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Recursive reduction of two-loop tensor integrals for automated NNLO calculations

High Precision for Hard Processes (HP2 2026) – Karlsruhe, Germany

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in collaboration with Max Zoller

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OpenLoops in a nutshell

OPENLOOPS [Cascioli, Maierhöfer, Pozzorini 2011; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller 2019] provides scattering probability densities ready for Monte-Carlo tools:

$$\mathcal{W} = \sum_{\substack{\bar{h}, \text{col} \\ \text{colour and helicity sum}}} |\mathbf{R}\bar{\mathcal{M}}|^2 = \sum_{\bar{h}, \text{col}} \left\{ \underbrace{|\bar{\mathcal{M}}_0|^2}_{\text{LO}} + \underbrace{2 \operatorname{Re} [\bar{\mathcal{M}}_0^* \mathbf{R}\bar{\mathcal{M}}_1]}_{\text{NLO virtual}} + \underbrace{|\mathbf{R}\bar{\mathcal{M}}_1|^2 + 2 \operatorname{Re} [\bar{\mathcal{M}}_0^* \mathbf{R}\bar{\mathcal{M}}_2]}_{\text{NNLO virtual-virtual}} + \dots \right\}$$

with

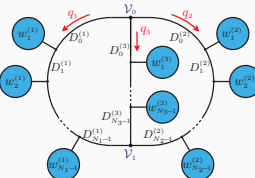
$$\bar{\mathcal{M}}_0 = \text{tree-level diagrams} + \dots, \quad \bar{\mathcal{M}}_1 = \text{one-loop diagrams} + \dots, \quad \bar{\mathcal{M}}_2 = \text{two-loop diagrams} + \dots$$

and renormalisation procedure

$$\mathbf{R}\bar{\mathcal{M}} = \bar{\mathcal{M}}_0 + \mathbf{R}\bar{\mathcal{M}}_1 + \mathbf{R}\bar{\mathcal{M}}_2 + \dots$$

- Full NLO QCD and NLO EW corrections available
- Excellent CPU performance and numerical stability $\Rightarrow$ crucial for real-virtual NNLO
- This talk: progress towards pure NNLO $2 \operatorname{Re} [\bar{\mathcal{M}}_0^* \mathbf{R}\bar{\mathcal{M}}_2]$

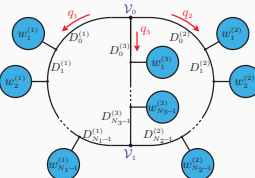
Towards OpenLoops@2L



The diagram shows a two-loop Feynman diagram with external momenta q_1, q_2, q_3 and internal masses $D_j^{(i)}$. The diagram is divided into two parts by a vertical line. The left part contains external momenta $w_1^{(1)}, w_2^{(1)}, w_{N_1-1}^{(1)}$ and internal masses $D_0^{(1)}, D_1^{(1)}, D_{N_3-1}^{(1)}$. The right part contains external momenta $w_1^{(2)}, w_2^{(2)}, w_{N_2-1}^{(2)}$ and internal masses $D_0^{(2)}, D_1^{(2)}, D_{N_2-1}^{(2)}$. The diagram is labeled with $\mathcal{V}_0$ and $\mathcal{V}_1$.

$$\sim \sum_{\mu_i, \nu_j} \underbrace{\mathcal{N}_{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s}}_{\text{4-dim coefficient}} \times \underbrace{\int d^D q_1 \int d^D q_2 \frac{q_1^{\mu_1} \dots q_1^{\mu_r} q_2^{\nu_1} \dots q_2^{\nu_s}}{\prod_{i,j} D_j^{(i)}(q_i)}}_{\text{tensor integral}} + \underbrace{\int d^D q_1 \int d^D q_2 \frac{\tilde{\mathcal{N}}}{\prod_{i,j} D_j^{(i)}(q_i)}}_{(D-4)\text{-dim numerator}}$$

Towards OpenLoops@2L

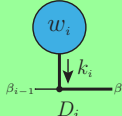


$$\sim \sum_{\mu_i, \nu_j} \underbrace{\mathcal{N}_{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s}}_{\text{4-dim coefficient}} \times \underbrace{\int d^D q_1 \int d^D q_2 \frac{q_1^{\mu_1} \dots q_1^{\mu_r} q_2^{\nu_1} \dots q_2^{\nu_s}}{\prod_{i,j} D_j^{(i)}(q_i)}}_{\text{tensor integral}} + \underbrace{\int d^D q_1 \int d^D q_2 \frac{\tilde{\mathcal{N}}}{\prod_{i,j} D_j^{(i)}(q_i)}}_{(D-4)\text{-dim numerator}}$$

Numerical construction of 4-dimensional tensor coefficients

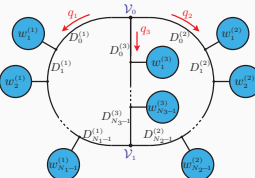
- Exploit factorisation into universal building blocks $\mathcal{K}_n \in \{S_n^{(i)}(\mathbf{q}_i), \mathcal{V}_{0,1}(\mathbf{q}_1, \mathbf{q}_2)\}$ with

numerator segments $S_i(\mathbf{q}) =$



- Recursively construct coefficients $\mathcal{N}_n(\mathbf{q}_1, \mathbf{q}_2) = \mathcal{N}_{n-1}(\mathbf{q}_1, \mathbf{q}_2) \cdot \mathcal{K}_n$
- ⇒ Highly efficient and fully general algorithm [Pozzorini, Schär, Zoller 2022]
- Implemented for QED and QCD corrections to the Standard Model

Towards OpenLoops@2L



$$\sim \sum_{\mu_i, \nu_j} \underbrace{\mathcal{N}_{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s}}_{\text{4-dim coefficient}} \times \underbrace{\int d^D q_1 \int d^D q_2 \frac{q_1^{\mu_1} \dots q_1^{\mu_r} q_2^{\nu_1} \dots q_2^{\nu_s}}{\prod_{i,j} D_j^{(i)}(q_i)}}_{\text{tensor integral}} + \underbrace{\int d^D q_1 \int d^D q_2 \frac{\tilde{\mathcal{N}}}{\prod_{i,j} D_j^{(i)}(q_i)}}_{(D-4)\text{-dim numerator}}$$

Restoration of $(D-4)$ -dimensional numerators:

- As at one-loop [Ossola, Papadopoulos, Pittau 2008], small set of **universal two-loop rational terms** from interplay of $\tilde{\mathcal{N}}$ with UV or IR divergences [Lang, Pozzorini, Zhang, Zoller 2020, 2020, 2021]

$$\mathbf{R} \left[\text{diagram} \right]_{D\text{-dim}} = \left[\text{diagram} + \text{diagram} \times \underbrace{\left(\delta Z_{1,\gamma} + \delta \tilde{Z}_{1,\gamma} + \delta \mathcal{R}_{1,\gamma} \right)}_{\text{subdiagrams}} + \text{diagram} \times \underbrace{\left(\delta Z_{2,\gamma} + \delta \mathcal{R}_{2,\gamma} \right)}_{\text{global}} \right]_{4\text{-dim}}$$

- Rational terms of UV origin: **computed and implemented for QED and QCD corrections**
- IR origin: under investigation



Towards OpenLoops@2L

$$\sim \sum_{\mu_i, \nu_j} \underbrace{\mathcal{N}_{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s}}_{\text{4-dim coefficient}} \times \underbrace{\int d^D q_1 \int d^D q_2 \frac{q_1^{\mu_1} \dots q_1^{\mu_r} q_2^{\nu_1} \dots q_2^{\nu_s}}{\prod_{i,j} D_j^{(i)}(q_i)}}_{\text{tensor integral}} + \underbrace{\int d^D q_1 \int d^D q_2 \frac{\tilde{\mathcal{N}}}{\prod_{i,j} D_j^{(i)}(q_i)}}_{(D-4)\text{-dim numerator}}$$

Calculation of tensor integrals:

Strategy:

$$\underbrace{\int d^D q_1 \int d^D q_2 \frac{q_1^{\mu_1} \dots q_1^{\mu_r} q_2^{\nu_1} \dots q_2^{\nu_s}}{\prod_{i,j} D_j^{(i)}(q_i)}}_{\text{tensor integrals}} \rightarrow \underbrace{\sum_k B_k^{\mu_1 \dots \mu_r \nu_1 \dots \nu_s} \mathcal{I}_k}_{\text{scalar integrals}} \rightarrow \underbrace{\sum_{k,l} B_k^{\mu_1 \dots \mu_r \nu_1 \dots \nu_s} C_{kl} \mathcal{M}_l}_{\text{master integrals}}$$

- Currently the bottleneck of NNLO automation

⇒ Progress report in this talk

- Alternative approach: loop-loop duality ⇒ see Stefano Pozzorini's and Nicolò Giraudo's talks

Numerical tensor integral reduction at one loop

- Consider one-loop tensor integral of rank r with D -dimensional loop momentum $\bar{q} = q + \tilde{q}$:

$$I^{\mu_1 \dots \mu_r} = \int d\bar{q} \frac{q^{\mu_1} \dots q^{\mu_r}}{\prod_{a=0}^{N_1-1} D_a(\bar{q})} \quad \text{with} \quad \int d\bar{q} = \mu^{2\epsilon} \int \frac{d^D q}{(2\pi)^D}$$

- Integrand level identity** [del Aguila, Pittau 2004]

$$q^\mu q^\nu = \sum_{a=-2}^3 \left(A_a^{\mu\nu} + B_{a,\lambda}^{\mu\nu} q^\lambda \right) D_a(\bar{q}) \quad \text{with} \quad D_a(\bar{q}) = \begin{cases} (\bar{q} + p_a)^2 - m_a^2, & a \geq 0 \\ 1, & a = -1 \\ \tilde{q}^2, & a = -2 \end{cases}$$

expresses **four-dimensional rank-two tensor** in terms of rank-zero and rank-one terms with coefficients $A_a^{\mu\nu}$, $B_{a,\lambda}^{\mu\nu}$ and **D -dimensional** denominators $D_a(\bar{q})$

- Requires three external momenta p_1 , p_2 , and p_3 to construct four denominators
- Price to pay: -2ϵ -dimensional terms $\tilde{q}^2$ contribute to divergent integrals

⇒ Implemented as on-the-fly reduction [Buccioni, Pozzorini, Zoller 2017] in OPENLOOPS 2

Closed form

$$q^\mu q^\nu = \sum_{a=-2}^3 \left(A_a^{\mu\nu} + B_{a,\lambda}^{\mu\nu} q^\lambda \right) D_a(\bar{q})$$

- We can also derive a closed form for higher ranks:

$$q^{\mu_1} \cdots q^{\mu_r} = \sum_{\Omega} \left[A_{\Omega}^{\mu_1 \cdots \mu_r} + B_{\Omega,\lambda}^{\mu_1 \cdots \mu_r} q^\lambda \right] D_{\Omega}(\bar{q})$$

$$\text{with } \Omega = \{\omega_{-2}, \dots, \omega_3\} \quad \text{with } D_{\Omega}(\bar{q}) = \prod_{a=-2}^3 [D_a(\bar{q})]^{\omega_a} \quad \text{and } \sum_{i=-2}^3 \omega_i = r - 1$$

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where

$$A_{\Omega}^{\mu_1 \dots \mu_r} = \sum_{a=-2}^3 \theta(\omega_a - 1) B_{\hat{n}_a \Omega, \lambda'}^{\mu_1 \dots \mu_{r-1}} A_a^{\lambda' \mu_r}, \quad \hat{n}_a \Omega = \Omega|_{\omega_a \rightarrow \omega_a - 1}$$

$$B_{\Omega, \lambda}^{\mu_1 \dots \mu_r} = \sum_{a=-2}^3 \theta(\omega_a - 1) \left[B_{\hat{n}_a \Omega, \lambda'}^{\mu_1 \dots \mu_{r-1}} B_{a, \lambda}^{\lambda' \mu_r} + A_{\hat{n}_a \Omega}^{\mu_1 \dots \mu_{r-1}} \delta^{\mu_r \lambda} \delta_{a, -1} \right]$$

- Recursive formulae between rank r and $r - 1$ (proof by induction, of course)
- ⇒ Efficient formulae without any big system of equations

Recursive reduction at two loops

- Apply this strategy to general two-loop integral

$$I^{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s} = \int d\bar{q}_1 \int d\bar{q}_2 \frac{q_1^{\mu_1} \dots q_1^{\mu_r} q_2^{\nu_1} \dots q_2^{\nu_s}}{\prod_{i=1}^3 \prod_{a=0}^{N_i-1} [D_a^{(i)}(\bar{q}_i)]^{n_{ia}}}$$

$$= \int d\bar{q}_1 \int d\bar{q}_2 \frac{\left(\sum_{\Omega_1} [A_{\Omega_1}^{\mu_1 \dots \mu_r} + B_{\Omega_1, \lambda_1}^{\mu_1 \dots \mu_r} q_1^{\lambda_1}] D_{\Omega_1}(\bar{q}_1) \right) \left(\sum_{\Omega_2} [A_{\Omega_2}^{\mu_1 \dots \mu_s} + B_{\Omega_2, \lambda_2}^{\mu_1 \dots \mu_s} q_2^{\lambda_2}] D_{\Omega_2}(\bar{q}_2) \right)}{\prod_{i=1}^3 \prod_{a=0}^{N_i-1} [D_a^{(i)}(\bar{q}_i)]^{n_{ia}}}$$

- Integrals remaining after recursive reduction:

$$\int d\bar{q}_1 \int d\bar{q}_2 \frac{(\tilde{q}_1^2)^{m_1} (\tilde{q}_2^2)^{m_2}}{\prod_{i=1}^3 \prod_{a=0}^{N_i-1} [D_a^{(i)}(\bar{q}_i)]^{n_{ia}}} \cdot \begin{cases} 1 \\ q_1^\mu \\ q_2^\nu \\ q_1^\mu q_2^\nu \end{cases}$$

Factorization of q and $\tilde{q}$ tensor structures

- Exploit the disjoint subspaces:

$$\begin{aligned}\int d\bar{q} \frac{\tilde{q}^2 q^\mu}{\mathcal{D}(\bar{q})} &= \underbrace{\int d\tilde{q} \tilde{q}^2}_{-2\epsilon - \dim} \underbrace{\int dq q^\mu}_{4 - \dim} \frac{1}{\mathcal{D}(\bar{q})} = \int d\tilde{q} \tilde{q}^2 \sum_a T_a^\mu I_{a,4} \\ &= \sum_a T_a^\mu \int d\tilde{q} \tilde{q}^2 \int dq \frac{\sum_k C_{ak} D_k(\bar{q})}{\mathcal{D}(\bar{q})} = \sum_{a,k} T_a^\mu C_{ak} \int d\bar{q} \frac{\tilde{q}^2 D_k(\bar{q})}{\mathcal{D}(\bar{q})}\end{aligned}$$

with 4-dimensional tensor structures T_i^μ and reduction coefficients C_{ak}

⇒ Can reduce tensor structures of remaining q_1^μ and q_2^ν separately from $\tilde{q}_1^2$ and $\tilde{q}_2^2$

Factorization of q and $\tilde{q}$ tensor structures

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with 4-dimensional tensor structures T_i^μ and reduction coefficients C_{ak}

- ⇒ Can reduce tensor structures of remaining q_1^μ and q_2^ν separately from $\tilde{q}_1^2$ and $\tilde{q}_2^2$
- ⇒ Use covariant decomposition for q_1^μ and q_2^ν which is cheap due to low ranks
- To be done: $\tilde{q}_1^2$ and $\tilde{q}_2^2$ reduction to D -dimensional scalar integrals

Implementation for amplitudes

$$\mathcal{M} = \sum \underbrace{\mathcal{N}_{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s}}_{\text{tensor coefficients}} \times \underbrace{\int d\bar{q}_1 \int d\bar{q}_2 \frac{q_1^{\mu_1} \dots q_1^{\mu_r} q_2^{\nu_1} \dots q_2^{\nu_s}}{\prod_{i,j} D_j^{(i)}(\bar{q}_i)}}_{\text{tensor integrals}}$$

- Implementation in terms of Mathematica generator for analytic steps and Fortran code for phase-space points
- Two reduction modes:
 - Tensor integral mode:

- Reduce **individual tensor integrals** and then contract with tensor coefficients afterwards:

$$\mathcal{M} = I^{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s} \cdot \mathcal{N}_{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s} = \left(X_{\Omega_1}^{\mu_1 \dots \mu_r} \cdot Y_{\Omega_2}^{\nu_1 \dots \nu_s} \cdot I_{\Omega_1 \Omega_2} \right) \cdot \mathcal{N}_{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s}$$

with $X, Y = A, B$

- Allows checking individual tensor integrals

Implementation for amplitudes

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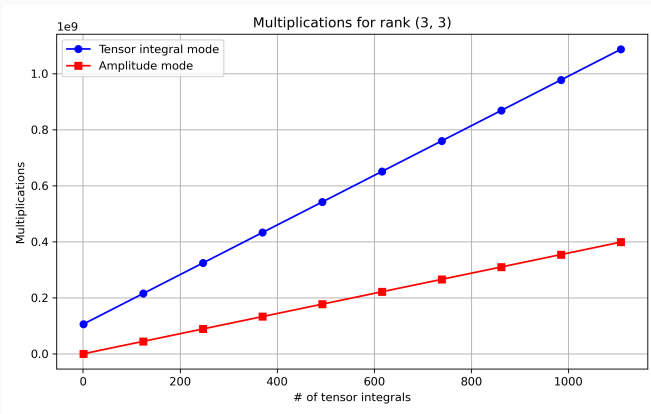
- Allows checking individual tensor integrals
- Amplitude mode:
 - **Contract** reduction coefficients with tensor coefficients **at intermediate steps**:

$$\mathcal{M} = \left(\left(\mathcal{N}_{\mu_1 \dots \mu_r; \nu_1 \dots \nu_s} \cdot X_{\Omega_1}^{\mu_1 \dots \mu_r} \right) \cdot Y_{\Omega_2}^{\nu_1 \dots \nu_s} \right) \cdot I_{\Omega_1 \Omega_2}$$

- **Avoids intermediate high-rank structures**

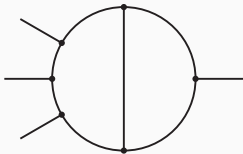
Performance considerations

- Multiplications are most expensive operations
- ⇒ Count multiplications to estimate performance of different reduction modes



- ⇒ Significantly higher performance in amplitude mode expected

Performance test of tensor reduction for pentagon-triangle



- Reduce integrals of maximum tensor ranks in (q_1, q_2) : (5, 1), (4, 2), and (3, 3)
- Contract with random tensor coefficients
- Replace scalar integrals by random values

Rank	Tensor integral mode	Amplitude mode
(5, 1)	4.7 ms per psp	0.48 ms per psp
(4, 2)	45 ms per psp	0.32 ms per psp
(3, 3)	65 ms per psp	0.32 ms per psp

⇒ Theoretically expected advantage of amplitude mode also achieved in practice

⇒ Promising performance

Numerical stability of tensor reduction for pentagon-triangle

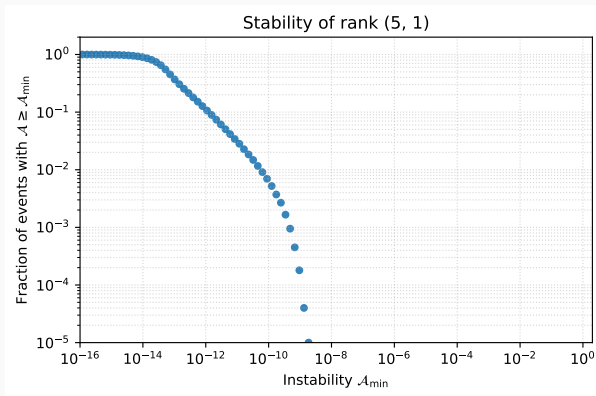
- Reduction down to rank one in each loop momentum on **integrand level**
- ⇒ Test numerical stability by comparing to direct evaluation of integrand at fixed values of q_1 and q_2

- Relative difference:

$$\mathcal{A} = \left| \frac{I_{\text{recursive}} - I_{\text{direct}}}{I_{\text{direct}}} \right|$$

- Test 100.000 random phase space points with $p_T \geq 0.05\sqrt{s}$

⇒ **Excellent stability**



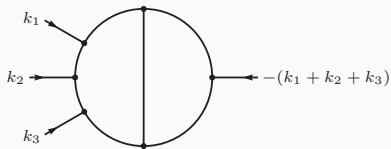
Handling scalar integrals

- Validation of full algorithm only possible on **integral level**

⇒ Handle scalar integrals:

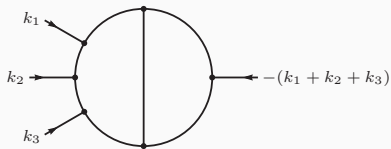
- Kira **[FL, Usovitsch, Wu 2025]** for analytic reduction at generator level
- Optimize master integral basis with `ImproveMasters.m` **[Smirnov, Smirnov 2020]** to remove spurious poles and large denominators
- Translate reduction tables to Fortran code
- AMFlow **[Huang, Liu, Ma 2026]** to compute master integrals at generator level
- Read results from disk at runtime
- Alternative option for master integrals: PYSECDEC **[Heinrich, Jones, Kerner, Magerya, Olsson, Schlenk 2023]** at runtime

Stability of full reduction for pentagon-triangle



- Since $\tilde{q}^2$ reduction not yet implemented, need finite integrals
- ⇒ Regulate IR divergences with off-shell masses for external legs: $k_i^2 = m^2 = 0.1s$
- ⇒ Rank-(4, 1) integral also UV finite

Stability of full reduction for pentagon-triangle



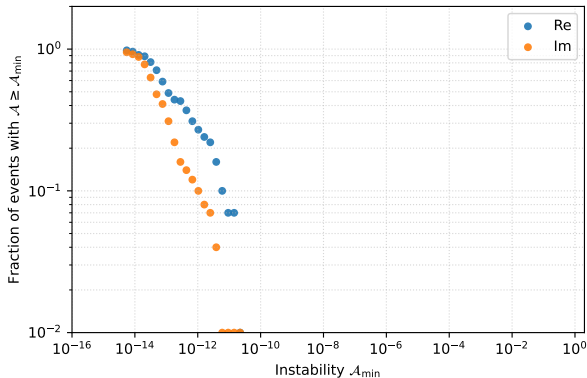
- Test 100 random phase space points with $p_T \geq 0.05\sqrt{s}$
- Master integrals evaluated with 32 digits (AMFlow)
- Contract tensor integral with suitable projector analytically for reference

⇒ Still excellent stability

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⇒ Regulate IR divergences with off-shell masses for external legs: $k_i^2 = m^2 = 0.1s$

⇒ Rank-(4, 1) integral also UV finite



Summary and outlook

- Two-loop automation highly desired for phenomenology at LHC and future colliders
- OPENLOOPS exploits splitting into different building blocks: process dependent coefficients, process-class dependent tensor integrals, and general counterterms
- Presented completely general algorithm for (numerical) reduction of tensor integrals
- Advantage: most complicated part (many propagators and high rank) with local integrand-level method avoiding big systems of equations
- First benchmarks of implementation promising in terms of speed and stability

Summary and outlook

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- Presented completely general algorithm for (numerical) reduction of tensor integrals
- Advantage: most complicated part (many propagators and high rank) with local integrand-level method avoiding big systems of equations
- First benchmarks of implementation promising in terms of speed and stability
- Next steps for proof-of-concept:
 - Implement $\tilde{q}^2$ reduction
 - Look into numerical integral reduction per phase-space point, see e.g. [Agarwal, Heinrich, Jones, Kerner, Klein, Lang, Magerya, Olsson 2024; Becchetti, Canko, Chen, Chestnov, Delto, Ditsch, Peraro, Pozzoli, Savoini, Zoia 2026] $\Rightarrow$ see Mattia Pozzoli's and Xiang Chen's talks
 - Master integrals will be the bottleneck for some time

Numerical tensor integral reduction in OpenLoops 2

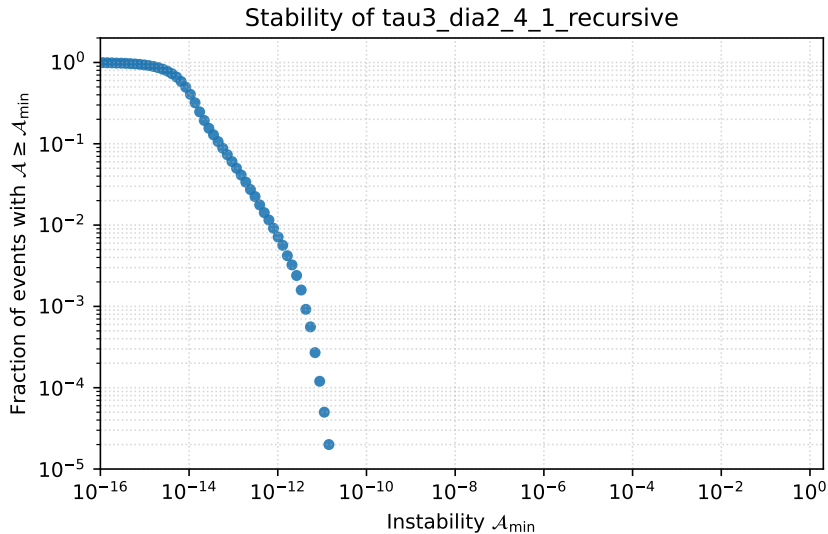
$$\text{Diagram} \sim \underbrace{\mathcal{N}_{\mu_1 \dots \mu_N}}_{\text{4-dim coefficient}} \underbrace{\int d\bar{q} \frac{q^{\mu_1} \dots q^{\mu_N}}{D_0(\bar{q}) \dots D_{N-1}(\bar{q})}}_{\text{tensor integral}},$$

$$q^\mu q^\nu = \sum_{a=-2}^3 \left(A_a^{\mu\nu} + B_{a,\lambda}^{\mu\nu} q^\lambda \right) D_a(\bar{q})$$

On-the-fly reduction [Buccioni, Pozzorini, Zoller 2017]:

- Perform reduction for contraction of tensor coefficients and tensor integrand
 - Implemented at level of tensor coefficients:
 - Interleave segment construction steps with on-the-fly tensor reduction steps
- ⇒ Avoids high-rank objects both in coefficients and tensor integrals
- Remaining integrals:
 - Ranks one and zero: covariant decomposition and OPP method [Ossola, Papadopoulos, Pittau 2006]
 - $\tilde{q}^2$: only four for all processes [del Aguila, Pittau 2004]
- ⇒ Highly efficient numerical tool

Rank (4, 1) recursive



Rank (1,1) full

