

HyperPrecision

High-precision numerical evaluation of multivariate hypergeometric functions

Souvik Bera¹ Sumit Banik²

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S. Banik, S. Bera, *Comput. Phys. Commun.* 328 (2026) 110328, [arXiv:2605.30216](#)

 github.com/HyperPrecision/HyperPrecision

¹Asia Pacific Center for Theoretical Physics, Pohang

²SLAC National Accelerator Laboratory, Stanford University

From Gauss to many variables

Pochhammer symbol: a ratio of two gamma functions,

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = a(a+1)\cdots(a+n-1), \quad (a)_0 = 1, \quad (1)_n = n!.$$

Gauss ${}_2F_1(a, b; c; z)$:

$${}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}, \quad |z| < 1.$$

satisfies a second-order differential equation with singular points at $0, 1, \infty$.

Multivariate generalizations: Appell, Horn, Lauricella, Kampé de Fériet ...

e.g. Appell F_1

$$F_1(a; b_1, b_2; c; x, y) = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n} (b_1)_m (b_2)_n}{(c)_{m+n}} \frac{x^m y^n}{m! n!}, \quad |x| < 1, |y| < 1.$$

satisfies systems of PDEs, singular *curves* instead of points.

Why analytic continuation? Lessons from our earlier work

A *series* defines the function only near the origin; continuation is classical for ${}_2F_1$, largely **open** beyond.

- **MBConicHulls** [Ananthanarayan, Banik, Friot, Ghosh 2012.15108], **triangulations** [Banik, Friot 2309.00409]: MB integral $\rightarrow$ sets of multiple series. **White zones** where none converges.
- **Olsson.wl**, **ROC2.wl** [Ananthanarayan, Bera, Friot, Pathak 2201.01189]: transformation theory, regions of convergence (two variables), **case by case**.
- **MultiHypExp** [Bera 2208.01000, 2306.11718]: analytic ϵ -expansion in multiple polylogarithms. **Restricted** parameter classes, continued via MPLs.
- **AppellF2.wl** [Ananthanarayan, Bera, Friot, Marichev, Pathak 2111.05798]; **AppellF1.wl**, **AppellF3.wl**, **LauricellaFD.wl** [$F_D^{(3)}$], **LauricellaSaranFS.wl** [$F_S^{(3)}$] [Bera, Pathak 2403.02237]: explicit continuations covering the real plane, with numerics. **One function per package**; singular lines excluded.

Lesson: continuations by hand, one function at a time; coverage incomplete; ϵ -expansion separate.

Goal: automatic, numerical: *any* complete Horn-type series, *any* real point, with ϵ -expansion.

Closest in spirit: PrecisionLauricella systems **hard-coded** ($F_A, F_B, F_D, n \leq 3$), HAPC [Bezuglov, Kniehl, Onishchenko, Veretin 2502.07935, 2605.31553].

Where do these functions appear?

Feynman integrals

One-loop with masses $\rightarrow F_1, F_D^{(3)}, F_S^{(3)}$;
bubble $\rightarrow$ Appell F_4 ; N -loop banana $\rightarrow 2^N$
Lauricella $F_C^{(N)}$;
in general MB representation or GKZ
system $\rightarrow$ Horn-type series

Angular integrals in $D = 4 - 2\epsilon$

Two massive denominators $\rightarrow$ Lauricella
 $F_B^{(3)}$ [Lyubovitskij et al. 2102.08943]; massless
 $\Omega_{1,1,1} \rightarrow$ five Horn-type triple series
[Somogyi 1101.3557]

Cosmological correlators

The in-in time integrals over de Sitter mode functions;
bootstrap equations and Mellin–Barnes give hypergeometric
functions of momentum ratios [Arkani-Hamed et al. 1811.00024;
Sleight 1906.12302; Chen, Feng, Tao 2411.03088]. Massive
exchange: single $\rightarrow {}_2F_1, {}_3F_2$; double $\rightarrow F_4(\dots; u^2, v^2) +$
Kampé de Fériet, $u + v > 1$ [Aoki et al. 2404.09547]

Holographic correlators

Witten diagrams: exchange $\rightarrow D$ -functions = Appell F_4 of
cross-ratios; conformal blocks in general $d \rightarrow$ Appell / Kampé
de Fériet [Bissi, Sinha, Zhou 2202.08475]. Non-conformal
 Dp -branes: 3-point functions $\rightarrow$ four $F_4(\dots; A, B)$,
 $\sqrt{A} + \sqrt{B} \geq 1$ [Bobev et al. 2503.18770]

A concrete problem

The one-loop bubble with masses $m_1, m_2 = 1$ has a closed form in Appell F_4 :

$$I_{1,1}^{\text{bub}}(p^2, m_1^2, 1) = z_1^{1-\epsilon} \Gamma(\epsilon - 1) F_4(1, 2 - \epsilon; 2 - \epsilon, 2 - \epsilon; z_1, z_2) \\ + \frac{\Gamma(\epsilon) \Gamma(1-\epsilon)}{\Gamma(2-\epsilon)} F_4(1, \epsilon; \epsilon, 2 - \epsilon; z_1, z_2)$$

with $z_1 = m_1^2, z_2 = p^2$, and

$$F_4(a, b; c_1, c_2; x, y) = \sum_{m,n \geq 0} \frac{(a)_{m+n} (b)_{m+n}}{(c_1)_m (c_2)_n} \frac{x^m y^n}{m! n!} \quad \text{converges only for } \sqrt{|x|} + \sqrt{|y|} < 1.$$

So the series is useless where we need it

above threshold, $p^2 > (m_1 + 1)^2$, we have $\sqrt{p^2} + m_1 > 1$: the physical region lies entirely **outside** the convergence domain, and we also want the ϵ -expansion.

The same story for every function on the previous slide: *the physics lives where the series does not converge.*

This talk

a tool that evaluates *any* such series anywhere on the real axis, automatically, including the ϵ -expansion.

Multivariate hypergeometric functions (MHFs)

Generalizing Gauss ${}_2F_1$:

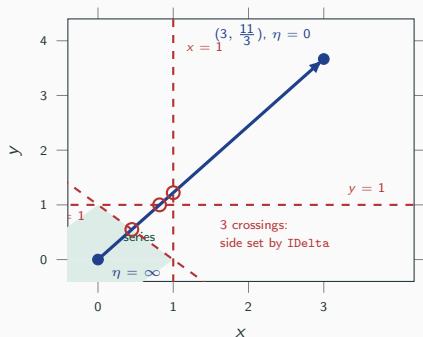
$$F(\mathbf{a}; \mathbf{b}; \mathbf{x}) = \sum_{\mathbf{m} \in \mathbb{N}_0^p} \frac{(\mathbf{a})_{\boldsymbol{\mu} \cdot \mathbf{m}}}{(\mathbf{b})_{\boldsymbol{\nu} \cdot \mathbf{m}}} \frac{\mathbf{x}^{\mathbf{m}}}{\mathbf{m}!}, \quad (a)_k = \frac{\Gamma(a+k)}{\Gamma(a)}$$

$\mathbf{a} \in \mathbb{C}^p$, $\mathbf{b} \in \mathbb{C}^q$ are the Pochhammer parameters; the integer matrices $\boldsymbol{\mu}$, $\boldsymbol{\nu}$ fix the family: Appell $F_1, \dots, F_4$; Horn $G_1, \dots, G_3$, $H_1, \dots, H_7$; Lauricella $F_A^{(n)}, F_B^{(n)}, F_C^{(n)}, F_D^{(n)}$; Kampé de Fériet; Lauricella - Saran; ...

Convergence (Horn's theorem): a neighbourhood of the origin, e.g. F_2 : $|x| + |y| < 1$; F_4 : $\sqrt{|x|} + \sqrt{|y|} < 1$; $F_B^{(3)}$: $|x_i| < 1$. In the physical applications, the arguments sit **outside** these regions, often with $|x_i| \gg 1$.

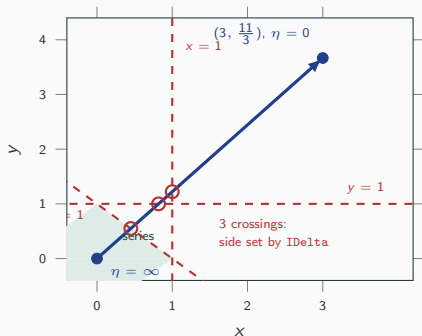
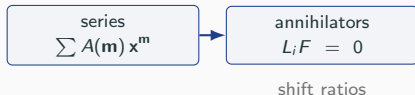
The idea: the DE method for Feynman integrals, applied to the function itself

$$\text{series} \\ \sum A(\mathbf{m}) x^{\mathbf{m}}$$



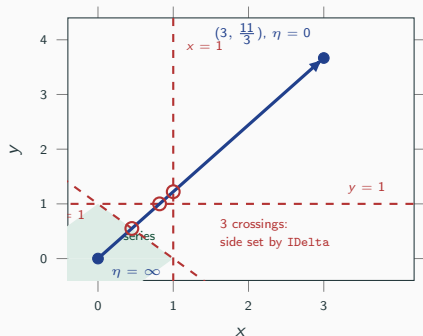
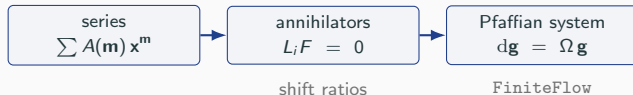
- Horn-type MHFs are *holonomic*: a finite-rank Pfaffian system exists
- on a line it becomes an ODE: solve numerically, as for Feynman integrals [DiffExp, AMFlow, SeaSyde, LINE ...]
- real singular curves on the way: side fixed by IDelta
- ϵ -expansion from an ϵ -grid by interpolation

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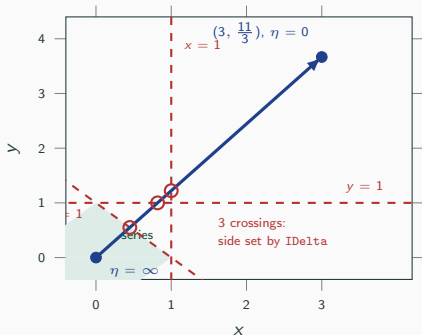
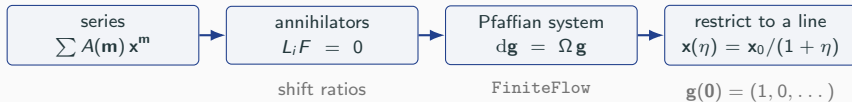
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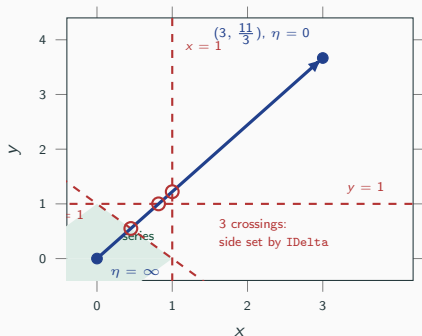
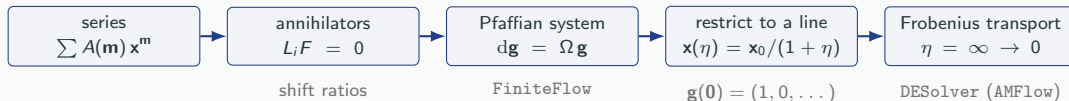
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Key point: everything is derived *on the fly* from the series, no hand-coded Pfaffian systems.

Ratios of neighbouring coefficients are rational in the indices:

$$P_i(\mathbf{m}) = \frac{A(\mathbf{m} + \mathbf{e}_i)}{A(\mathbf{m})} = \frac{g_i(\mathbf{m})}{h_i(\mathbf{m})}, \quad \text{complete if } \Phi_i(\mathbf{m}) = \lim_{t \rightarrow \infty} P_i(t\mathbf{m}) \text{ is finite and non-zero.}$$

HyperPrecision handles complete Horn-type series (Appell, Lauricella, Kampé de Fériet, most Feynman-integral MHFs). The series is annihilated by

$$L_i = h_i(\theta) \frac{1}{x_i} - g_i(\theta), \quad \theta_i = x_i \partial_{x_i} \quad (\text{Euler operators}).$$

$$\text{Appell } F_2(a, b_1, b_2; c_1, c_2; x, y) = \sum_{m,n} \frac{(a)_{m+n} (b_1)_m (b_2)_n}{(c_1)_m (c_2)_n} \frac{x^m y^n}{m! n!}$$

$$P_1 = \frac{(b_1 + m)(a + m + n)}{(m + 1)(c_1 + m)}, \quad \Phi_1 = \frac{m + n}{m} \quad (\text{complete}), \quad L_2 : x \leftrightarrow y, \quad b_1 \leftrightarrow b_2, \quad c_1 \leftrightarrow c_2$$

$$L_1 = -ab_1 + (c_1 - x(a + b_1 + 1))\partial_x - b_1 y \partial_y - xy \partial_x \partial_y - (x - 1)x \partial_x^2$$

Vector of F and finitely many θ -derivatives, length $r = \text{holonomic rank}$:

$$d\mathbf{g} = \Omega \mathbf{g}, \quad \Omega = \sum_{i=1}^n \Omega_i dx_i, \quad \mathbf{g} = (F, \theta_i F, \theta_i \theta_j F, \dots)^T$$

Ω_i rational in $\mathbf{x}$ and the parameters; integrable, $d\Omega + \Omega \wedge \Omega = 0$.

Appell F_2 : rank 4

$$\mathbf{g} = (F_2, \theta_x F_2, \theta_y F_2, \theta_x \theta_y F_2)^T$$

$$\Omega_1 = \begin{pmatrix} 0 & \frac{1}{x} & 0 & 0 \\ * & * & * & * \\ 0 & 0 & 0 & \frac{1}{x} \\ * & * & * & * \end{pmatrix}$$

poles at $x = 0$, $x = 1$, $x + y = 1$ full matrices in
backup

Derivative basis: parameters only in numerators

$\Rightarrow \epsilon$ never enters denominators of Ω_i
 $\Rightarrow$ singular locus in η is ϵ -independent
 $\Rightarrow$ stable transport, clean ϵ -reconstruction
 Observed for every MHF we tried; only simple poles so far.

Ranks: $F_A^{(n)}, F_B^{(n)}, F_C^{(n)}: 2^n; \quad F_D^{(n)}: n + 1$.

Step 1: the Pfaffian system from FiniteFlow, not from Gröbner bases

1. differentiate $\{L_i F = 0\}$ repeatedly $\rightarrow$ large linear system among derivatives of F
2. finite rank $\Rightarrow$ higher derivatives reduce to a finite set of *master* derivatives = basis $\mathbf{g}$
3. reduce $\partial_{x_j} \mathbf{g}$ onto $\mathbf{g} \Rightarrow$ entries of Ω_j

Linear algebra **over finite fields** with

FiniteFlow, then rational reconstruction:
exact Ω_i , no expression swell.

[Peraro 1608.01902, 1905.08019]

[cf. Chestnov et al. 2204.12983; Henn et al. 2303.11105]

Time to build the connection matrices

	rank	HyperPrec.	HolonomicF.	M2
${}_2F_1$	2	0.11 s	0.0012 s	0.015 s
F_1	3	0.15 s	0.036 s	0.2 s
$F_D^{(3)}$	4	0.3 s	0.6 s	52 s
$F_D^{(4)}$	5	1.4 s	6 s	> 24 h
$F_D^{(5)}$	6	25 s	73 s	—

HolonomicF.: HolonomicFunctions, OreGroebnerBasis

step only; M2: ConnectionMatrices in Macaulay2.

Single target point (default)

PreRestriction $\rightarrow$ **True**: build the restricted 1-d matrix $M(\eta)$ directly, never the full Ω . Cheaper at high rank.

Step 2: transport along a straight contour

Boundary: analytic, at the origin

$$F(\mathbf{0}) = 1, \quad \theta_i F|_0 = 0 \Rightarrow \mathbf{g}(\mathbf{0}) = (1, 0, \dots, 0)^T$$

Contour and ODE

$$x_i(\eta) = \frac{x_{0,i}}{1 + \eta} : \quad \eta = \infty \text{ (origin)} \rightarrow \eta = 0 \text{ (target)}$$

$$\frac{d\mathbf{g}}{d\eta} = M(\eta) \mathbf{g}, \quad M(\eta) = - \sum_i \frac{x_{0,i}}{(1 + \eta)^2} \Omega_i(\mathbf{x}(\eta))$$

As in auxiliary mass flow: boundary at $\eta = \infty$,
answer at $\eta = 0$ [Liu, Ma, Wang 1711.09572]

Solver: DESolver from AMFlow

- Frobenius series around each singular point of $M(\eta)$, matched in overlapping discs; truncation set by the requested precision
- singular point on the contour: side chosen by $\text{IDelta} = \pm i$
- target *on* the singular locus: asymptotic expansion (warning issued)

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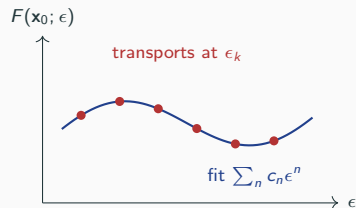
Step 3: Laurent expansion in ϵ by interpolation

Parameters linear in ϵ , e.g. $F_4(1, 2 - \epsilon; 2 - \epsilon, 2 - \epsilon; z_1, z_2)$.

No symbolic expansion of the system; instead

1. choose an ϵ -grid $\{\epsilon_k\}$ (size from order and precision)
2. one transport per ϵ_k : independent, **parallel** sub-kernels
3. fit $\sum_n c_n \epsilon^n$ through the samples (EpsFit[] of DESolver); poles $1/\epsilon^k$ included

[as in AMFlow and PrecisionLauricella]



Needs FiniteFlow, bundles DESolver (from AMFlow). Target $F_2(2, \frac{3}{2}, 1 + \epsilon; 4, -1 - \epsilon; 3, \frac{11}{3})$:

```
SetDirectory["/path/to/HyperPrecision"]; Get["HyperPrecision`"]
F2Par = {a → 2, b1 → 3/2, b2 → 1 +  $\epsilon$ , c → 4, d → -1 -  $\epsilon$ }; F2Var = {x → 3, y → 11/3};
F2Series = (Pochhammer[a, m + n] Pochhammer[b1, m] Pochhammer[b2, n] x^m y^n)/(Pochhammer[c, m]
  Pochhammer[d, n] m! n!);

(* 1: any series *)      HypExpand[F2Series, F2Par, F2Var, { $\epsilon$ , 1}, 10]
(* 2: predefined head *) HypFunctionExpand[AppellF2[2, 3/2, 1 +  $\epsilon$ , 4, -1 -  $\epsilon$ , 3, 11/3], { $\epsilon$ , 1}, 10]
(* 3: inspect first *)  F2Pfaffian = FindPfaffianSystem[F2Series, {m, n}, {x, y}];
                        TransportDE[F2Pfaffian, F2Par, F2Var, { $\epsilon$ , 1}, 10]

Out[] = 0.5149686376/ $\epsilon$  + (0.528662817 - 4.194390019 I) - (10.978138236 + 4.834942296 I)  $\epsilon$ 
```

Options: `IDelta` = $\pm I$, side of the cut when the contour crosses a real singularity (default $-I$ gives the complex value above); `PreRestriction`: $M(\eta)$ directly (default) or the full Ω , reusable for many targets; `ParallelRun`, `WPrecision`, `SummedBoundary`, `VerboseMode`.

How we tested it

test	functions	reference	
other packages	$F_1, F_2, F_3; F_A^{(3)}, F_B^{(3)}, F_D^{(3)}, F_S^{(3)}$	PrecisionLauricella; C++ Bulirsch–Stoer; AppellF1/F2/F3.wl; LauricellaFD/FS.wl	✓
Feynman integrals	bubble (F_4), sunset ($F_C^{(3)}$)	AMFlow	✓
reduction formulae	$F_1, \dots, F_4, F_D^{(3)}, F_S^{(3)}$ at special parameters	closed forms: log, $1/\sqrt{\cdot}$	✓
connection formulae	Euler transformation of ${}_2F_1; F_3 \rightarrow F_2$	IDelta flipped on the transformed side	✓
	$F_3 \rightarrow F_1$, one argument transformed	needs complex arguments	✗
singular points	F_1 on $x = y$; F_4 on $\sqrt{x} + \sqrt{y} = 1$; $F_C^{(3)}$ Bessel moments	$1/(1-x)$; $\pi/\Gamma(\frac{3}{4})^4$; elliptic K	✓
physics applications	angular integrals; bispectrum shape; Dp-brane 3-pt	Lyubovitskij, Ahmed, Haug et al.; CosmoFlow; Bobev et al.	✓

Agreement to all requested digits wherever the reference returns a result. The ✗ is expected: a limitation of real-argument transport.

Angular integrals I: two massive denominators

$$\Omega_{j_1 \dots j_n} = \int d\Omega_{D-1}(q) \prod_i (p_i \cdot q)^{-j_i}, \quad D = 4 - 2\epsilon \quad [\text{van Neerven; Somogyi 1101.3557; Lyubovitskij et al. 2102.08943}]$$

Two massive denominators $\rightarrow$ one Lauricella $F_B^{(3)}$, $x_1 = 1 - \frac{v_{11}}{v_{12}}$, $x_2 = 1 - \frac{v_{22}}{v_{12}}$, $x_3 = 1 - v_{12}$:

$$\Omega_{j,\ell} = \frac{2\pi v_{12}^{1-j-\ell-\epsilon}}{1-2\epsilon} F_B^{(3)}\left(\frac{j}{2}, \frac{\ell}{2}, \frac{3-j-\ell}{2} - \epsilon; \frac{j+1}{2}, \frac{\ell+1}{2}, \frac{1-j-\ell}{2} - \epsilon; \frac{3}{2} - \epsilon; x_1, x_2, x_3\right)$$

$j = \ell = 1$, $(v_{11}, v_{12}, v_{22}) = (\frac{1}{3}, \frac{28}{5}, \frac{21}{11}) \Rightarrow (x_1, x_2, x_3) = (\frac{79}{84}, \frac{29}{44}, -\frac{23}{5})$, outside $|x_i| < 1$:

```
HypVal = HypFunctionExpand[LauricellaFB[{1/2, 1/2, 1/2 - \epsilon, 1, 1, -\epsilon}, {3/2 - \epsilon}, {79/84, 29/44, -(23/5)}], {\epsilon, 4}, 30];
Series[(2 \pi)/(1 - 2 \epsilon) v12^(1 - j - 1 - \epsilon) HypVal /. {v12 -> 28/5, j -> 1, 1 -> 1}, {\epsilon, 0, 4}] // Normal
(* = Omega_{1,1} *)
Out[] = 2.98895792833890493764 + 5.02623791539371567057 \epsilon + 9.72395042780000306338 \epsilon^2 + ...
```

Check

Reproduces the analytic $\mathcal{O}(\epsilon)$ result of Lyubovitskij et al. (logs and Li_2 of v_{ij} and $\sqrt{v_{12}^2 - v_{11}v_{22}}$), and gives four more orders for free.

Angular integrals II: three massless denominators, $\Omega_{1,1,1}$

Not a single MHF. Start from the MB representation [Somogyi 1101.3557]

$$\Omega_{1,1,1} = \frac{2^{-1-2\epsilon} \pi^{1-\epsilon}}{\Gamma(-1-2\epsilon)} \int \frac{dz_{12} dz_{13} dz_{23}}{(2\pi i)^3} \Gamma(-z_{12}) \Gamma(-z_{13}) \Gamma(-z_{23}) \Gamma(-2-\epsilon-z_{12}-z_{13}-z_{23}) \\ \times \Gamma(1+z_{12}+z_{13}) \Gamma(1+z_{12}+z_{23}) \Gamma(1+z_{13}+z_{23}) v_{12}^{z_{12}} v_{13}^{z_{13}} v_{23}^{z_{23}}$$

MBConicHulls turns it into **five** Horn-type triple series, $S_k = \sum_{n_i} \frac{(-1)^{n_1+n_2+n_3}}{n_1! n_2! n_3!} \mathcal{T}_k$, e.g.

$$\mathcal{T}_1 = v_{12}^{n_1} v_{13}^{n_2} v_{23}^{n_3} \Gamma(1+n_1+n_2) \Gamma(1+n_1+n_3) \Gamma(1+n_2+n_3) \Gamma(-2-\epsilon-n_1-n_2-n_3),$$

$$\mathcal{T}_2 = v_{12}^{n_1} v_{13}^{n_2} v_{23}^{-2-\epsilon-n_1-n_2+n_3} \Gamma(1+n_1+n_2) \Gamma(2+\epsilon+n_1+n_2-n_3) \Gamma(-1-\epsilon-n_1+n_3) \Gamma(-1-\epsilon-n_2+n_3),$$

and $\mathcal{T}_3, \mathcal{T}_4, \mathcal{T}_5$ of the same type (non-integer powers of v_{ij} , four Γ 's). Each S_k goes through [HypExpand](#) unchanged; the prefactors are ordinary Γ 's.

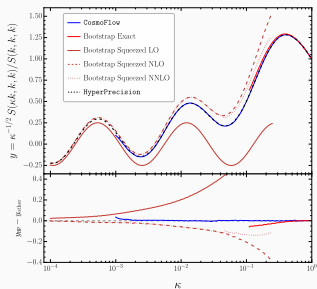
Result

$\Omega_{1,1,1}$ to $\mathcal{O}(\epsilon^{10})$ with 50 digits at random (v_{12}, v_{13}, v_{23}); agrees with Ahmed et al. 2410.18886 through ϵ^2 and with the all-order result of Haug et al. 2508.00693 through ϵ^{10} .

Beyond colliders: same code, no manual continuation

Cosmological bispectrum, double massive exchange

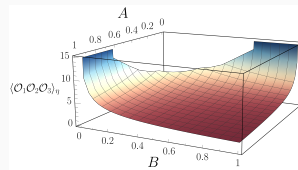
Aoki, Pinol, Sano, Yamaguchi, Zhu 2404.09547



Seed integrals: $F_4(\dots; u^2, v^2) + \text{series}$, physical region $u + v > 1$. As 27 bivariate series, each continued automatically; matches bootstrap-exact and CosmoFlow.

Holographic 3-pt functions, non-conformal Dp-branes

Bobev, Mera Álvarez, Paul 2503.18770



Four $F_4(\dots; A, B)$ with $A = d_{xy}^2/d_{yz}^2$, $B = d_{xz}^2/d_{yz}^2$.
Triangle inequality: $\sqrt{A} + \sqrt{B} \geq 1$, the *whole* physical region is outside convergence.

Summary and outlook

HyperPrecision v1.2

- input: any complete Horn-type series (or a predefined head), real target point, order, digits
- Pfaffian system on the fly with `FiniteFlow`; analytic boundary at the origin; straight-line transport with `DESolver`; ϵ -expansion by interpolation
- tested to six variables; bubble, sunset, angular integrals; many singular points

Limitations and next steps

- **rank drops** at special parameters not handled automatically yet
- cost dominated by the ODE solve in `MATHEMATICA` → native C++ Bulirsch–Stoer integrator
- high precision → analytic reconstruction with `PSLQ` / `LLL`
- `IDelta` is one global choice set by hand → **complex target points**: $p^2 \rightarrow p^2 + i\delta$ picks the branch automatically



Any complete Horn-type series, any real point, any precision. Thank you!

 github.com/HyperPrecision/HyperPrecision, with `Examples.nb`

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