

NNLO QCD predictions for $t\bar{t}j$ production at the LHC

with Simon Badger, Matteo Becchetti, Colomba Brancaccio, Michal Czakon,
Rene Poncelet and Simone Zoia

based on arXiv:2511.11424 and arXiv:2511.11431

Heribertus Bayu Hartanto

Asia Pacific Center for Theoretical Physics (APCTP) Pohang, South Korea

HP2 2026, Karlsruhe

October 8th, 2026

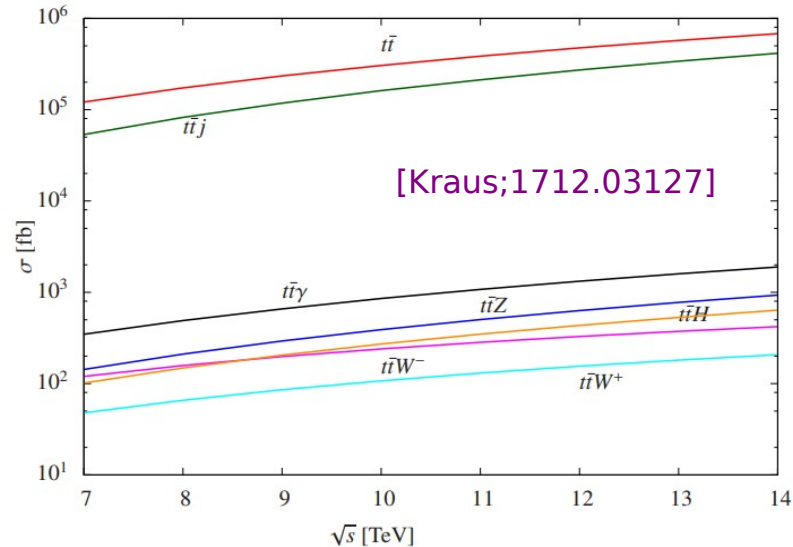
$t\bar{t} + (j/\gamma/H/W/Z)$ production at the LHC

Precision tests of SM

Sensitivity to m_t , α_s , PDFs and $t\bar{t}V$ vertex

Signal/background processes in Higgs studies and BSM searches

Beyond NLO: massive two-loop five-point amplitudes are the bottleneck



NNLO QCD calculation
with approximated
two-loop amplitudes
(soft Higgs/W,
massification)

Higgs boson production in association with a top-antitop quark pair in next-to-next-to leading order QCD

Stefano Catani,^{1,*} Simone Devoto,^{2,†} Massimiliano Grazzini,^{3,‡}
Stefan Kallweit,^{4,§} Javier Mazzitelli,^{5,6,¶} and Chiara Savoini^{3,**}

Precise predictions for the associated production of a W boson with a top-antitop quark pair at the LHC

Luca Buonocore^a, Simone Devoto^b, Massimiliano Grazzini^a, Stefan Kallweit^c,
Javier Mazzitelli^d, Luca Rottoli^a and Chiara Savoini^a

Next-to-next-to-leading order event generation for $t\bar{t}H$ production with approximate two-loop amplitude

Christian Biello,^a Chiara Savoini,^b Chiara Signorile-Signorile^c and Marius Wiesemann^d

NNLO QCD with
massive two-loop five-
point amplitudes in the
leading colour approx

NNLO QCD predictions for $t\bar{t}W$ production at hadron colliders

Matteo Becchetti^(a), Dhimiter Canko^(a), Xiang Chen^(b), Vsevolod Chestnov^(c),
Maximilian Delto^(d), Sara Ditsch^(e,f), Massimiliano Grazzini^(b), Stefan Kallweit^(b),
Tiziano Peraro^(a), Mattia Pozzoli^(a), Chiara Savoini^(f), Lorenzo Tancredi^(f)
and Simone Zoia^(b)

Higher-order QCD corrections to top-quark pair production in association with a jet

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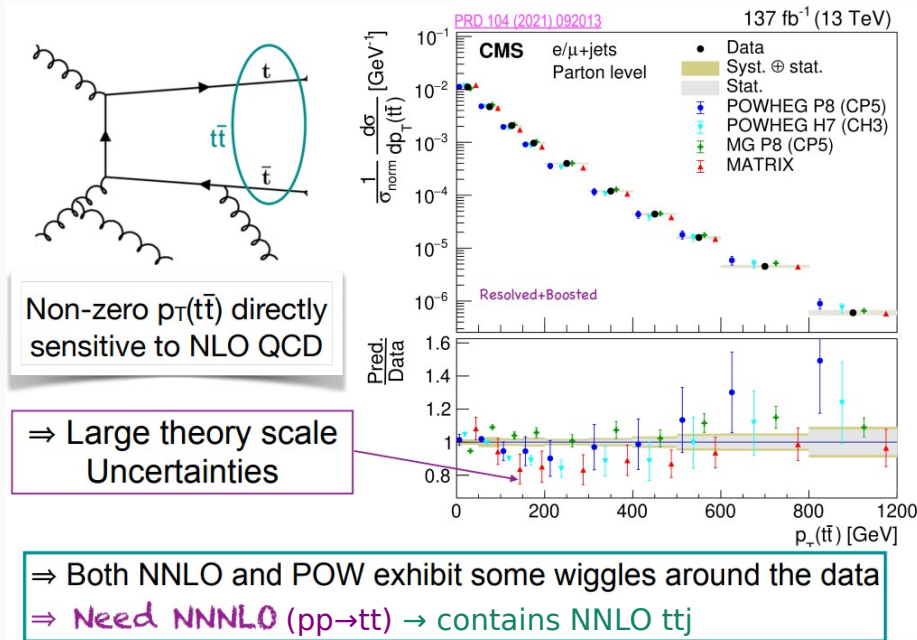
Rene Poncelet^{**}
Institute of Nuclear Physics PAN, ul. Radzikowskiego 152, 31-342 Kraków

Simone Zoia^{††}
Physik-Institut, Universität Zürich, Winterthurerstrasse 190, 8057 Zürich, Switzerland
(Dated: November 18, 2025)

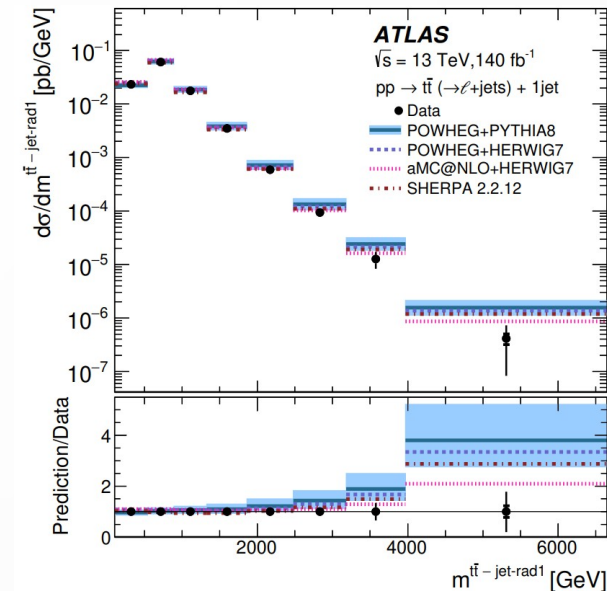
ttj production at the LHC

- large fraction of tt events accompanied by additional *jets* ($\sim 40\%$ for $p_{Tj} > 40$ GeV)
- wide range of phenomenological studies due to richness of 2→3 kinematics
- improve the description of $pp \rightarrow tt$ (theoretically and experimentally)

Olaf Behnke's talk@TOP2024



measurements available from ATLAS+CMS



[ATLAS:JHEP08(2024)182]

⇒ background for SM Higgs production in VBF, BSM searches with $\ell_1^+ \ell_2^- + \text{MET} + \text{jets}$

ttj production at the LHC

m_{top} measurements $\Rightarrow$ assess EW vacuum stability

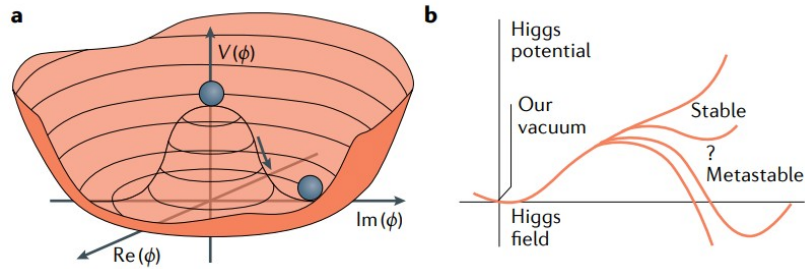
m_{top} extraction with ttj final state

$$\mathcal{R}(m_t^{\text{pole}}, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{-jet}}} \frac{d\sigma_{t\bar{t}+1\text{-jet}}}{d\rho_s}(m_t^{\text{pole}}, \rho_s) \quad \rho_s = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}}$$

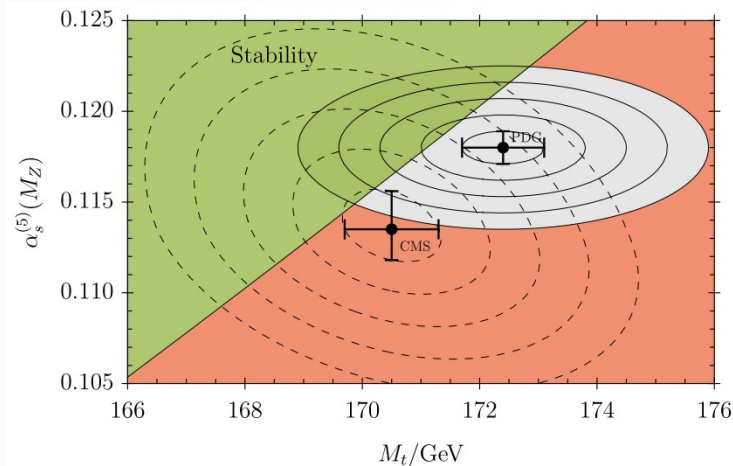
[Alioli, Fernandez, Fuster, Irlles, Moch, Uver, Vos; 1303.6415]

[Bevilacqua, **HBH**, Kraus, Schulze, Worek(2017)]

[Alioli, Fuster, Garzelli, Gavardi, Irlles, Melini, Moch, Uwer, Vos; 2202.07975]

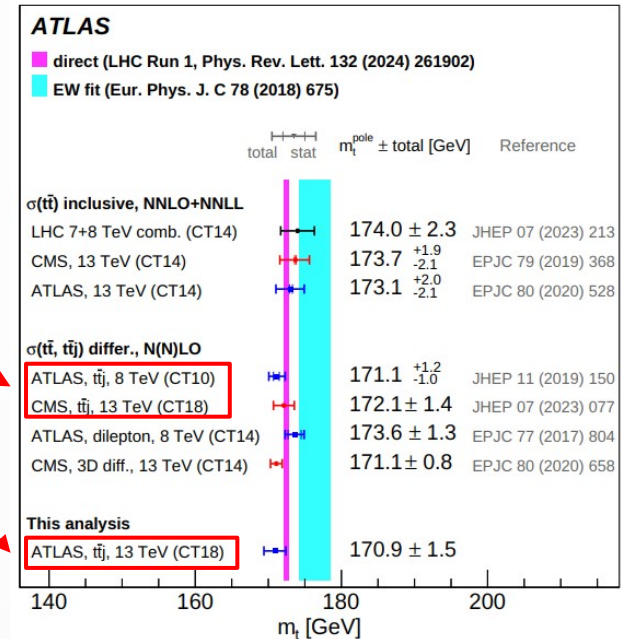


[Bass, DeRoeck, Kado; Nat Rev Phys 3, 608–624(2021)]



[Hiller, Hohne, Litim, Steudtner; 2401.08811]

NLO ttj predictions
used for theory
templates



[ATLAS; 2507.02632]

Theoretical predictions for $pp \rightarrow tt\bar{t}\bar{t}$

$$d\sigma = \underbrace{d\sigma^{(0)}}_{\text{LO}} + \underbrace{\alpha_s d\sigma^{(1)}}_{\delta\text{NLO}} + \underbrace{\alpha_s^2 d\sigma^{(2)}}_{\delta\text{NNLO}} + \dots$$

NLO QCD (stable top) [Dittmaier,Uwer,Weinzierl;(2007,2008)]

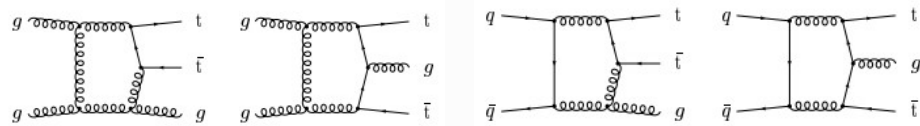
NLO QCD (NWA) [Melnikov,(Scharf),Schulze;(2010,2011)]

NLO QCD (off-shell) [Bevilacqua,**HBH**,Kraus,Worek;(2015,2016)]

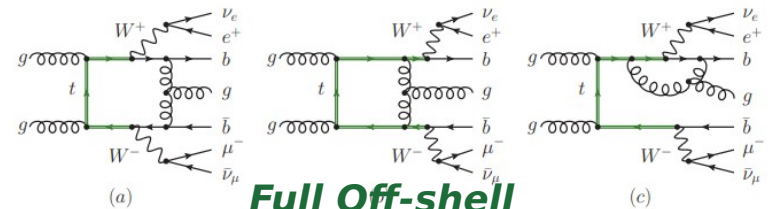
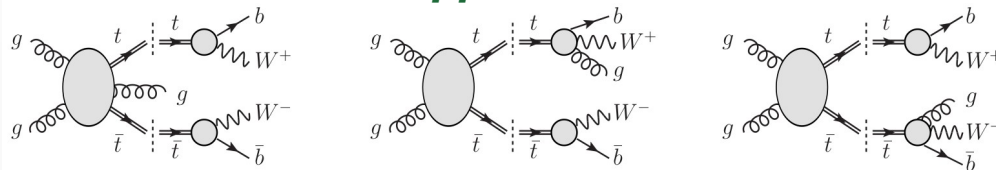
NLO QCD + PS [Kardos,Papadopoulos,Trocsanyi;(2011)][Alioli,Moch,Uwer;(2011)][Czakon,**HBH**,Kraus,Worek;(2015)]

Merged NLO QCD+EW (stable top) [Gutschow,Lindert,Schönherr;(2018)]

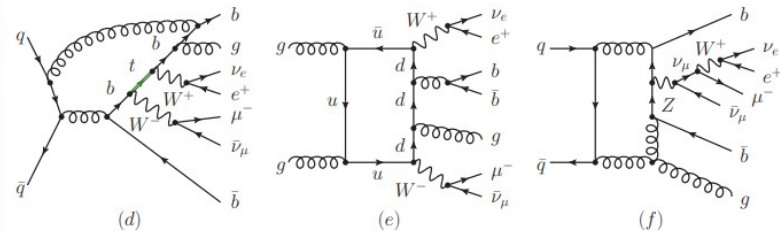
Stable top



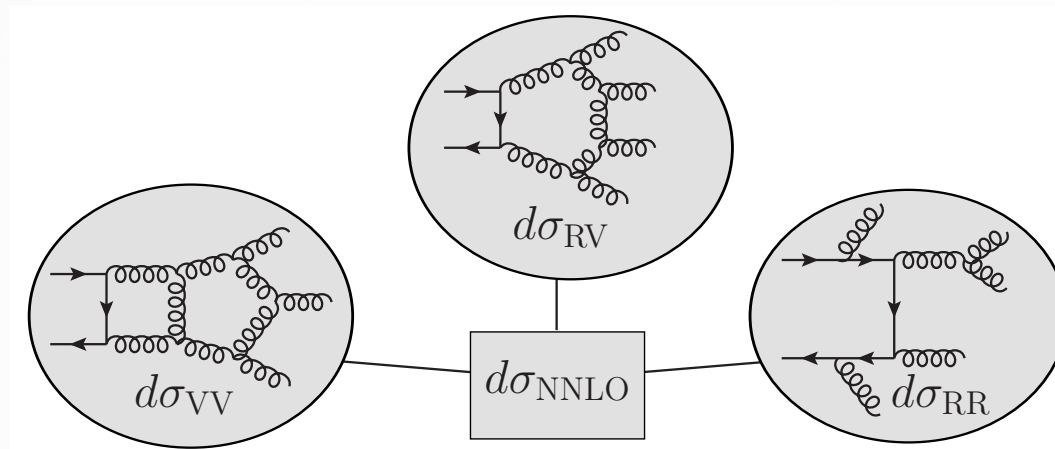
Narrow-width Approximation (NWA)



Full Off-shell



NNLO QCD corrections to $pp \rightarrow ttj$



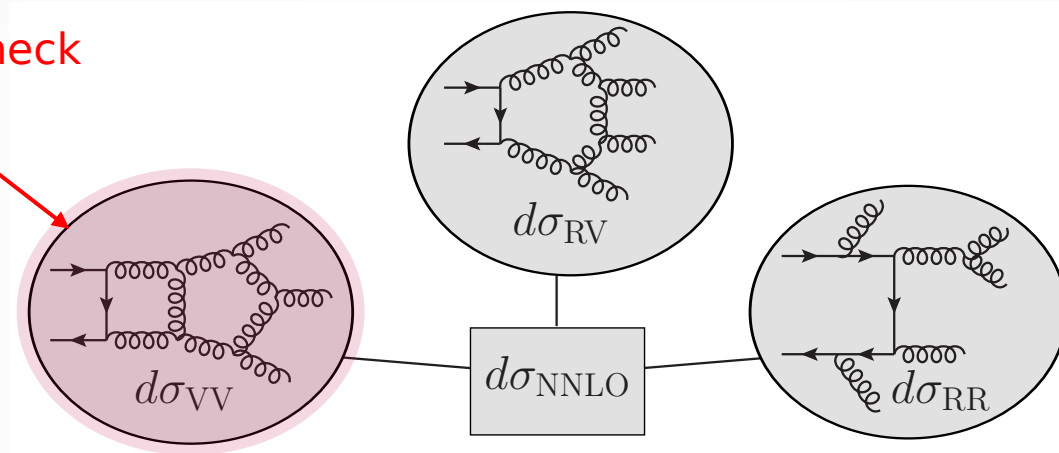
Work with **stable** top quarks

- Amplitudes:**
- Tree-level $pp \rightarrow ttjj$: AvH [Bury, van Hameren(2015)]
 - One-loop $pp \rightarrow ttjj$: OpenLoops [Buccioni, Lang, Lindert, Maierhofer, Pozzorini, Zhang, Zoler(2018,2019)]
 - Two-loop $pp \rightarrow ttj$: [this work](#)

NNLO subtraction scheme: Sector Improved Residue Subtraction Scheme (STRIPPER)
[Czakon(2010)][Czakon, Heymes(2014)][Czakon, van Hameren, Mitov, Poncelet(2019)]

NNLO QCD corrections to $pp \rightarrow ttj$

main bottleneck



Work with **stable** top quarks

- Amplitudes:**
- Tree-level $pp \rightarrow ttjjj$: AvH [Bury, van Hameren(2015)]
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NNLO subtraction scheme: Sector Improved Residue Subtraction Scheme (STRIPPER)
[Czakon(2010)][Czakon, Heymes(2014)][Czakon, van Hameren, Mitov, Poncelet(2019)]

Two-loop amplitudes for $pp \rightarrow ttj$

[Badger, Becchetti, Brancaccio, Czakon, **HBH**, Poncelet, Zoia; 2511.11424]

⇒ State-of-the-art loop amplitude computation: $2 \rightarrow 3$ scattering with **external & internal masses**

$$\mathcal{A}^{(2)} = \int d^d k_1 d^d k_2 \sum_{i=1}^{N_{\text{diag}}} \frac{\mathcal{N}_i(\{k\}, \{p\})}{\mathcal{D}_i(\{k\}, \{p\})} = \sum_i c_i(\vec{x}, \epsilon) \text{MI}_i(\vec{x}, \epsilon)$$

$$\vec{x} = (d_{12}, d_{23}, d_{34}, d_{45}, d_{15}, m_t^2)$$

$$d_{ij} = p_i \cdot p_j$$

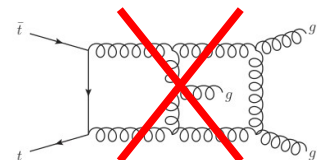
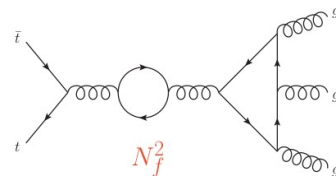
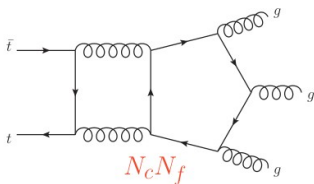
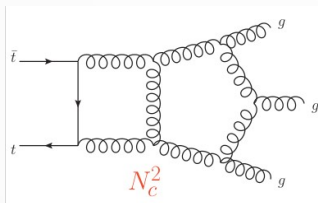
High-degree rational functions (6 kinematic variables + ϵ), massive spinors

Internal massive propagators, functions beyond polylogarithms

⇒ Two scattering processes: $gg \rightarrow ttj$ and $qq \rightarrow ttj$

⇒ Work in the **leading colour** approximation: only planar diagrams

$gg \rightarrow ttj$



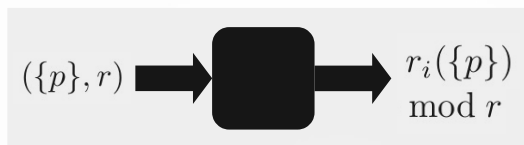
keep the computation tractable!!

Two-loop amplitudes for $pp \rightarrow ttj$

[Badger, Becchetti, Brancaccio, Czakon, **HBH**, Poncelet, Zoia; 2511.11424]

- ⇒ Finite remainder → subtract universal UV/IR singularities
- construct special function basis

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\}) \text{mon}_i(f)$$



Algebraic operations → numerical evaluations over finite fields
Reconstruct analytic expressions from numerical samples

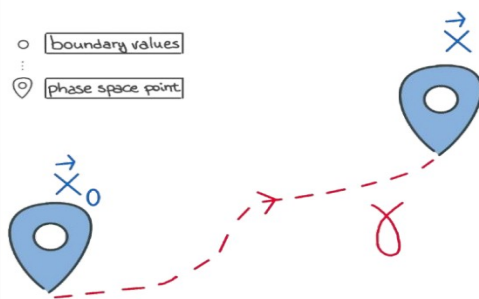
- ⇒ Construct DEs for special functions and solve numerically

$$d\vec{f}(\vec{x}) = M(\vec{x}) \cdot \vec{f}(\vec{x})$$

C++ implementation

~20 seconds per point

double-double for
rational coefficients,
double for special functions



$$A^{(2)}(\{p\}, \epsilon) = \sum_i (\text{Feynman diagram})_i$$

↓ map loop numerators to $\mathcal{I}$

$$A^{(2),h}(\{p\}, \epsilon) = \sum_i c_i^h(\{p\}, \epsilon) \mathcal{I}_i(\{p\}, \epsilon)$$

↓ IBP reduction

$$A^{(2),h}(\{p\}, \epsilon) = \sum_i d_i^h(\{p\}, \epsilon) \text{MI}_i(\{p\}, \epsilon)$$

↓ map to special functions

↓ subtract UV/IR poles

↓ ϵ expansion

$$F^{(2),h}(\{p\}) = \sum_i r_i^h(\{p\}) \text{mon}_i(f) + \mathcal{O}(\epsilon)$$

Two-loop amplitudes for $pp \rightarrow ttj$: reconstruction data

[Badger,Becchetti,Brancaccio,Czakon,**HBH**,Poncelet,Zoia;2511.11424]

independent helicities
evaluated simultaneously

$$\vec{h} = (+ + +, + + -, + - +)$$

mom. twistor variables

$$(s_{34}, t_{12}, t_{23}, t_{45}, t_{51}, x_{5123})$$

Helicity amplitude construction

$$\mathcal{A}^{(L)h_3h_4h_5} = \sum_{i=1}^4 \Gamma_i \mathcal{G}_i^{(L)h_3h_4h_5}$$

$$\mathcal{G}_i^{(L)h_3h_4h_5} = \sum_{j=1}^4 (\Gamma^\dagger \Gamma)_{ij}^{-1} [\Gamma_j^\dagger \mathcal{A}^{(L)}]^{h_3h_4h_5}$$

“contracted helicity amplitude”

$$\Gamma_1 = m_t^2 \bar{u}_2 v_1$$

$$\Gamma_2 = m_t \bar{u}_2 \not{p}_3 v_1$$

$$\Gamma_3 = m_t \bar{u}_2 \not{p}_4 v_1$$

$$\Gamma_4 = \bar{u}_2 \not{p}_3 \not{p}_4 v_1$$

	MI	SF	#1	#2	#3	#4	# of points
$[\Gamma_1^\dagger \mathcal{A}^{(2),N_c^2}]^{\vec{h}}$	404/393	314/303	291/280	291/0	44/40	44/0	137076
$[\Gamma_2^\dagger \mathcal{A}^{(2),N_c^2}]^{\vec{h}}$	398/389	304/293	291/280	291/0	44/40	44/0	89624
$[\Gamma_3^\dagger \mathcal{A}^{(2),N_c^2}]^{\vec{h}}$	411/402	320/311	291/280	291/0	44/40	44/0	161482
$[\Gamma_4^\dagger \mathcal{A}^{(2),N_c^2}]^{\vec{h}}$	420/411	320/311	291/280	291/0	44/40	44/0	179838

N/D: max polynomial degrees
in the numerator/denominator

mass-renormalised
amplitude
all-order ϵ dependence

finite
remainder

linear
relations

denominator
guessing

partial fraction
in x_{5123}
(no x_{5123} dependence)

more
denominator
guessing

Two-loop amplitudes for $pp \rightarrow ttj$: reconstruction data

[Badger,Becchetti,Brancaccio,Czakon,**HBH**,Poncelet,Zoia;2511.11424]

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Results: $pp \rightarrow ttj$ @ NNLO QCD

[Badger,Becchetti,Brancaccio,Czakon,**HBH**,Poncelet,Zoia;2511.11431]

- LHC@13 TeV
- PDF4LHC21
- **Stable top**, $m_t = 172.5$ GeV
- anti- k_T jet algorithm, $\Delta R = 0.4$
- Scales:

$$\mu_R = \mu_F = \mu_0 = H_T/n \quad \text{with } n \in \{1, 2, 4\}$$

$$H_T = M_T(t) + M_T(\bar{t}) + p_T(j_1)$$

“soft” phase-space:

$$p_T(j) \geq 30 \text{ GeV and } |y(j)| \leq 2.5$$

“hard” phase-space:

follow setup used in ongoing CMS analysis

$$p_T(j) \geq 120 \text{ GeV and } |y(j)| \leq 2.4$$

$$p_T(j) \geq 0.4 \cdot (p_T(t) + p_T(\bar{t}))$$

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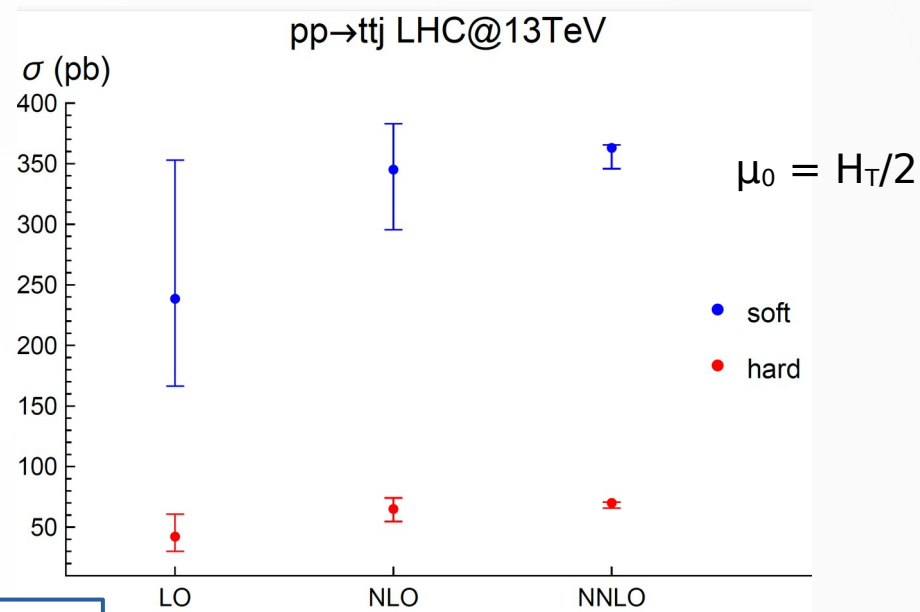
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$$K_{\text{NLO}} = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$$
$$K_{\text{NNLO}} = \frac{\sigma_{\text{NNLO}}}{\sigma_{\text{NLO}}}$$

soft: $K_{\text{NLO}} \sim 1.45$, $K_{\text{NNLO}} \sim 1.05$

hard: $K_{\text{NLO}} \sim 1.54$, $K_{\text{NNLO}} \sim 1.08$

Theory uncertainties from 7-point scale variations

$$\left(\frac{\mu_R}{\mu_0}, \frac{\mu_F}{\mu_0} \right) = (0.5, 0.5), (0.5, 1), (1, 0.5), (1, 1), (1, 2), (2, 1), (2, 2)$$

Results: $pp \rightarrow ttj$ @ NNLO QCD

[Badger, Becchetti, Brancaccio, Czakon, **HBH**, Poncelet, Zoia; 2511.11431]

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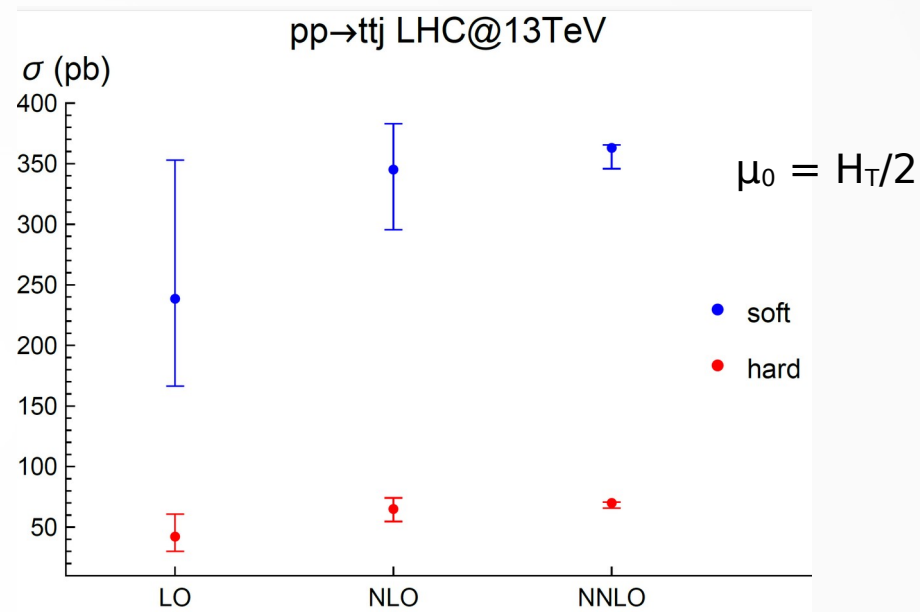
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Size of double virtual corrections

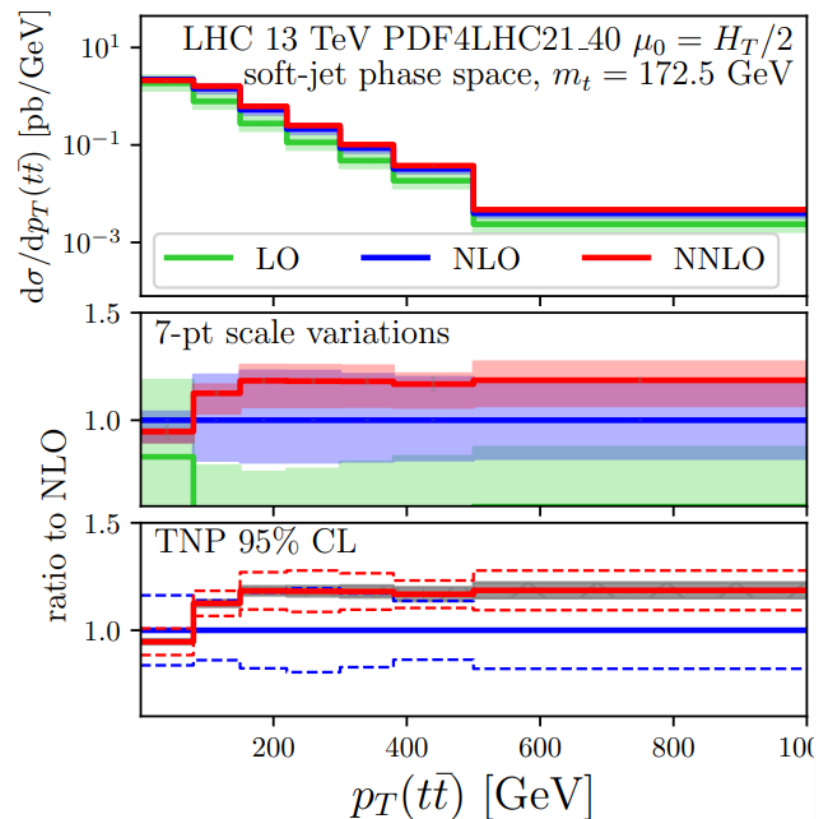
$$F_{\text{l.c. resc.}}^{(2)}(\mu^2) = \frac{F^{(0)}}{F_{\text{l.c.}}^{(0)}} F_{\text{l.c.}}^{(2)}(Q^2) + \sum_{i=0}^4 c_i \log^i(\mu^2/Q^2)$$

6% - 10% depending on scale choice 11

Results: $pp \rightarrow t\bar{t}$ @ NNLO QCD

[Badger,Becchetti,Brancaccio,Czakon,**HBH**,Poncelet,Zoia;2511.11431]

Differential cross sections



Missing higher-order uncertainties (MHOU) are estimated using **7-point scale variations** and **Theory Nuisance Parameters (TNP)** method

[Tackmann;2411.18606][Lim,Poncelet;2412.14910]

$$\frac{d\bar{\sigma}_{\text{TNP}}^{(N+1)}}{d\bar{\sigma}^{(0)}} = \sum_{j=1}^N f_k^{(j)}(\vec{\theta}, x) \left(\frac{d\bar{\sigma}^{(j)}}{d\bar{\sigma}^{(0)}} \right)$$

Impact of missing subleading colour terms is also estimated using TNP method

$$\frac{F_{l.c.}^{(2)}}{F_{l.c.}^{(0)}} \rightarrow \frac{F_{l.c.}^{(2)}}{F_{l.c.}^{(0)}} + \frac{\theta}{N_c^2} \frac{F_{l.c.}^{(2)}}{F_{l.c.}^{(0)}} \quad \theta \in [-1, 1]$$

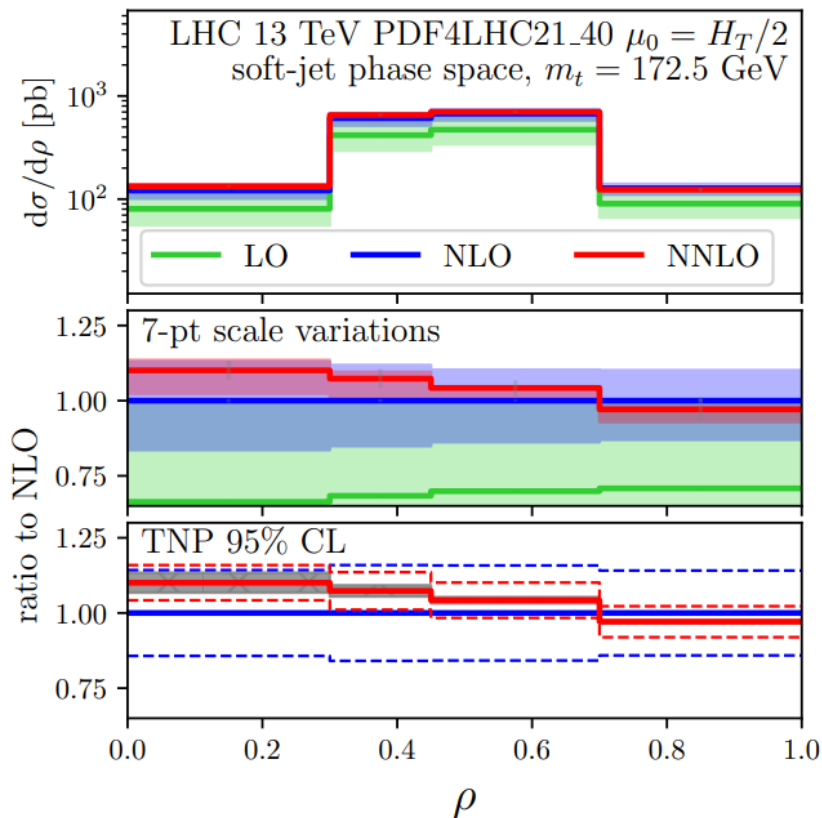
~ 1% - 4% in the high- p_T region

see also [Czakon,Poncelet;2512.17591]

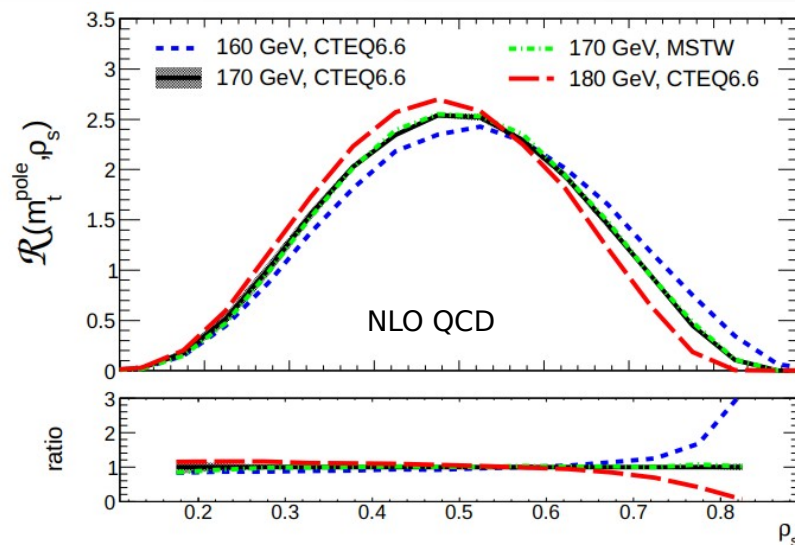
Results: $pp \rightarrow t\bar{t}j$ @ NNLO QCD

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Differential cross sections

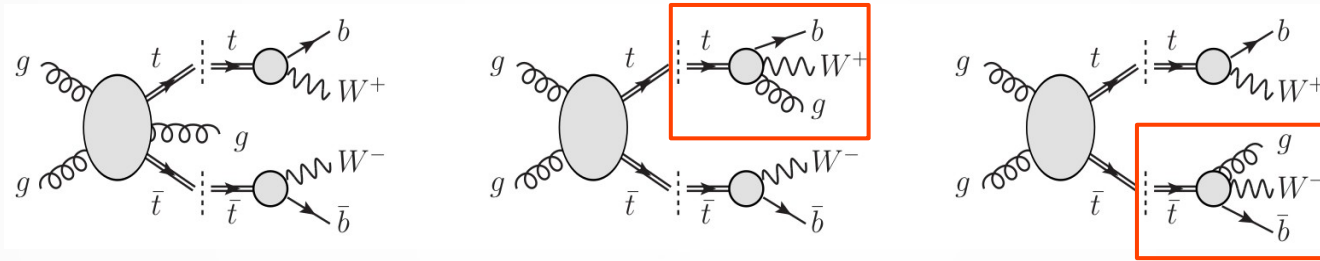


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[Alioli,Fernandez,Fuster,Irles,Moch,Uver,Vos;1303.6415]

Towards $pp \rightarrow ttj$ @NNLO QCD with top decays in NWA



Two-loop QCD amplitudes for $t \rightarrow bgv$

with C. Brancaccio [arXiv:26xx.xxxxx]

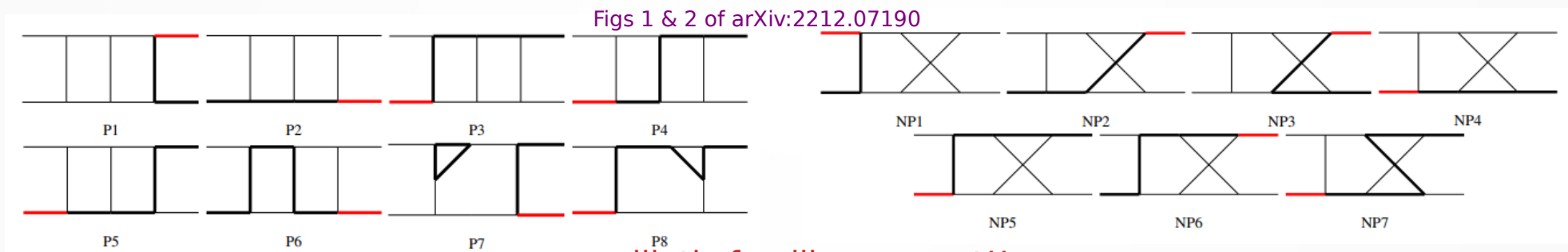
Two-loop amplitudes for $t \rightarrow b g \nu_l$

Full two-loop QCD amplitudes for $gb \rightarrow tW$ available [Chen,Dong,Li,Li,Wang;Wang;arXiv:2212.07190]

⇒ Numerical grid for unpolarised squared amplitude

⇒ “Good” basis of MIs: factors of $(1 + n \varepsilon)$ in the denominator (n : rational number)

⇒ Two-loop MIs evaluated by solving DEs using AMFlow solver + boundary points from AMFlow



many elliptic families present!!

Our goal:

⇒ analytic helicity amplitudes in terms of MIs

⇒ include the decay of W boson

⇒ a routine for numerical evaluation of the hard function for the decay kinematics

Two-loop amplitudes for $t \rightarrow b g \nu \ell$

$$0 \rightarrow t(p_1) + \bar{b}(p_2) + g(p_3) + \bar{\nu}(p_4) + \ell(p_5)$$

Separate W production and decay amplitudes:

$$\mathcal{A}_5^{(L)}(1_t^{h_1}, 2_{\bar{b}}^{h_2}, 3_g^{h_3}, 4_{\bar{\nu}}^{h_4}, 5_{\ell}^{h_5}) = \mathcal{A}_4^{(L)\mu}(1_t^{h_1}, 2_{\bar{b}}^{h_2}, 3_g^{h_3}, W) \mathcal{D}_\mu(W \rightarrow \bar{\nu} \ell) P_W(s_{45})$$

W-production currents in terms of form factors α

$$\mathcal{A}_4^{(L)\mu}(1_t^{h_1}, 2_{\bar{b}}^{h_2}, 3_g^{h_3}, W) = \sum_{i=1}^4 \alpha_i^{(L)}(1_t^{h_1}, 2_{\bar{b}}^{h_2}, 3_g^{h_3}, W) \xi_i^\mu$$

$$\alpha_i^{(L)} = \sum_j (\Delta^{-1})_{ij} \tilde{\mathcal{A}}_{4,j}^{(L)}$$

$$\begin{aligned} \xi_1^\mu &= p_1^\mu, & \xi_2^\mu &= p_2^\mu, & \xi_3^\mu &= p_3^\mu, \\ \xi_4^\mu &= \frac{\langle 3|1|2\rangle\langle 2|\gamma^\mu|3\rangle}{4d_{23}} - \frac{\langle 2|1|3\rangle\langle 3|\gamma^\mu|2\rangle}{4d_{23}}. \end{aligned}$$

Spin-basis decomposition of the contracted helicity amplitudes

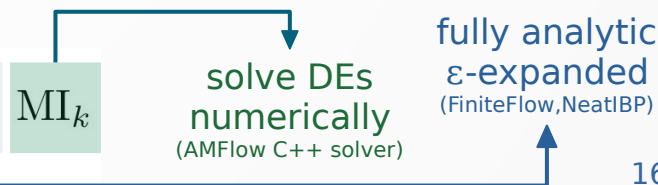
$$\begin{aligned} \tilde{\mathcal{A}}_{4,i}^{(L)}(1_t^+, 2_{\bar{b}}^+, 3_g^{h_3}, W; \eta) &= m_t^2 \frac{\Phi^{h_3}}{\langle 1^b \eta \rangle} \frac{[2|3|1|2]}{s_{23}^2} \left[\langle \eta 2 \rangle \tilde{\mathcal{A}}_{4,i}^{(L),[2]}(1_t^+, 2_{\bar{b}}^+, 3_g^{h_3}, W) \right. \\ &\quad \left. + \frac{\langle \eta 3 \rangle \langle 2|1|3\rangle}{s_{23}} \tilde{\mathcal{A}}_{4,i}^{(L),[3]}(1_t^+, 2_{\bar{b}}^+, 3_g^{h_3}, W) \right]. \end{aligned}$$

$$\tilde{\mathcal{A}}_{4,i}^{(L),[2]} \text{ and } \tilde{\mathcal{A}}_{4,i}^{(L),[3]}$$

are functions of s_{12}, s_{23}, s_{45} and m_t^2

Analytic mass-renormalised amplitudes
Compute finite remainders numerically

$$\tilde{\mathcal{A}}_{4,i}^{(L),[j]} = \sum_{k=1} c_{i,k}^{(L),[j]}(\epsilon, \vec{s}, m_t^2)$$



Two-loop amplitudes for $t \rightarrow b g \nu l$

Mathematica routine for the numerical evaluation of the hard functions

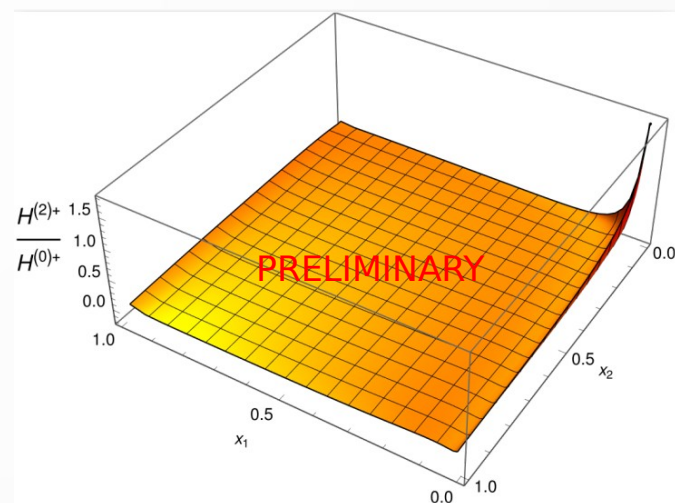
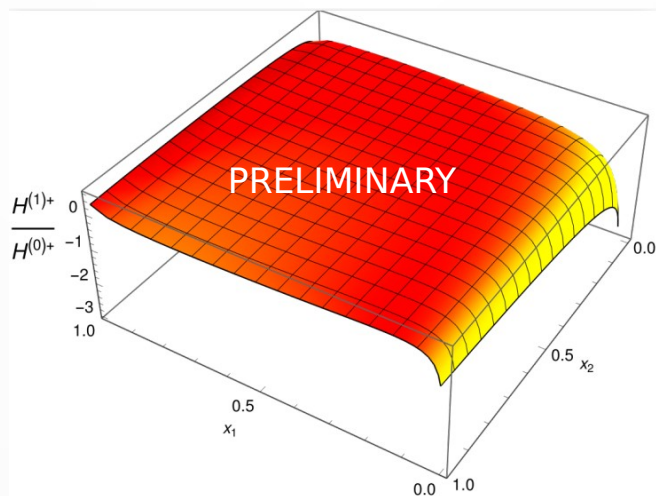
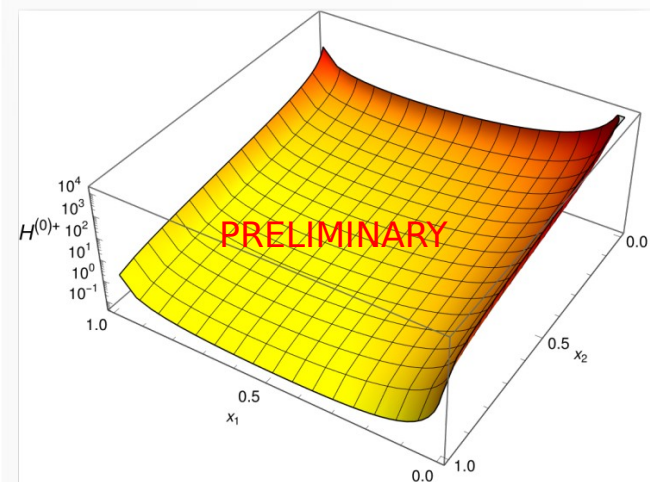
$$\mathcal{H}^{(0)} = \overline{\mathcal{F}^{(0)*}} \mathcal{F}^{(0)} \quad \mathcal{H}^{(1)} = 2 \operatorname{Re} \overline{\mathcal{F}^{(0)*}} \mathcal{F}^{(1)} \quad \mathcal{H}^{(2)} = 2 \operatorname{Re} \overline{\mathcal{F}^{(0)*}} \mathcal{F}^{(2)} + \overline{\mathcal{F}^{(1)*}} \mathcal{F}^{(1)}$$

Numerical results: visualisation on phase-space slices

→ PS parametrisation from Bruscheifer, Caola, Melnikov [arXiv:1301.7133] → $(x_1, x_2, x_3, x_7, x_8, x_9)$

Plots on (x_1, x_2) surface

$$E_3 = x_1 E_{\max}, \quad \cos \theta_3 = 1 - 2x_2, \quad \varphi_3 = 2\pi x_3.$$



Two-loop amplitudes for $t \rightarrow b g \nu l$

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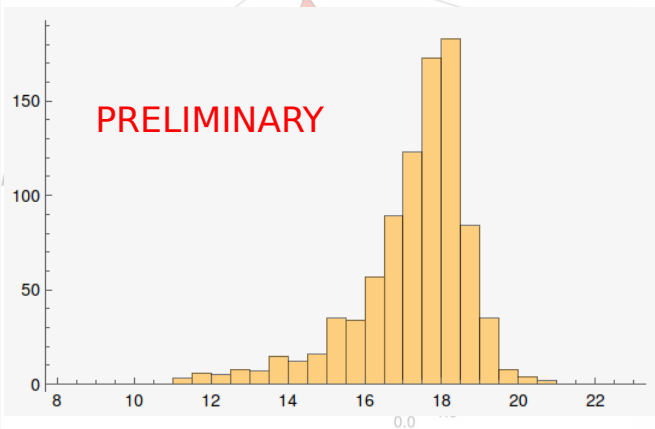
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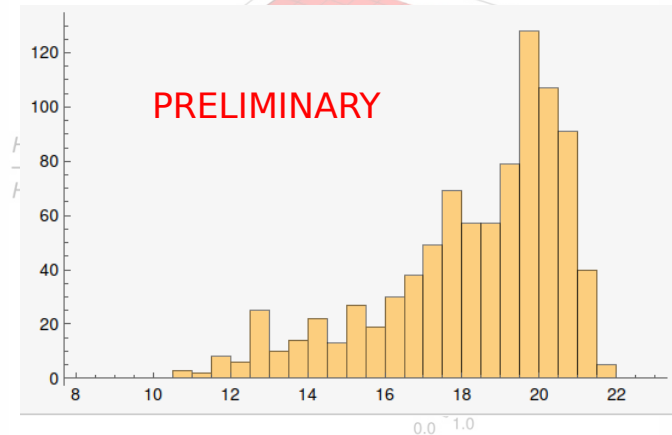
Plots on (x_1, x_2) surface

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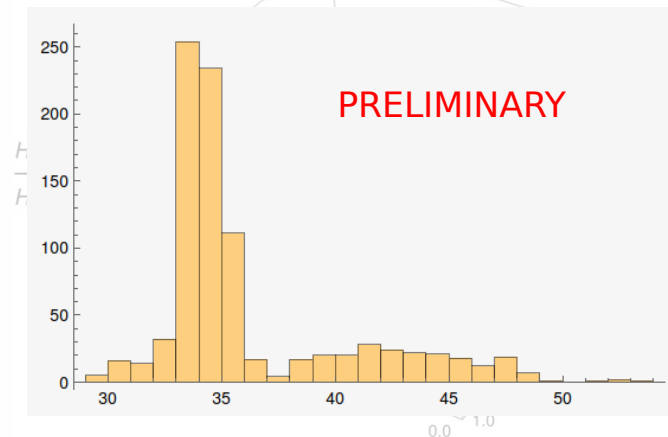
Estimated correct digits for $H^{(2)}$



Cancellation digits for $1/\varepsilon$ pole in $H^{(2)}$



Evaluation time (minutes)



Two-loop amplitudes for $t \rightarrow b g \nu l$

Mathematica routine for the numerical evaluation of the hard functions

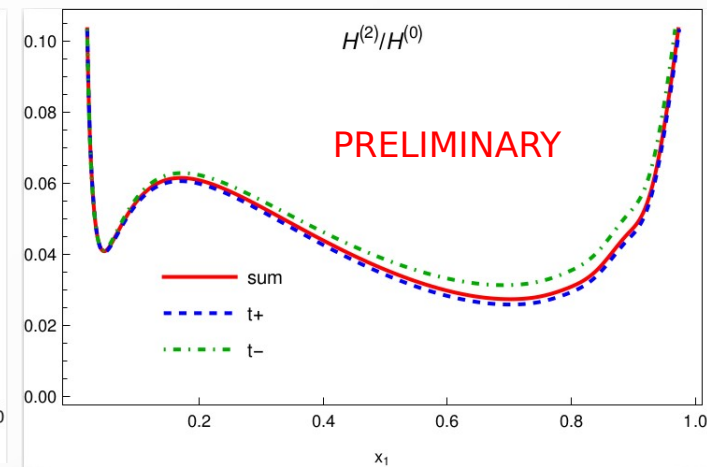
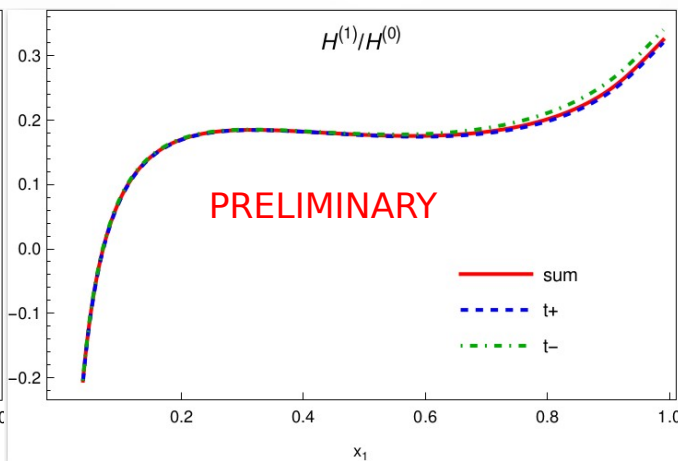
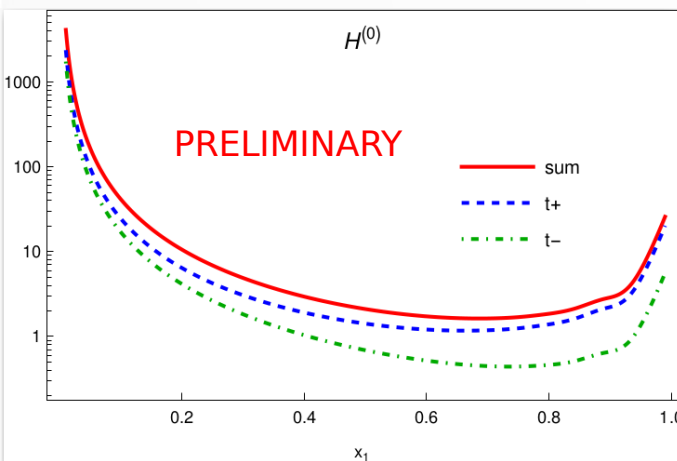
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Numerical results: visualisation on phase-space slices

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Plots on x_1 slice

$$E_3 = x_1 E_{\max}, \quad \cos \theta_3 = 1 - 2x_2, \quad \varphi_3 = 2\pi x_3.$$



Summary

- ✓ First results for $t\bar{t}j$ production at NNLO QCD accuracy (stable top)
 - STRIPPER subtraction scheme + two-loop leading colour amplitudes
- ✓ Analytic computation for two-loop helicity amplitudes: $gg \rightarrow t\bar{t}g$ and $qq \rightarrow t\bar{t}g$
 - good enough for pheno, but not the simplest form
- ✓ Two-loop amplitudes for $t \rightarrow b g \nu l$
 - ingredients of $pp \rightarrow t\bar{t}j$ NNLO QCD calculation with top decays in NWA
 - analytic form in terms of MIs, numerical evaluation of MIs through DEs

Backup slides

NLO QCD $pp \rightarrow ttj$: NWA vs Full off-shell calculations

[Bevilacqua, **HBH**, Kraus, Schulze, Worek; 1710.07515]

$$pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} j + X$$

