

Recent developments in NNLOCAL

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work done in collaboration with V. Del Duca, C. Duhr, L. Fekésházy, F. Guadagni, P. Mukherjee, F. Tramontano and S. Van Thurenhout, based on 2412.21028, 2512.05192, 2512.24403 (JHEP) and 2601.02033, 2602.24257 (proceedings)

Public tools for fully differential NNLO

Several excellent public tools exist to compute fully differential cross sections up to second order radiative corrections

Slicing codes

- MATRIX
- MCFM

Subtraction codes

- NNLOJET
- HISTORY

- Formal accuracy must be matched by numerical precision: codes must aim for independence from numerical cutoffs.
- Slicing schemes: the cutoff is explicit but power corrections can be computed systematically.
- Even so, the numerics can get heavy and reaching full numerical control over the lowest order logarithms of the cutoff can become non-trivial.
- Subtraction schemes: the cutoff is implicit but required ultimately due to the finite precision of floating point arithmetic.

Is there really a need for a new code? (Spoiler: we think options are always good!)

Why a new code?

Is there really a need for a new code? ... and to do what?

- A natively faster code would improve profiling of uncertainties (scale/PDF/ α_s)
- The precision frontier for LHC experiments relies on the use of fully-differential parton-level events matched with parton showers (NNLO+PS matching such as MiNNLO_{PS} or Geneva)
- Those frameworks are computationally expensive
- A speedup in the core of the differential NNLO calculation would translate directly into a much more efficient NNLO+PS generation: important to produce Monte Carlo samples with adequate statistics for HL-LHC runs

NNLO matched to parton showers not impossible, but complex:

- Avoid global phase space cutoffs (challenging slicing methods)
- Provide a well-defined, on-shell event kinematics (challenging sector subtraction schemes)
- Preserve local spin and azimuthal correlations (challenging antenna subtraction setups)

Color singlets are standard candles

PDF fitting constraints and grids

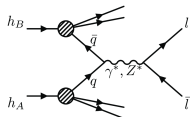
- Fitters cannot run full differential codes directly inside the fitting loop for PDF determinations. Instead, they rely on precomputed interpolation grids (e.g., FastNLO, APPLgrid) combined with K -factors
- Significantly faster NNLO differential calculations would eliminate the approximations introduced by fixed grids and K -factors. This would allow dynamic fits directly at NNLO with a far more rigorous and systematic treatment of theoretical uncertainties (e.g., scale variation).

Precision electroweak measurements

- The extraction of the W boson mass and the effective electroweak mixing angle from data require comparing differential distributions – such as the leptonic p_T or the Collins-Soper angular distribution – against theoretical templates generated across a range of parameter values. Scanning this parameter space to find the minimum χ^2 requires thousands of distinct configurations.
- An ultra-fast NNLO differential code would allow the direct generation of high-statistics templates at fixed-order. This would bypass the need for complex re-weighting of samples generated at NLO, significantly reducing the systematic uncertainties associated with the theoretical fit.

Cross sections for hadron collisions

Hadronic cross sections can be computed as convolutions of parton distribution functions (PDFs) with hard scattering partonic cross sections: **collinear factorization** theorem.



$$\sigma_{AB} = \sum_{a,b} \int dx_a \int dx_b \underbrace{f_{a/A}(x_a, \mu_F) f_{b/B}(x_b, \mu_F)}_{\text{PDF's}} \underbrace{\sigma_{ab}(x_a, x_b, \mu_F)}_{\text{partonic x-sect}} \left[1 + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^n}{Q^n} \right) \right]$$

- Non-perturbative PDFs $f_{a/A}(x_a, \mu_F)$, $f_{b/B}(x_b, \mu_F)$ to be measured in experiment or computed on the lattice
- Hard partonic cross section $\sigma_{ab}(x_a, x_b, \mu_F)$ can be computed in perturbation theory, e.g., at NNLO

$$\sigma_{ab}(x_a, x_b, \mu_F) = \sigma_{ab}^{\text{LO}}(x_a, x_b, \mu_R, \mu_F) + \sigma_{ab}^{\text{NLO}}(x_a, x_b, \mu_R, \mu_F) + \sigma_{ab}^{\text{NNLO}}(x_a, x_b, \mu_R, \mu_F)$$

The NNLO cross section

NNLO cross section: must account for extra radiation, both real and virtual

		Real	Virt.	
$\sigma_{ab}^{\text{NNLO}} = \int_{m+2} d\sigma_{ab}^{\text{RR}} J_{m+2}$		2	0	∞
$+ \int_{m+1} \left(d\sigma_{ab}^{\text{RV}} + d\sigma_{ab}^{\text{C}_1} \right) J_{m+1}$		1	1	∞
$+ \int_m \left(d\sigma_{ab}^{\text{VV}} + d\sigma_{ab}^{\text{C}_2} \right) J_m$		0	2	∞

- Three contributions separately divergent due to IR singularities: i) Real: divergent phase space integrals ii) Virt.: explicit ϵ -poles in loop integrals
- The issue of IR divergences can conceptually be solved by **local subtraction** [Ellis, Ross, Terrano 1980]

Local subtraction: use appropriately chosen **subtraction terms** to reshuffle divergences between the contributions such that each one becomes finite after the reshuffling.

CoLoRFuINNLO is a particular realization of this idea. NNLO cross section with subtractions

$$\begin{aligned}\sigma_{ab}^{\text{NNLO}} = & \int_{m+2} \left[d\sigma_{ab}^{\text{RR}} J_{m+2} - d\sigma_{ab}^{\text{RR},A1} J_{m+1} - d\sigma_{ab}^{\text{RR},A2} J_m + d\sigma_{ab}^{\text{RR},A12} J_m \right] \\ & + \int_{m+1} \left\{ \left[d\sigma_{ab}^{\text{RV}} + d\sigma_{ab}^{\text{C1}} + \int_1 d\sigma_{ab}^{\text{RR},A1} \right] J_{m+1} - \left[d\sigma_{ab}^{\text{RV},A1} + d\sigma_{ab}^{\text{C1},A1} + \left(\int_1 d\sigma_{ab}^{\text{RR},A1} \right)^{A1} \right] J_m \right\} \\ & + \int_m \left\{ d\sigma_{ab}^{\text{VV}} + d\sigma_{ab}^{\text{C2}} + \int_2 \left[d\sigma_{ab}^{\text{RR},A2} - d\sigma_{ab}^{\text{RR},A12} \right] + \int_1 \left[d\sigma_{ab}^{\text{RV},A1} + d\sigma_{ab}^{\text{C1},A1} \right] + \int_1 \left(\int_1 d\sigma_{ab}^{\text{RR},A1} \right)^{A1} \right\} J_m\end{aligned}$$

- Due to the presence of subtraction terms that remove non-integrable singularities, the phase space integral on each line is finite in $d = 4$ dimensions.
- The explicit IR ε -poles in loop corrections are cancelled by the integrated forms of subtraction terms, so each line is also free from ε -poles.
- Standard numerical methods can be used to compute each contribution.

Role of individual terms

$$\left\{ \begin{array}{l} d\sigma_{ab}^{\text{RR},A_1} \\ d\sigma_{ab}^{\text{RR},A_2} \\ d\sigma_{ab}^{\text{RR},A_{12}} \\ d\sigma_{ab}^{\text{RV},A_1} \\ d\sigma_{ab}^{\text{C}_1,A_1} \\ \left(\int_1 d\sigma_{ab}^{\text{RR},A_1} \right)^{A_1} \end{array} \right\} \text{ regularizes } \left\{ \begin{array}{l} d\sigma_{ab}^{\text{RR}} \\ d\sigma_{ab}^{\text{RR}} \\ d\sigma_{ab}^{\text{RR},A_2}, d\sigma_{ab}^{\text{RR},A_1} \\ d\sigma_{ab}^{\text{RV}} \\ d\sigma_{ab}^{\text{C}_1} \\ \int_1 d\sigma_{ab}^{\text{RR},A_1} \end{array} \right\} \text{ as } \left\{ \begin{array}{l} 1 \\ 2 \\ 1, 2 \\ 1 \\ 1 \\ 1 \end{array} \right\} \text{ partons are unresolved}$$

Accounting for all limits: the inclusion-exclusion principle

Obviously all limits must be accounted for precisely once and multiple subtraction must be avoided.

Overlapping singularities can be addressed by the **inclusion-exclusion principle**: subtract each limit once, add back pairwise overlaps, subtract triple overlaps and so on.

$$A_1 = \sum (C_{ir} + S_r - C_{ir} \cap S_r)$$

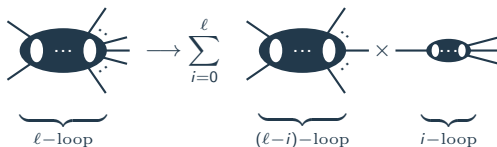
$$A_2 = \sum \left[C_{irs} + C_{ir,js} + CS_{ir,s} + S_{rs} \right. \\ \left. - (C_{irs} \cap CS_{ir,s} + C_{irs} \cap S_{rs} + C_{ir,js} \cap CS_{ir,s} + C_{ir,js} \cap S_{rs} + CS_{ir,s} \cap S_{rs}) \right. \\ \left. + (C_{irs} \cap CS_{ir,s} \cap S_{rs} + C_{ir,js} \cap CS_{ir,s} \cap S_{rs}) \right]$$

- The overlap of a triple and double collinear limit leads to a configuration with at least 3 unresolved partons.
- Not each limit corresponds to a distinct formula. E.g., explicit calculation shows that $C_{ir,js} \cap CS_{ir,s} \cap S_{rs} = C_{ir,js} \cap S_{rs}$ at the level of symbolic operators.

Construction of the subtraction terms

Subtraction terms are built from IR factorization formulae

$$U_j |\mathcal{M}_{ab,m+j}(\{p\}_{m+j})|_{\ell\text{-loop}}^2 = \left(\frac{\alpha_s}{2\pi}\right)^j \sum_{i=0}^{\ell} \text{Sing}_j^{(i)} \times \underbrace{|\mathcal{M}_{\hat{a}\hat{b},m}(\{\hat{p}\}_m)|_{(\ell-i)\text{-loop}}^2}_{j \text{ partons removed}}$$



IR factorization formulae cannot directly be used as subtraction terms:

1. Must avoid multiple subtraction in overlapping singular regions (inclusion-exclusion).
2. The formulae must be extended away from the IR limits over the full phase space.
3. Subtractions must be integrated over momenta of unresolved radiation.

The structure of cancellations implied by the inclusion-exclusion principle must be preserved when extending the IR limit formulae over full phase space

- **Extend** $|\mathcal{M}_{\hat{a}\hat{b},m}(\{\hat{p}\}_m)|^2_{(l-i)-\text{loop}} \Rightarrow$ specify precisely the momenta that enter the reduced matrix elements via momentum mappings which implement momentum conservation and mass-shell conditions

$$\{p\}_{m+j} \rightarrow \{\tilde{p}\}_m, \quad j = 1, 2$$

- **Extend** $\text{Sing}_j^{(i)} \Rightarrow$ define precisely the various quantities entering the singular structures such as momentum fractions, transverse momenta and eikonal factors.

Organizing principles

- Internal overlaps in $\mathbf{A}_1$ and $\mathbf{A}_2$: take the soft(er) limit first. E.g., $\mathbf{C}_{ir} \cap \mathbf{S}_r$ is the collinear limit of the soft factorization formula.
- External overlap among $\mathbf{A}_1$ and $\mathbf{A}_2$ cured by $\mathbf{A}_{12}$: start from the most unresolved ($\mathbf{A}_2$) to obtain iterated limits: $\mathbf{A}_{12} = \mathbf{A}_1 \mathbf{A}_2$.

Full set of counterterms for color-singlet production

$$\begin{aligned}
\mathcal{A}_1^{(0)} &= \sum_{r \in F} \left[S_r^{(0,0)} + \sum_{\substack{i \in F \\ i \neq r}} \left(\frac{1}{2} c_{ir}^{FF(0,0)} - c_{ir}^{FF} S_r^{(0,0)} \right) + \sum_{c \in I} \left(c_{cr}^{IF(0,0)} - c_{cr}^{IF} S_r^{(0,0)} \right) \right] \\
\mathcal{A}_2^{(0)} &= \frac{1}{2} \sum_{r \in F} \sum_{\substack{s \in F \\ s \neq r}} \left\{ S_{rs}^{(0,0)} + \sum_{c \in I} \left[c_{crs}^{IFF(0,0)} - c_{crs}^{IFF} S_{rs}^{(0,0)} + \sum_{\substack{d \in I \\ d \neq c}} \left(c_{cr,ds}^{IF,IF(0,0)} + c_{cr,ds}^{IF,IF} S_{rs}^{(0,0)} \right) \right] \right\} \\
\mathcal{A}_{12}^{(0)} &= \sum_{s \in F} \left[\mathcal{A}_2^{(0)} S_s + \sum_{\substack{r \in F \\ r \neq s}} \left(\frac{1}{2} \mathcal{A}_2^{(0)} c_{rs}^{FF} - \mathcal{A}_2^{(0)} c_{rs}^{FF} S_s \right) + \sum_{c \in I} \left(\mathcal{A}_2^{(0)} c_{cs}^{IF} - \mathcal{A}_2^{(0)} c_{cs}^{IF} S_s \right) \right]
\end{aligned}$$

with

$$\begin{aligned}
\mathcal{A}_2^{(0)} S_s &= \sum_{\substack{r \in F \\ r \neq s}} \left[S_{rs}^{(0,0)} S_s + \sum_{c \in I} \left(c_{crs}^{IFF(0,0)} S_s - c_{crs}^{IFF} S_{rs}^{(0,0)} S_s \right) \right] \\
\mathcal{A}_2^{(0)} c_{rs}^{FF} &= S_{rs}^{(0,0)} c_{rs}^{FF} + \sum_{c \in I} \left(c_{crs}^{IFF(0,0)} c_{rs}^{FF} - c_{crs}^{IFF} S_{rs}^{(0,0)} c_{rs}^{FF} \right) \\
\mathcal{A}_2^{(0)} c_{rs}^{FF} S_s &= \sum_{c \in I} c_{crs}^{IFF(0,0)} c_{rs}^{FF} S_s \\
\mathcal{A}_2^{(0)} c_{cs}^{IF} &= \sum_{\substack{r \in F \\ r \neq s}} \left(c_{csr}^{IFF(0,0)} c_{cs}^{IF} + \sum_{\substack{d \in I \\ d \neq c}} c_{cs,dr}^{IF,IF(0,0)} c_{cs}^{IF} \right) \\
\mathcal{A}_2^{(0)} c_{cs}^{IF} S_s &= \sum_{\substack{r \in F \\ r \neq s}} \left(S_{rs}^{(0,0)} c_{cs}^{IF} S_s + c_{csr}^{IFF(0,0)} c_{cs}^{IF} S_s - c_{csr}^{IFF} S_{rs}^{(0,0)} c_{cs}^{IF} S_s \right)
\end{aligned}$$

Full set of counterterms for color-singlet production

General remarks

- I and F denote sets of initial-state and final-state partons.
- The various terms correspond to the limit implied by the notation.
- The $(0,0)$ superscript signals that the original IR limit formulae involved tree-level singular structures multiplying tree-level reduced matrix elements.
- Notice no explicit subtraction term for one collinear and one soft parton: a simple consequence of the choice for momentum fractions and the simplicity of the color-singlet process.

Counter-events

- Not all subtraction terms require separate momentum mappings: the number of distinct momentum configurations is less than the number of subtraction terms.
- Subtraction terms corresponding to overlapping limits (e.g., $\mathcal{C}_{ir}^{FF} \mathcal{S}_r^{(0,0)}$, $\mathcal{C}_{rs}^{IFF} \mathcal{S}_{rs}^{(0,0)}$) use the mapping corresponding to the **rightmost** limit (e.g., $\mathcal{S}_r^{(0,0)}$, $\mathcal{S}_{rs}^{(0,0)}$).
- Iterated single unresolved terms (those in $\mathcal{A}_{12}^{(0)}$) use convolutions of single unresolved mappings. The mapping appropriate to the single unresolved limit involved in the definition is performed first. For consistency with the overlapping case, the notation is chosen such that this limit appears as the **rightmost** one.

Full set of double real subtraction terms

Consider $gg \rightarrow Hgg$ as double real for $gg \rightarrow H$, representative of the complexity

$$\int_{m+2} \left[d\sigma_{ab}^{\text{RR}} J_{m+2} - d\sigma_{ab}^{\text{RR}, A_1} J_{m+1} - d\sigma_{ab}^{\text{RR}, A_2} J_m + d\sigma_{ab}^{\text{RR}, A_{12}} J_m \right]$$

	Types	Terms	Counter events	Master integrals
A_1	5	13	7	11
A_2	5(11)	9(29)	5	42
A_{12}	13(23)	39(81)	17	104
Total	23(39)	61(123)	29	157

- Numbers shown with(without) accounting for cancellation of soft-collinear type terms

Full set of real-virtual subtraction terms

Consider $gg \rightarrow Hg$ as real-virtual for $gg \rightarrow H$, representative of the complexity

$$\int_{m+1} \left\{ \left[d\sigma_{ab}^{RV} + d\sigma_{ab}^{C_1} + \int_1 d\sigma_{ab}^{RR, A_1} \right]_{J_{m+1}} - \left[d\sigma_{ab}^{RV, A_1} + d\sigma_{ab}^{C_1, A_1} + \left(\int_1 d\sigma_{ab}^{RR, A_1} \right)^{A_1} \right]_{J_m} \right\}$$

	Types	Terms	Counter events	Master integrals
RV, A_1	2	3	2	24
C_1, A_1	3	6	2	10
A_1, A_1	5	13	2	65
Total	10	22	2	99

- In the real-virtual contribution only single unresolved real radiation $\Rightarrow$ construction of subtraction terms is only “NLO complexity” (integrals are NNLO complexity though)

Integrating the subtraction terms

To cancel virtual poles, **subtraction terms must be integrated** over the momenta of unresolved emission(s).

The required phase space integrals were computed with a **combination of methods**

- IBP reduction + DEs: $\int_2 d\sigma_{ab}^{\text{RR}, A_2}$
- Direct integration of parametric representations: $\int_2 d\sigma_{ab}^{\text{RR}, A_{12}}$
- MB methods: elements of the distributional expansions

Results are **fully analytic**

- All integrated subtraction terms can be expressed in terms of GPLs of at most weight 2 and classical polylogarithms
- Algebraic letters are present (square roots as well as nested square roots coming from quartic polynomials) in finite parts
- Can check cancellation of double virtual poles analytically

Integrating the subtraction terms

Integrated subtraction terms are distributions of the momentum fractions x_a and x_b of incoming momenta.

- Distributional expansion challenging due to the “overlap” $1/[(1-x_a)(1-x_b)(1-x_ax_b)]$
- Compute parametric limiting behaviour as one or both variables go to 1.
- Build (integrated) subtractions pretty much the same way we built A_1 , A_2 and A_{12} .

$$\begin{aligned} [CT] \otimes d\sigma_{AB} = & \sum_{a,b} \int_0^1 d\eta_a d\eta_b d\sigma_{\bar{a}\bar{b}}(\eta_a p_A, \eta_b p_B) \int_0^1 dx_a dx_b \left\{ (x_a x_b)^{2\varepsilon} [CT(x_a, x_b; \varepsilon)] \frac{f_{a/A}(\eta_a/x_a)}{x_a} \frac{f_{b/B}(\eta_b/x_b)}{x_b} \right. \\ & + (x_b)^{2\varepsilon} (-\mathbf{L}_a + [\mathbf{L}_a])[CT(x_a, x_b; \varepsilon)] f_{a/A}(\eta_a) \frac{f_{b/B}(\eta_b/x_b)}{x_b} + (x_a)^{2\varepsilon} (-\mathbf{L}_b + [\mathbf{L}_b])[CT(x_a, x_b; \varepsilon)] \frac{f_{a/A}(\eta_a/x_a)}{x_a} f_{b/B}(\eta_b) \\ & \left. + (-\mathbf{L}_{ab} + \mathbf{L}_a \mathbf{L}_{ab} + \mathbf{L}_b \mathbf{L}_{ab} + [\mathbf{L}_{ab}] - [\mathbf{L}_a \mathbf{L}_{ab}] - [\mathbf{L}_b \mathbf{L}_{ab}])[CT(x_a, x_b; \varepsilon)] f_{a/A}(\eta_a) f_{b/B}(\eta_b) \right\}. \end{aligned}$$

$$\mathbf{L}_a[CT(x_a, x_b; \varepsilon)] = \sum_i (1-x_a)^{a_i+b_i\varepsilon} g_i(x_b; \varepsilon)$$

$$[\mathbf{L}_a][CT(x_a, x_b; \varepsilon)] = \int_0^1 dx_a \mathbf{L}_a[CT(x_a, x_b; \varepsilon)]$$

$$\mathbf{L}_{ab}[CT(x_a, x_b; \varepsilon)] = \sum_i (1-x_a)^{a_i+b_i\varepsilon} (1-x_b)^{c_i+d_i\varepsilon} g_i\left(\frac{x_a}{x_b}; \varepsilon\right)$$

$$[\mathbf{L}_{ab}][CT(x_a, x_b; \varepsilon)] = \int_0^1 dx_a dx_b \mathbf{L}_{ab}[CT(x_a, x_b; \varepsilon)]$$

$$\mathbf{L}_a \mathbf{L}_{ab}[CT(x_a, x_b; \varepsilon)] = \sum_i (1-x_a)^{a_i+b_i\varepsilon} (1-x_b)^{c_i+d_i\varepsilon} g_i(\varepsilon)$$

$$[\mathbf{L}_a \mathbf{L}_{ab}][CT(x_a, x_b; \varepsilon)] = \int_0^1 dx_a \mathbf{L}_a \mathbf{L}_{ab}[CT(x_a, x_b; \varepsilon)]$$

The NNLOCAL code

```
[ ~/-research/colorsinglet/runs_for_nnlocalgg/nnlocal/bin/testrun-H
, ../nnlocal
***** version beta *****
*
* ##      ## ##      ## ##      #####      #####      ###      ##      *
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***** December 20th, 2024 *****
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*
* https://github.com/nnlocal/nnlocal.git
*
*****
* Using input file named input.DAT
```

NNLOCAL: a **Monte Carlo program** implementing the described scheme

- Proof-of-concept version available at <https://github.com/nnlocal/nnlocal.git>
- Limitations: gluon-fusion Higgs production in HEFT ($m_t \rightarrow \infty$) with no light quarks ($n_f = 0$); minimal optimization (especially in VV part); dynamical switching to quadruple precision sometimes necessary

NNLOCAL-dev

- Implementation of integrated counterterms optimized: no longer needs quadruple precision
- New file structure for easy extensibility
- Better analysis routine (regular and variable bin size, multi-dimensional histograms), support for scale and pdf variations
- Full gluon-fusion Higgs production in HEFT

Optimization of integrated counterterms

A straightforward collection of integrated counterterms leads to a large expression of $\sim \mathcal{O}(100\text{k terms})$. Workable for proof-of-concept implementation, but needs optimization.

1. Gather the expression by special functions ($\log$, Li , G)
2. Partial fraction their rational coefficients
3. Recombine and gather by rational structures
 - Powers of some denominator factors reduced $\Rightarrow$ better stability
 - Some denominator structures disappear completely
4. Numerical checks on the coefficients of independent rational structures
 - Analytic zeros found
 - Some big expressions coincide with other very small expressions
 - Note: analytic simplification (Hopf-algebra, etc.) not straightforward (roots)
5. Avoid catastrophic cancellation
 - $x \rightarrow 0$ not exactly possible, but low values accessible for low mass color singlet

$$\log\left(2 - x^2 - 2\sqrt{1 - x^2}\right) = 4\log(x) - 2\log\left(1 + \sqrt{1 - x^2}\right), \dots$$

The size expression is reduced by an order of magnitude. The simplified expression is much more stable numerically $\Rightarrow$ quadruple precision evaluation no longer necessary.

Optimization of integrated counterterms

[illegible]

...and many more such relations.

New file structure for easy extensibility: (color singlet) processes can be user defined by building the following (13) routines

- Process setup: physical parameters, phase space dimension

`setup_process.f`

- Matrix elements:

`gg_cs_r0v0.f` `gg_cs_r1v0.f` `gg_cs_r1v0_vec.f` `gg_cs_r2v0.f`
`gg_cs_r0v1.f` `gg_cs_r1v1.f` `gg_cs_r0v1_eps.f` `gg_cs_r0v2_eps.f`

- Analysis routine: regular and variable bin sizes, multi-dimensional histograms

`analysis_core.f90`

- Phase spaces for Born, Born+1 and Born+2 particle kinematics:

`gen_rad_0.f` `gen_rad_1.f` `gen_rad_2.f`

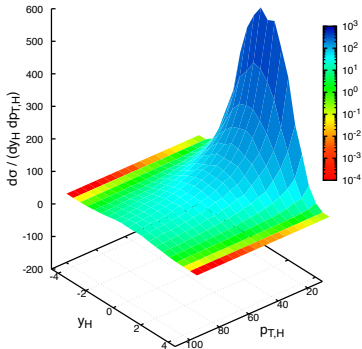
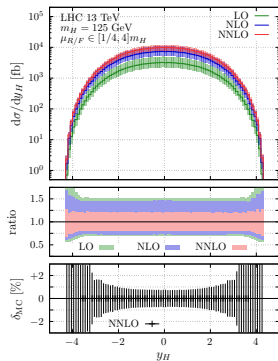
Higgs production in gluon fusion

Inclusive cross section: comparison with an implementation of the fully-inclusive formulae [Anastasiou, Melnikov 2003] directly in NNLOCAL

m_H [GeV]	NNLOCAL (incl.)	NNLOCAL
100	61.91 pb	61.88(3) pb
125	40.33 pb	40.32(2) pb
250	9.046 pb	9.046(4) pb
500	1.489 pb	1.489(1) pb
1000	154.9 fb	155.0(1) fb
2000	7.462 fb	7.464(3) fb

- NNLOCAL (incl.) in perfect agreement with n3l0xs [Baglio, Duhr, Mistlberger, Szafron 2022]
- Values for $\mu_R = \mu_F = m_H$ shown, but several choices were checked.
- Runtime per mass value: ~ 70 sec on a MacBook Pro M2 with 8 CPUs.

Differential distributions ($\frac{d\sigma}{dy_H}$ and $\frac{d\sigma}{dy_H dp_{T,H}}$) at the 13 TeV LHC, $m_H = 125$ GeV



- Standard 7-point scale variation with $\mu_{R/F} \in [1/4, 4]m_H$.
- Bottom panel of left figure shows estimated Monte Carlo integration uncertainty.
- Runtime ~ 15 mins. on a MacBook Pro M2 with 8 CPUs.

A fast and stable local subtraction method for higher-order radiative corrections is desirable to meet the precision demands of future collider data.

The CoLoRFulNNLO framework, already successfully implemented for both massless and massive final-state radiations, has also proven effective when extended to initial-state radiation.

The NNLOCAL public code features a dedicated implementation of these subtraction techniques.

The upcoming version of the code features many refinements and optimizations and is designed to be user-friendly and easily extendable.

A public release is imminent, we look forward to its deployment in realistic experimental analyses and phenomenology studies.

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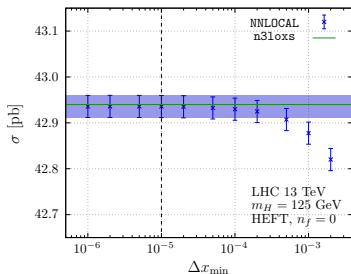
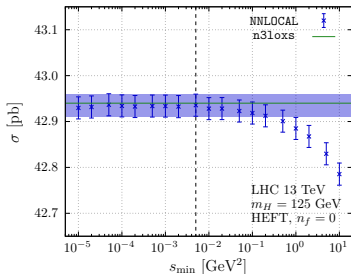
Thank you for your attention and stay tuned!

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Backup material

Technical cuts

Due to finite precision arithmetics, instabilities eventually set in as IR limits are approached $\Rightarrow$ technical cuts



- $s_{\min}$: smallest allowed 2-particle invariant in real emission
- $\Delta x_{\min}$: closest approach to critical ("0/0") lines in integrated subtraction terms

The pure NNLO contribution

The pure NNLO contribution for the total cross section

m_H [GeV]	NNLO only	double virtual	real-virtual	double real
100	19.76(5) pb	20.76(5) pb	-1.318(1) pb	0.321(9) pb
125	12.64(1) pb	13.16(1) pb	-0.7492(4) pb	0.235(3) pb
250	2.724(2) pb	2.781(2) pb	-0.10475(4) pb	0.0474(2) pb
500	0.4420(3) pb	0.4460(3) pb	-0.009645(4) pb	0.00564(1) pb
1000	47.23(3) fb	47.30(3) fb	-0.4699(2) fb	0.3982(8) fb
2000	2.496(2) fb	2.492(2) fb	-0.007597(2) fb	0.01220(3) fb

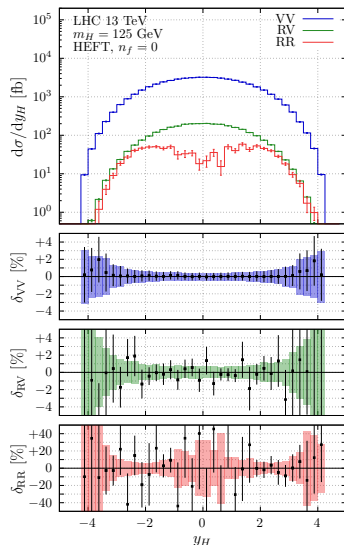
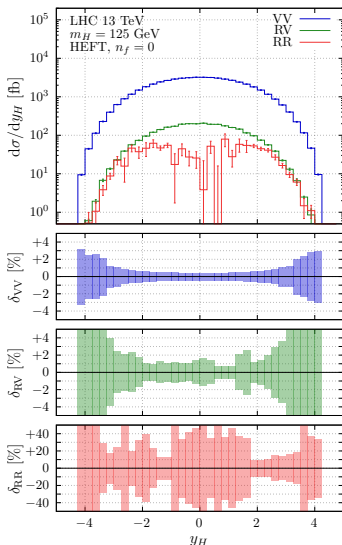
Recall the total NNLO gluons only cross section

m_H [GeV]	n3l0xs (gg)	NNLOCAL (gg)
100	65.72 pb	65.74(4) pb
125	42.94 pb	42.94(2) pb
250	9.730 pb	9.733(5) pb
500	1.626 pb	1.626(1) pb
1000	173.7 fb	173.7(1) fb
2000	8.794 fb	8.790(5) fb

So $VV \sim \mathcal{O}(1)$, $RV \sim \mathcal{O}(0.1)$ and $RR \sim \mathcal{O}(0.01)$ of the total NNLO cross section

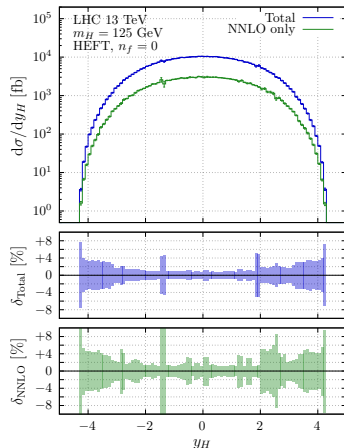
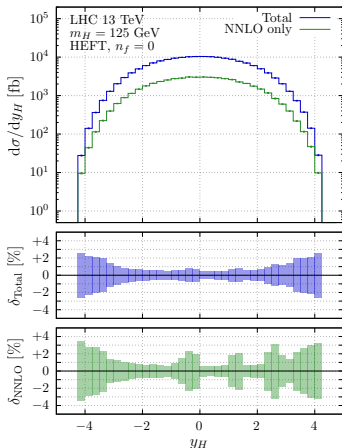
The pure NNLO contribution

The untrimmed (left) and trimmed (right) VV, RV and RR parts of the NNLO correction



Mitigating misbinning with robust statistics

Rapidity distribution of a Higgs boson of mass $m_H = 125$ GeV at the 13 TeV LHC



- Finer binning to test numerical stability under more demanding conditions
- Overall good stability and convergence, but some spikes appear: **misbinning**

Misbinning: well-known and more or less unavoidable consequence of using a subtraction scheme to compute binned distributions.

- The weights of the double real matrix element and the subtraction end up in different bins.
- This leads to an apparent uncanceled singularity as far as the bin is concerned.
- Misbinning introduces unphysical outliers into the set of bin value estimates.
- A single extreme value can change the arithmetic mean by a large amount.
- So use an estimator that is robust to such extreme data.

Robust statistics: the study of statistical methods that remain reliable and accurate even when data contain outliers or deviate from standard assumptions.

- One of the simplest methods for building robust estimators is trimming. E.g., the α -trimmed mean is defined as

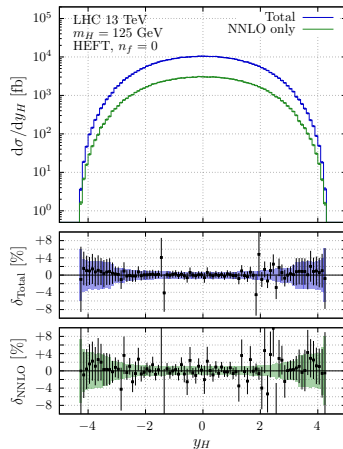
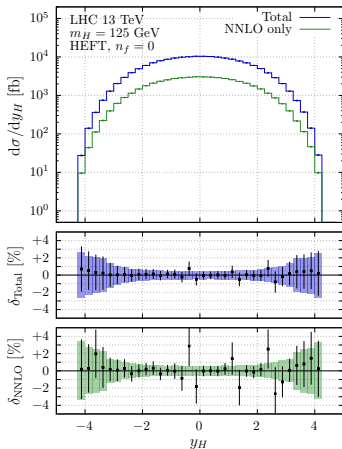
$$\bar{x}_\alpha = \frac{1}{n - 2m} \sum_{i=m+1}^{n-m} x_{(i)}$$

where $m = \lfloor n\alpha \rfloor$ (here $\lfloor \dots \rfloor$ denotes the integer part) and $x_{(i)}$ is the order statistics $\{x_{(1)}, x_{(1)}, \dots, x_{(n)}\}$, obtained by sorting the n numbers such that

$$x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$$

- In words, it is the arithmetic mean of the $(n - 2m)$ numbers obtained by removing the smallest and largest $m = \lfloor n\alpha \rfloor$ entries.
- Typically results are obtained by combining several statistically independent runs by computing bin-by-bin averages of bin value estimates from the individual runs. It is trivial to implement trimming on these bin value estimates.

Trimmed rapidity distribution



- α -trimming with $\alpha = 0.015$ used on data containing 256 separate runs