

Higgs boson decay to massive bottom quarks at $\mathcal{O}(\alpha_s^4)$

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High Precision for
Hard Processes 2026



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Zürich** ^{UZH}

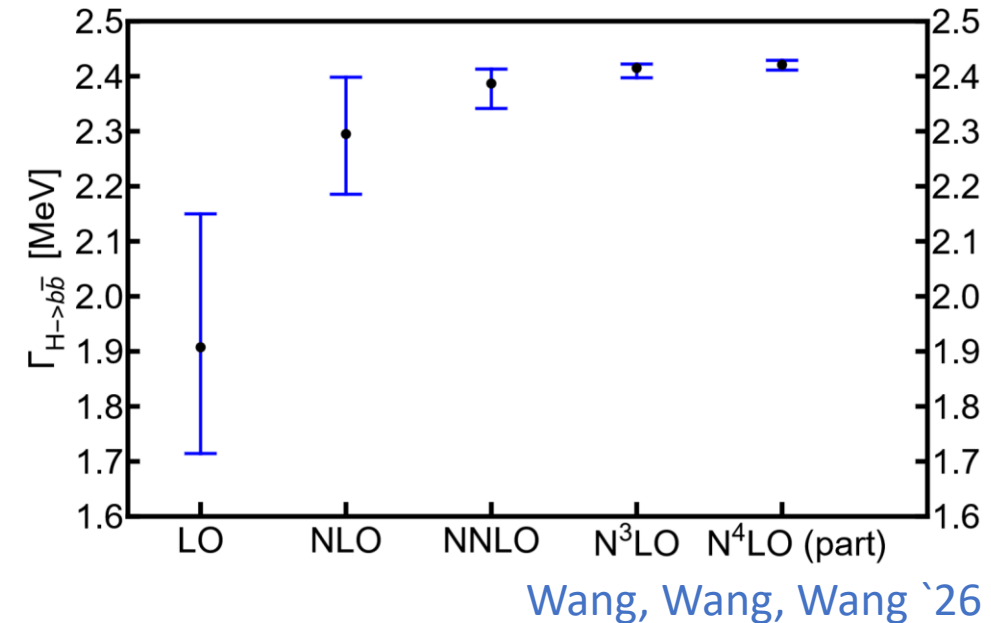
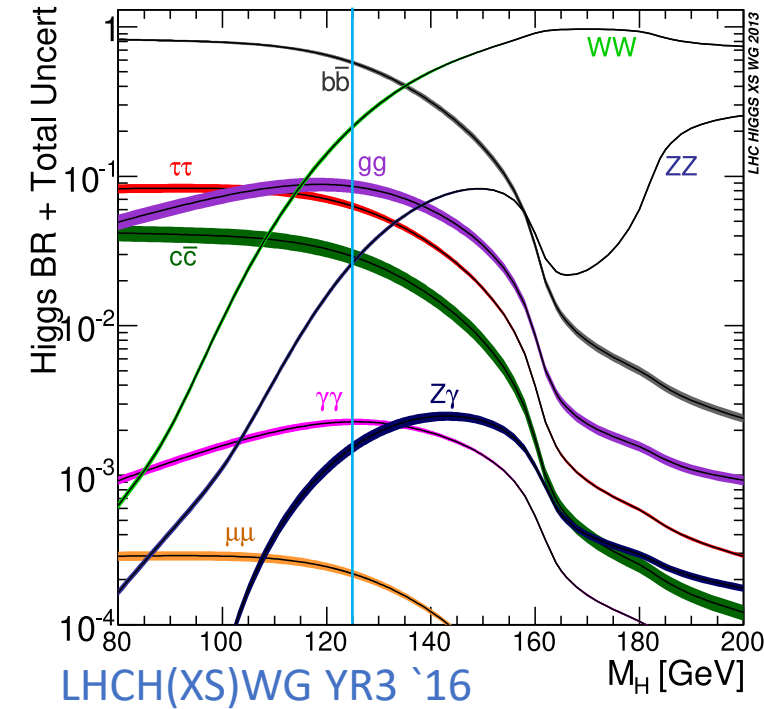


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Motivation

- Higgs boson plays unique role in the SM:
 - Only scalar particle
 - Only particle with Yukawa interactions to fermions
- $H \rightarrow b\bar{b}$ is dominant decay channel in the SM
- Predictions for total decay width directly impact extraction of Higgs couplings from experimental measurements
- Maintaining good control over theory uncertainty is crucial for facilitating high precision measurements of Higgs couplings at the HL-LHC and future e^+e^- Higgs factories
- HL-LHC projection anticipates uncertainty $\mathcal{O}(2\%)$ and permille-level measurements at FCC-ee or CEPC expected
 - ATL-PHYS-PUB-2018-054 '18, CEPC Physics Study Group '22,
The FCC integrated programme: a physics manifesto '25
- Large scale dependence even at N3LO (much worse for m_b renormalized on-shell)
- Probe higher-order structure of soft massive quark logarithms in y_t -induced terms



Contributions to $H \rightarrow b\bar{b}$

- NLO QCD and EW: Braaten, Leveille `80, Sakai `80, Janot `89, Drees, Hikasa `90, Dabelstein, Hollik `92, Kniehl `92
- NNLO QCD:
 - ↪ total rate: Chetyrkin, Kwiatkowski `95, Harlander, Steinhauser `97, Primo, Sasso, Somogyi, Tramontano `19, Wang, Wang, Zhang `23
 - ↪ differential: Bernreuther, Chen, Si `18, Behring, Bizon `19, Somogyi, Tramontano `20
- Higher-order QCD (massless bottom-quarks): Gorishnii, Kataev, Larin, Surguladze `90, Chetyrkin `96, Baikov, Chetyrkin, Kühn `05, Herzog, Riuji, Ueda, Vermaseren, Vogt `17, Anastasiou, Herzog, Lazopoulos `11, Del Duca, Duhr, Somogyi, Tramontano, Trócsányi `15, Mondini, Schiavi, Williams `15, Chen, Jakubcik, Marcoli, Stagnitto `23, Yan, Wu, Shen, Huang, Wu `24, Fox, Gehrmann-De Ridder, Gehrmann, Glover, Marcoli, Preuss `25
- Mixed QCDxEW: Kataev `97, Mihaila, Schmidt, Steinhauser `15
- y_t induced contribution:
 - ↪ $\mathcal{O}(\alpha_s^2)$ Primo, Sasso, Somogyi, Tramontano `19,
 - ↪ $\mathcal{O}(\alpha_s^3)$ Mondini, Schubert, Williams `20
- For the dominant y_b^2 contribution, corrections beyond NNLO were found to be below 0.2% when assuming massless final-state b-quarks: Baikov, Chetyrkin, Kühn `05, Herzog, Riuji, Ueda, Vermaseren, Vogt `17
- $1/m_t^2$ corrections start at $\mathcal{O}(\alpha_s^2)$ and modify the $\mathcal{O}(\alpha_s^2)$ corrections by 1 permille: Primo, Sasso, Somogyi, Tramontano `19

Computational framework

- Integrate out the top quark:
 ↪ very small $1/m_t^2$ corrections

$$\mathcal{L}_{\text{eff}} = -\frac{H}{v} (C_1 \mathcal{O}_1^R + C_2 \mathcal{O}_2^R) + \mathcal{L}_{\text{QCD}}$$

$$\mathcal{O}_1 = (G_{a,\mu\nu}^0)^2, \quad \mathcal{O}_2 = m_b^0 \bar{b}^0 b^0$$

- Wilson coefficients:
- $$C_1 = -\left(\frac{\alpha_s}{\pi}\right) \frac{1}{12} - \left(\frac{\alpha_s}{\pi}\right)^2 \frac{11}{48} - \left(\frac{\alpha_s}{\pi}\right)^3 \left[\frac{137}{576} L_t + \frac{443}{864} \right] + \mathcal{O}(\alpha_s^4)$$
- $$C_2 = 1 + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{1}{3} L_t + \frac{5}{18} \right] + \left(\frac{\alpha_s}{\pi}\right)^3 \left[\dots \right] + \mathcal{O}(\alpha_s^4)$$

Chetyrkin, Kniehl, Steinhauser '97,
 Chetyrkin, Kühn, Sturm '05,
 Liu, Steinhauser '15,
 Davies, Steinhauser, Wellmann '17

- Decompose decay width: $\Gamma_{H \rightarrow b\bar{b}} = \Gamma_{H \rightarrow b\bar{b}}^{C_2 C_2} + \Gamma_{H \rightarrow b\bar{b}}^{C_1 C_2} + \Gamma_{H \rightarrow b\bar{b}}^{C_1 C_1}$

$$\Gamma_{H \rightarrow b\bar{b}}^{C_2 C_2} = C_2 C_2 \left[\Delta_{0,b\bar{b}}^{C_2 C_2} + \left(\frac{\alpha_s}{\pi}\right) \Delta_{1,b\bar{b}}^{C_2 C_2} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta_{2,b\bar{b}}^{C_2 C_2} + \left(\frac{\alpha_s}{\pi}\right)^3 \Delta_{3,b\bar{b}}^{C_2 C_2} + \left(\frac{\alpha_s}{\pi}\right)^4 \Delta_{4,b\bar{b}}^{C_2 C_2} + \mathcal{O}(\alpha_s^5) \right]$$

$$\Gamma_{H \rightarrow b\bar{b}}^{C_1 C_2} = C_1 C_2 \left[\left(\frac{\alpha_s}{\pi}\right) \Delta_{1,b\bar{b}}^{C_1 C_2} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta_{2,b\bar{b}}^{C_1 C_2} + \left(\frac{\alpha_s}{\pi}\right)^3 \Delta_{3,b\bar{b}}^{C_1 C_2} + \mathcal{O}(\alpha_s^4) \right]$$

$$\Gamma_{H \rightarrow b\bar{b}}^{C_1 C_1} = C_1 C_1 \left[\left(\frac{\alpha_s}{\pi}\right) \Delta_{1,b\bar{b}}^{C_1 C_1} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta_{2,b\bar{b}}^{C_1 C_1} + \mathcal{O}(\alpha_s^3) \right]$$

$\mu = m_H$	[MeV]	$\Gamma_{Hb\bar{b}}^{C_2 C_2}$	$\Gamma_{Hb\bar{b}}^{C_1 C_2}$	$\Gamma_{Hb\bar{b}}^{C_1 C_1}$
$\Gamma_{H \rightarrow b\bar{b}} (\overline{\text{MS}})$	$\mathcal{O}(\alpha_s^0)$	1.9076	-	-
	$\mathcal{O}(\alpha_s^1)$	0.3873	-	-
	$\mathcal{O}(\alpha_s^2)$	0.0735	0.0183	-
	$\mathcal{O}(\alpha_s^3)$	0.0048	0.0142	0.0090
	$\mathcal{O}(\alpha_s^4)$	-0.0025	*	0.0087

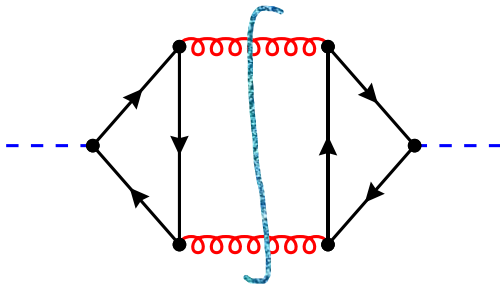
Wang, Wang, Wang '26

Two possible strategies to obtain $\Gamma_{H \rightarrow b\bar{b}}$

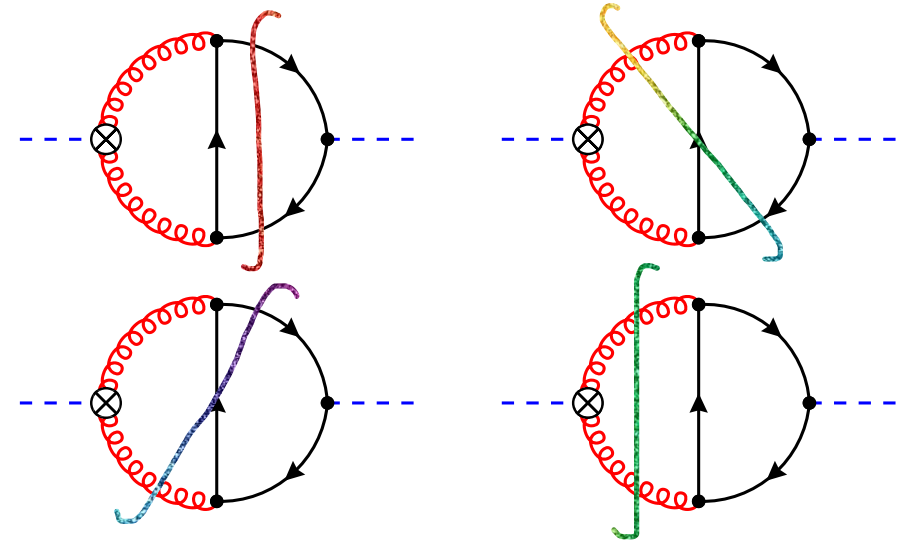
Extract **imaginary part** of the forward scattering amplitude

$$\Gamma_{H \rightarrow b\bar{b}} = \frac{\text{Im}_{b\bar{b}} \Sigma(H \rightarrow H)}{m_H}$$

- Sectors without imaginary part can be set zero for amplitude reduction
- Large- m_b^2 or large-momentum expansion suitable to get boundary value for ODEs
- Care must be taken with manually setting zero sectors when solving the ODEs
- Cannot disentangle different final states
- Need to subtract contributions to $H \rightarrow gg$



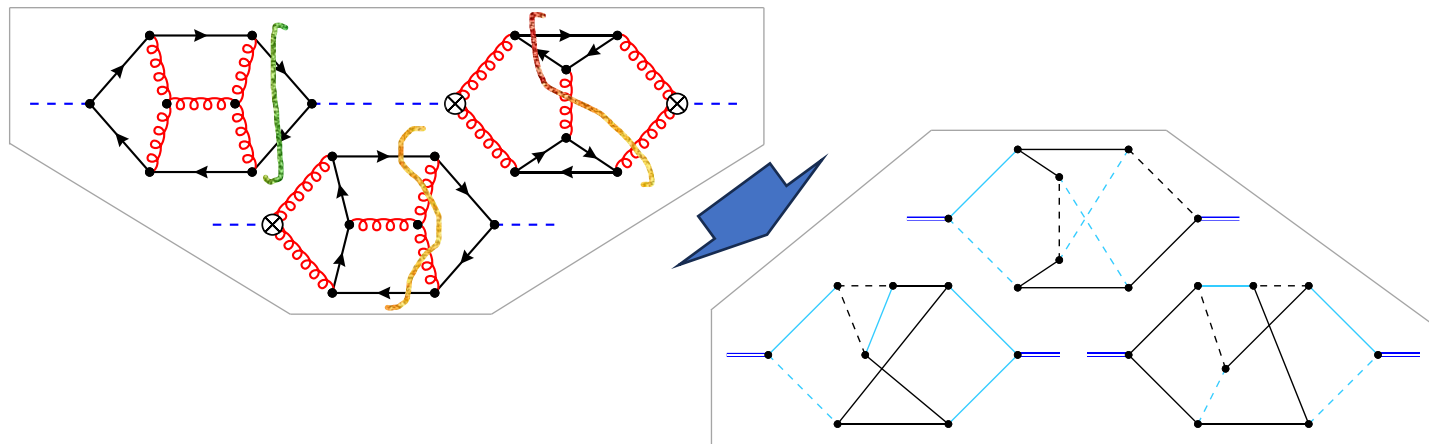
Directly compute **individual cuts** of the forward scattering amplitude



- Simplified amplitude and ODEs reduction due to many trivial sectors
- Transparent categorization of contributions by different final-state multiplicities
- Evaluation of boundary value for ODEs difficult

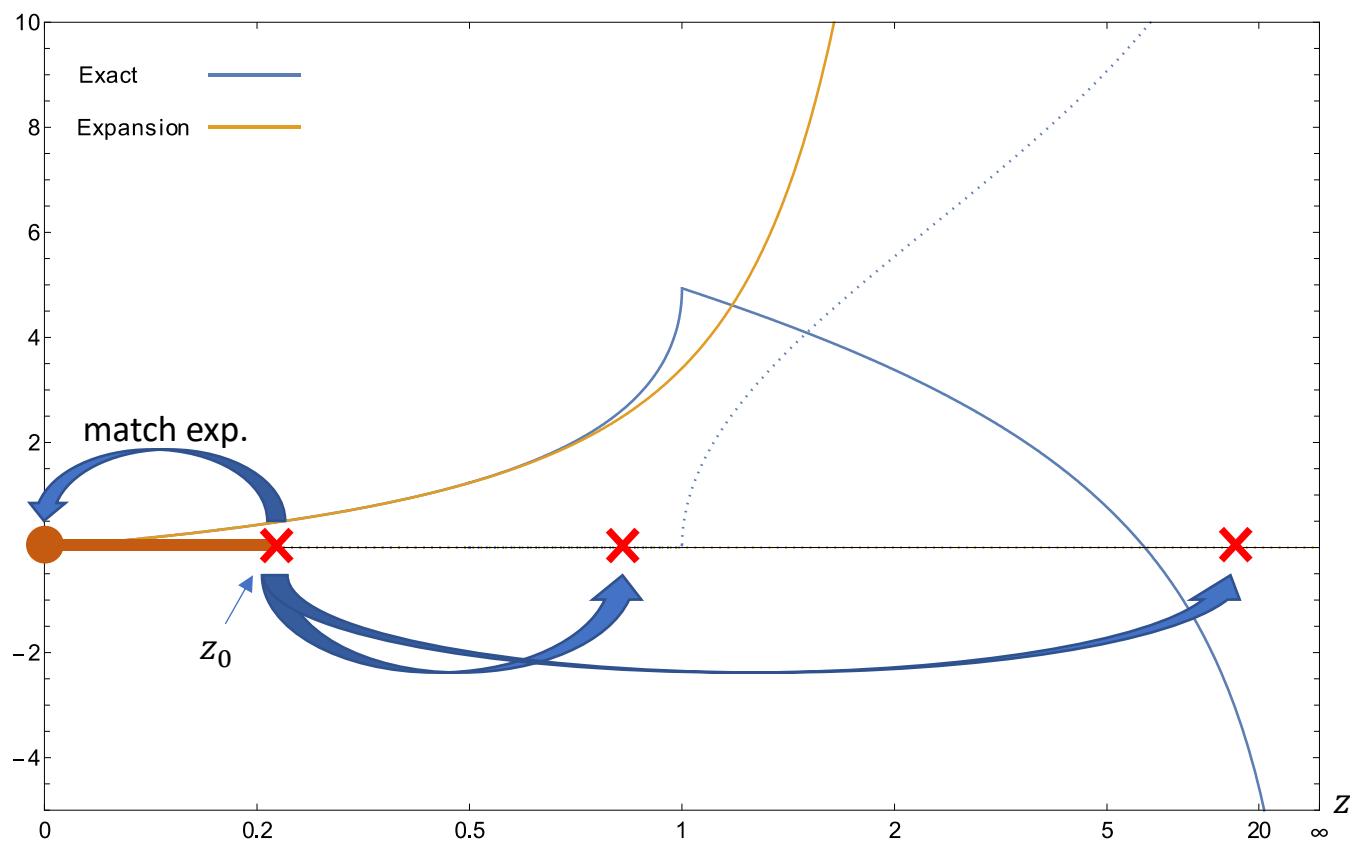
Roadmap of the computation in a nutshell

- Bookkeeping and construction of integral families
↳ CalcLoop package [Ma '22](#)

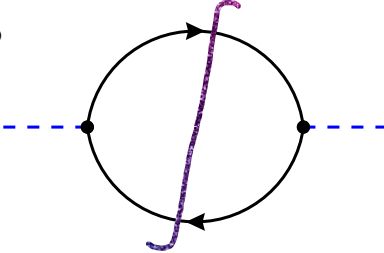
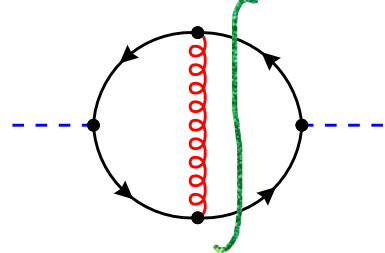
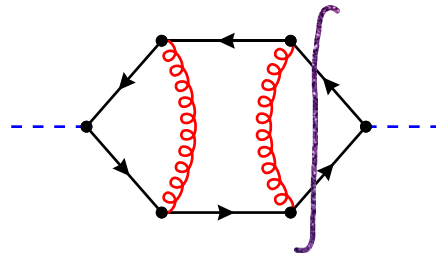
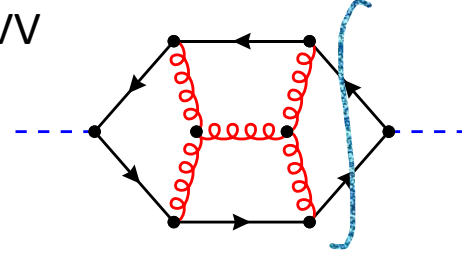
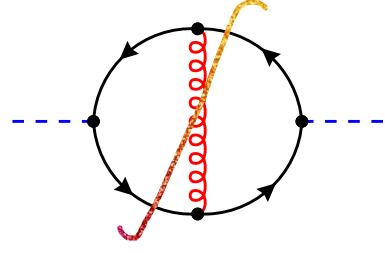
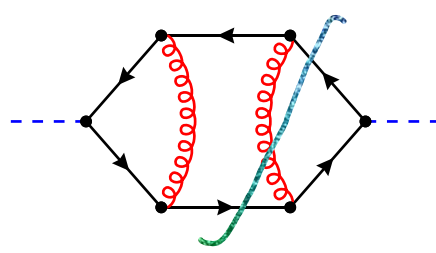
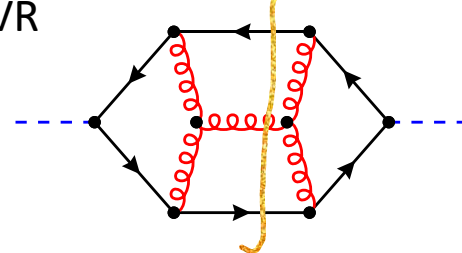
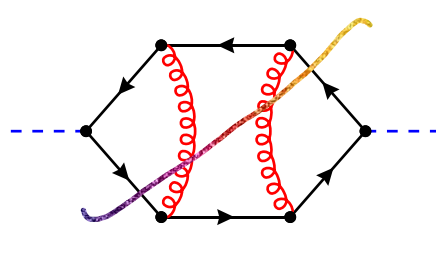
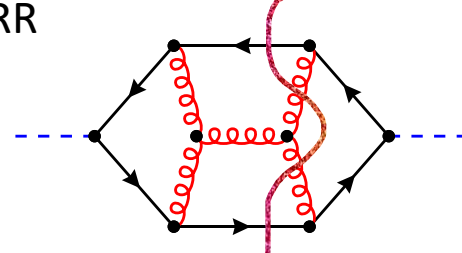
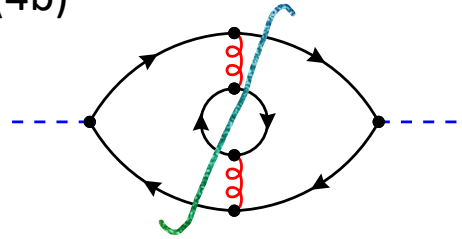
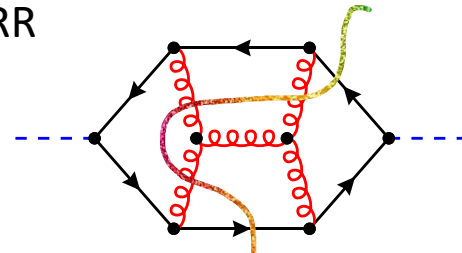


- Reduce integrals to master integrals
↳ Kira [Lange, Usovitsch, Wu '25](#)
+ FireFly [Klappert, Klein, Lange '20](#)
↳ Blade [Guan, Liu, Ma, Wu '24](#)
+ FiniteFlow [Peraro '19](#)

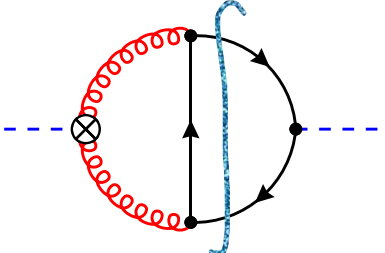
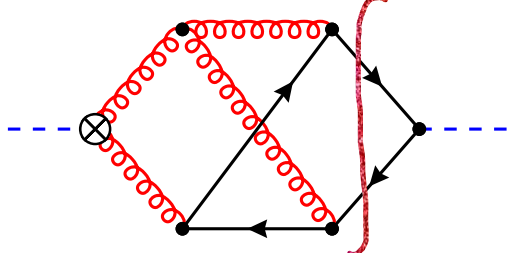
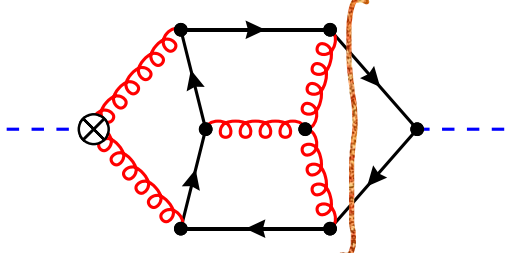
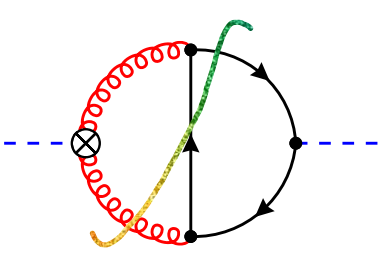
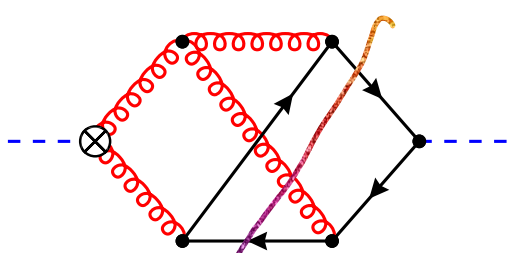
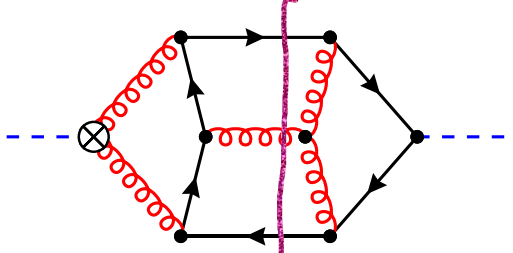
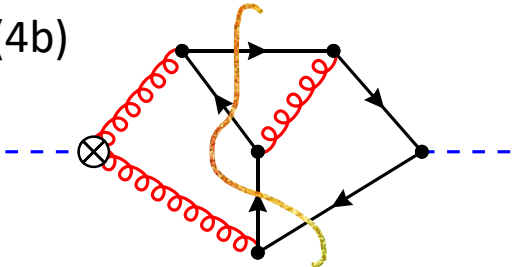
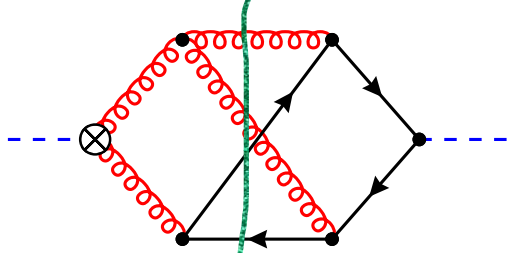
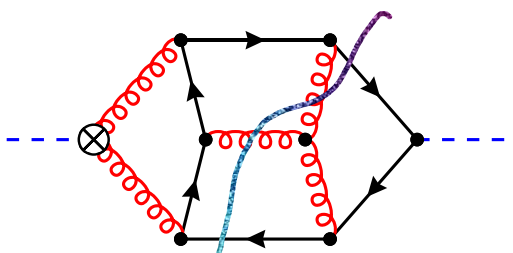
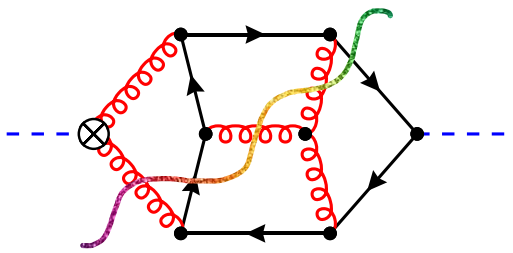
- Derive ODEs in $z = m_b^2/m_H^2$ and solve master integrals at some boundary point z_0 with AMFlow [Liu, Ma '22 \(+ Huang '26\)](#)
↳ sample the parameter space and obtain asymptotic expansions



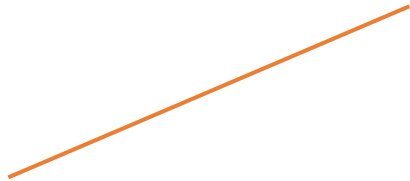
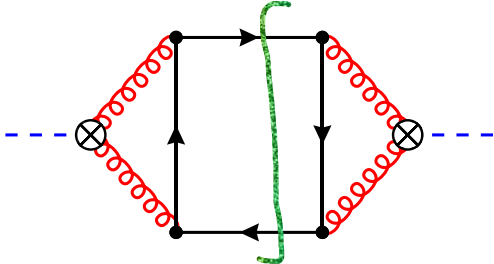
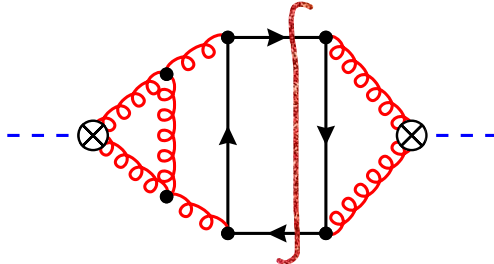
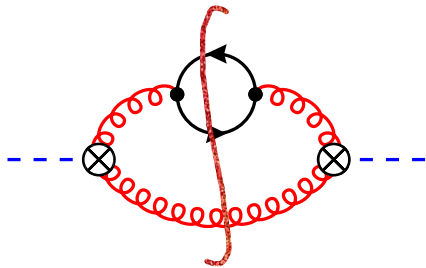
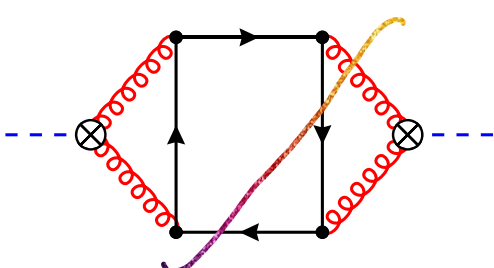
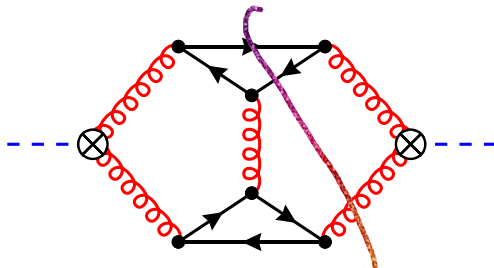
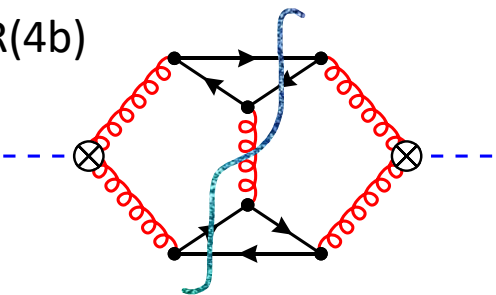
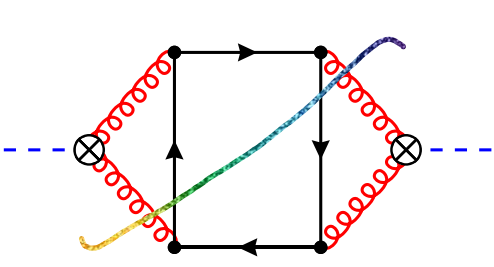
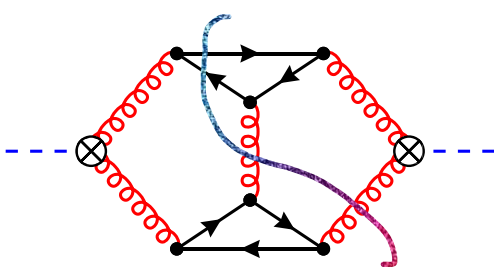
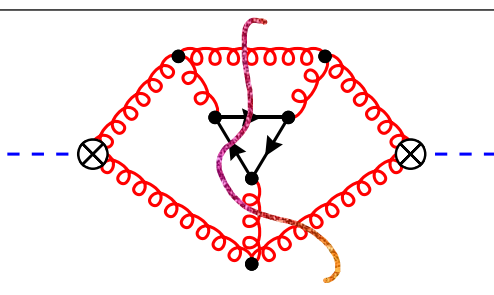
Ingredients $\Gamma_{H \rightarrow b \bar{b}}^{C_2 C_2}$

$\mathcal{O}(\alpha_s^0)$	$\mathcal{O}(\alpha_s^1)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$
B 	V 	VV 	VVV 
	R 	VR 	VVR 
		RR 	VRR 
RR(4b) 	Analytically to $\mathcal{O}(\alpha_s^2)$ Wang, Wang, Zhang '23 Numerically to $\mathcal{O}(\alpha_s^3)$ from $e^+e^- \rightarrow t\bar{t}$ Chen, Guan, He, Liu, Ma '22		
			RRR 

Ingredients $\Gamma_{H \rightarrow b \bar{b}}^{C_1 C_2}$

$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$
V 	VV 	VVV 
R 	VR 	VVR 
RR(4b) 	RR 	VRR 
	Analytically to $\mathcal{O}(\alpha_s^3)$ Wang, Wang, Wang '24	RRR 

Ingredients $\Gamma_{H \rightarrow b \bar{b}}^{C_1 C_1}$

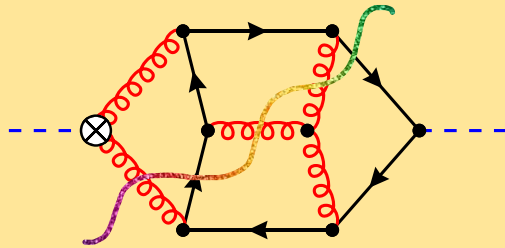
$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
V 	VV 	VVV 
R 	VR 	VVR 
RRR(4b) 	RR 	VRR 
	Analytically to $\mathcal{O}(\alpha_s^4)$ Wang, Wang, Wang '26	RRR 

Integral families and master integrals for $\Gamma_{H \rightarrow b \bar{b}}^{C_1 C_2}$ and $\Gamma_{H \rightarrow b \bar{b}}^{C_1 C_1}$

RRR

↪ 315 families | 68614 integrals

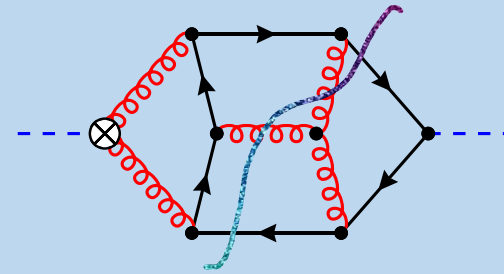
↪ 1905 masters in 189 families



VRR

↪ 401 families | 101716 integrals

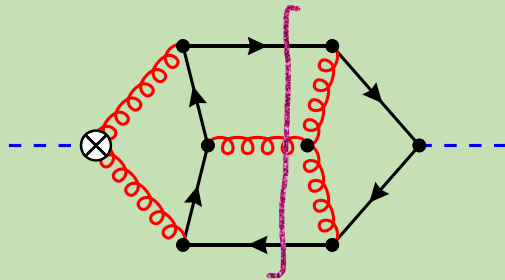
↪ 2562 masters in 176 families



VVR

↪ 158 families | 56007 integrals

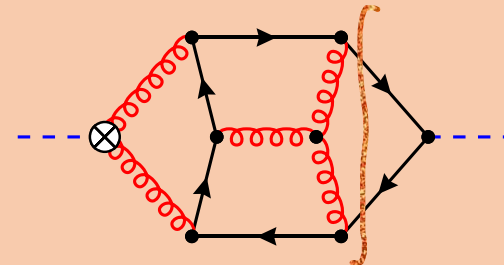
↪ 1329 masters in 73 families



VVV

↪ 47 families | 17304 integrals

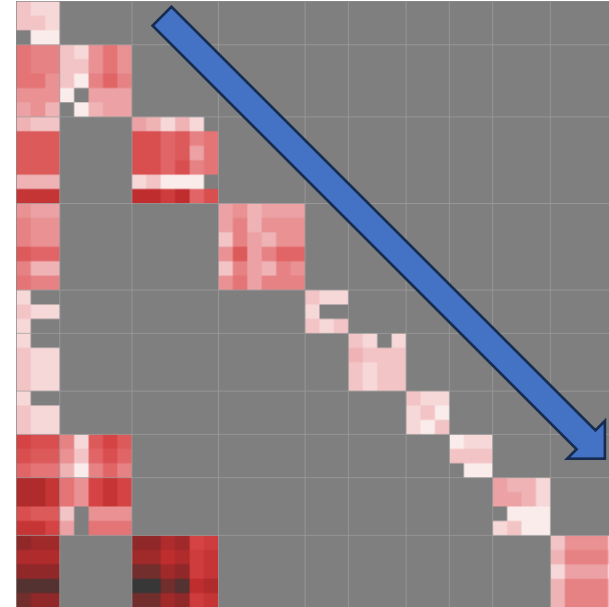
↪ 578 masters in 28 families



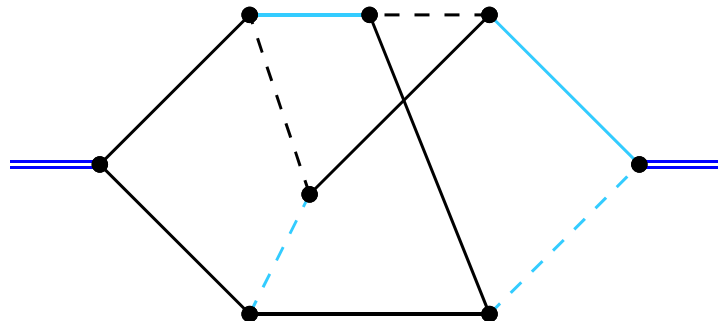
Derivation of the ODEs

- Carefully choose a suitable set of master integrals before performing a complete reduction of amplitudes and ODEs

- *Suitable* → Size of the ODEs should be as *small* as possible
 - ↳ run reduction of ODEs and sample integrals to default set of master integrals at fixed values for the variables
 - ↳ sector-by-sector: replace default master integrals by sample integrals and inspect size of ODEs
 - ↳ run complete reduction to improved master integral basis
- Improved master integrals satisfy simpler ODEs
 - ↳ coefficients have smaller degree in numerators and denominators and shorter integer prefactors
 - ↳ reduced number of probes and prime fields needed for interpolation
 - ↳ renders denominators d-factorizing in some cases

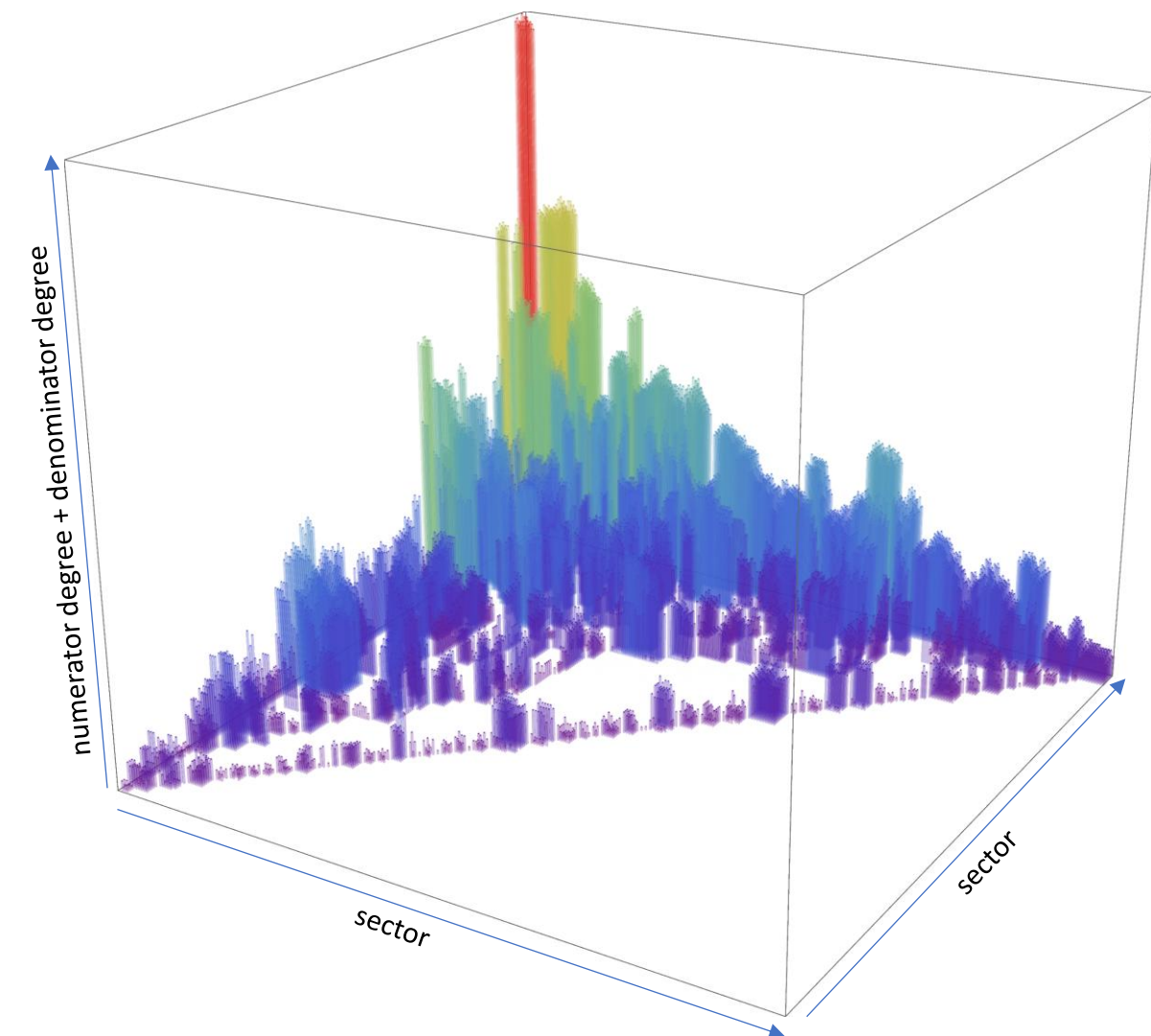


- Example: VRR non-planar family

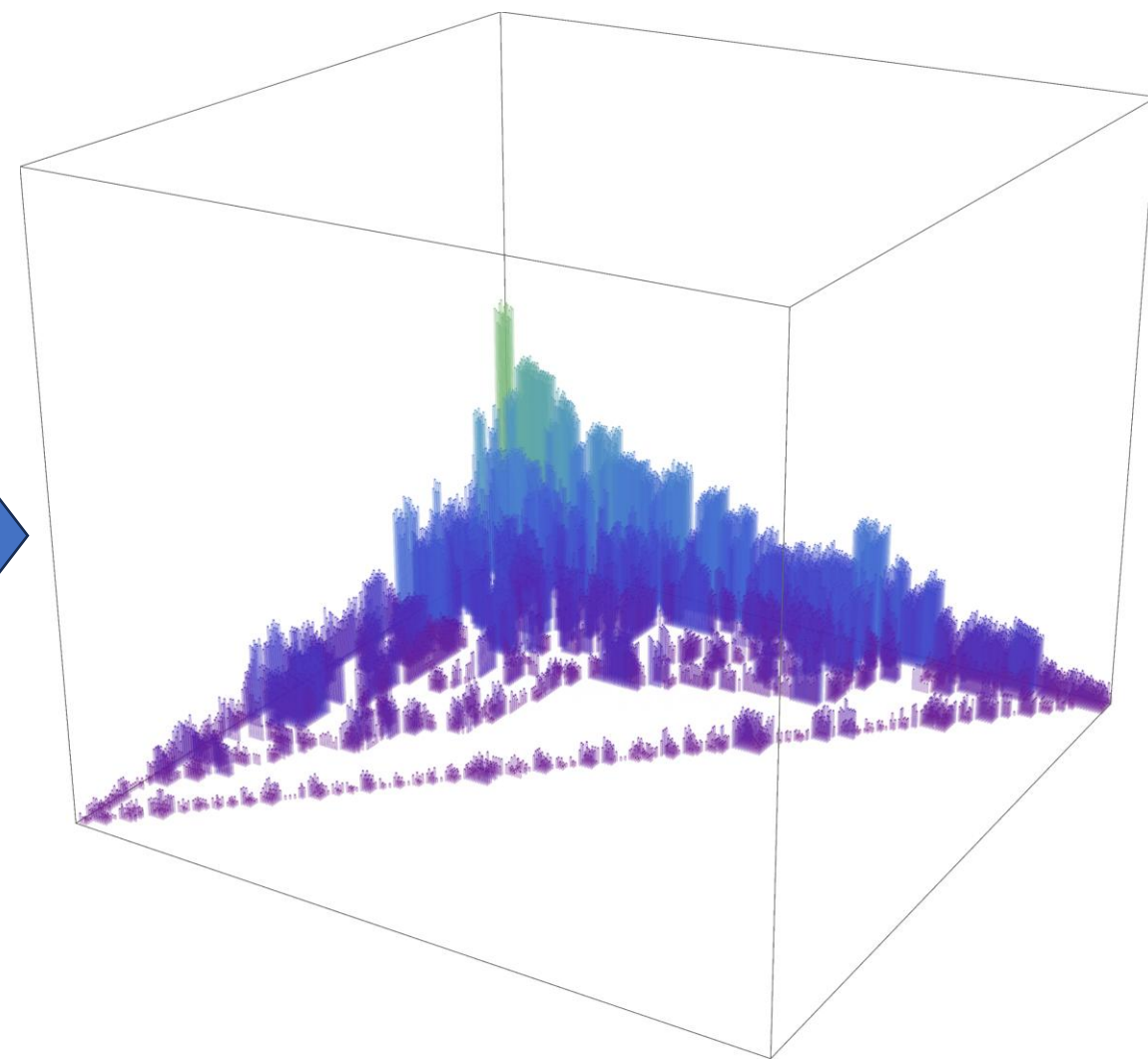


228 master integrals
10 masters in largest sector

Improving the basis of master integrals



$\frac{1}{4}$

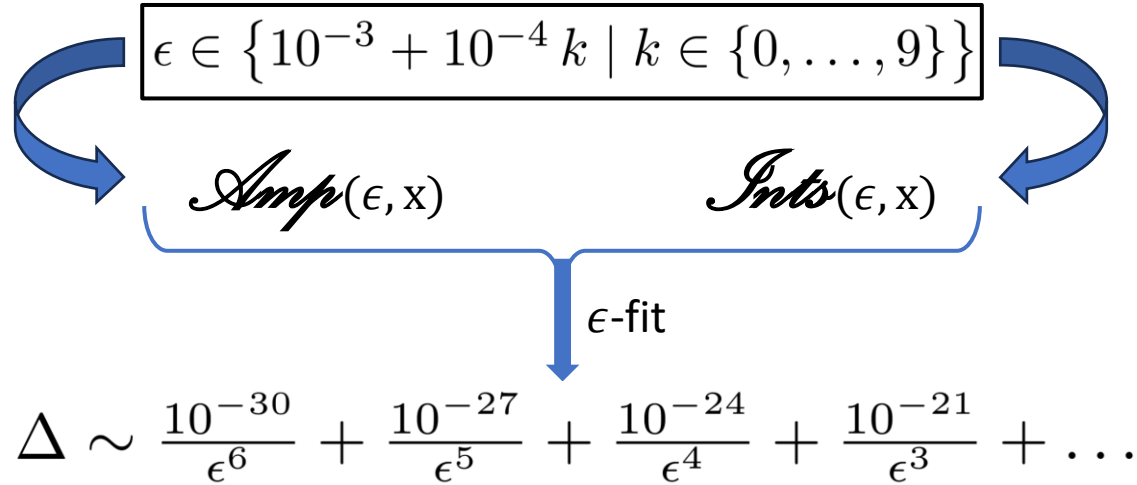


Maximum degree of numerator: 98
Maximum degree of denominator: 27
Completed reconstruction in 3122 s | 17341 probes
Required prime fields: 7 + 1

Maximum degree of numerator: 49
Maximum degree of denominator: 0
Completed reconstruction in 917 s | 4656 probes
Required prime fields: 4 + 1

Fitting the ϵ dependence

- Amplitudes and master integrals are evaluated at **fixed ϵ values**



Benefits

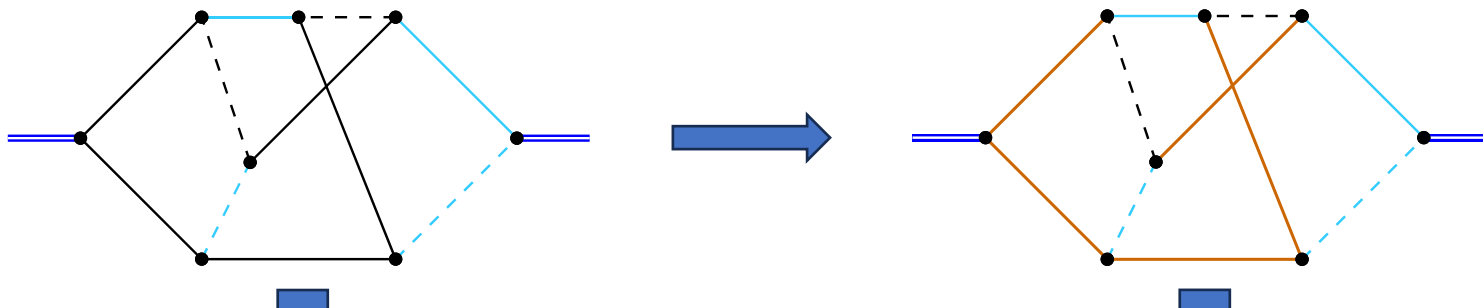
- Eliminates one variable from calculation
- Better numerical stability
- No need to keep track of ϵ poles
- Whole calculation is exact in ϵ
↳ restore ϵ dependence at the end
- Straightforward to parallelize
- Pole cancellation after renormalization allows to estimate precision of finite part

Drawbacks

- Closed form in ϵ of boundaries for ODEs desired
- Quality of ϵ -fit is source of uncertainty on top of other sources, e.g. numerical cancellations or loss of precision during numerical evolutions of ODEs

Evaluation of master integrals with AMFlow method

- Implement η -parameter by shifting the mass of uncut propagators $m_b^2 \rightarrow m_b^2 + i\eta$



- Number of master integrals and size of coupled blocks in ODEs becomes significantly larger

228 master integrals
10 masters in largest sector

552 master integrals
21 masters in largest sector

- Directly derive the ODEs in η with Kira

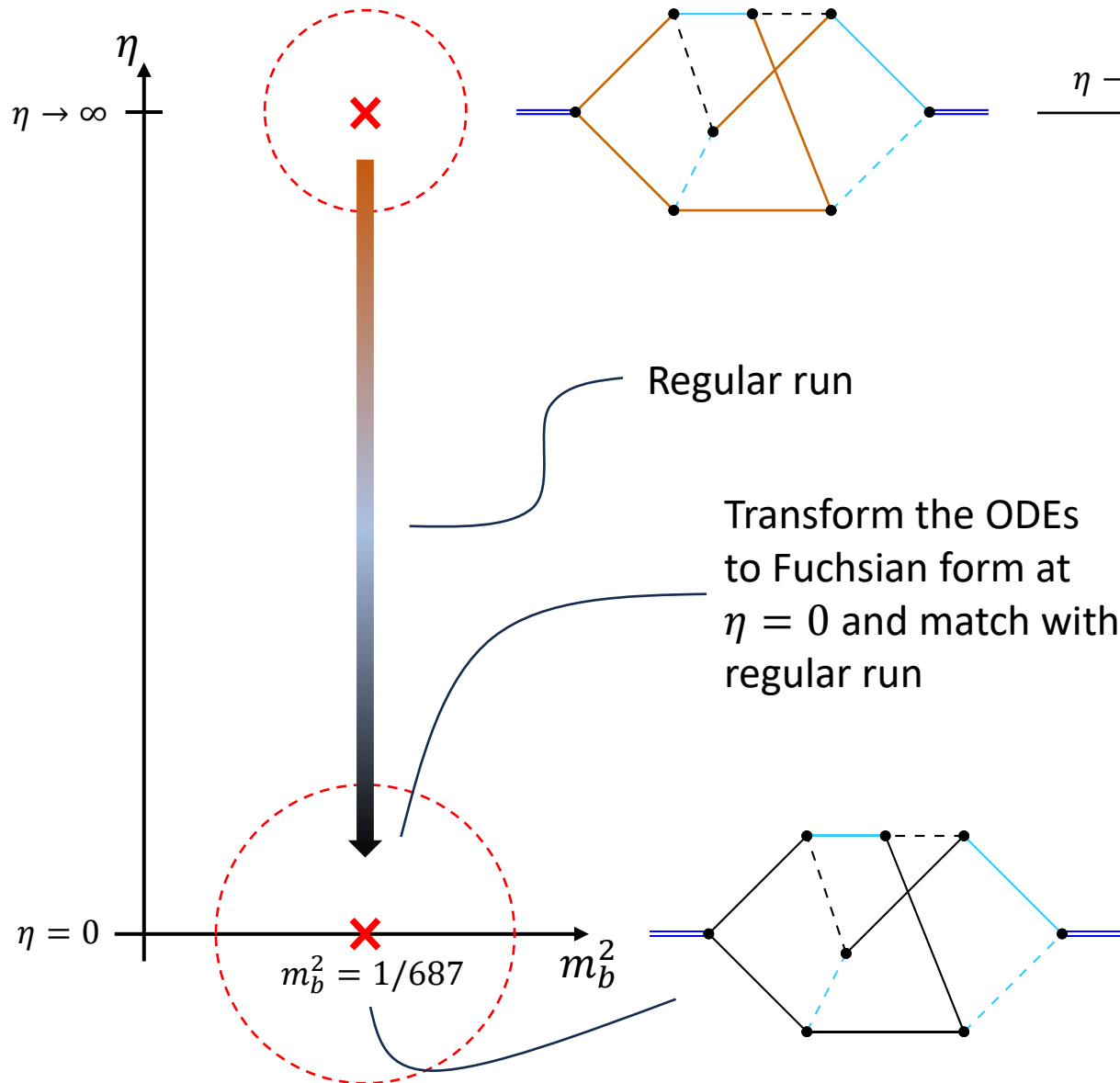
$\hookrightarrow$ Maximum degree of d : 47
 $\hookrightarrow$ Maximum degree of m_b^2 : 231
 $\hookrightarrow$ Maximum degree of η : 257

Maximum degree of numerator: 309
 Maximum degree of denominator: 259

$\Rightarrow$ #probes $\approx 8 \times 10^6$

- Reduce at fixed ϵ values and fixed $m_b^2 = 1/687$

Evaluation of master integrals with AMFlow method



$\eta \rightarrow \infty$

Evaluate boundary fully automated at large η with AMFlow at fixed ϵ values

$$\frac{d}{d\eta} I(\eta, \epsilon) = \mathbb{M}(\eta, \epsilon) I(\eta, \epsilon)$$

$$\mathbb{M}(\eta, \epsilon) = \mathbb{A}_0(\epsilon) \eta^{-p-1} + \mathbb{A}_1(\epsilon) \eta^{-p} + \mathcal{O}(\eta^{-p+1})$$

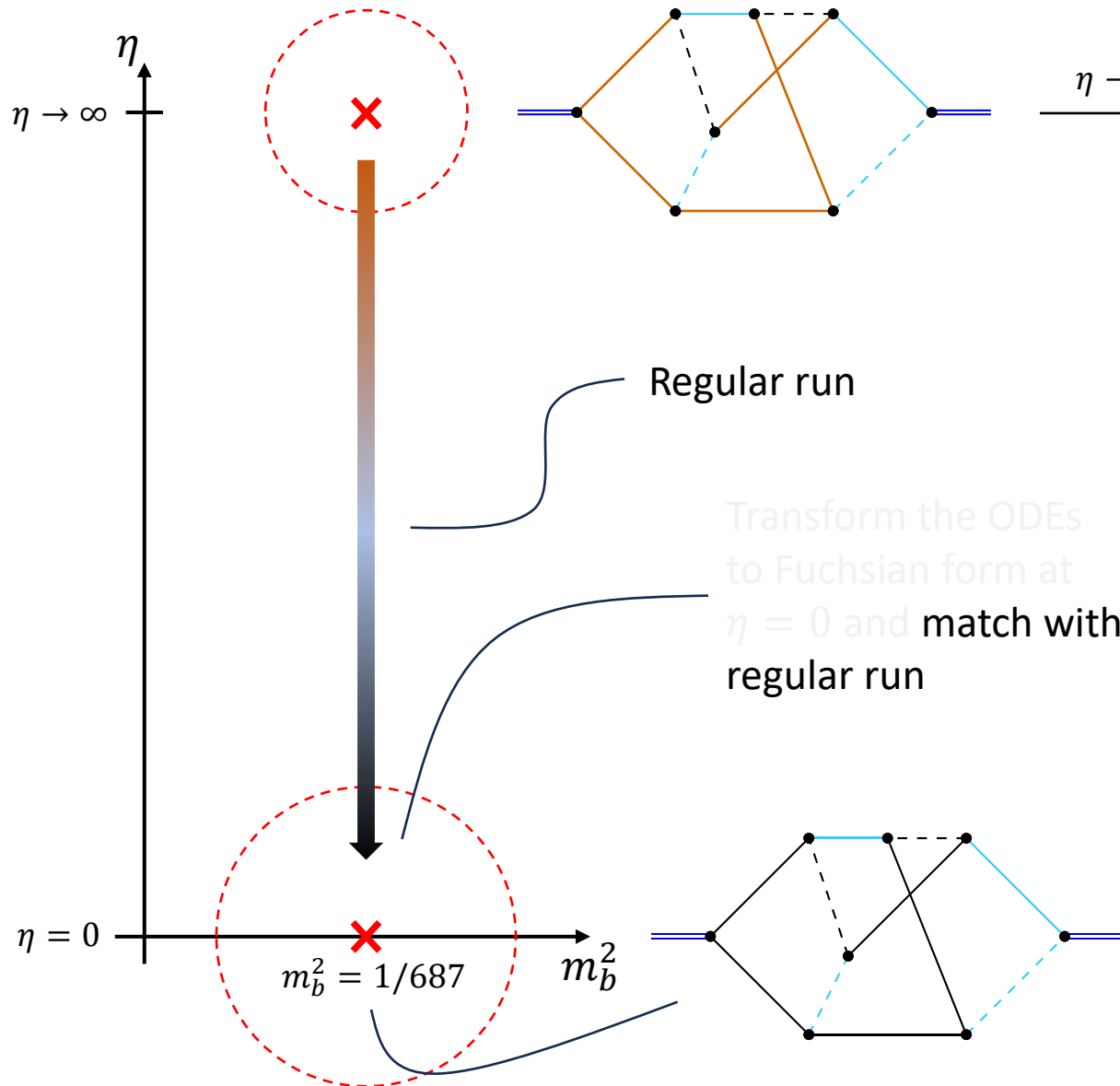
➤ Reduce Poincaré rank at $\eta = 0$ to zero [Lee '14](#)

$$\frac{d}{d\eta} I(\eta, \epsilon) = \frac{\mathbb{A}_0(\epsilon)}{\eta} I(\eta, \epsilon) \implies I(\eta, \epsilon) = e^{\mathbb{A}_0(\epsilon) \log(\eta)} I_0(\epsilon)$$

➤ Obtain scalings at from Jordan normal form of $\mathbb{A}_0$

$$I(\eta, \epsilon) = \sum_{\lambda \in S} \eta^\lambda \sum_{j=0}^{\infty} \sum_{n=0}^{L_\lambda} c_{j,\lambda,n} \log^n(\eta) \eta^j$$

Evaluation of master integrals with AMFlow method



$\eta \rightarrow \infty$

Evaluate boundary fully automated at large η with AMFlow at fixed ϵ values

$$\frac{d}{d\eta} I(\eta, \epsilon) = \mathbb{M}(\eta, \epsilon) I(\eta, \epsilon)$$

$$\mathbb{M}(\eta, \epsilon) = \mathbb{A}_0(\epsilon) \eta^{-p-1} + \mathbb{A}_1(\epsilon) \eta^{-p} + \mathcal{O}(\eta^{-p+1})$$

- Bottleneck: Fuchsification of ODEs with large coupled blocks leads to expression swell
- Circumvent bottleneck using Frobenius method
↳ implemented in AMFlow 2.0 [Huang, Liu, Ma '26](#)
- Successfully applied to all integral families contributing to $H \rightarrow b\bar{b}b\bar{b} + X$

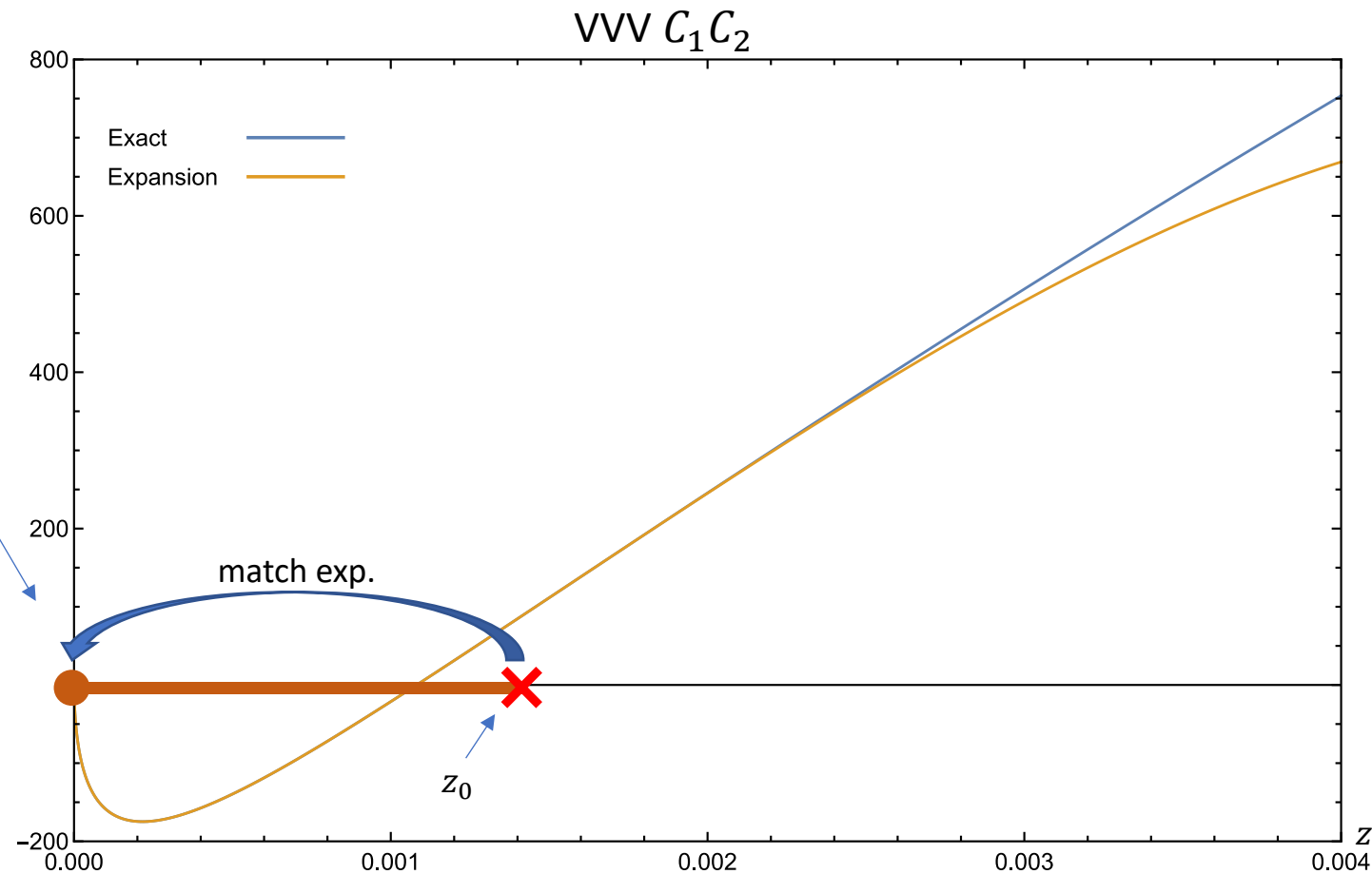
$$I(\eta, \epsilon) = \sum_{\lambda \in S} \eta^\lambda \sum_{j=0}^{\infty} \sum_{n=0}^{L_\lambda} c_{j,\lambda,n} \log^n(\eta) \eta^j$$

Collecting results for the amplitudes

- All master integrals have been evaluated at $z_0 = 1/687$ at fixed ϵ values
 $\hookrightarrow$ second evaluation with doubled precision has been performed: > 50 correct digits at fixed ϵ
- Numerically sampled the parameter space $z_0 \in (0, 1/687)$
- Expand ODEs at $z_0 = 0$ in analogy to AMFlow method

$$Amp(z, \epsilon) = \sum_{\lambda \in S} z^\lambda \sum_{j=0}^{\infty} \sum_{n=0}^{L_\lambda} c_{j,\lambda,n} \log^n(z) z^j$$

- ϵ dependence of the expansion coefficients is reconstructed at the amplitude level
- at $\mathcal{O}(\alpha_s^4)$ we obtain up to $\log^6(z)$
- variation of the ϵ -fit indicates 10 correct digits for the finite part



Results for $\Gamma_{H \rightarrow b\bar{b}}$ in $\overline{\text{MS}}$ scheme

- Include contributions from $H \rightarrow b\bar{b} + X$ and $H \rightarrow b\bar{b}b\bar{b} + X$
- Work in 5-flavor QCD with $m_H=125.13$ GeV, and $m_b(m_H)=2.79$ GeV, and $\alpha_s(m_H)=0.11253$, and $m_t=172.6$ GeV

$\mu = m_H$	[MeV]	$\Gamma_{Hb\bar{b}}^{C_2C_2}$	$\Gamma_{Hb\bar{b}}^{C_1C_2}$	$\Gamma_{Hb\bar{b}}^{C_1C_1}$
$\Gamma_{H \rightarrow b\bar{b}}(\overline{\text{MS}})$	$\mathcal{O}(\alpha_s^0)$	1.9121	-	-
	$\mathcal{O}(\alpha_s^1)$	0.3879	-	-
	$\mathcal{O}(\alpha_s^2)$	0.0736	0.0183	-
	$\mathcal{O}(\alpha_s^3)$	0.0048	0.0142	0.0090
	$\mathcal{O}(\alpha_s^4)$	-0.0024	0.0064	0.0087

Chen, Chen, Guan,
Niggetiedt (preliminary)

$$\Gamma_{H \rightarrow b\bar{b}}(\mu = m_H) = (2.3760 + 0.0389 + 0.0177) \text{ MeV} = 2.4326 \text{ MeV}$$

- Known contributions agree with [Wang, Wang, Wang '26](#)
- $y_b y_t$ -induced terms change decay width by 2.6‰ at $\mathcal{O}(\alpha_s^4)$
- Still 2.5‰ permille uncertainty from scale variation $\leftrightarrow \mathcal{O}(10)$ times larger for m_b renormalized in OS scheme
- Good perturbative convergence in y_b^2 -part $\leftrightarrow$ not as good in y_t -induced terms due to logarithmic enhancement

Summary of the current status

- Decay width of Higgs to bottom quarks is fundamental quantity that will be measured at future colliders
 - To match future experimental accuracy, $\mathcal{O}(\alpha_s^4)$ corrections for the $C_1 C_2$ contribution are required
 - The computation of ingredients relevant to $\Gamma_{H \rightarrow b \bar{b}}^{C_1 C_1}$, $\Gamma_{H \rightarrow b \bar{b}}^{C_1 C_2}$, and $\Gamma_{H \rightarrow b \bar{b}}^{C_2 C_2}$ has been completed by numerically evaluating the corresponding propagator-type cut integrals up to 4-loop order (retaining the mass of the bottom quark everywhere)
 - The evaluation of master integrals has been carried out in the AMFlow framework
 - ↳ ODEs in η have been derived at fixed mass and fixed ϵ values
 - ↳ suitable bases of master integrals have been found
 - ↳ master integrals satisfy ODEs with smaller degrees in rational function coefficients
 - ↳ problematic large sectors with relatively high Poincaré rank are under control
 - Expansions of amplitudes around the massless limit feature terms up to $\log^6(z)$
 - $C_1 C_2$ contribution shifts $\Gamma_{H \rightarrow b \bar{b}}$ by 2.6‰ at $\mathcal{O}(\alpha_s^4)$ for m_b renormalized in $\overline{\text{MS}}$ scheme
 - All contributions to $\Gamma_{H \rightarrow gg}$ have been obtained for completeness
-

Ongoing work:

- Cross-check against literature
- Assess perturbative convergence for m_b renormalized in different scheme