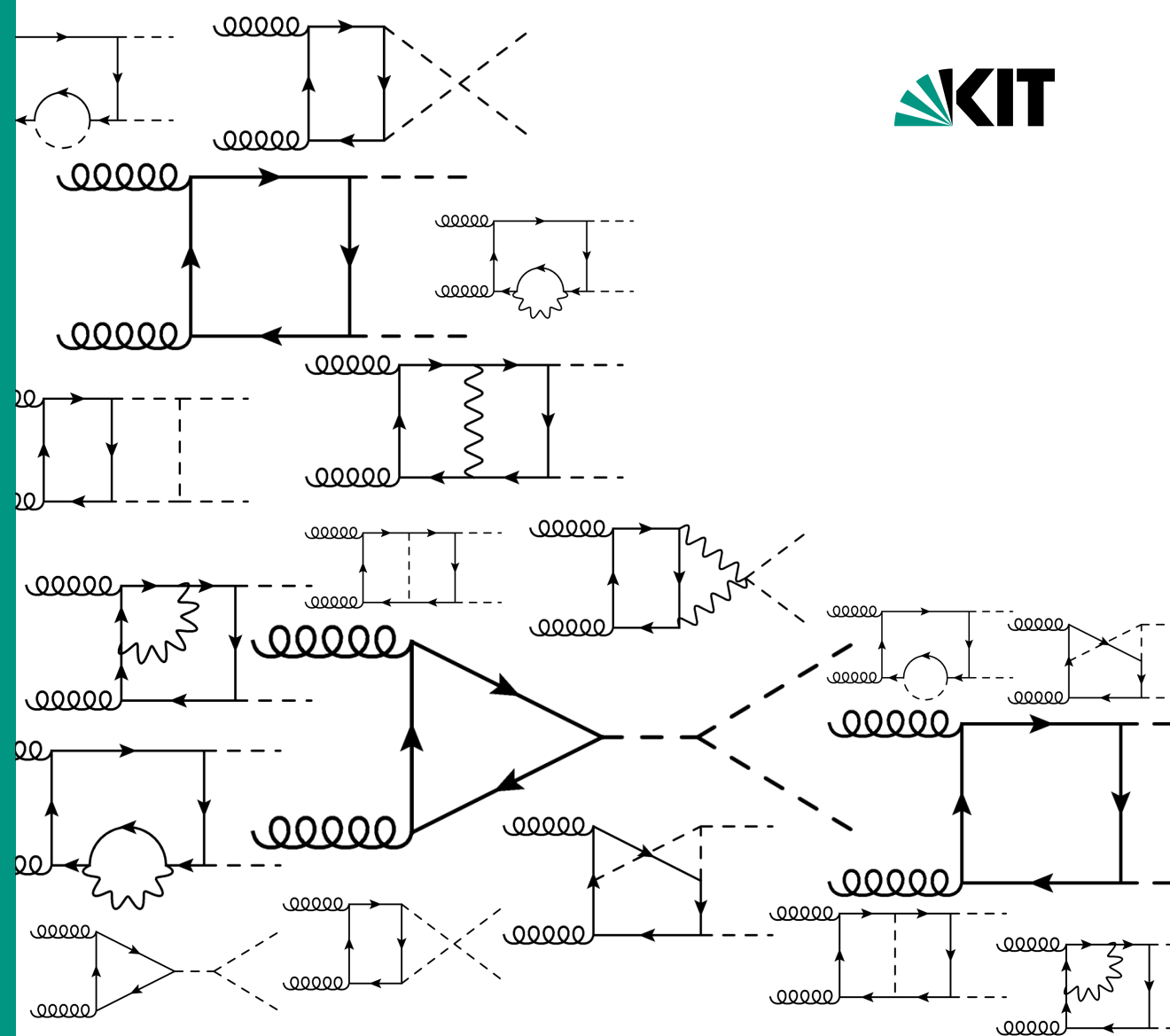


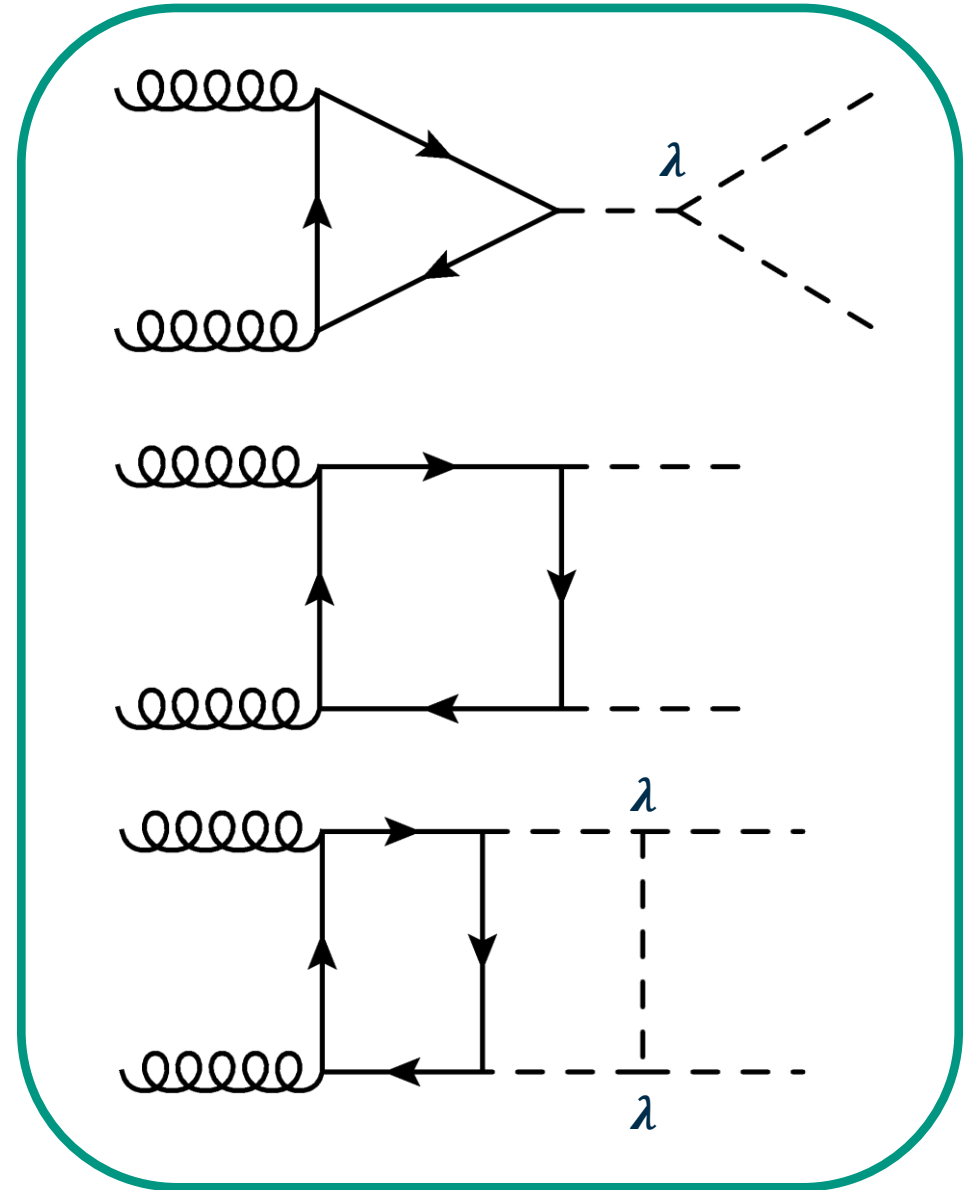
NLO ELECTROWEAK CORRECTIONS to HH PRODUCTION via GLUON FUSION



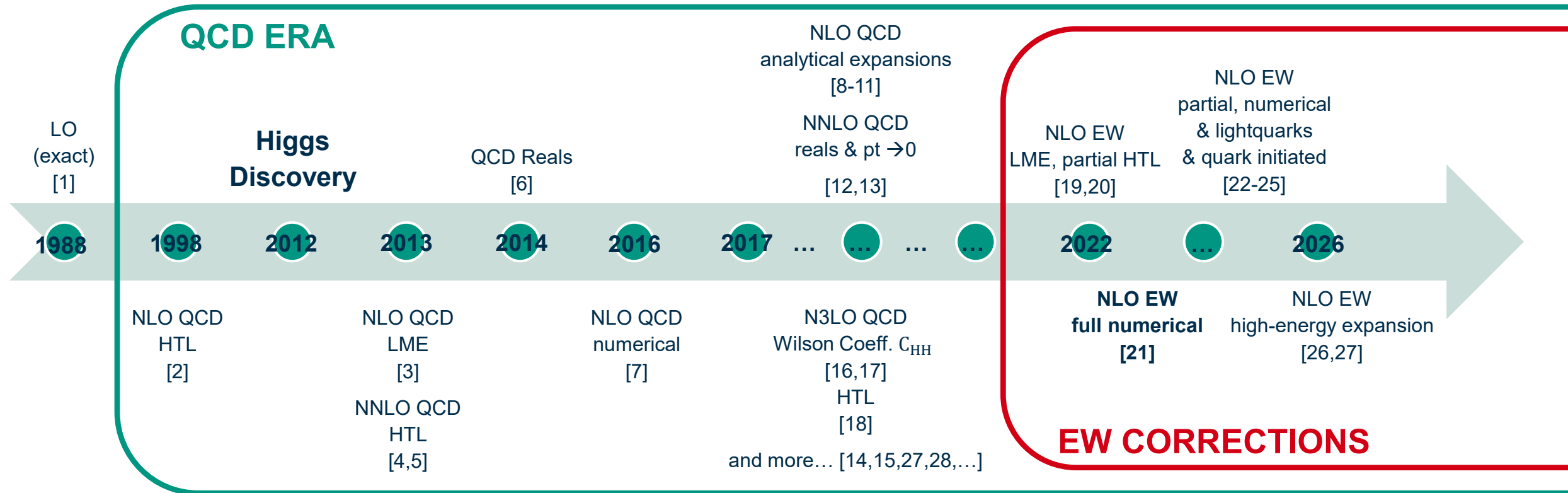
Dominik Grau, Matthias Steinhauser, Daniel Stremmer,
Joshua Davies, Kay Schönwald

Motivation

- Most promising process to probe the Higgs self-coupling in the Standard Model (SM)
 - $V_{\text{SM}}(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{1}{4} \lambda H^4$
- Current experimental constraints for $\kappa_\lambda = \frac{\lambda_{\text{obs}}}{\lambda}$
 - $-0.71 < \kappa_\lambda < 6.1$, ATLAS&CMS[2602.23991]
- Projected uncertainty for high luminosity LHC
 - $\Delta\kappa_\lambda \approx 30\%$, ATLAS&CMS[2504.00672]



The Theory Side - Progression



Inspired by Sauro Carlotti

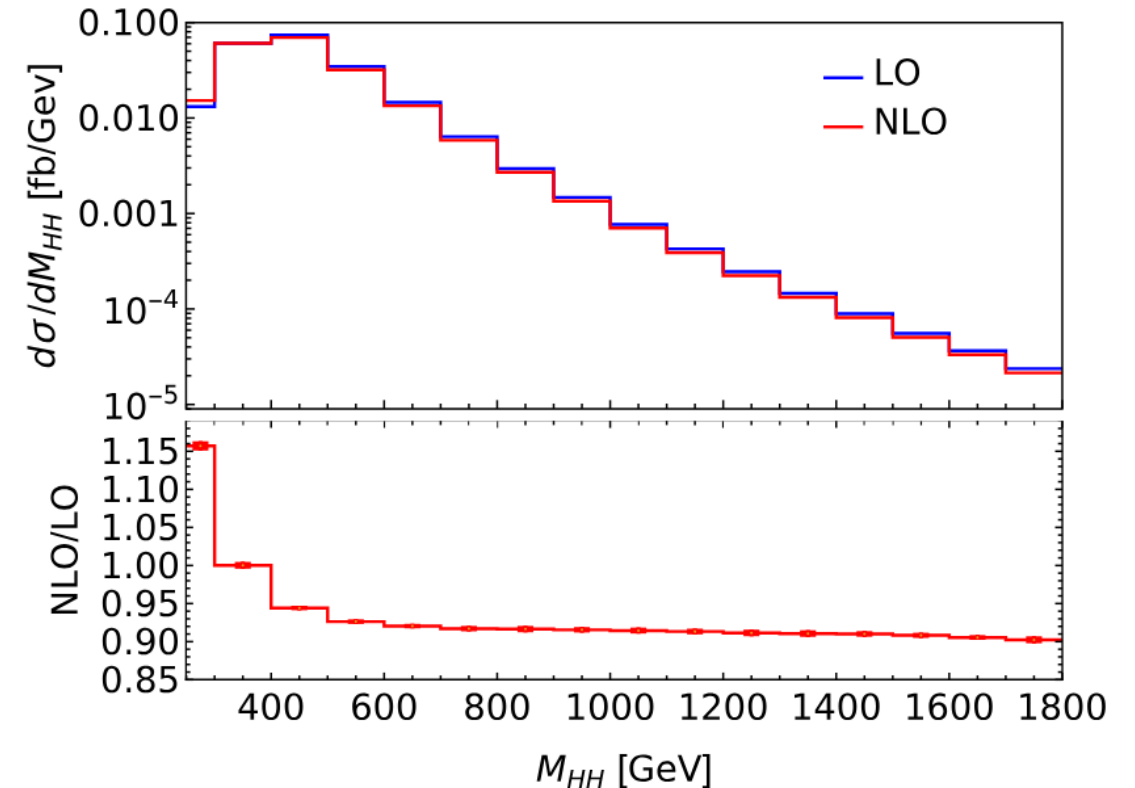
[1] Glover, van der Bij '88 [2] Dawson, Dittmaier, Spira '98 [3] Grigo, Hoff, Melnikov, Steinhauser '13 [4] de Florian, Mazzitelli '13 [5] Grigo, Melnikov, Steinhauser '14 [6] Maltoni, Vryonidou, Zaro '14 [7] Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16 [8] Gröber, Maier, Rauh '17 [9] Davies, Mishima, Steinhauser, Wellmann '18 '19 [10] Bonciani, Degraasi, Giardino, Gröber '18 [11] Bagnaschi, Degraasi, Gröber '23 [12] Davies, Herren, Mishima, Steinhauser '19 '21 [13] Davies, Schönwald, Steinhauser '23 '25 [14] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19 [15] Davies, Steinhauser '19 [16] Spira '16 [17] Gerlach, Herren, Steinhauser '18 [18] Chen, Li, Shao, Wang '19 [27] Davies, Schönwald, Steinhauser, Vitti '24 [28] Davies, Stremmer, Schönwald, Steinhauser '25

[19] Mühlleitner, Schlenk, Spira '22 [20] Davies, Mishima, Schönwald, Steinhauser, Zhang '22 [21] Bi, Huang, Huang, Ma, Yu '23 [22] Heinrich, Jones, Kerner, Stone, Vestner '24 [23] Bonetti, Rendler, Torres Bobadilla '25 [24] Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, Spira '25 [25] Bonetti, Heinrich, Rendler, Torres Bobadilla '26 [26] Davies, Mishima, Schönwald, Steinhauser, Zhang '25 [27] Davies, Schönwald, Steinhauser, Zhang '26

Recap: NLO Electroweak Corrections

- QCD scale uncertainty is now a few percent, top mass scheme dependency $\sim 20\%$ to NLO
 - NLO EW corrections are substantial source of theoretical uncertainty, especially at the differential level
 - NLO EW full numerical exist [Bi, Huang, Huang, Ma, Yu '23], but:
 - They need to be crosschecked
 - The top-quark mass is fixed!
- Keeping full top-quark mass dependence allows convenient study of mass effects

Differential cross sections:



Two-Loop Diagrams

[illegible]

Computing the amplitude – A textbook way

See talk by
Philipp Rendler

1. Amplitude

Generate all diagrams and sum them

2. IBP

Use the method of **Integration by Parts (IBP)** reduction to express all appearing Feynman integrals in terms of a minimal basis of master integrals $\vec{M}$.

3. DEs

Setup a system of **differential equations (DE)** for the master integrals and solve it.

Computing the amplitude – A textbook way ... ☹ ?

See talks by
Sauro Carlotti
and
Augustin
Vestner

1. Amplitude

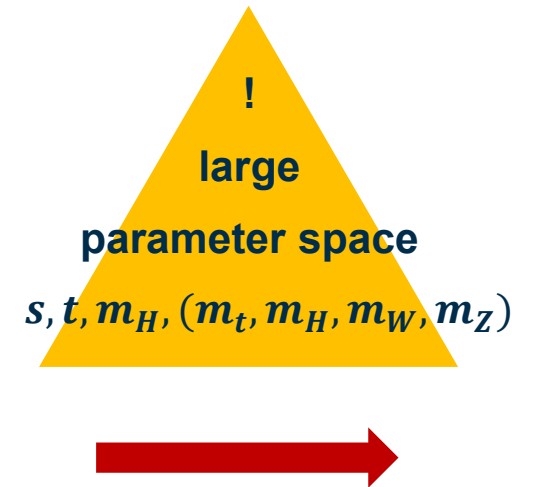
Generate all diagrams and sum them

2. IBP

Use the method of **Integration by Parts (IBP)** reduction to express all appearing Feynman integrals in terms of a minimal basis of master integrals $\vec{M}$.

3. DEs

Setup a system of **differential equations (DE)** for the master integrals and solve it.



Computing the integrals is not that easy

3. DEs

Setup a system of **differential equations (DE)** for the master integrals and solve it.

$$\partial_x \vec{M} = A_x \vec{M}$$

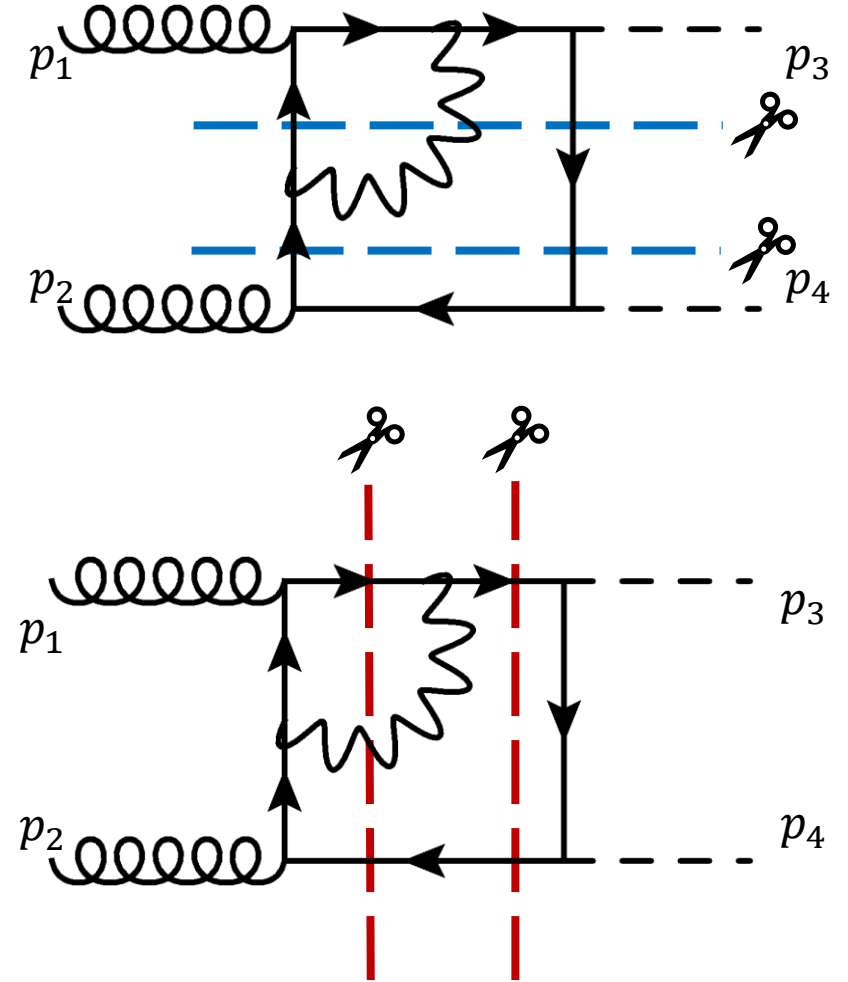
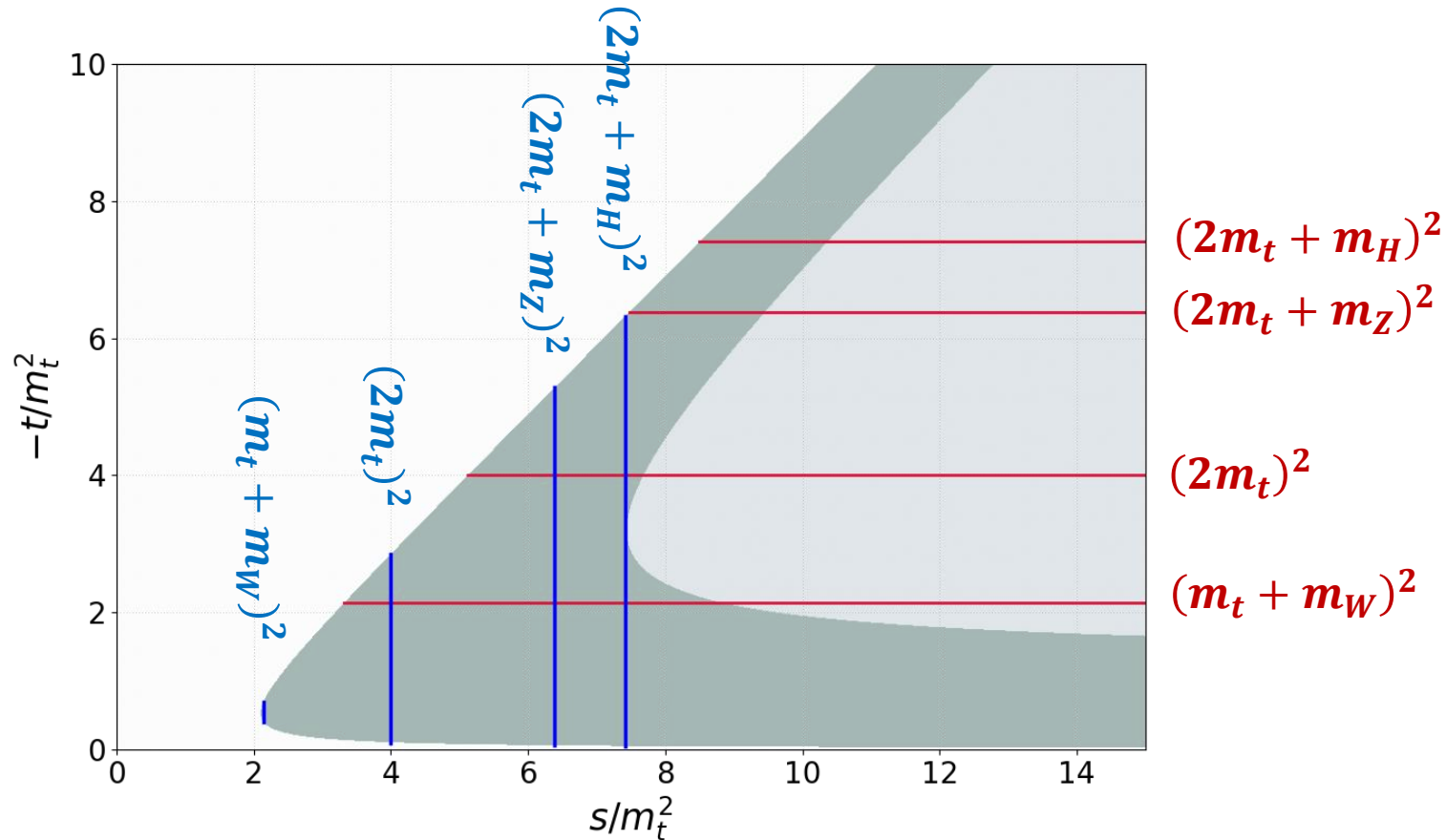
$$x \in \{s, t, m_H, (m_H, m_W, m_Z)\}$$

The **coefficient matrices** A_x are rational functions of the kinematic variables and scales which **exhibit singular loci** in the phase space.

The most complicated master integrals contain elliptic sectors for which no efficient analytical closed form exists so far.

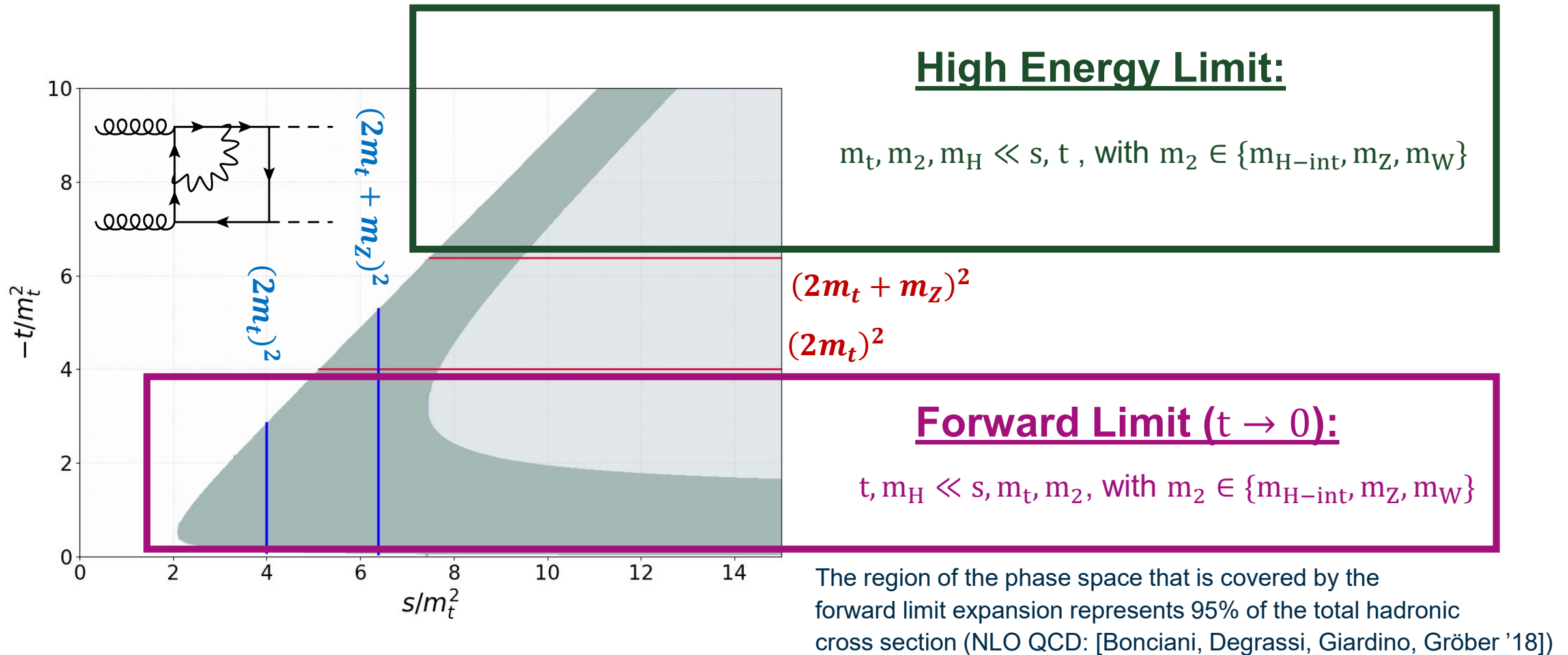
The problematic Singular Loci

agree with the Particle Cuts that can be made at diagram level



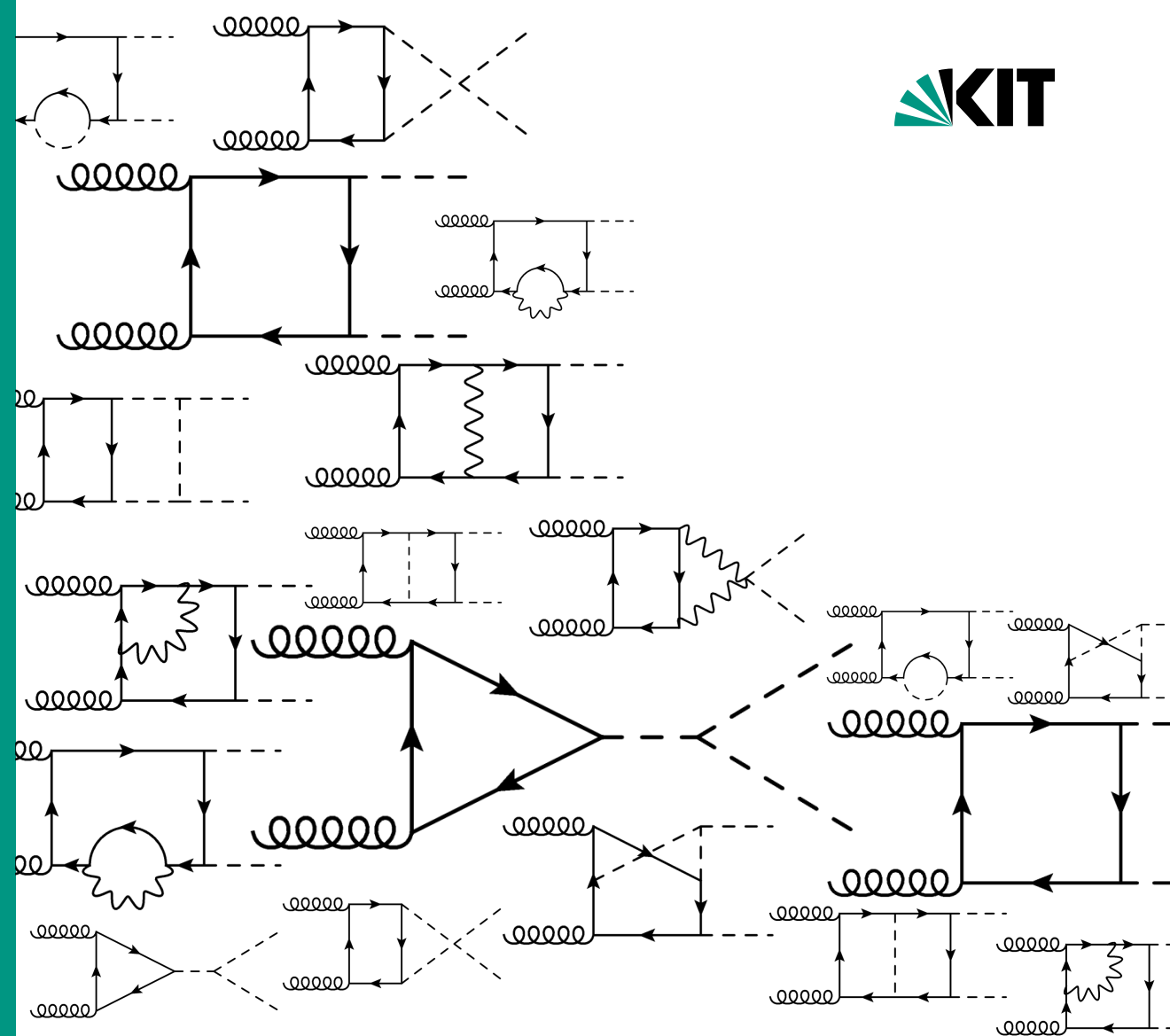
Introducing Analytic Approximations

to reduce the parameterspace/complexity of the IBP & DE to be solved



NLO ELECTROWEAK CORRECTIONS to HH PRODUCTION via GLUON FUSION in the FORWARD LIMIT

Dominik Grau, Matthias Steinhauser, Daniel Stremmer,
Joshua Davies, Kay Schönwald



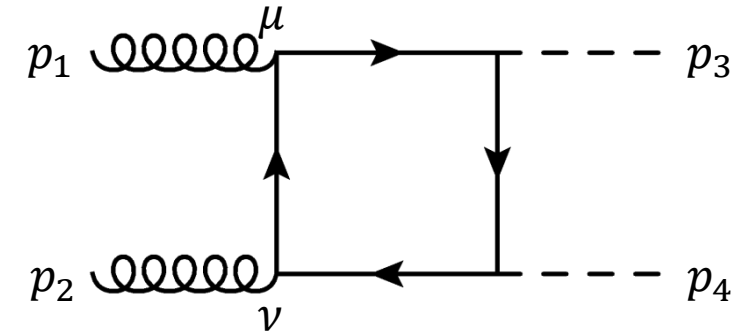
Workflow

Amplitude Generation

$t \rightarrow 0$ Expansion

IBP & DE

1. Generate diagrams with **QGRAF** [Nogueira '91]
2. Generate minimal set of scalar integral families using **TAPIR** [Gerlach, Herren, Lang '22]
 - We use Feynman-Gauge for EW gauge parameter $\xi_{\text{EW}} = 1$
3. Map diagrams on integral families with **EXP** [Harlander, Seidensticker, Steinhauser '98]
4. Generate amplitude and apply projectors using in house **FORM** [Davies, Kaneko, Marinissen, Ueda, Vermaseren '26] code



$$A = \delta_{ab} \epsilon_\mu^a(p_1) \epsilon_\nu^b(p_2) A^{\mu\nu} \quad \text{with} \quad A^{\mu\nu} = F_1 M_1^{\mu\nu} + F_2 M_2^{\mu\nu} \quad \text{and} \quad P_{1/2, \mu\nu} A^{\mu\nu} = F_{1/2}$$

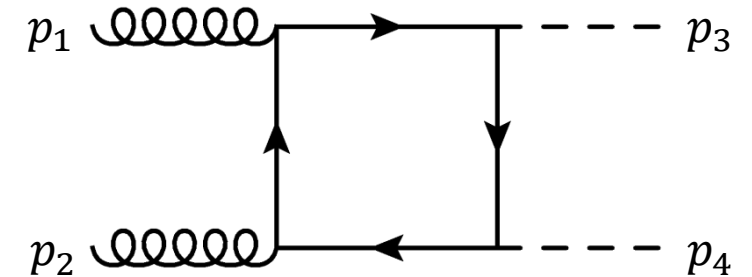
Workflow

$t \rightarrow 0$ Expansion

IBP & DE

Solving the DE

1. Consider scaling $t, m_H \ll s, m_t, m_2$, with $m_2 \in \{m_{H\text{-int}}, m_Z, m_W\}$
2. Expand all appearing scalar integrals $I(s, t, m_H, m_t, m_2)$ in linear combination $(p_1 + p_3)$ of the external momenta
3. Perform tensor reduction and obtain:



$$I(s, t, m_H, m_t, m_2) = \sum_{i,j,k} c_{i,j,k} I_{0,i}(s, m_t, m_2) t^j m_H^{2k} \text{ for } j + k \leq 5$$

Identical to pt-expansion, see. [Bonciani, Degrassi, Giardino, Gröber '18] for the NLO QCD amplitude

Workflow

IBP & DE

Solving the DE

- After the expansion and tensor reduction there are $\#O(10^7)$ $I_{0,i}(s, m_t, m_2)$ integrals with $m_2 \in \{m_{H\text{-int}}, m_Z, m_W\}$
- Use **KIRA3** [Maierhoefer, Usovitsch, Uwer '17; Lange, Usovitsch, Wu '25] to perform Integration by Parts (IBP) method to reduce them to a basis of #200 master integrals $M_i(s, m_t, m_2)$

- Use **LiteRed** [R.N. Lee '12] to setup the DE:

$$\partial_x \vec{M} = A_x \vec{M}$$
$$x \in \{s, m_2\}$$

The DE still exhibits elliptic behaviour, but by reducing the parameterspace the complexity has significantly decreased

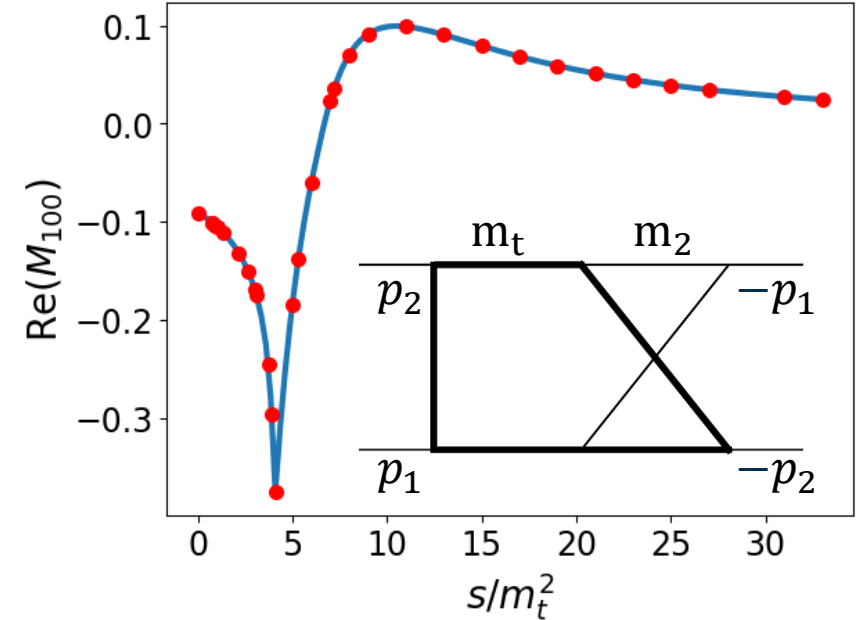
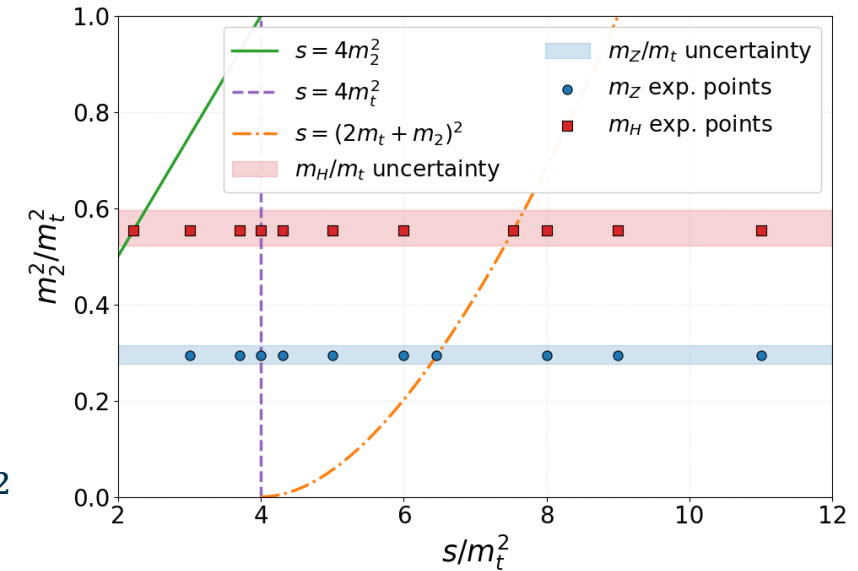
Workflow

Solving the DE

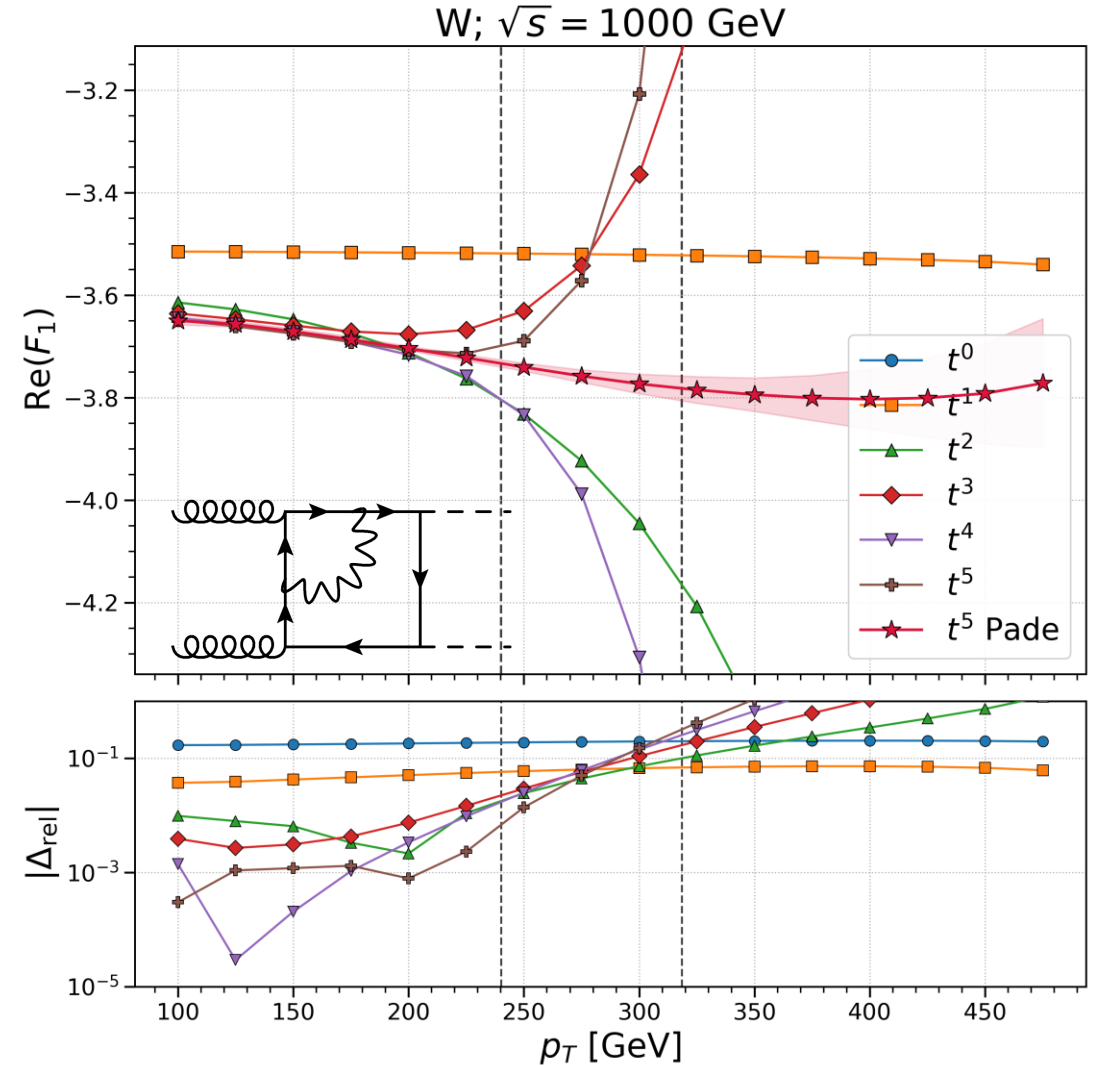
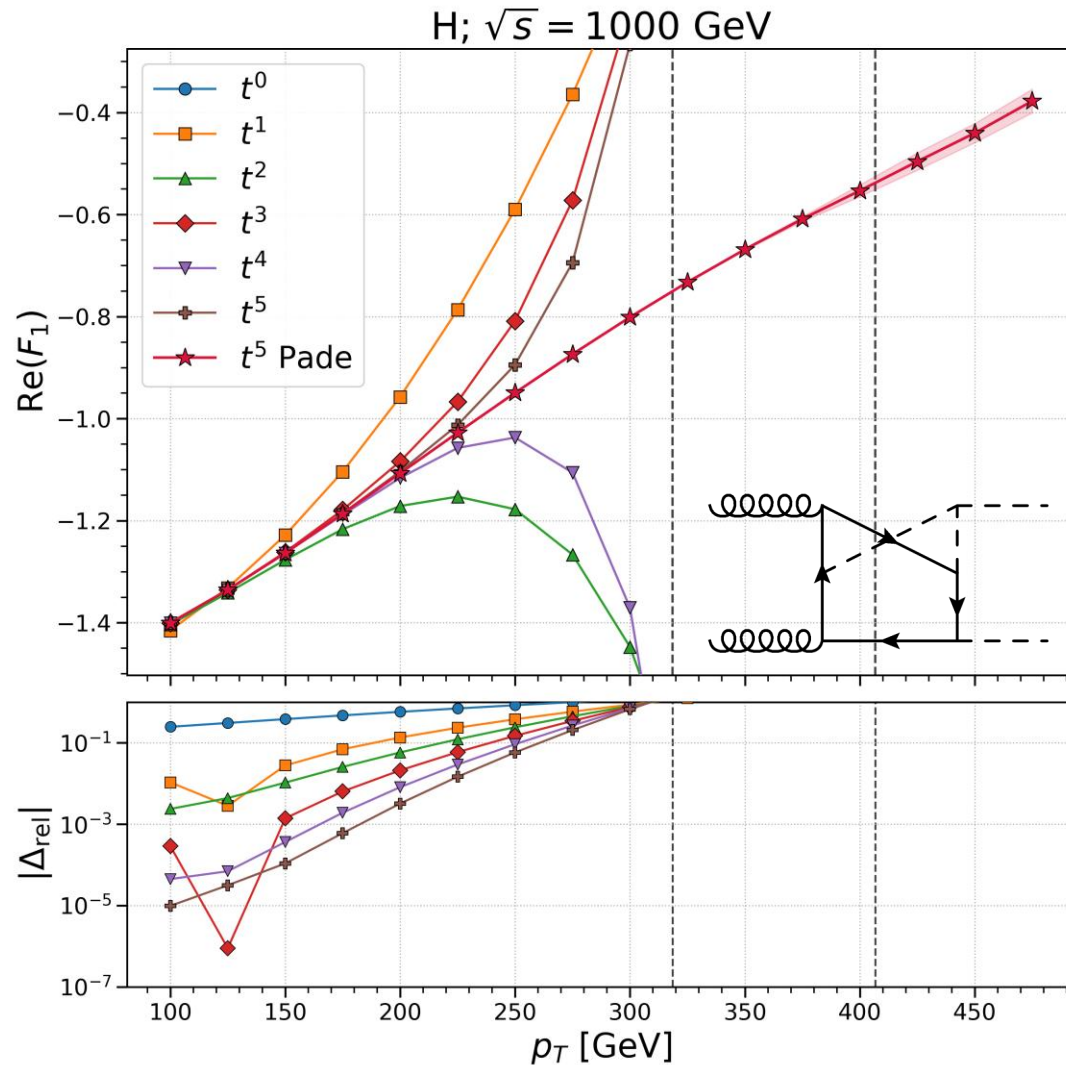
- Using **Expand and Match**[Fael, Lange, Schönwald, Steinhauser '21]
 - Expand the DE around fixed values of $x = s/m_t^2$ and $y = m_2^2/m_t^2$
 - If $\{x,y\}$ =(physical threshold) choose ansatz:

$$\sum_{i,j,k,n} c_{i,j,k,n} \epsilon^i (\sqrt{x})^j (\ln \sqrt{x})^k y^n$$
 else choose

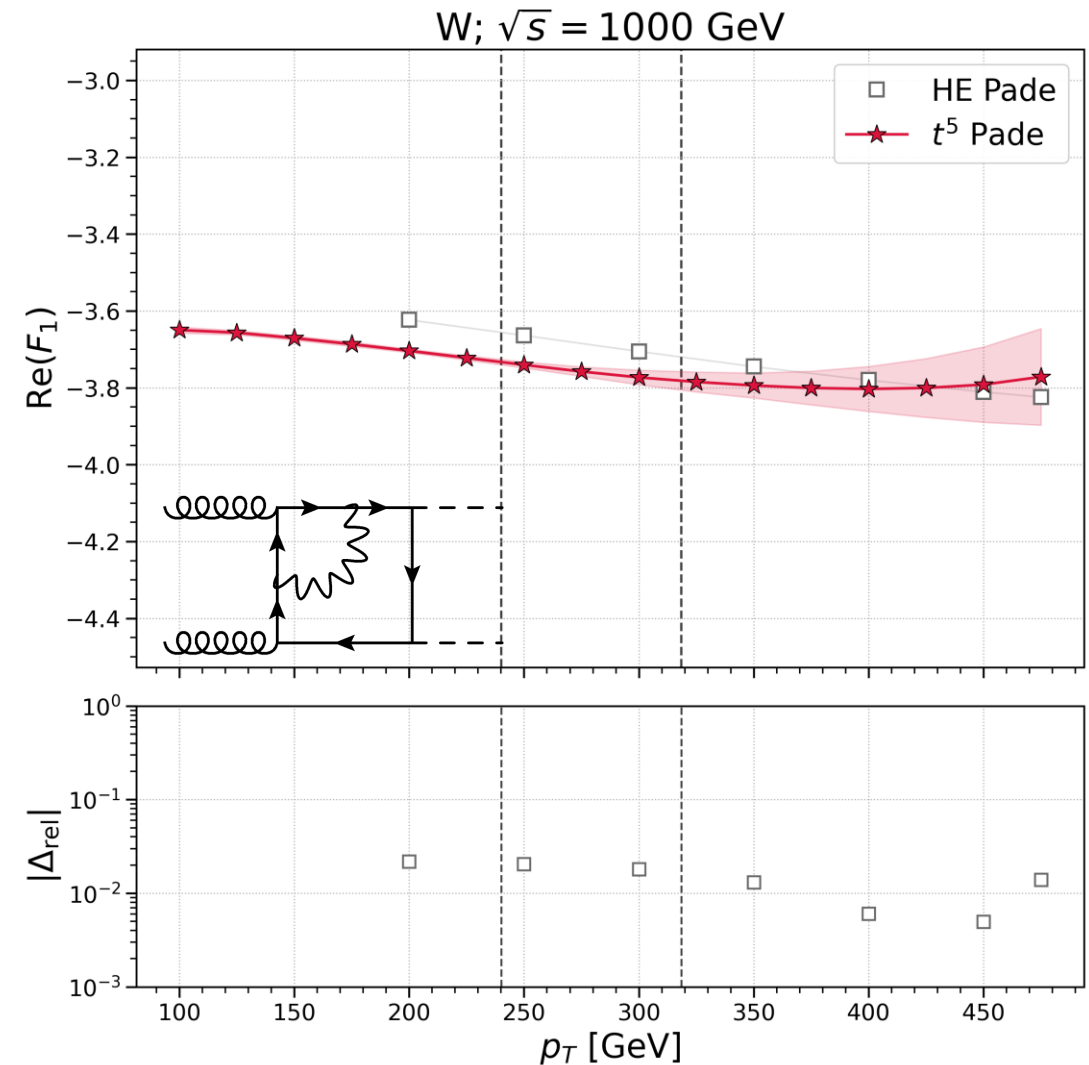
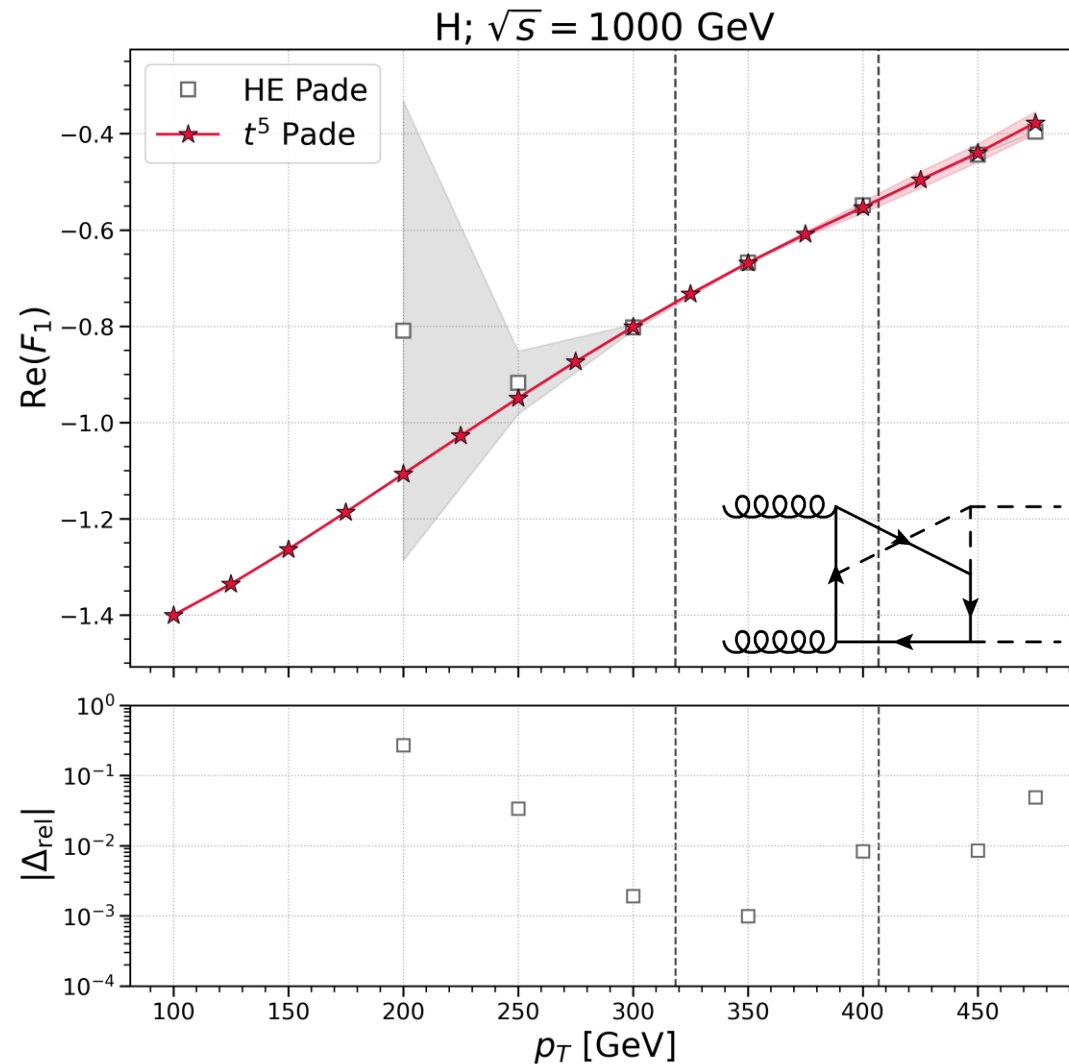
$$\sum_{i,j,k} c_{i,j,k} \epsilon^i x^j y^k.$$
 - Match against numerical results (**AMFLOW**[Xiao Liu, Yan-Qing Ma '22]) to fix boundary conditions



Particle Contributions to the bare Amplitude



Particle Contributions to the bare Amplitude



Summary

- Subpercent precision for small p_T phase space using forward limit expansions
- Coverage of full phase space by combining with High-Energy expansion

Whats next?

- Finalize and publish
- Implementation into the C++ library *ggxy*[Davies, Schönwald, Steinhauser, Stremmer '25]
 - $\mathcal{O}(ms)$ runtime per phase space points



Renormalization

- Tadpole contributions are consistently included in the bare two-loop amplitude and the Z factors according to the so-called Fleischer–Jegerlehner tadpole scheme
- Since there is no real-radiation contribution to the NLO EW corrections all $1/\epsilon$ are of ultraviolet nature. We remove them by using the on-shell renormalization scheme for the following quantities:
 - Higgs-Boson mass: $m_{H,0} = Z_{m_H} m_H$
 - W-Boson mass $m_{W,0} = Z_{m_W} m_W$
 - Top-Quark mass $m_{t,0} = Z_{m_t} m_t$
 - Electron charge $e_0 = Z_e e$
 - Cosine of the Weinberg angle $cw_0 = Z_{cw} cw$

And the wave-function of the Higgs-Boson:

$$H_0 = \sqrt{Z_H} H$$

Computer Programs

- Amplitude Generation & Projection
 - QGRAF [Nogueira'91]
 - TAPIR [Gerlach, Herren, Lang – 2201.05618]
 - EXP [Harlander, Seidensticker, Steinhauser'98]
 - in house FORM code
- IBP-Reduction:
 - KIRA [Maierhoefer, Usovitsch, Uwer - 1705.05610]
 - FIRE [Smirnov, Chukharev - 1901.07808]
 - LITERED [Lee – 1212.2685]
 - REDUZE [Manteuffel, Studerus – 1201.4330]

IBP – Integration by Parts

- **Example:** Massive Tadpole Integral:

$$F(a) = \int \frac{d^d k}{(m^2 - k^2)}$$

- The IBP relation is as follows:

$$\int d^d k \frac{\partial}{\partial k_\mu} \frac{k_\mu}{(m^2 - k^2)} = 0$$

- This leads to:

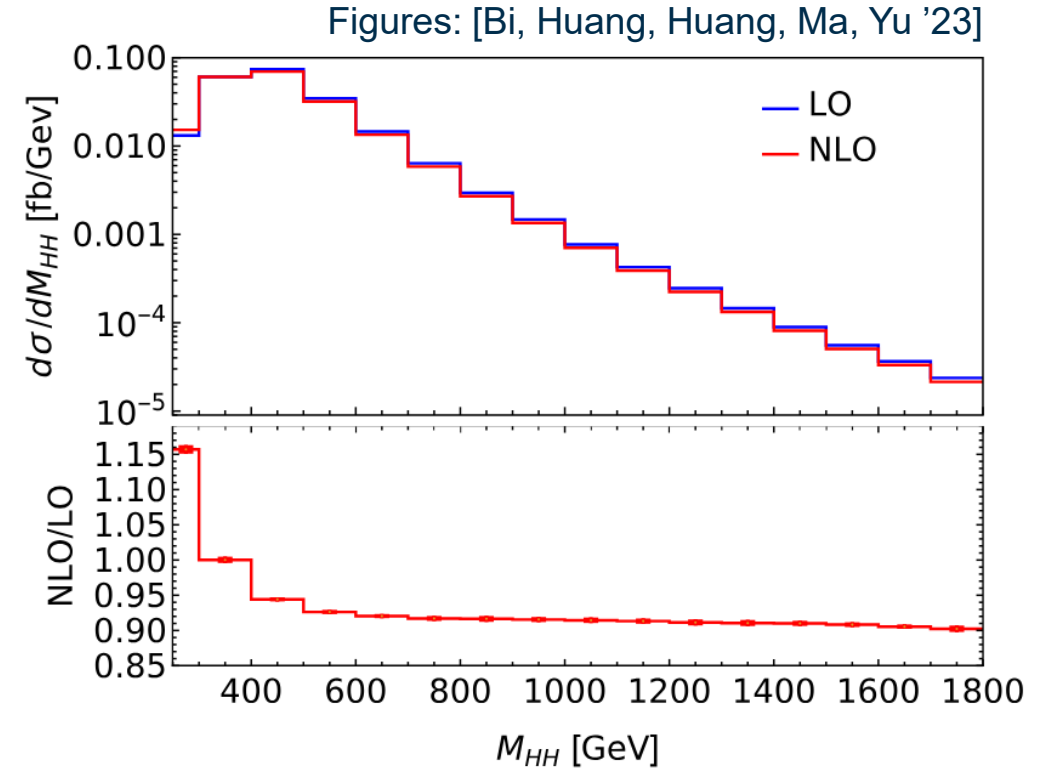
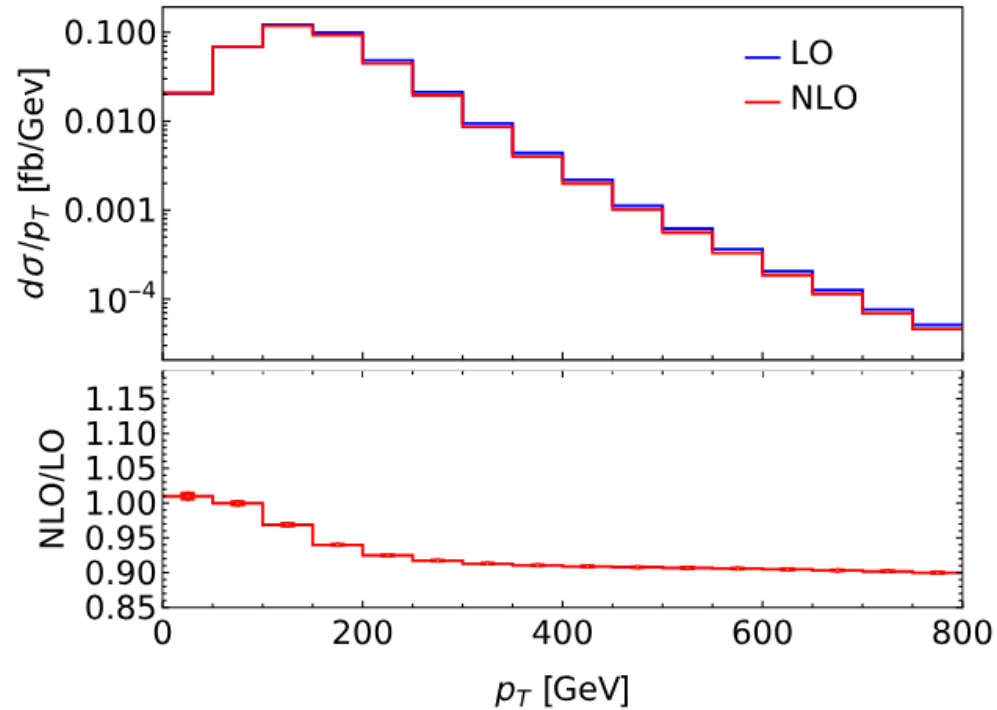
$$\begin{aligned} (d - 2a)F(a) - 2am^2 F(a + 1) &= 0 \\ \Rightarrow F(a) &= \frac{d - 2a + 2}{2(a - 1)m^2} F(a - 1) \end{aligned}$$

→ From knowing the Master Integral $F(1)$ all other integrals can be calculated recursively

Generalization:

- Derivatives w.r.t. all loop momenta
 - Integrand multiplied with all loop + external momenta
- System of linear Equation
- Solve with Laporta Algorithm

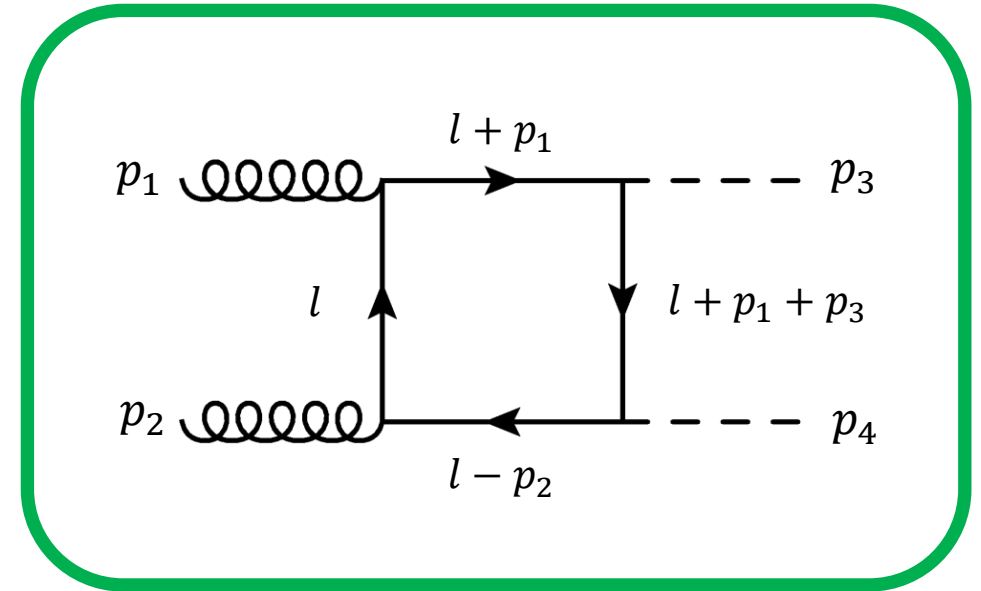
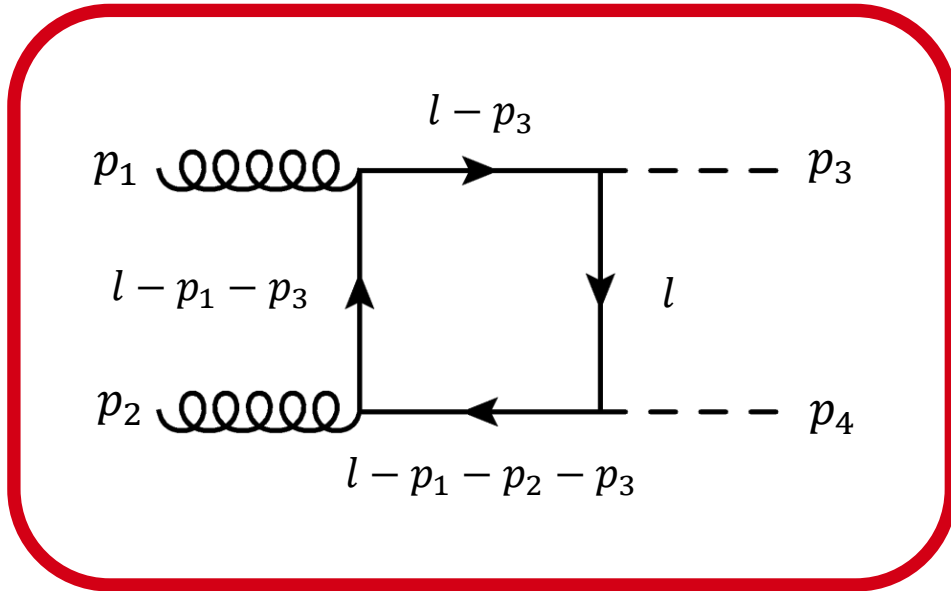
Differential Crosssections



- The region of the phase space that is covered by the forward limit expansion represents 95% of the total hadronic cross section (NLO QCD: [Bonciani, Degrandi, Giardino, Gröber '18])

Runtime & Filesize Optimization: Fixing Momentum routing

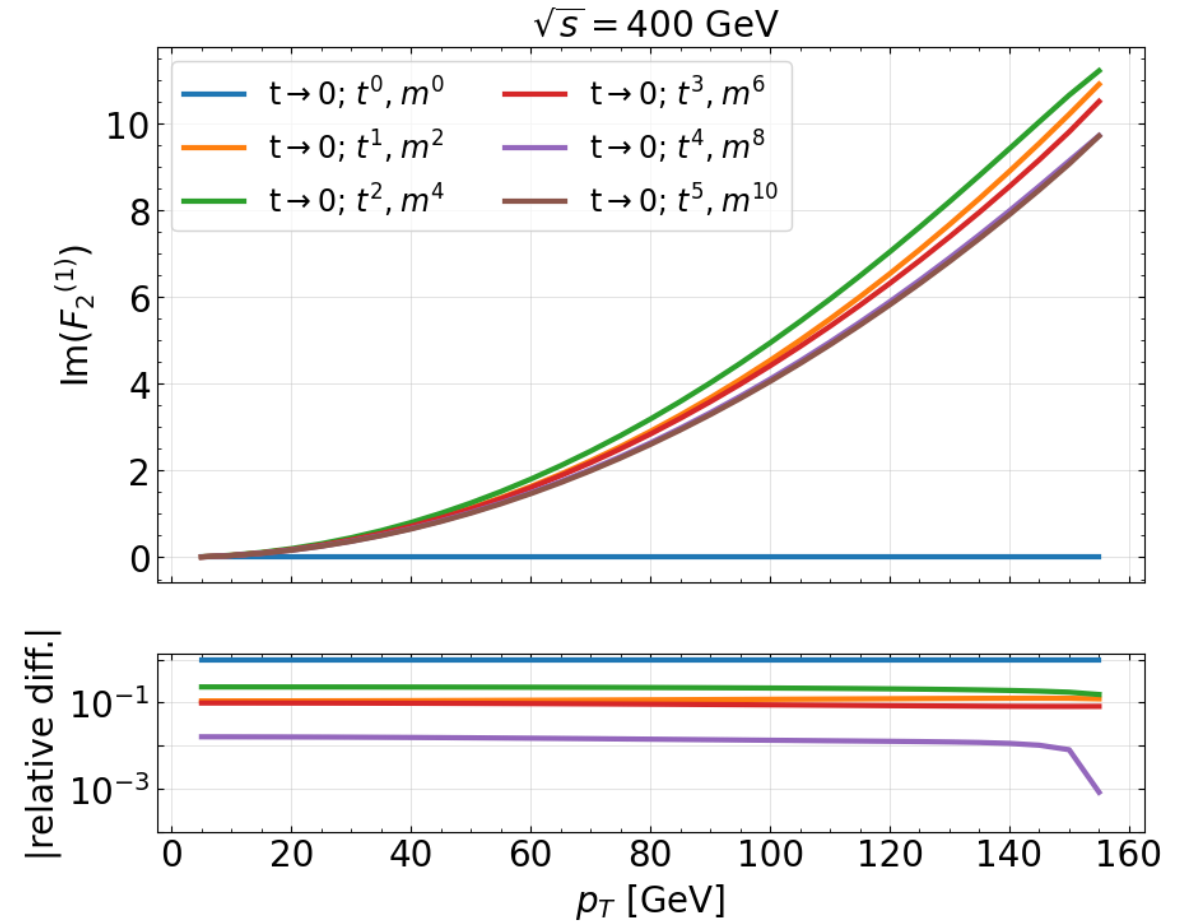
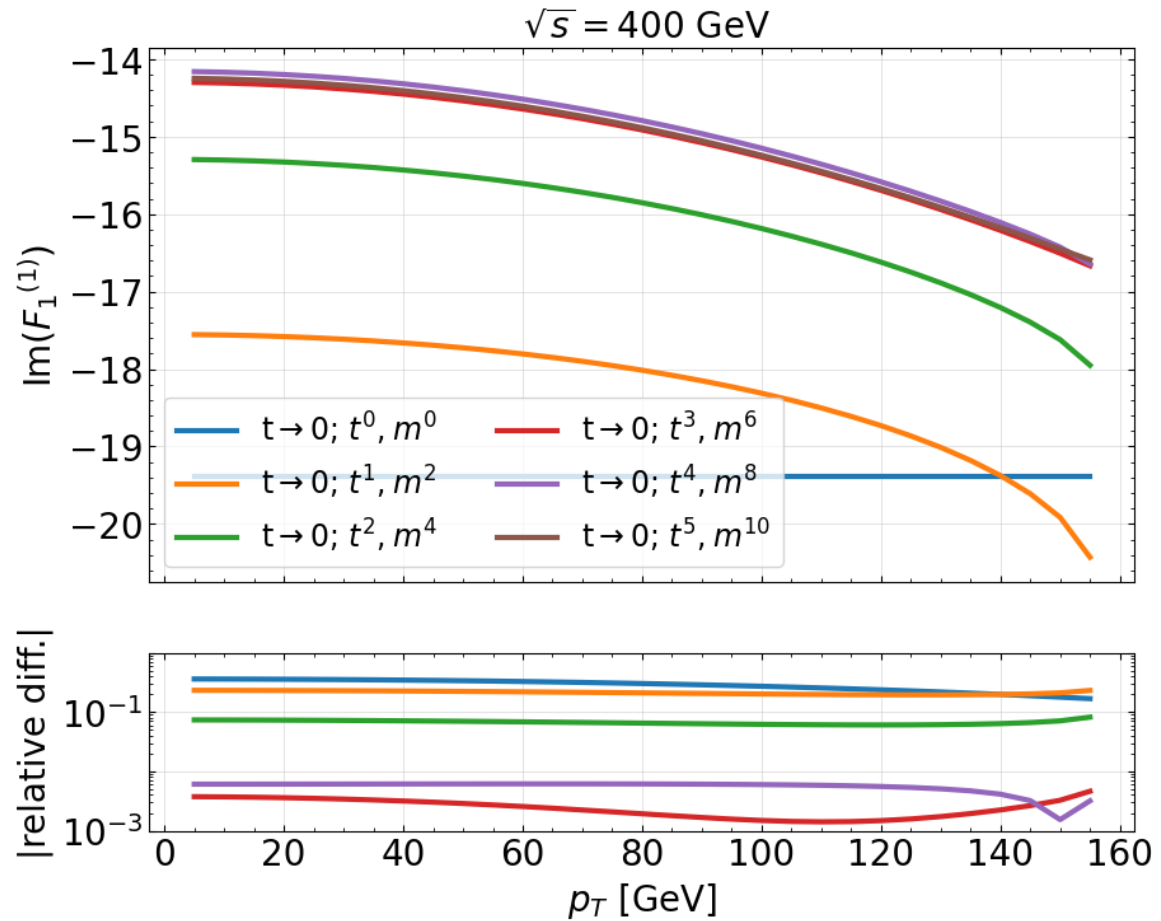
TAPIR output



After Expansion:	140 mio. terms	$\left[\text{For full NLO EW amplitude expanded to } t^j m_H^{2k} \text{ for } j + k \leq 2 \right]$		35 mio. terms
After Tensor Reduction:	9 mio. terms			6 mio. terms

- Minimize occurrences of $\delta q = p_1 + p_3, p_1, p_2, l_1, l_2$
- Minimize occurrences of p_1, p_2, l_1, l_2 in δq dependent denominators

The renormalized Amplitude



Radius of Convergence – Particle Cuts – The H,Z Case

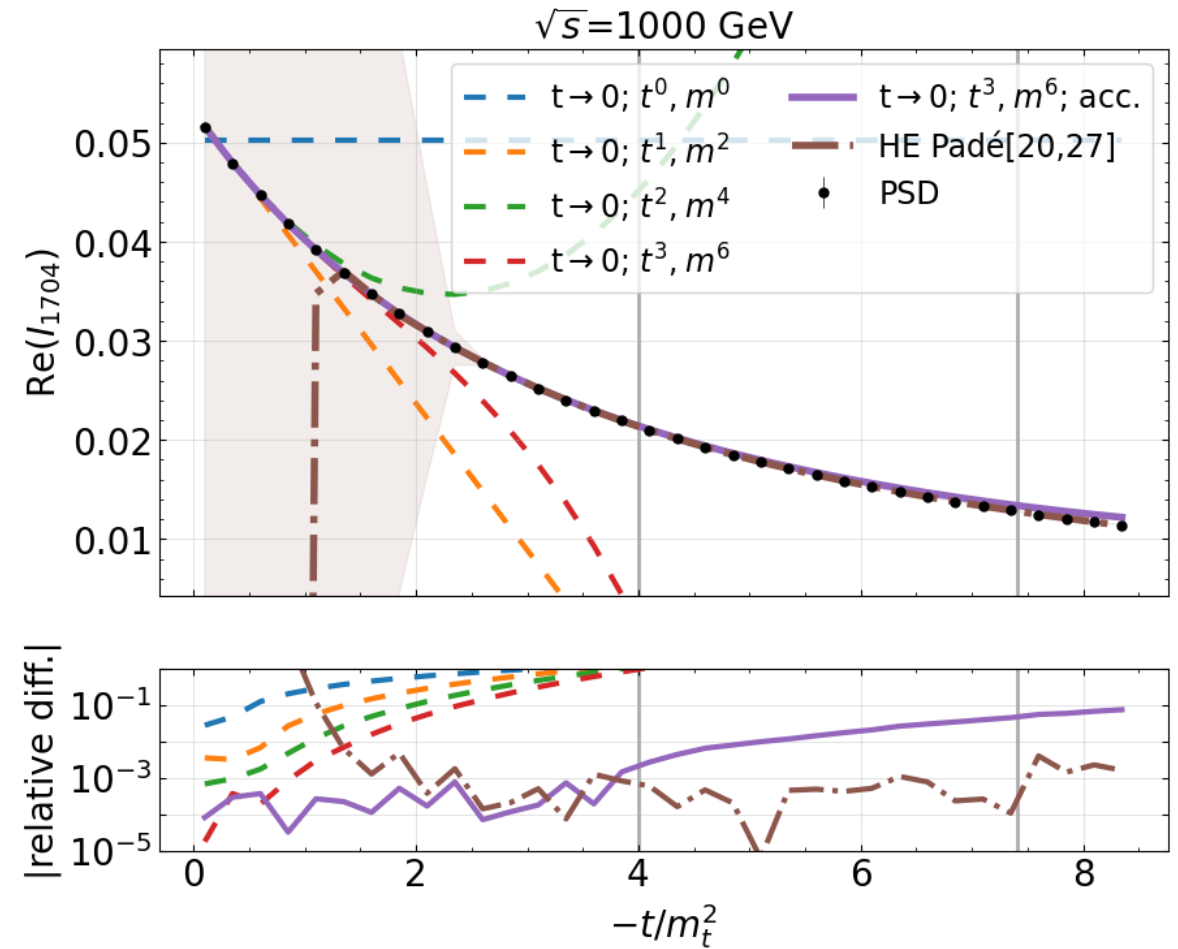
t-channel cuts:

1. $\sqrt{-t} = 2m_t$
2. $\sqrt{-t} = 2m_t + m_{H,Z}$

The t-channel cuts can be covered by using:

- Padé-Approximation:

$$\left[\frac{n}{m}\right](x) = \frac{a_0 + a_1x + \dots + a_nx^n}{1 + b_1x + \dots + b_mx^m}$$



$$I(s, t, m_H, m_t, m_2) = \sum_{i,j,k} c_{i,j,k} I_{0,i}(s, m_t, m_2) t^j m_H^{2k} \text{ for } j + k \leq 3$$