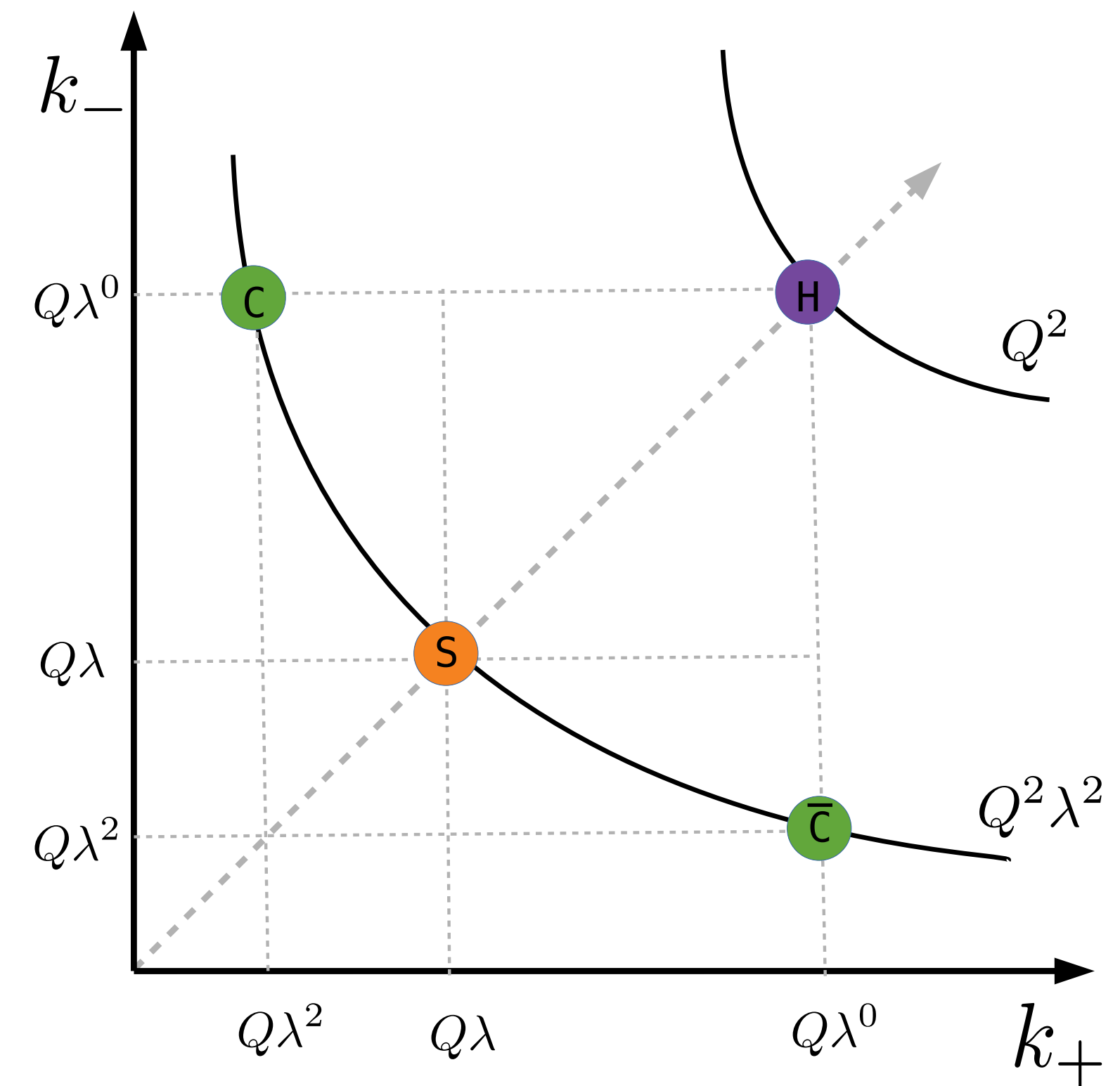
- Soft region: $k_S^\mu \sim (\lambda,\lambda,\lambda) Q$

- Soft and collinear modes have same virtuality

⇒ Additional rapidity divergences

- Introduce additional regulator [Becher, Bell; 12]

$$\prod_i \int \frac{d^d k_i}{(2\pi)^d} \left(\frac{\nu}{k_i^+ + k_i^-} \right)^\alpha \delta(k_i^2) \theta(k_i^0)$$



Factorisation

- Generic structure for global observables (without CVL)

$$d\sigma = \sum_{i,j,k} B_{i/h}(\mu) \otimes B_{j/h}(\mu) \otimes J_k \otimes \text{Tr} \left[H_{ij \rightarrow k}(\mu) \cdot S_{ij \rightarrow k}(\mu) \right]$$

- Hard functions: $H_{ij \rightarrow k}(\mu)$
 - QCD-SCET matching $\rightarrow$ need virtual amplitudes with full color information

Factorisation

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- Hard functions: $H_{ij \rightarrow k}(\mu)$
- Soft functions: $S_{ij \rightarrow k}(\mu)$
 - Vacuum matrix elements of Wilson lines
 - Public code for NNLO soft functions with two massless legs [Bell, Rahn, Talbert; 18,20]
 - Extension for arbitrary number of massless legs [Bell, Dehnadi, Mohrmann, Rahn; 23]
 - NNLO soft function for massive partons [Bell, Broggio, Edelmann, Lim, Rahn; 26]



Factorisation

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- Hard functions: $H_{ij \rightarrow k}(\mu)$
- Soft functions: $S_{ij \rightarrow k}(\mu)$
- Beam functions: $B_{i/h}(\mu)$
 - Proton matrix elements of collinear leading-twist operators
 - Perturbative matching kernels from matching onto pdfs
 - Automated NNLO framework for all partonic channels [Bell, KB, Das, Wald; 22,24]

Factorisation

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- Hard functions: $H_{ij \rightarrow k}(\mu)$
- Soft functions: $S_{ij \rightarrow k}(\mu)$
- Beam functions: $B_{i/h}(\mu)$
- Jet functions: $J_k(\mu)$
 - Vacuum matrix elements of collinear leading-twist operators → [this talk](#)
 - Quark jet function → talk by Bell (LL 2026) [Bell, KB, Das, Wald; 21][KB; 22]
 - Transverse-momentum-like variables → talk by Giovanbattista

Gluon jet functions

- Definitions

$$g_s^2 J_g^{\mu\nu}(\tau, \mu) = \frac{Q}{\pi} \sum_X (2\pi)^d \delta\left(Q - \sum_i k_i^-\right) \delta^{d-2}\left(\sum_i k_i^\perp\right) \langle 0 | \mathcal{A}_\perp^\mu | X \rangle \langle X | \mathcal{A}_\perp^\nu | 0 \rangle \mathcal{M}(\tau; \{k_i\})$$

- ▶ **Collinear matrix element** corresponds to timelike splitting functions
- ▶ **Phase-space constraints** fix jet energy and jet axis
- ▶ **Generic measurement function** $\mathcal{M}(\tau; \{k_i\})$
- Structure of divergences is independent of the observables
 - ▶ Isolate singularities with universal phase-space parametrisation
 - ▶ Compute observable-dependent integrations numerically

Gluon jet functions

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- Decompose in two Lorentz structures

$$J_g^{\mu\nu}(\tau, \mu) = -g_\perp^{\mu\nu} J_g^T(\tau, \mu) + \left[g_\perp^{\mu\nu} + (d-2) \frac{x_\perp^\mu x_\perp^\nu}{x_T^2} \right] J_g^L(\tau, \mu)$$

$x_\perp^\mu$ external reference vector

$$J_g^T(\tau, \mu) = \frac{g_\perp^{\mu\nu}}{2-d} J_{g,\mu\nu}(\tau, \mu) \qquad J_g^L(\tau, \mu) = \frac{1}{(d-2)(d-3)} \left[g_\perp^{\mu\nu} + (d-2) \frac{x_\perp^\mu x_\perp^\nu}{x_T^2} \right] J_{g,\mu\nu}(\tau, \mu)$$

Approach at NLO

- Parametrisation:

$$k_- = zQ, \quad p_- = \bar{z}Q$$

$$|\vec{k}_\perp| = |\vec{p}_\perp| = k_T$$

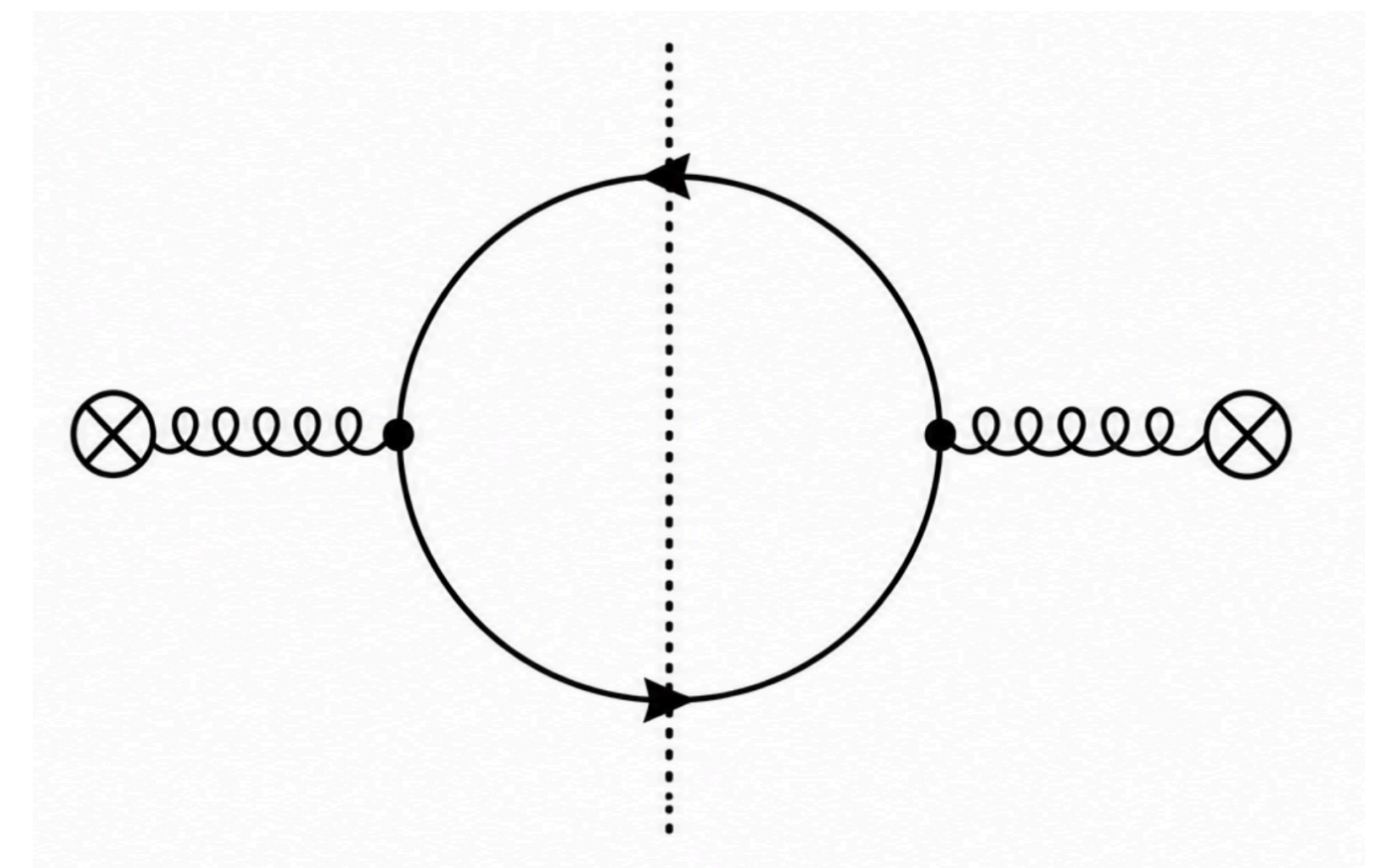
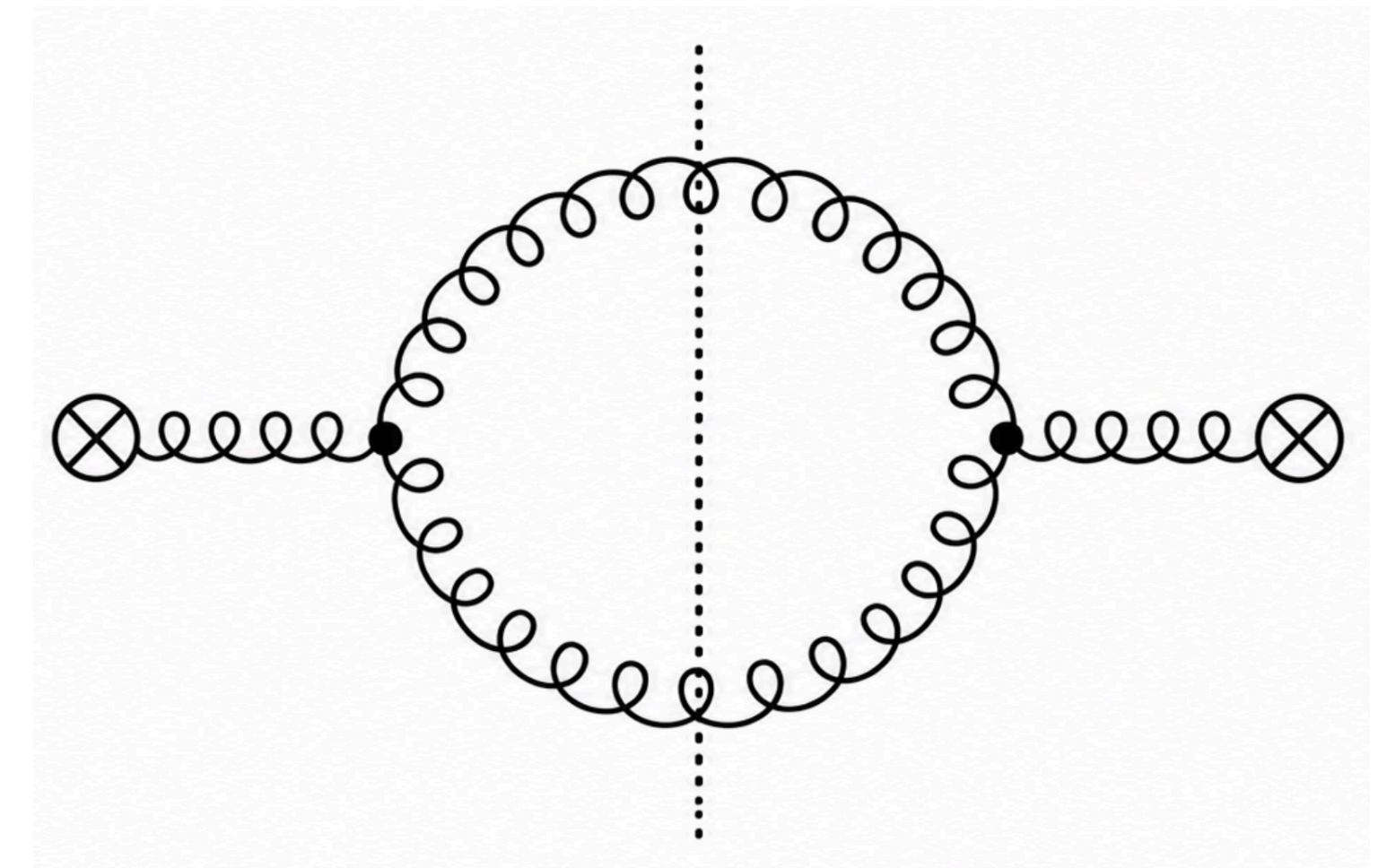
$$\cos \theta_k = 1 - 2t_k$$

- Measurement function:

$$\mathcal{M}_1(\tau; k, p) = \exp \left[-\tau k_T^m \left(\frac{k_T}{Q} \right)^n \Omega_1^{-\frac{m+n}{1+n}n} f(z, t_k) \right]$$

$$\Omega_1 = \begin{cases} z(1-z) & \text{for } C_A \\ 1 & \text{else} \end{cases}$$

- Ω_1 is chosen to ensure $f(z, t_k)$ finite in critical limits



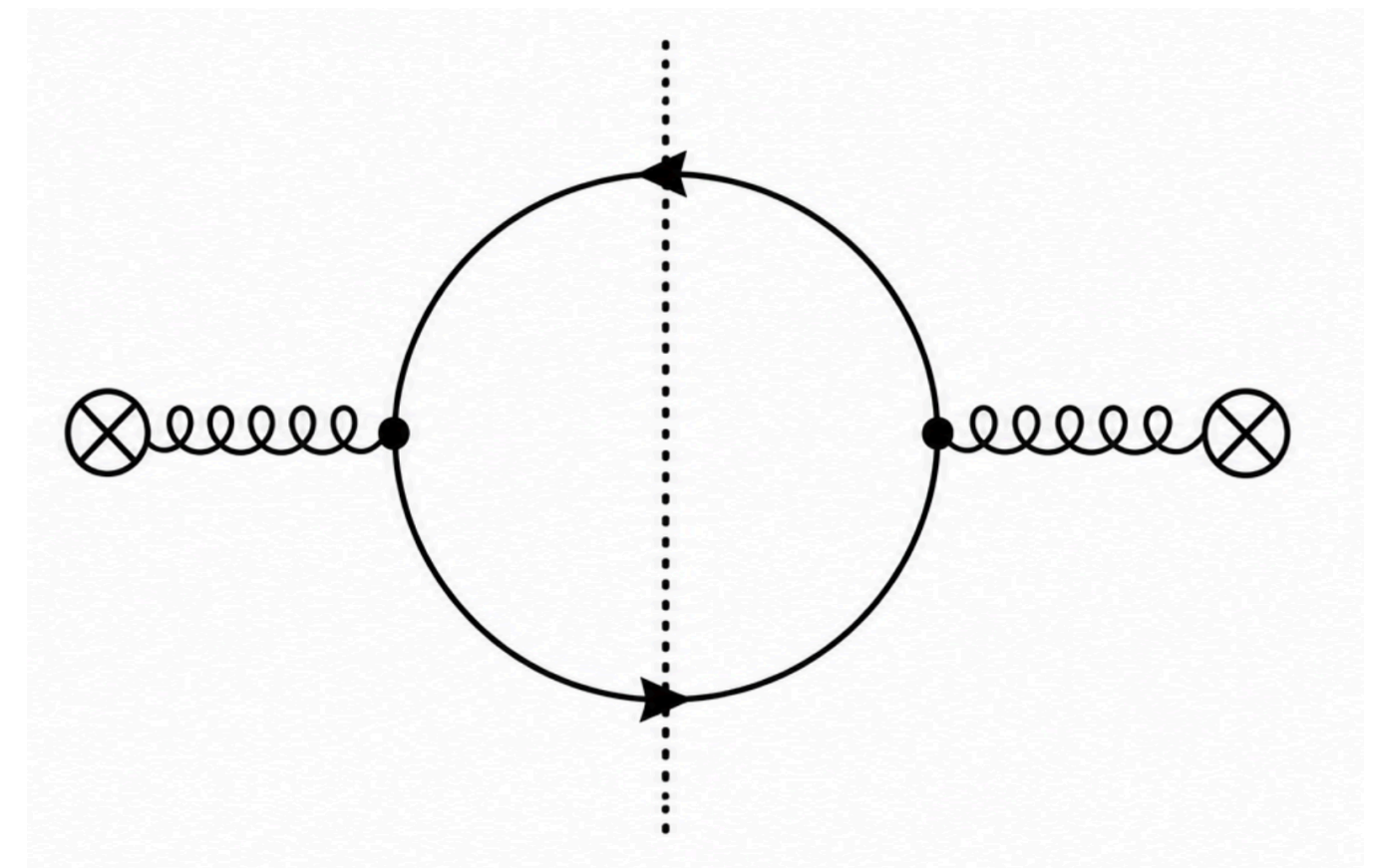
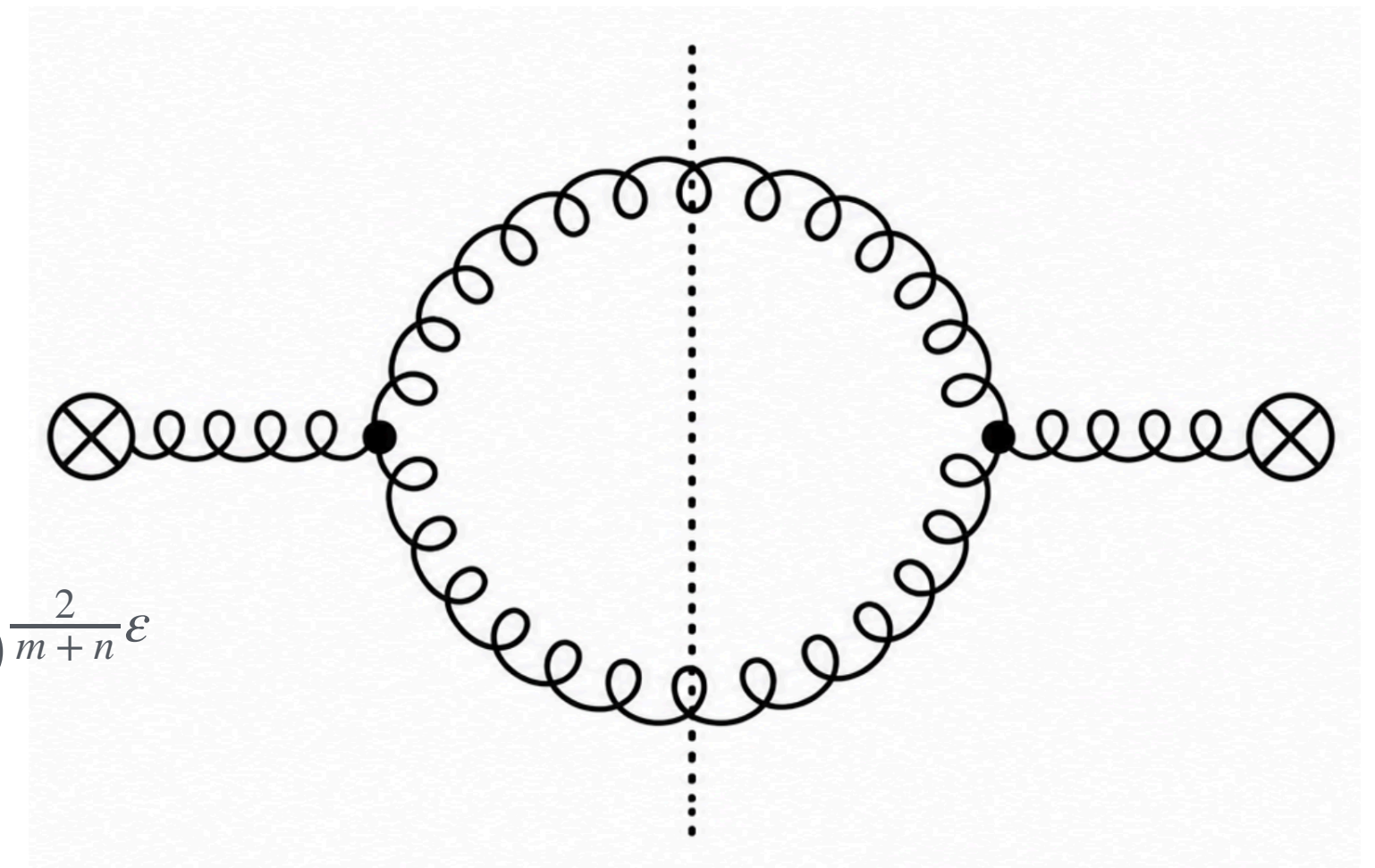
Approach at NLO

- Master formulas:

$$J_{g \rightarrow gg}^{T,1} \sim C_A \Gamma \left(\frac{-2\varepsilon}{m+n} \right) \int_0^1 dz \, (z\bar{z})^{-1-2\frac{n}{m+n}\varepsilon-\alpha} (z\bar{z} P_{g \rightarrow gg}^0) \int_0^1 dt_k (4t_k \bar{t}_k)^{-\frac{1}{2}-\varepsilon} f(z, t_k)^{\frac{2}{m+n}\varepsilon}$$

$$J_{g \rightarrow q\bar{q}}^{T,1} \sim T_F \Gamma \left(\frac{-2\varepsilon}{m+n} \right) \int_0^1 dz \, P_{g \rightarrow q\bar{q}}^0 \int_0^1 dt_k (4t_k \bar{t}_k)^{-\frac{1}{2}-\varepsilon} f(z, t_k)^{\frac{2}{m+n}\varepsilon}$$

$$J_g^{L,1} \sim \Gamma \left(\frac{-2\varepsilon}{m+n} \right) \int_0^1 dz \, (z\bar{z}) \int_0^1 dt_k (4t_k \bar{t}_k)^{-\frac{1}{2}-\varepsilon} (1 - 2\bar{\varepsilon}(1 - 4t_k \bar{t}_k)) f(z, t_k)^{\frac{2}{m+n}\varepsilon}$$



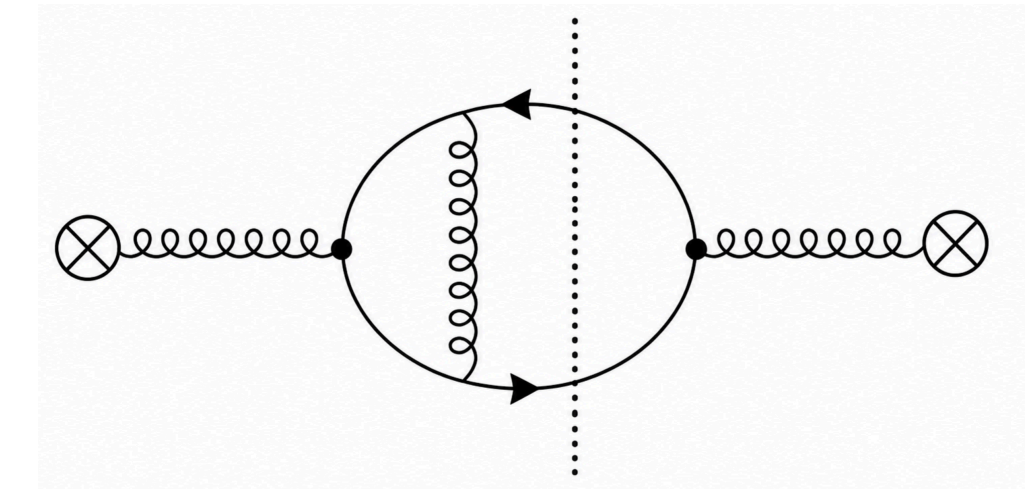
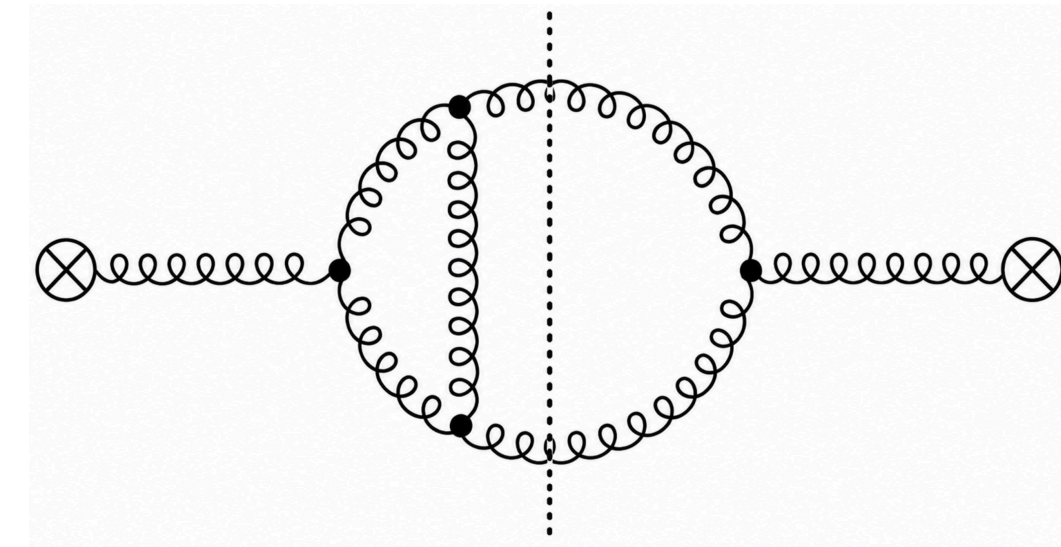
- Additional regulator is needed for $n = 0$ (SCET-II)

Approach at NNLO

- At NNLO we need to consider two different contributions :

1. Real-Virtual contribution

- ▶ Matrix element: NLO collinear splitting functions
- ▶ Same parametrisation as NLO
- ▶ Divergency structure same as NLO + $\mathcal{O}(\varepsilon^{-2})$ effect



Approach at NNLO

- At NNLO we need to consider two different contributions :

1. Real-Virtual contribution

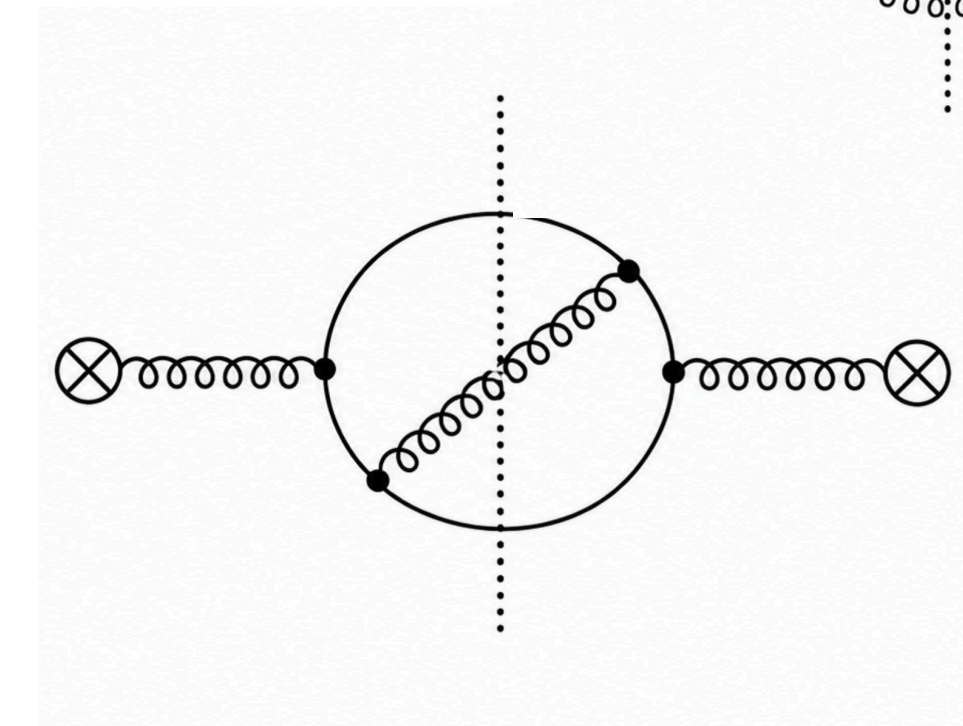
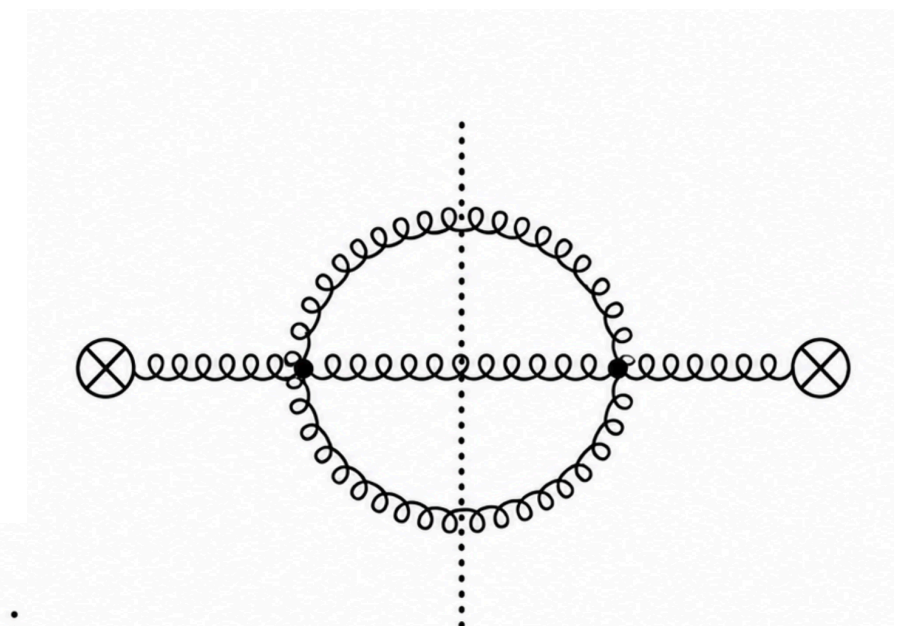
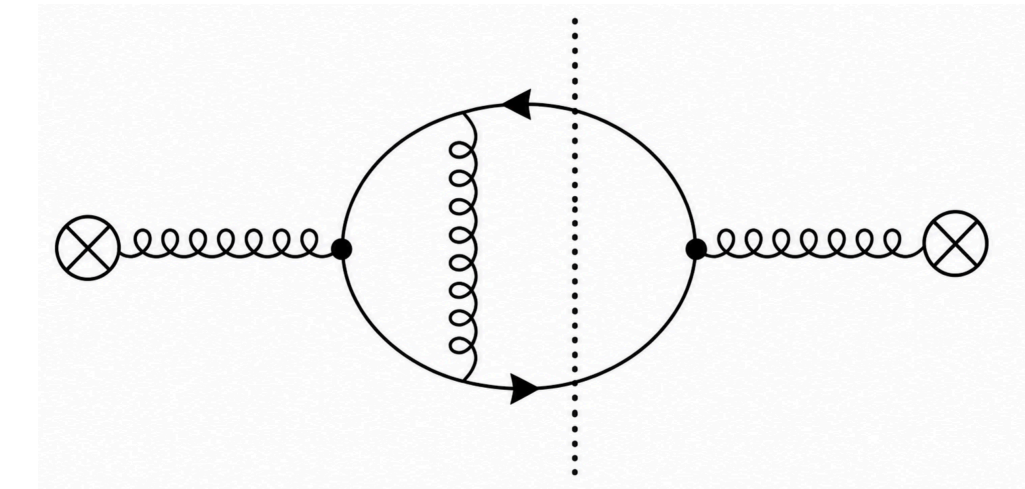
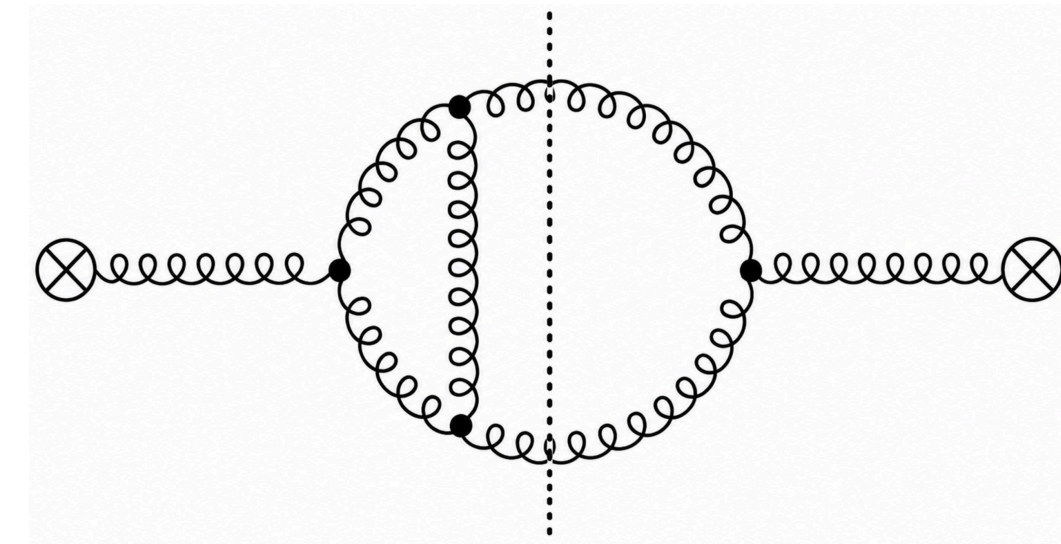
- ▶ Matrix element: NLO collinear splitting functions
- ▶ Same parametrisation as NLO
- ▶ Divergency structure same as NLO + $\mathcal{O}(\varepsilon^{-2})$ effect

2. Real-Real contribution

- ▶ Matrix element: LO triple collinear splitting functions

- ▶ Parametrisation: $q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}, \quad z = \frac{k_- + l_-}{Q}$

$$a = \sqrt{\frac{k_- l_+}{k_+ l_-}}, \quad b = \sqrt{\frac{k_- k_+}{l_- l_+}} \quad \theta_k, \theta_l, \theta_{kl} \rightarrow \cos(\theta_k) = 1 - 2t_k, \text{ etc}$$



Approach at NNLO (RR)

- Measurement function:

$$\mathcal{M}_2(\tau; k, l, p) = \exp \left[-\tau q_T^{\textcolor{red}{m}} \left(\frac{q_T}{Q} \right)^{\textcolor{red}{n}} \Omega_2^{-\frac{m+n}{1+n} \textcolor{red}{n}} \textcolor{red}{F}(z, a, b, t_{kl}, t_k, t_l) \right]$$

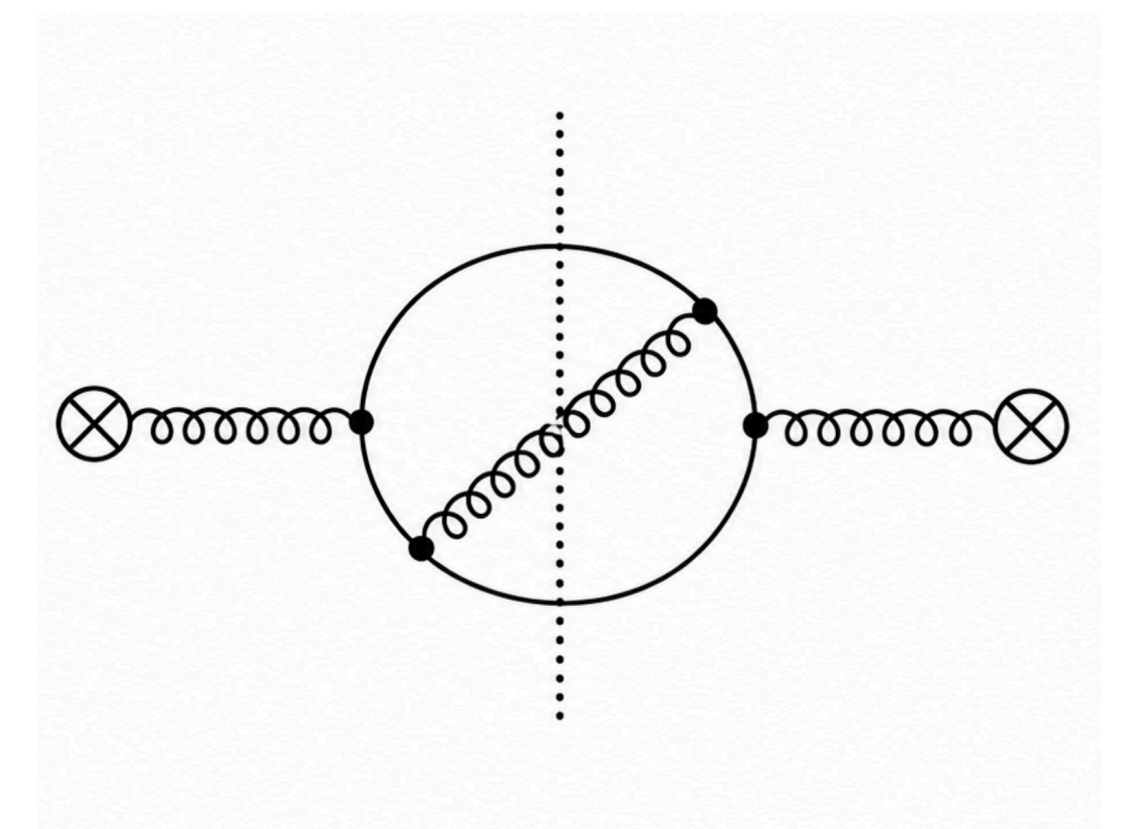
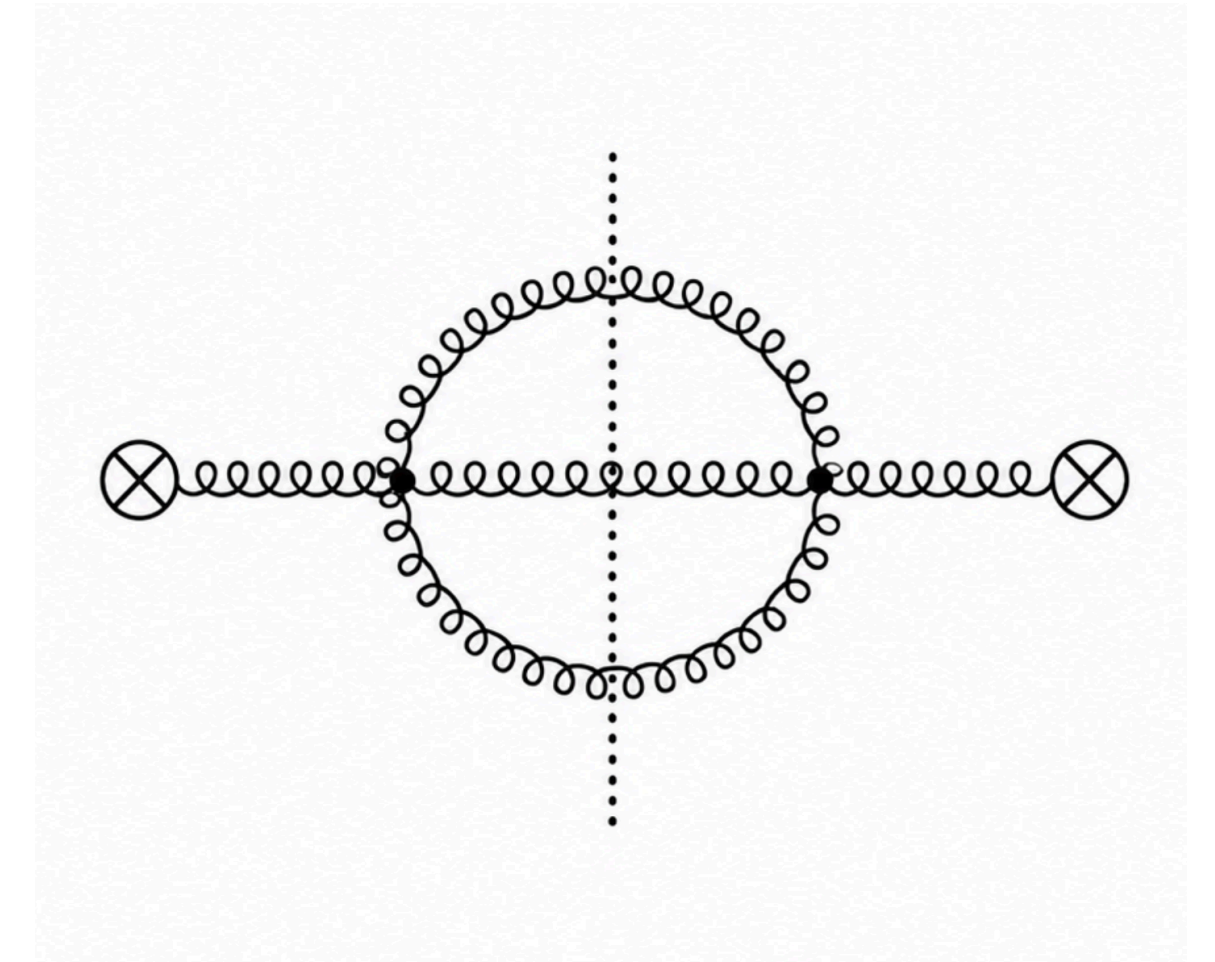
- Ω_2 depends on the specific divergency structure

- Many overlapping singularities remain

- Sector decomposition
- Selector functions
- Non-linear transformations

} All singularities factorised

- Further n -dependent rescaling to keep F finite $\rightarrow$ $\mathcal{O}(110)$ for $J_g^{T,2}$ sectors
 $\mathcal{O}(50)$ for $J_g^{L,2}$ sectors



SCET-I renormalisation

- Jet functions follow the RGE

$$L = \ln \left(\frac{\mu \bar{\tau}^{\frac{1}{1+n}}}{Q^{\frac{n}{1+n}}} \right)$$

$$\frac{dJ_g^{T,L}(\tau, \mu)}{d \ln \mu} = 2 \left[\frac{1+n}{n} \Gamma_{\text{Cusp}} L + \gamma_J \right] J_g^{T,L}(\tau, \mu)$$

- One-loop solution

$$J_g^T(\tau, \mu) = 1 + \left(\frac{\alpha_s}{4\pi} \right) \left[\frac{1+n}{n} \Gamma_0 L^2 + 2\gamma_0 L + c_1^T \right] + \mathcal{O}(\alpha_s^2)$$

$$J_g^L(\tau, \mu) = \left(\frac{\alpha_s}{4\pi} \right) [c_1^L] + \mathcal{O}(\alpha_s^2)$$

- Extraction of γ_J and $c^{T,L}$ at NNLO

SCET-II renormalisation

- Additional scale $\nu \rightarrow$ Additional large logarithms
 - Resum via collinear anomaly approach [Becher,Neubert; 13]

$$\left[J_g^{T,L}(\tau, \mu, \nu) \bar{J}_g^T(\tau, \mu, \nu) S(\tau, \mu, \nu) \right] \underset{\alpha=0}{\equiv} (Q^2 \bar{\tau})^{-F_g(\tau, \mu)} W_g^{T,L}(\tau, \mu)$$

- Anomaly exponent $F_g(\tau, \mu)$ and remainder $W_g^{T,L}(\tau, \mu)$ fulfil RGEs

$$\frac{dF_g(\tau, \mu)}{d \ln \mu} = 2\Gamma_{\text{Cusp}} \quad \frac{dW_g^{T,L}(\tau, \mu)}{d \ln \mu} = 2 \left[\Gamma_{\text{Cusp}} L - \gamma_H \right] W_g^{T,L}(\tau, \mu)$$

- Extraction of d_g and $c^{T,L}$ at NNLO

Results

- All sectors implemented into pySecDec → Numerical integration via CUBA library

[Heinrich et.al.; 17,18,21,23]

[Hahn; 05,16]

SCET-I

- ▶ Thrust

[Becher, Bell; 10]

- ▶ Transverse Thrust

SCET-II

- ▶ Jet broadening (recoil-free)

- ▶ WTA transverse-momentum resummation

- Decomposition

$$c_2 = c_2^{T_F} (T_F n_f)^2 + c_2^{C_F} (C_F T_F n_f) + c_2^{C_A} (C_A T_F n_f) + c_2^{C_A^2} (C_A)^2$$

- For SCET-II observables we obtain soft function via SoftSERVE

Thrust [Farhi; 77]

- Relevant for inclusive B-decays, jettiness slicing,....

- Input values: $n = m = 1, \quad F(z, a, b, t_{kl}, t_k, t_l) = \frac{1}{z} + \frac{a(\bar{b}^2 + 4b\bar{t}_{kl})}{\bar{z}(1 + ab)(a + b)}$

- Individual sectors at sub-percent accuracy

	$c_2^{T_F}$	$c_2^{C_F}$	$c_2^{C_A}$	$c_2^{C_A^2}$
This work	2.014 ± 0.001	0.904 ± 0.050	-13.727 ± 0.069	3.195 ± 0.168
[Becher, Bell; 10]	2.014	0.900	-13.725	3.197

Transverse Thrust

- Similar to standard thrust [Becher, Garcia i Tormo; 15]
 - Transverse to beam direction
- Input values:

$$n = m = 1, \quad F(z, a, b, t_{kl}, t_k, t_l) = \frac{16s_\theta}{z\bar{z}(1+ab)(a+b)} \left[t_k \bar{t}_k (a\bar{z} + b)ab + t_l \bar{t}_l (a + b\bar{z}) + (t_k \bar{t}_l + t_l \bar{t}_k - t_{kl})abz \right]$$

- s_θ angle between beam and jet axis

	$c_2^{T_F}$	$c_2^{C_F}$	$c_2^{C_A}$	$c_2^{C_A^2}$
c_2^T	7.863 ± 0.001	-47.210 ± 0.116	30.6910 ± 0.192	172.918 ± 0.815
c_2^L	1.185 ± 0.008	21.968 ± 0.050	-17.784 ± 0.106	-2.392 ± 0.049

Winner-Take-All axis

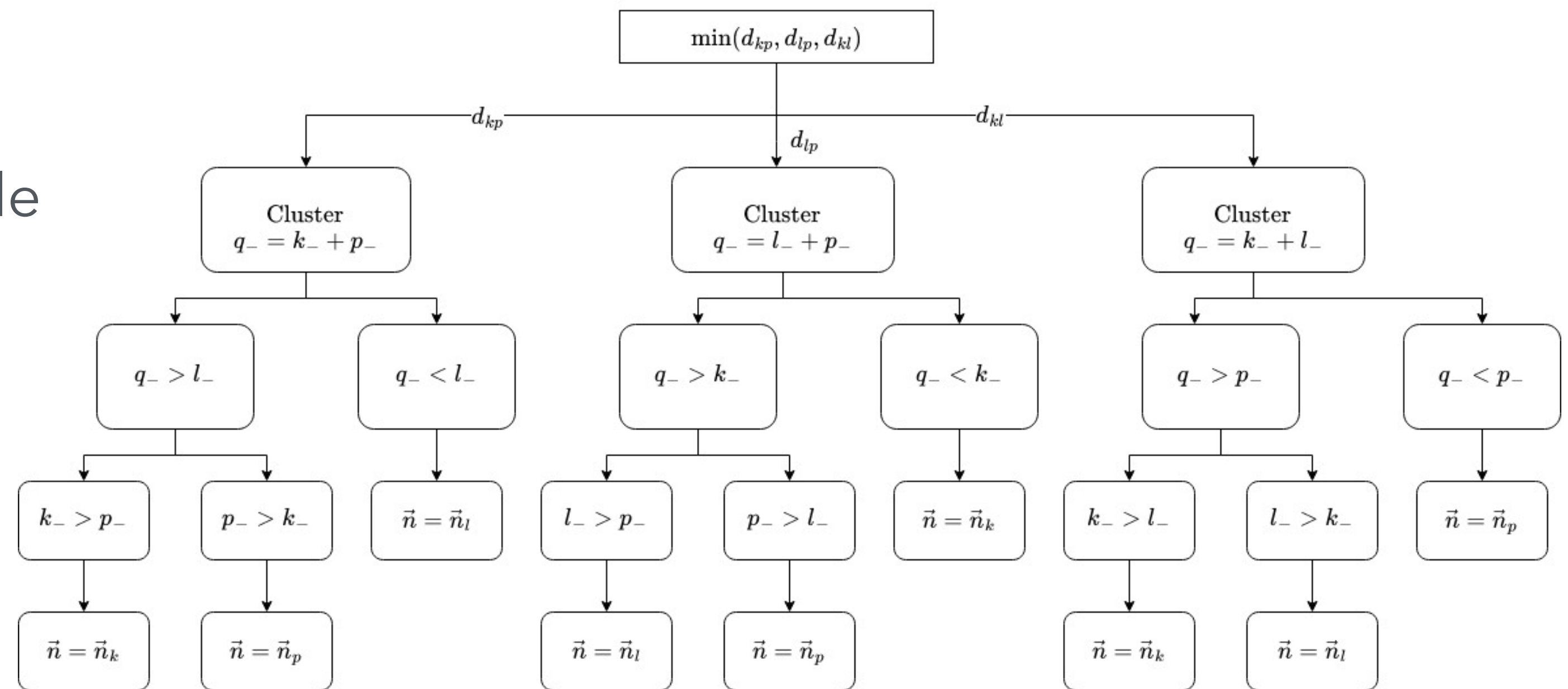
- Jet axis is determined in a pairwise recombination scheme

- k_T -type distance measure $d_{ij} = 2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})$

- θ_{ij} angle between particles

- Finally most energetic particle sets the axis

[Bertolini, Chan, Thaler; 13]



Jet Broadening

- Scalar sum of transverse momenta with respect to winner-take-all axis
- SCET-II observable

	$d_2^{T_F}$	$d_2^{C_F}$	$d_2^{C_A}$	$d_2^{C_A^2}$
This work	0	0	-18.737 ± 0.007	15.976 ± 0.015
[Bell, et al; 18]	0	0	-18.737 ± 0.001	15.981 ± 0.002
	$c_2^{T_F}$	$c_2^{C_F}$	$c_2^{C_A}$	$c_2^{C_A^2}$
This work	17.435 ± 0.001	-85.222 ± 0.058	135.965 ± 0.021	293.955 ± 0.071

WTA transverse-momentum resummation

- Vector sum of transverse momenta with respect to winner-take-all axis
- Azimuthal decorrelation [Chien et al.; 24]

	$d_2^{T_F}$	$d_2^{C_F}$	$d_2^{C_A}$	$d_2^{C_A^2}$
This work	0	0	-8.301 ± 0.026	-3.846 ± 0.151
[Becher, Neubert; 11]	0	0	-8.296	-3.732
	$c_2^{T_F}$	$c_2^{C_F}$	$c_2^{C_A}$	$c_2^{C_A^2}$
c_2^T	-5.961 ± 0.008	170.483 ± 0.055	9.194 ± 0.051	25.126 ± 0.486
c_2^L	1.596 ± 0.004	-2.203 ± 0.028	-1.807 ± 0.036	1.459 ± 0.021

Summary

- Numerical framework for automated computation of SCET ingredients
 - SoftSERVE extension under development
 - Beam-function framework fully developed
 - Presented gluon jet function framework and first results

Summary & Outlook

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 - SoftSERVE extension under development
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Outlook

- Development of JetSERVE and BeamSERVE
 - First tests show large reduction in uncertainty
- Phenomenological applications