

Collinear functions up to NNLO in QCD

Prasanna Kumar Dhani

University of Zurich

Based on arXiv: 2208.05840 + work in preparation

HP2 2026·KIT Karlsruhe·5 October 2026



University of
Zurich^{UZH}

* Introduction & Motivation

Fixed Order Vs Resummed

Transverse Momentum Resummation

* Computation of Collinear Functions

Step 1: Differential

Step 2: Integrated

* Perturbative Results up to NNLO

* Summary & Outlook

Theoretical separation

$$\frac{d\sigma_F}{dq_T} = \frac{d\sigma_F^{\text{sing.}}}{dq_T} (\equiv [d\sigma_F]) + \frac{d\sigma_F^{\text{reg.}}}{dq_T}$$

Large contributions! $\left\{ \left[\frac{1}{q_T^2} \ln^n \left(\frac{M^2}{q_T^2} \right) \right]_+, \delta(q_T^2) \right\}$ $\lim_{Q_0 \rightarrow 0} \int_0^{Q_0} dq_T \frac{d\sigma_F^{\text{reg.}}}{dq_T} = 0$

Process independent: blind to F Depend on F!

Good news: singular contributions are of infrared (soft and collinear) origin. Hence, they have some degree of universality that leads to Resummation of these terms.

$$[g(\mathbf{q_T})]_+ = \lim_{q_0 \rightarrow 0} \left[\theta(q_T - q_0) g(\mathbf{q_T}) - \delta^{(2)}(\mathbf{q_T}) \int d^2 \mathbf{k_T} g(\mathbf{k_T}) \theta(\mu_0 - k_T) \theta(k_T - q_0) \right]$$

Singular part of the differential cross section in $\mathbf{b}$ -space has the following structure

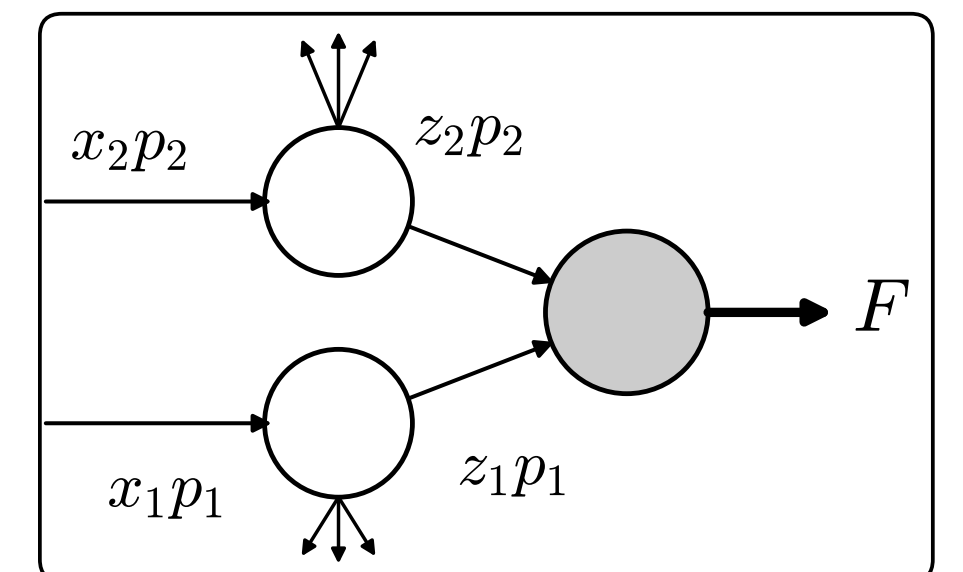
$$[d\sigma_F] \propto \overset{\text{Soft contribution}}{S_c} \times \underset{\text{Hard collinear factor}}{[H^F C_1 C_2]} \otimes f_{a_1/h_1} \otimes f_{a_2/h_2}$$

For the gluon gluon fusion channel

$$[H^F C_1 C_2]_{gg;a_1 a_2} = \overset{\text{Hard functions}}{H_{\mu_1 \nu_1, \mu_2 \nu_2}^F} (x_1 p_1, x_2 p_2; \Omega; \alpha_s(M^2)) \times C_{ga_1}^{\mu_1 \nu_1} (z_1; p_1, p_2, \mathbf{b}; \alpha_s(b_0^2/b^2)) C_{ga_2}^{\mu_2 \nu_2} (z_2; p_1, p_2, \mathbf{b}; \alpha_s(b_0^2/b^2))$$

Collinear functions

This Talk!



[Collins, Soper, Sterman \(1985\)](#); [Catani, Cieri, de Florian, Ferrera, Grazzini \(1311.1654\)](#)

[Becher, Neubert \(1007.4005\)](#); [Echevarria, Idilbi, Scimemi \(1111.4996\)](#); [Chiu, Jain, Neill, Rothstein \(1202.0814\)](#)

where Catani, Grazzini (1011.3918)

$$C_{ga}^{\mu\nu}(z; p_1, p_2, \mathbf{b}; \alpha_s) = d^{\mu\nu}(p_1, p_2) C_{ga}(z, \alpha_s) + D^{\mu\nu}(p_1, p_2; \mathbf{b}) G_{ga}(z; \alpha_s)$$

$d^{\mu\nu}(p_1, p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2}$
Average part
Correlation part

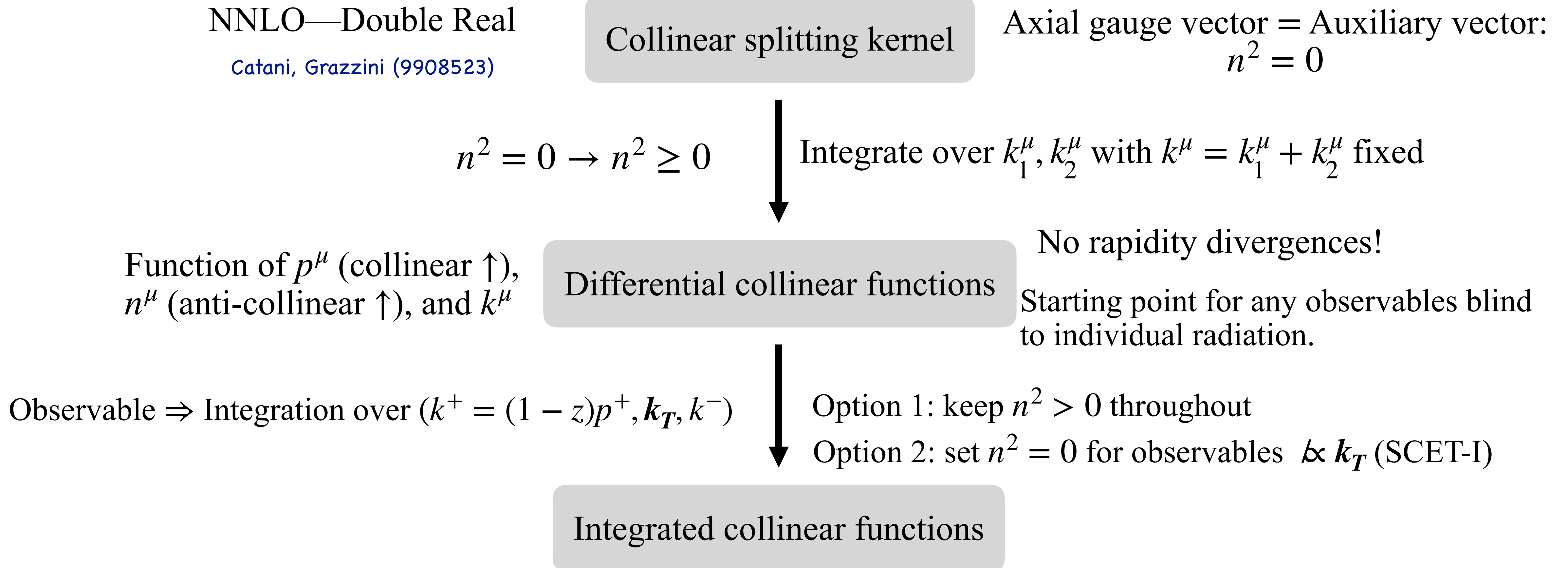
$D^{\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\mu\nu}(p_1, p_2) - 2 \frac{b^\mu b^\nu}{\mathbf{b}^2}$
 $b^\mu = (0, \mathbf{b}, 0)$

For quark $\propto \delta^{ss'} C_{qa}(z, \alpha_s)$

Both azimuthally independent and dependent collinear functions are known to N3LO in QCD coupling.

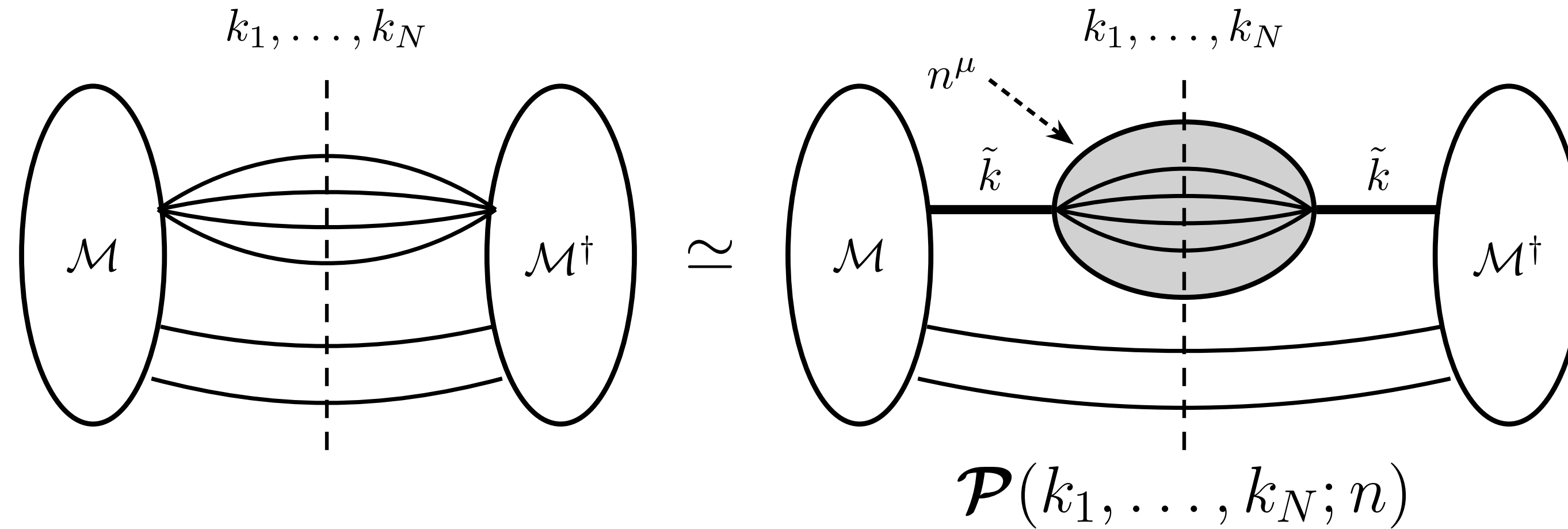
de Florian, Grazzini (0108273); Catani, Grazzini (1011.3918); Catani, Grazzini (1106.4652); Catani, Cieri, de Florian, Ferrera, Grazzini (1209.0158); Gehrmann, Lubbert, Yang (1209.0682, 1403.6451); Echevarria, Scimemi, Vladimirov (1604.07869); Luo, Wang, Xu, Yang, Yang, Zhu (1908.03831); Luo, Yang, Zhu, Zhu (1909.13820); Luo, Yang, Zhu, Zhu (1909.13820); Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov (1907.03780); Catani, Dhani (2208.05840); Luo, Yang, Zhu, Zhu (1912.05778); Ebert, Mistlberger, Vita (2006.05329); Luo, Yang, Zhu, Zhu (2012.03256); Zhu (2509.01703)

So why compute them again? Because the question isn't how far, but how: can we define, at an intermediate level, a collinear function of the total radiation momentum that is free of rapidity divergences from the start?



Goal: differential collinear functions are exact in ϵ ; we plan to achieve the same for integrated collinear functions at the bare level!

$\hookrightarrow d = 4 - 2\epsilon$



$$\begin{array}{c}
 \text{Collinear partons} \\
 \underbrace{\hspace{10em}} \\
 |\mathcal{M}(k_1, \dots, k_N)|^2 = \langle \mathcal{M}(\tilde{k}) | \underbrace{\mathcal{P}(k_1, \dots, k_N; n)}_{\text{Splitting kernel}} | \mathcal{M}(\tilde{k}) \rangle + \dots
 \end{array}$$

Collinear limit of $\sum_{i=1}^N k_i$ Auxiliary vector n

Bern, Chalmers, Dixon, Kosower, Dunbar (9312333, 9403226, 9503236); Kosower (9901201); Bern, Del Duca, Kilgore, Schmidt (9903516); Catani, Grazzini (9908523); Catani, de Florian, Rodrigo (0312067, 1112.4405)

In addition to spin and color indices, the splitting kernel depends on scalar functions of collinear momenta of the form

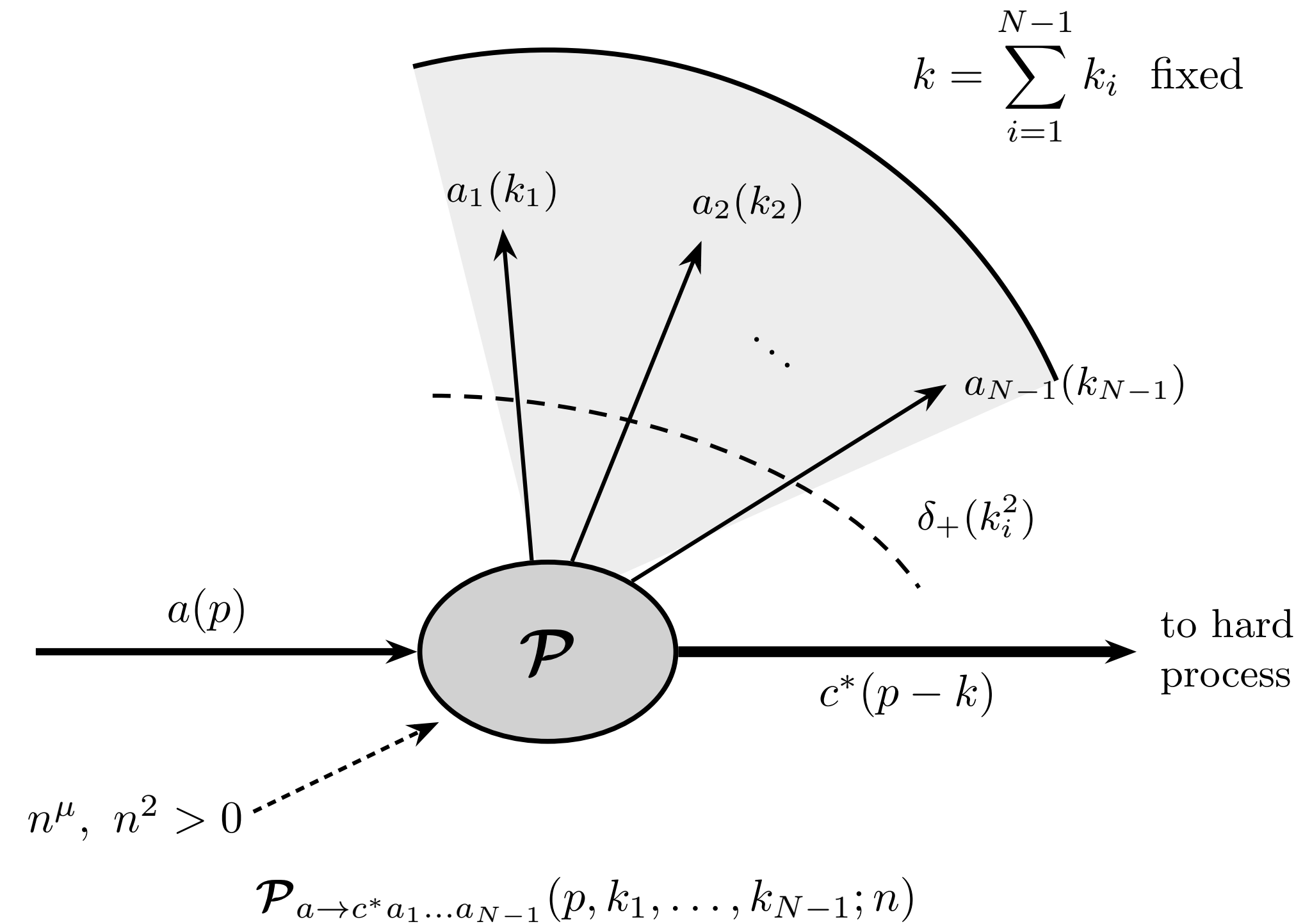
$$s_{ij} = 2k_i \cdot k_j \qquad \frac{x_i}{x_j} = \frac{n \cdot k_i}{n \cdot k_j}$$


 Usually light-like: $n^\mu = (0, \mathbf{0}_T, n^-)$

We emphasize that one can also set

$$n^2 \neq 0 \quad (\text{Corrections of the } \mathcal{O}(k_{iT} \text{ or } k_{jT}), \text{ and hence neglected in the collinear limit})$$


This work: $n^2 > 0$ (Time-like: $n^\mu = (n^+, \mathbf{0}_T, n^-)$)



Observables blind to individual radiations.

$$\mathcal{F}_{ca}(p, k; n) = \sum_{N=2}^{+\infty} \left[\prod_{m=1}^{N-1} \int \frac{d^d k_m}{(2\pi)^{d-1}} \delta_+(k_m^2) \right] \delta^{(d)} \left(k - \sum_{i=1}^{N-1} k_i \right) \sum_{a_1, \dots, a_{N-1}} \frac{\mathcal{P}_{a \rightarrow c^* a_1 \dots a_{N-1}}(p, k_1, \dots, k_{N-1}; n)}{\text{SF}(a_1, \dots, a_{N-1})}, c = q, \bar{q}, g$$

Symmetry factor

$$\mathcal{F}(p, k; n) = \mathcal{F}^{(R)}(p, k; n) + [\mathcal{F}^{(RR)}(p, k; n) + \mathcal{F}^{(RV)}(p, k; n)] + \mathcal{O}(\alpha_S^3)$$

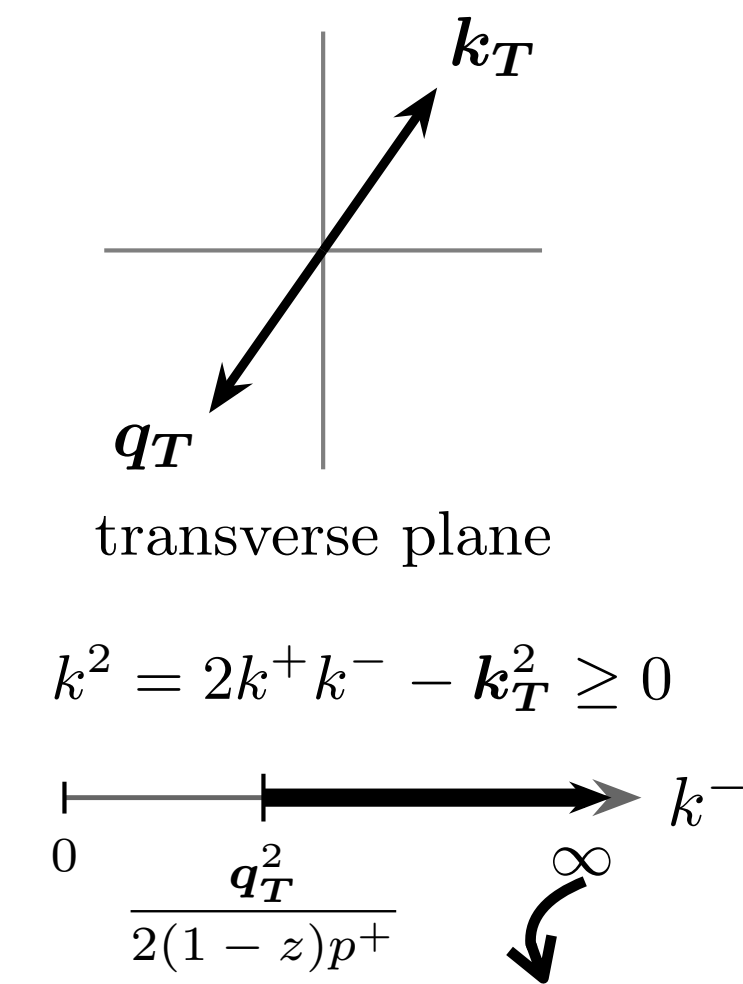
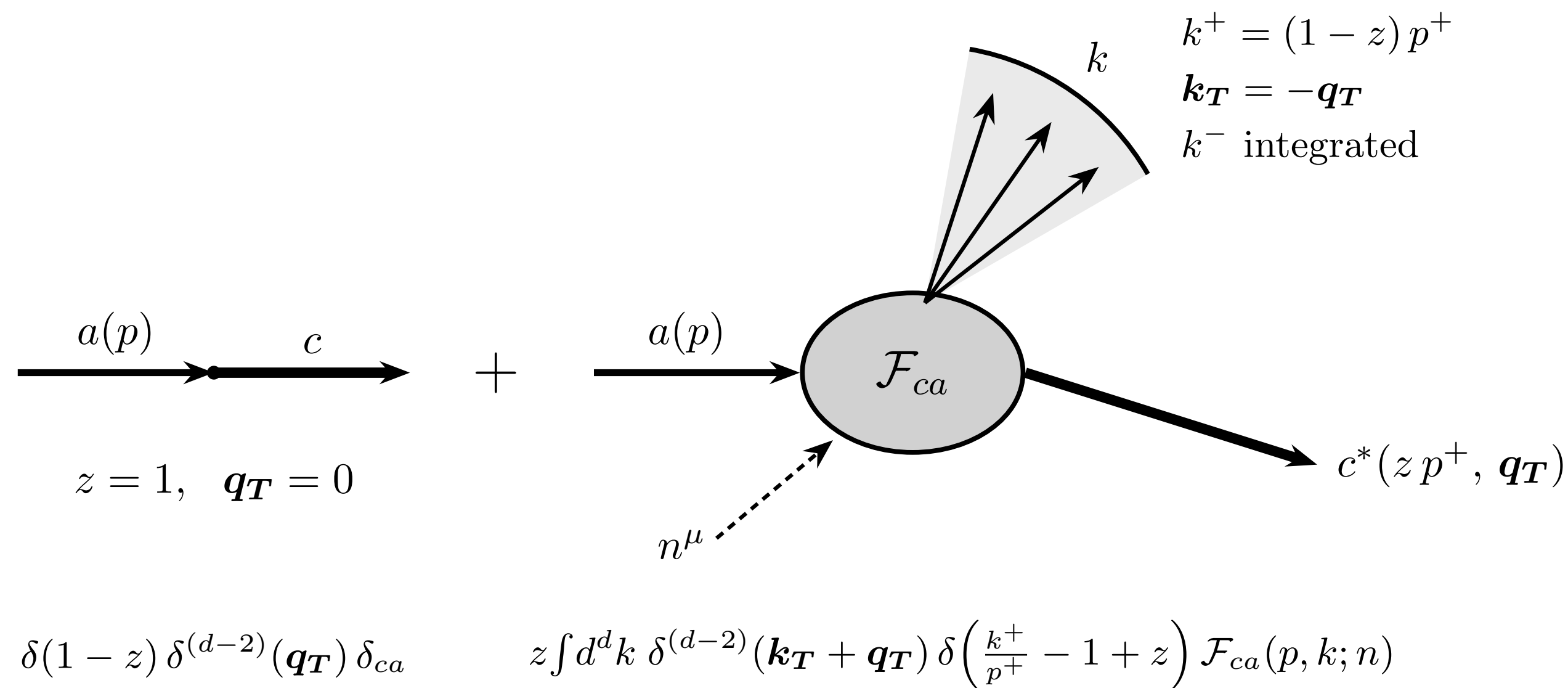
For transverse momentum, we have

Catani+PKD (2208.05840)

Defines the observable

$$F_{ca}(z; zp, \mathbf{q}_T; n) = \delta(1 - z) \delta^{(d-2)}(\mathbf{q}_T) \delta_{ca} + z \int d^d k \delta^{(d-2)}(\mathbf{k}_T + \mathbf{q}_T) \delta\left(\frac{k^+}{p^+} - 1 + z\right) \mathcal{F}_{ca}(p, k; n)$$

Phase space in the collinear limit



Origin of rapidity divergences!

For Zero-Jettiness, we have

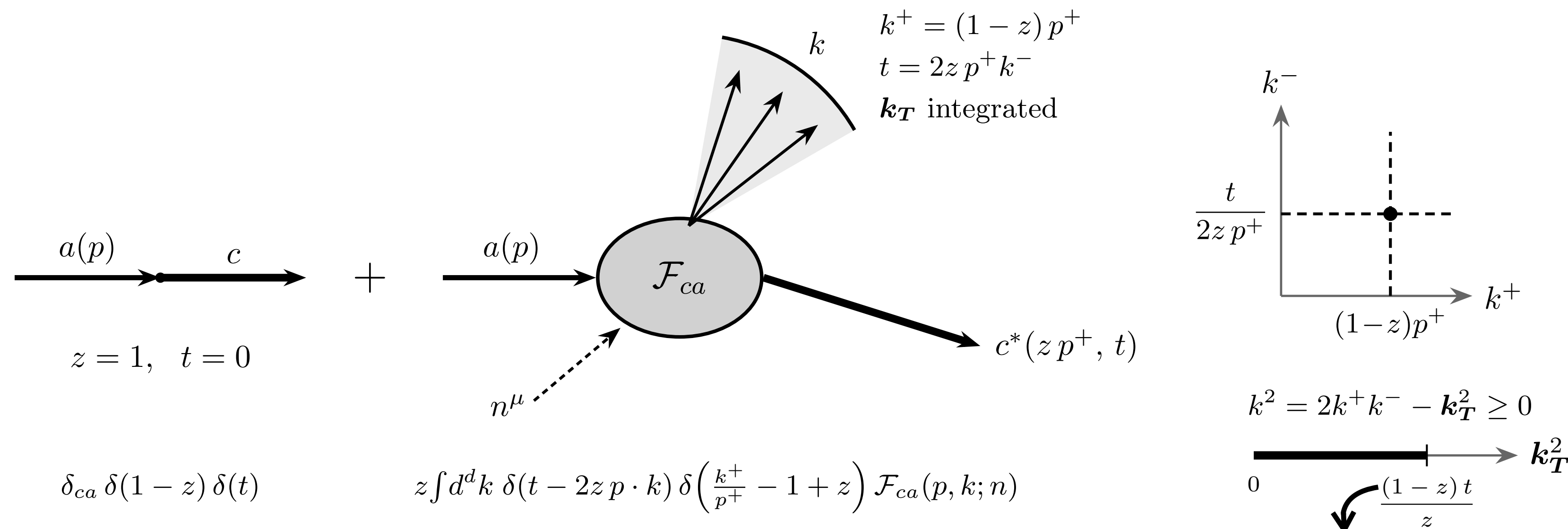
Catani+PKD (2208.05840)

$$\mathcal{B}_{ca}(z; t) = \delta_{ca} \delta(1 - z) \delta(t) + z \int d^d k \delta(t - 2z p \cdot k) \delta\left(\frac{k^+}{p^+} - 1 + z\right) \mathcal{F}_{ca}(p, k; n)$$

Defines the observable

Transverse virtuality $\propto \tau_0$

Phase space in the collinear limit



No rapidity divergences — ϵ is sufficient!

Consider the singular term from $\mathcal{F}_{ca}^{(R)}$

Catani+PKD (2208.05840)

$$\frac{1}{1-z_n} = \frac{np}{nk} = \frac{p^+}{k^+ + \frac{n^2}{2np} \frac{k^-}{n^-} p^+} = \frac{1}{1-z + \frac{n^2 \mathbf{q}_T^2}{(1-z)(2np)^2}} \xrightarrow{n^2=0} \frac{1}{1-\underbrace{z}_{\text{Not well defined!}}}$$

$\lambda = \frac{n^2 \mathbf{q}_T^2}{(2np)^2}$

Using a time-like auxiliary vector

$$\frac{1}{1-z + \frac{\lambda}{1-z}} = \left(\frac{1-z}{(1-z)^2 + \lambda} \right)_+ + \delta(1-z) \int_0^1 dz' \frac{1-z'}{(1-z')^2 + \lambda} = \frac{1}{2} \ln \left(\frac{1}{\lambda} \right) \delta(1-z) + \left(\frac{1}{1-z} \right)_+ + \mathcal{O}(\underbrace{\sqrt{\lambda}}_{\text{Sub-dominant!}})$$

Neglected terms are always of order $\mathcal{O}(\lambda^{\frac{n \in N}{2} + m\epsilon})$, and they are perfectly legitimate in the collinear limit!

No additional power corrections because in our calculation, n^2 is kept finite!

Consider again the singular term from $\mathcal{F}_{ca}^{(R)}$

Catani+PKD (2208.05840)

$$\frac{1}{1-z_n} = \frac{np}{nk} = \frac{p^+}{k^+ + \frac{n^2}{2np} \frac{k^-}{n^-} p^+} = \frac{1}{1-z + \frac{n^2 t}{z(2np)^2}} \xrightarrow{n^2=0} \frac{1}{1-z} \xrightarrow{(1-z)^{-\epsilon} \text{ from } k_T \text{ integral}} \frac{(1-z)^{-\epsilon}}{1-\overline{z}}$$

$\lambda' = \frac{n^2 t}{(2np)^2}$

Well defined!

Using a time-like auxiliary vector

$$\frac{(1-z)^{-\epsilon}}{1-z + \frac{\lambda'}{z}} = \left(\frac{(1-z)^{-\epsilon}}{1-z + \frac{\lambda'}{z}} \right)_+ + \delta(1-z) \int_0^1 dz' \frac{(1-z')^{-\epsilon}}{1-z' + \frac{\lambda'}{z'}} = \left[(\lambda')^{-\epsilon} \Gamma(1+\epsilon) \Gamma(1-\epsilon) - 1 \right] \frac{1}{\epsilon} \delta(1-z) + \left(\frac{(1-z)^{-\epsilon}}{1-z} \right)_+ + \mathcal{O}(\lambda')$$

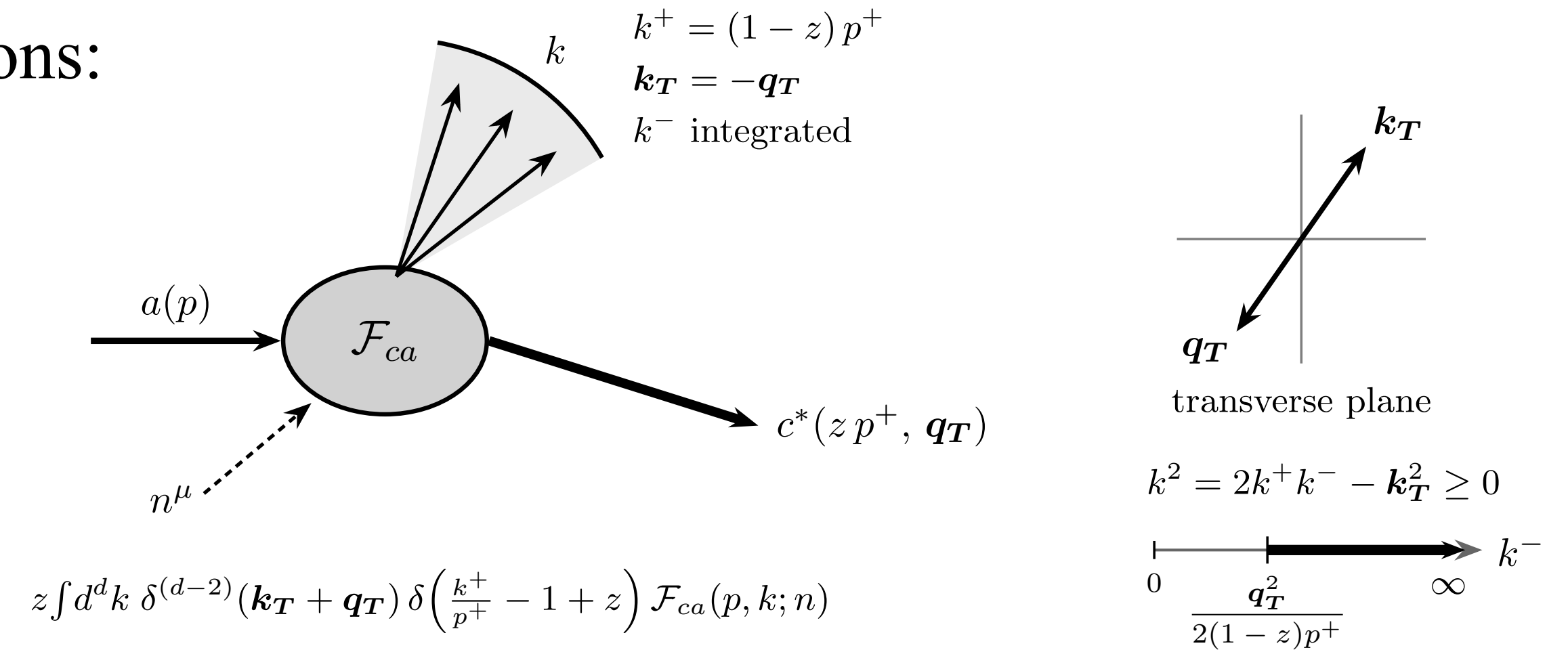
Sub-dominant!

Zero-jettiness beam functions can be computed either by using a light-like or a time-like auxiliary vector!

Rapidity divergences are artifact of a priori approximations:

$$F_{ca}(z; zp, \mathbf{q}_T; n) \propto z \int d^d k \delta^{(d-2)}(\mathbf{k}_T + \mathbf{q}_T) \delta\left(\frac{k^+}{p^+} - 1 + z\right) \mathcal{F}_{ca}(p, k; n)$$

In the collinear limit!



Collinear factorization: $k^- \lesssim \mathcal{O}(k^+)$ $\xrightarrow[\text{Simplification}]{\text{Phase space}}$ $0 < k^- < +\infty$ Only if $\mathcal{F}_{ca}$ is well behaved in the limit $k^- \rightarrow \infty$

Many rapidity regulators exist in the literature for computing individual components like soft functions and collinear functions. They are introduced at the integrated level.

[Chiu, Jain, Neill, Rothstein \(1104.0881\)](#); [Becher, Bell \(1112.3907\)](#); [Echevarria, Scimemi, Vladimirov \(1509.06392\)](#); [Li, Neill, Zhu \(1604.00392\)](#)

As we discussed, we ensure that our calculation is rapidity divergence-free at the matrix element level!

There are 10 integrals to compute out of which 3 integrals (in red) needs to be computed for $n^2 > 0$.

$$\frac{1}{p \cdot k_1}, \frac{1}{(p \cdot k_1)^2}, \frac{1}{n \cdot k_1}, \frac{1}{n \cdot (k_2 - p)}, \frac{1}{n \cdot k_1 p \cdot k_1}, \frac{1}{n \cdot k_1 p \cdot k_2}, \frac{1}{p \cdot k_1 n \cdot (k_2 - p)}, \frac{1}{p \cdot k_2 n \cdot (k_2 - p)}, \frac{1}{(p \cdot k_2)^2 n \cdot (k_2 - p)}, \frac{1}{(p \cdot k_2)^2 (n \cdot (k_2 - p))^2}$$

Somogyi (1101.3557); Lyubovitskij, Wunder, Zhevlakov (2102.08943)

$$\begin{aligned} \text{S1: } \frac{1}{n \cdot k_1} &\propto \int d^d k \frac{(k^2)^{-\epsilon}}{n \cdot k} {}_2F_1\left(\frac{1}{2}, 1, \frac{3}{2} - \epsilon, 1 - \frac{n^2 k^2}{(n \cdot k)^2}\right) \left| {}_2F_1(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 dt t^{b-1} (1-t)^{c-b-1} (1-zt)^{-a} \right. \\ \text{S2: } \frac{1}{n \cdot k_1 p \cdot k_1} &\propto \frac{1}{n \cdot p} \int d^d k (k^2)^{-1-\epsilon} F_1\left(1, -\epsilon, -\epsilon, 1-2\epsilon, 1 - \frac{1 + \sqrt{1 - \frac{n^2 k^2}{(n \cdot k)^2}}}{\frac{n \cdot p}{n \cdot k} \frac{k^2}{p \cdot k}}, 1 - \frac{1 - \sqrt{1 - \frac{n^2 k^2}{(n \cdot k)^2}}}{\frac{n \cdot p}{n \cdot k} \frac{k^2}{p \cdot k}}\right) \left| F_1(a; b_1, b_2; c; x, y) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_0^1 dt t^{a-1} (1-t)^{c-a-1} (1-xt)^{-b_1} (1-yt)^{-b_2} \right. \\ \text{S3: } \frac{1}{n \cdot k_1 p \cdot k_2} &\propto \frac{1}{n \cdot p} \int d^d k \frac{(k^2)^{-\epsilon}}{k_T^2 + \frac{n^2(p \cdot k)^2}{(n \cdot p)^2}} F_1\left(1, -\epsilon, -\epsilon, 1-2\epsilon, 1 - \frac{1 + \sqrt{1 - \frac{n^2 k^2}{(n \cdot k)^2}}}{\frac{n \cdot p}{n \cdot k p \cdot k} \left(k_T^2 + \frac{n^2(p \cdot k)^2}{(n \cdot p)^2}\right)}, 1 - \frac{1 - \sqrt{1 - \frac{n^2 k^2}{(n \cdot k)^2}}}{\frac{n \cdot p}{n \cdot k p \cdot k} \left(k_T^2 + \frac{n^2(p \cdot k)^2}{(n \cdot p)^2}\right)}\right) \end{aligned}$$

Depending on the prefactor, these terms can generate rapidity divergences! So, it is useful to study them in the limit

$$v_{11} \equiv \frac{n^2 k^2}{(n \cdot k)^2} \rightarrow 0$$

Problem. v_1 massive, v_2 massless: the angular integral of $1/(v_1 \cdot k \ v_2 \cdot k)$ is non-analytic as $v_{11}(v_{ij} \equiv v_i \cdot v_j) \rightarrow 0$ ($\propto v_{11}^{-\epsilon}$)

Lemma. For any $v_3 = (1 - \rho)v_1 + \rho v_2$ on the line through v_1, v_2 :

$$\frac{1}{v_1 \cdot k \ v_2 \cdot k} = \frac{\rho}{v_1 \cdot k \ v_3 \cdot k} + \frac{1 - \rho}{v_2 \cdot k \ v_3 \cdot k}$$

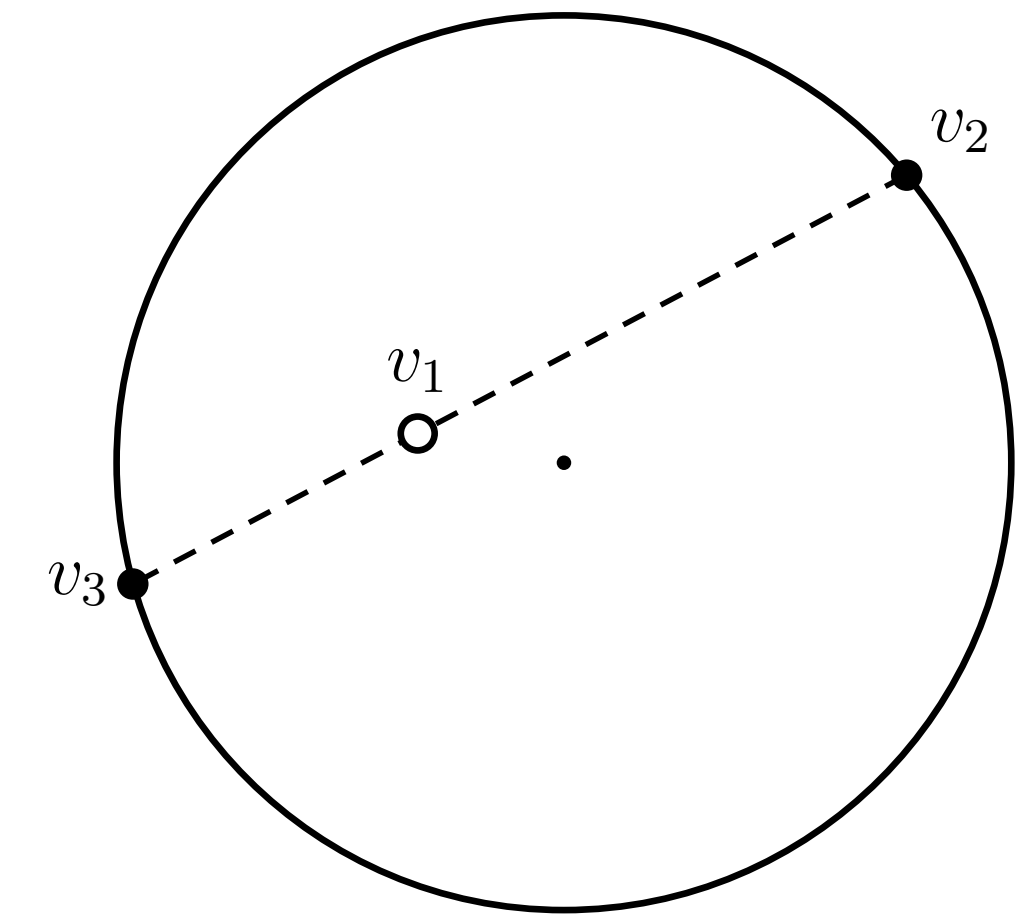
since $v_3 \cdot k = (1 - \rho)v_1 \cdot k + \rho v_2 \cdot k$ — ordinary partial fractions.

Choice. Put v_3 on the light cone: $v_{33} = 0 \Rightarrow \rho = -\frac{v_{11}}{2v_{12} - v_{11}}$

Result. One hard integral $\rightarrow$ two simple ones:

(v_1, v_3) : quasi-collinear, $v_{13} \propto v_{11} \Rightarrow$ carries the entire $v_{11}^{-\epsilon}$

(v_2, v_3) : both massless, $\Rightarrow$ smooth as $v_{11} \rightarrow 0$



massless: on the sphere; massive: inside

chord $v_2 \rightarrow v_1$ re-enters at v_3

$v_{11} \rightarrow 0 : v_3 \rightarrow v_1, \rho \rightarrow 0$

Massless limit (suitable form for the limit) can also be achieved by some complicated transformations!

Hypergeometric function

$${}_2F_1\left(\frac{1}{2}, 1, \frac{3}{2} - \epsilon, 1 - v_{11}\right) = -\frac{1-2\epsilon}{2\epsilon} \left[{}_2F_1\left(\frac{1}{2}, 1; 1 + \epsilon; v_{11}\right) - 2^{2\epsilon} \frac{\Gamma(1-2\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-\epsilon)} v_{11}^{-\epsilon} (1-v_{11})^{-\frac{1}{2}+\epsilon} \right] \Bigg|_{v_{11} = \frac{n^2 k^2}{(n \cdot k)^2}}$$

$$\xrightarrow{v_{11} \rightarrow 0} -\frac{1-2\epsilon}{2\epsilon} \left[1 - 2^{2\epsilon} \frac{\Gamma(1-2\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-\epsilon)} v_{11}^{-\epsilon} \right] + \mathcal{O}(v_{11})$$

Appell function

$$F_1\left(1, -\epsilon, -\epsilon, 1-2\epsilon, 1 - \frac{1+\sqrt{1-v_{11}}}{v_{12}}, 1 - \frac{1-\sqrt{1-v_{11}}}{v_{12}}\right) \Bigg|_{v_{12} = \frac{n \cdot p k^2}{n \cdot k p \cdot k} \text{ or } \frac{n \cdot p}{n \cdot k p \cdot k} \left(k_T^2 + \frac{n^2(p \cdot k)^2}{(n \cdot p)^2}\right)}$$

$$= \left(\frac{v_{11}}{v_{12}^2}\right)^\epsilon \left[-v_{11}^{-2\epsilon} (2v_{12} - v_{11})^{2\epsilon} F_1\left(-2\epsilon, -\epsilon, -\epsilon, 1-2\epsilon, \frac{v_{12}(1+\sqrt{1-v_{11}}) - v_{11}}{2v_{12} - v_{11}}, \frac{v_{12}(1-\sqrt{1-v_{11}}) - v_{11}}{2v_{12} - v_{11}}\right) + 2v_{11}^{-\epsilon} (2v_{12} - v_{11})^\epsilon {}_2F_1\left(-\epsilon, -\epsilon, 1-\epsilon, 1 - \frac{v_{12}^2}{2v_{12} - v_{11}}\right) \right]$$

$$\xrightarrow{v_{11} \rightarrow 0} 2^{1+\epsilon} v_{12}^{-\epsilon} {}_2F_1\left(-\epsilon, -\epsilon, 1-\epsilon, 1 - \frac{v_{12}}{2}\right) - 2^{2\epsilon} \frac{\Gamma(1-2\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-\epsilon)} v_{11}^{-\epsilon} + \mathcal{O}(v_{11})$$

Much simpler expressions! New results for the differential collinear functions.

Consider the following integral

$$\begin{aligned}
 I &\equiv \int_{k_{\min}^-}^{\infty} \frac{dk^-}{k^-} (k^2)^{-1-\epsilon} \frac{n \cdot p}{n \cdot k} F_{21}(v_{11}), \quad F_{21}(v_{11}) = {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2} - \epsilon; 1 - v_{11}\right), \quad v_{11} = \frac{n^2 k^2}{(n \cdot k)^2} \\
 &= (\mathbf{q}_T^2)^{-1-\epsilon} \int_0^1 dt \, t^\epsilon (1-t)^{-1-\epsilon} \frac{1-z}{(1-z)^2 + \lambda/t} F_{21}(v_{11}), \quad v_{11} = \frac{4\lambda (1-z)^2 (1-t)}{t [(1-z)^2 + \lambda/t]^2}, \quad \lambda = \frac{n^2 q_T^2}{(2n \cdot p)^2} \\
 &= \frac{(1-2\epsilon)(\mathbf{q}_T^2)^{-1-\epsilon}}{2\epsilon^2} \left\{ \Gamma_\epsilon \left[\frac{1}{(1-z)_+} + \frac{\delta(1-z)}{2} \left(\frac{\Gamma_{2\epsilon}}{2\epsilon} + 2\psi(1+\epsilon) - \psi(1+2\epsilon) + \gamma_E - \ln \lambda \right) \right] - \frac{\Gamma(1+\epsilon)\Gamma(1-2\epsilon)\Gamma_{2\epsilon}}{2\Gamma(1-\epsilon)} \lambda^{-\epsilon} \left[\frac{1}{(1-z)_+^{1-2\epsilon}} + \frac{\delta(1-z)}{2\epsilon} \right] \right\}
 \end{aligned}$$

$$\Gamma_\epsilon \equiv \Gamma(1-\epsilon)\Gamma(1+\epsilon), \quad \Gamma_{2\epsilon} \equiv \Gamma(1-2\epsilon)\Gamma(1+2\epsilon), \quad \psi(x) \equiv \frac{d}{dx} \ln \Gamma(x)$$

The result is exact in ϵ !

All the relevant integrals corresponding to both transverse momentum and zero-jettiness are completed!

Infrared factorization

$$\widetilde{F}_{ca}\left(z; \frac{\mathbf{b}^2}{b_0^2}, \frac{n^2 b_0^2}{(2zpn)^2 \mathbf{b}^2}\right) = Z_c\left(\alpha_S(b_0^2/\mathbf{b}^2), \frac{n^2 b_0^2}{(2zpn)^2 \mathbf{b}^2}\right) \sum_b \int_z^1 \frac{dx}{x} \widetilde{C}_{cb}\left(x; \alpha_S(b_0^2/\mathbf{b}^2), \epsilon, \frac{n^2 b_0^2}{(2zpn)^2 \mathbf{b}^2}\right) \widetilde{\Gamma}_{ba}(z/x; b_0^2/\mathbf{b}^2) \quad \left| \quad \int d^{d-2} \mathbf{q}_T e^{-i \mathbf{b} \cdot \mathbf{q}_T} \frac{\ln^m(\mathbf{q}_T^2)}{(\mathbf{q}_T^2)^{1+\delta}} \xrightarrow{d=4-2\epsilon} \frac{d^m}{d\rho^m} \Big|_{\rho=0} \pi^{1-\epsilon} \left(\frac{\mathbf{b}^2}{4}\right)^{\epsilon+\delta-\rho} \frac{\Gamma(\rho-\epsilon-\delta)}{\Gamma(1+\delta-\rho)}\right.$$

$$\widetilde{C}_{cb}\left(z; \alpha_S, \epsilon = 0, \frac{n^2 b_0^2}{(2zpn)^2 \mathbf{b}^2}\right) = C_{cb}(z; \alpha_S) \quad \widetilde{\Gamma}_{ba}(z; \mu_F^2) = \delta_{ba} \delta(1-z) - \frac{\alpha_S(\mu_F^2)}{\pi} \frac{P_{ba}^{(1)}(z)}{\epsilon} + \mathcal{O}(\alpha_S^2)$$

To be validated against the known results!

Infrared factorization factors for both gluon and quark are as follows

$$Z_g = 1 + \frac{\alpha_S}{\pi} \left[\frac{C_A}{2} \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln\left(\frac{n^2 b_0^2}{(2znp)^2 b^2}\right) \right) + \frac{\beta_0}{\epsilon} - C_A \frac{\pi^2}{24} \right] + \mathcal{O}(\alpha_S^2) \quad \left| \quad Z_q = 1 + \frac{\alpha_S}{\pi} \left[\frac{C_F}{2} \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln\left(\frac{n^2 b_0^2}{(2znp)^2 b^2}\right) \right) + \frac{3}{4} \frac{C_F}{\epsilon} - C_F \frac{\pi^2}{24} \right] + \mathcal{O}(\alpha_S^2) \right.$$

New results! New results!

$$\longrightarrow \frac{\partial Z_c}{\partial \ln n^2} \equiv \frac{\partial Z_c}{\partial \ln n^+} \propto \gamma_c \text{ (Rapidity anomalous dimension - to be validated against the known results!)}$$

SUMMARY & OUTLOOK

- * We have discussed in detail the use of a time-like auxiliary vector to address rapidity divergences in the computation of collinear functions up to NNLO in perturbative QCD.
- * One of the advantages of using a time-like auxiliary vector is that it can be kept finite throughout the computation, and hence no additional power corrections due to the regulator.
- * We achieved the computation in two steps — differential collinear functions — collinear functions.
- * Our differential collinear functions are well-defined objects for both SCET-I and SCET-II type observables. They can serve as a starting point for observables that are insensitive to individual radiations.
- * As first applications, we employ them to compute transverse momentum dependent and zero-jettiness dependent collinear functions.

- * Our formalism can be easily extended to compute soft functions that control soft dynamics.
- * Our treatment of rapidity divergences has recently been employed to compute jet functions for transverse-momentum-like variables in jet production processes at NNLO.

Buonocore, Grazzini, Guadagni, Haag, Rottoli, Kallweit (2508.19226, 2512.03954) + work in progress



See Giovanbattista Favorito's Talk

- * Our formalism can be easily extended to compute soft functions that control soft dynamics.
- * Our treatment of rapidity divergences has recently been employed to compute jet functions for transverse-momentum-like variables in jet production processes at NNLO.

Buonocore, Grazzini, Guadagni, Haag, Rottoli, Kallweit (2508.19226, 2512.03954) + work in progress

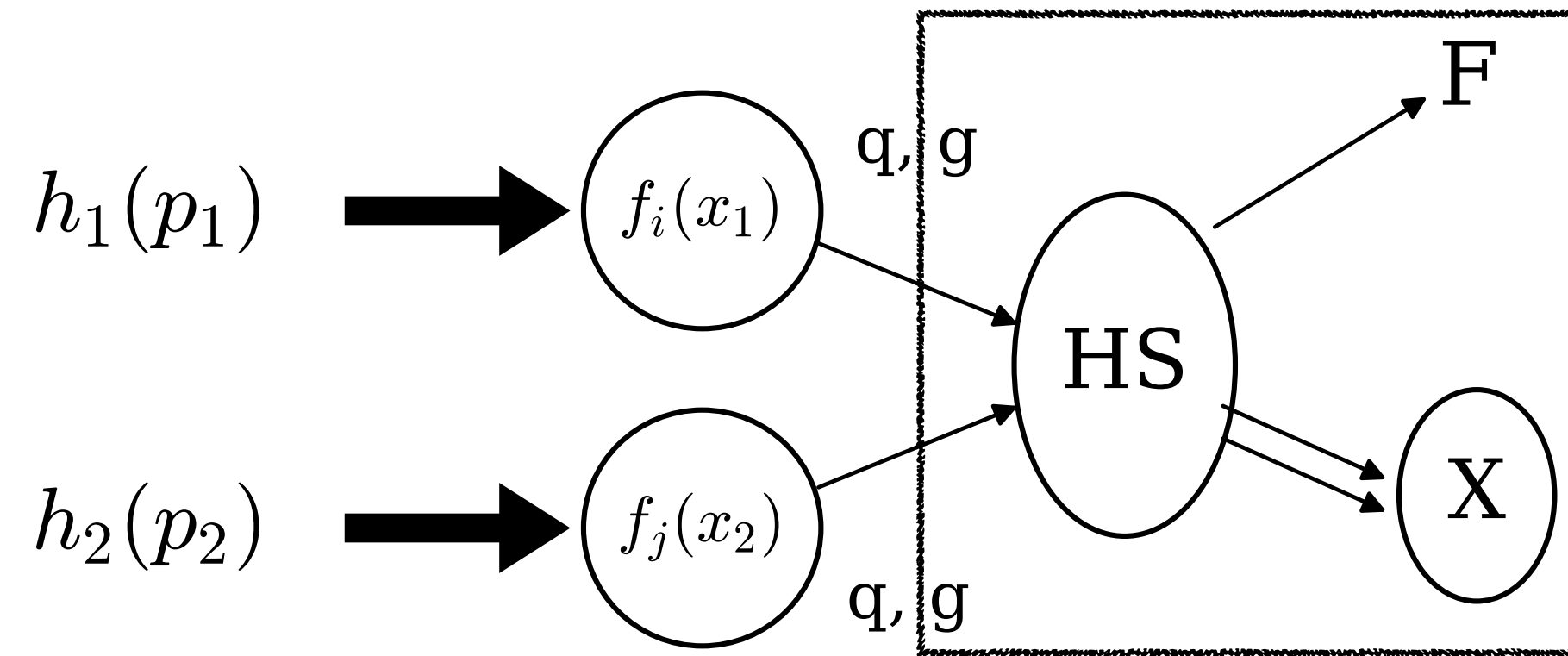


See Giovanbattista Favorito's Talk

Stay tuned! I am happy to answer your questions!

Thank you very much!

For a generic hadro production process



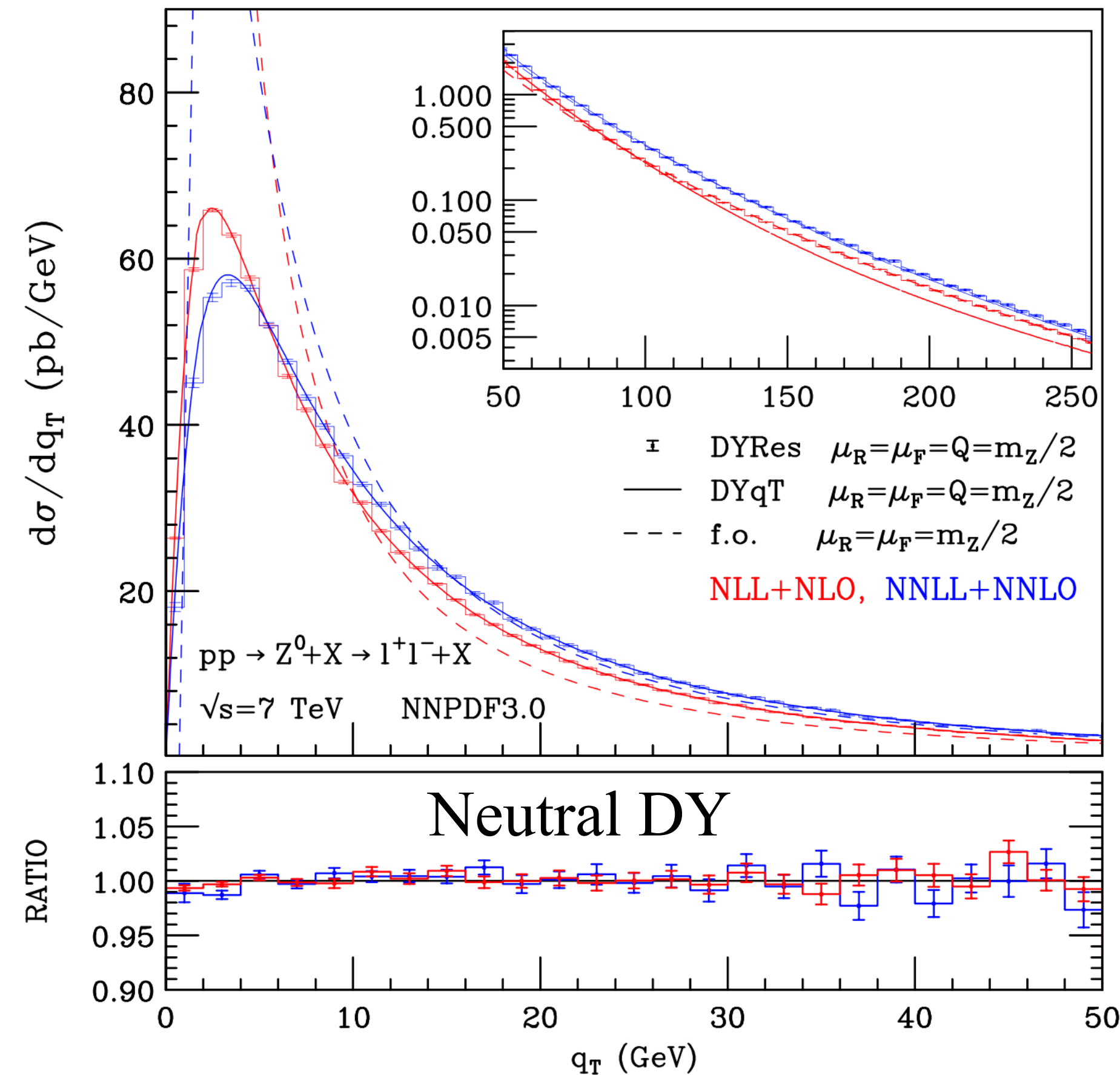
$$h_1(p_1) + h_2(p_2) \rightarrow F(\{q_i\}) + X$$

$$c_i(x_1 p_1) + c_j(x_2 p_2) \rightarrow F(q \equiv \sum_i q_i)$$

Parton model: $d\sigma_{h_1, h_2} = \sum_{i, j} \int dx_1 dx_2 \overbrace{f_i(x_1) f_j(x_2)}^{\text{Long distance}} \underbrace{d\hat{\sigma}_{ij}}_{\text{Short distance}} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)$

$$d\hat{\sigma}_{ij} = d\hat{\sigma}_{ij}^{(0)} + \alpha_S d\hat{\sigma}_{ij}^{(1)} + \dots$$

WHY RESUMMATION? FIXED ORDER VS RESUMED PREDICTIONS



Catani, de Florian, Ferrera, Grazzini (1507.06937)

For a relative order n compared to Born

$$\alpha_s^n \frac{1}{q_T^2} \ln^{2n-1} \left(\frac{M^2}{q_T^2} \right), \frac{1}{q_T^2} \ln^{2n-2} \left(\frac{M^2}{q_T^2} \right), \dots, \frac{1}{q_T^2} \ln \left(\frac{M^2}{q_T^2} \right), \frac{1}{q_T^2}$$

$$\ln^{2n} \left(\frac{M^2 b^2}{b_0^2} \right)$$

$$\text{LL: } \alpha_s^n \ln^{n+1} \left(\frac{M^2 b^2}{b_0^2} \right)$$

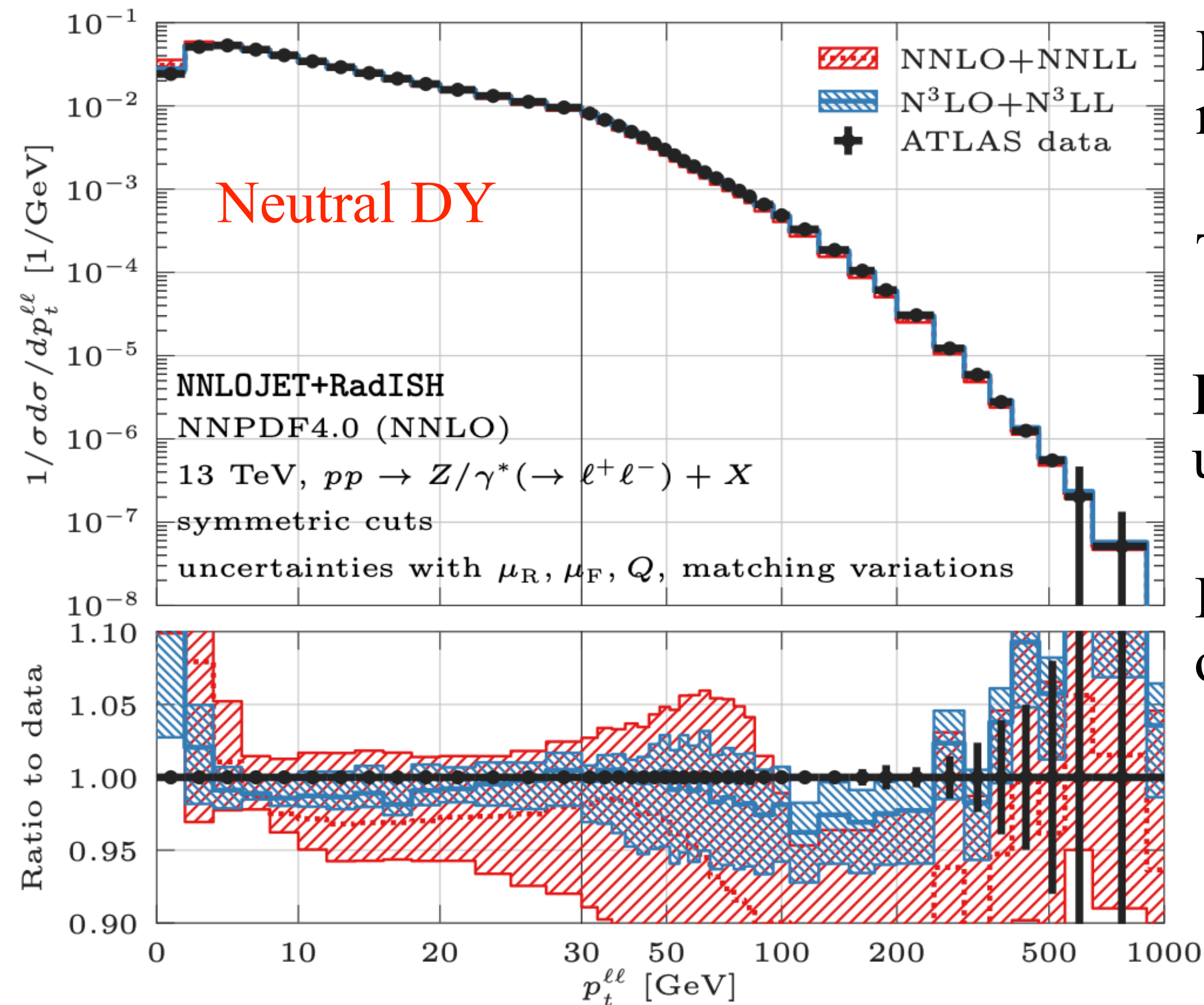
$$\text{NLL: } \alpha_s^n \ln^n \left(\frac{M^2 b^2}{b_0^2} \right)$$

$$\text{NNLL: } \alpha_s^n \ln^{n-1} \left(\frac{M^2 b^2}{b_0^2} \right)$$

Next-to-next-to-leading logarithmic accuracy

Resummation is effective in the low transverse momentum region!

RESUMMED PREDICTION VS DATA



Chen, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli
(2203.01565); Camarda, Cieri, Ferrera (2103.04974)

Bulk of the events are produced in the low transverse momentum region.

Theory is consistent with the data.

From NNLO+NNLL to N³LO+N³LL: reduction in uncertainties and improvement in data agreement.

Improved theory predictions helps in precise determination of crucial SM parameters.

How are these large logarithms resummed ?