

Automated matching of a QED parton shower with NLO EW at a future lepton collider

Based on 2603.05585 with Marek Schönherr

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Introduction and motivation

- Precision QED needed for low-energy e^+e^- colliders (see [RadioMonteCarLow2](#))
- Future electron-positron collider now the top priority of collider community
- See talks from M. Pellen and S. Devoto for plenty of theorist motivation
- FCC-ee will run at 5 different centre-of-mass energies
- ZH production will be studied at
 - 240 GeV – dedicated run just above threshold
 - 365 GeV – produced during the $t\bar{t}$ run

→ Beyond NLO EW corrections necessary for precision goals!

Do we need NNLO QED, especially for radiation off initial-state electrons?
What about resummation?

- YFS soft-photon resummation [Yennie, Frautschi, Suura '61](#)
 - Can be systematically matched to higher orders
 - Implemented in SHERPA¹ and as CEEX in KKMC² and upcoming Monte Carlo → [Jérémy Paltrinieri's talk](#)
- Alternatively, **NLO-matched parton shower but for QED**
 - Initial-state QED parton shower matched to NLO for $e^+e^- \rightarrow \ell^+\ell^-(\gamma)$ and recently $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ implemented in BABAYAGA
[Carloni Calame et al. '06, '25, '26](#)
- Here we will introduce a process-independent solution based on MC@NLO
[Frixione, Webber '02](#)

¹Krauss, Schönherr '08, Krauss, Lindert, Linten, Schönherr '18, Krauss, Price, Schönherr '22, LF, Schönherr '22

²Jadach, Ward, Was '00

The MC@NLO method

See Joanne Roper's talk for more details

$$\begin{aligned}
 \langle O \rangle^{\text{MC@NLO}} = & \int d\Phi_n \bar{B} \left[\underbrace{\bar{\Delta}_n(t_c, \mu_Q^2) O(\{p_n\})}_{\text{unresolved, singular}} + \underbrace{\sum_{ij,k} \int d\Phi_1 O(\{p_{n+1}\}) \frac{D_{ij,k}^A}{B} \bar{\Delta}_n(t_c, \mu_Q^2)}_{\text{resolved, singular}} \right] \\
 & + \underbrace{\int d\Phi_{n+1} \left[R - \sum_{ij,k} D_{ij,k}^A \right] O(\{p_{n+1}\})}_{\text{resolved, non-singular}}
 \end{aligned}$$

The MC@NLO method

$$\begin{aligned}
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 & + \int d\Phi_{n+1} \left[\underbrace{R - \sum_{ij,k} D_{ij,k}^A}_{\text{resolved, non-singular}} O(\{p_{n+1}\}) \right]
 \end{aligned}$$

S-events

H-events

1. Generate S-event (using the $\bar{B}$ ME squared) or H-event (using $R - D^A$) according to their relative probabilities
2. If H-event, pass straight to standard parton shower
3. If S-event, use one-step shower with D^A as kernels
 - If no emission, event has Born kinematics
 - If emission at $t_{ij,k}$, pass to standard shower for further showering $t < t_{ij,k}$

- Weighted veto algorithm to handle negative dipole splitting functions
- Suitable identification of scales μ_F, μ_Q
- Real photons always couple with $\alpha(0) \implies$ mixed EW input scheme best for high-energy processes
- Infrared cutoff has no natural scale: sensible value becomes observable-dependent
- Initial-state leptons have a divergent structure function

The electron structure function

The electron structure function

- To first approximation, the electron is "all electron" $f_e(x) = \delta(1 - x)$
- But with more resolving power, we see a shorter-distance electron
 - At shorter distances, the electron is less screened by the EM field
 - Effectively, some of the total energy of the electron is carried by its EM field
- We can use DGLAP to find the **leading-logarithmic electron structure function** at all x and Q^2 ,

$$f_e(x, Q^2) = \beta \frac{\exp(-\gamma_E \beta + \frac{3}{4}\beta)}{\Gamma(1 + \beta)} (1 - x)^{\beta-1} + \beta \sum_{n=0}^{\infty} \beta^n \mathcal{H}_n(x)$$

where $\beta = \frac{\alpha}{\pi} \left(\ln \left(\frac{Q^2}{m_e^2} \right) - 1 \right)$

Asymptotically, $f_e(x, Q^2) \xrightarrow{x \rightarrow 1} \beta(1 - x)^{\beta-1}$

Regulating the electron structure function

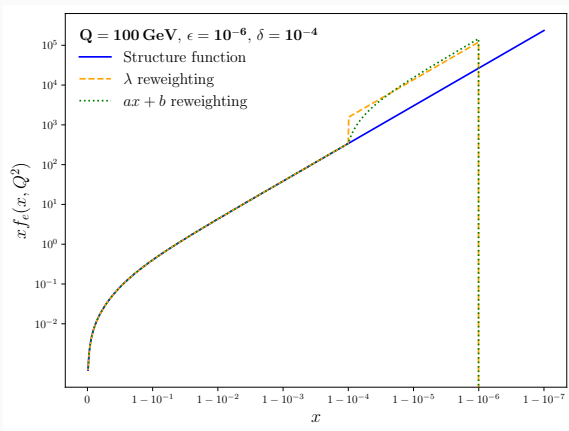
- This form of $f_e(x, Q^2)$ has an *integrable* singularity at $x = 1$
- Any numerical algorithm must have an upper limit $x_{\max} = 1 - \epsilon$
- Even when ϵ is very small, we miss contributions to the cross section
- Traditionally: rescale $f_e(x, Q^2)$ by a factor λ near the limit to preserve the integral and hence the XS
- This creates a discontinuity at $x = 1 - \delta$, which is unphysical

Instead, use a linear rescaling of $f_e(x, Q^2)$ in the region $x \in (1 - \delta, 1 - \epsilon)$

$$\int_{1-\delta}^{1-\epsilon} (ax + b) \beta (1-x)^{\beta-1} dx = \int_{1-\delta}^1 \beta (1-x)^{\beta-1} dx$$
$$\Rightarrow a = \frac{(1+\beta) \epsilon^\beta}{\delta \delta^\beta - (\delta + \beta(\delta - \epsilon)) \epsilon^\beta}; \quad b = 1 - a(1 - \delta)$$

The electron structure function

- This method is numerically stable and reproduces the λ result
- Consistent over a wide range of ϵ and δ choices, $\epsilon \ll \delta \ll 1$



Ideally reweight with something differentiable... WIP

Overestimating the electron structure function

In the initial-state parton shower we must undo the PDF evolution

Must include a ratio $\mathcal{L}(\{a\})/\mathcal{L}(\{\tilde{a}\})$ of particle luminosities. This needs an overestimate to be sampled using the veto algorithm.

If z is the splitting variable describing sharing of momenta:

$$\frac{\mathcal{L}(\{a\})}{\mathcal{L}(\{\tilde{a}\})} \xrightarrow{x \rightarrow 1} \frac{\frac{x}{z}(1-x/z)^{\beta-1}}{x(1-x)^{\beta-1}}.$$

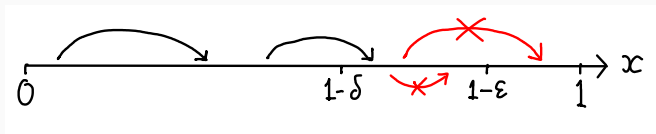
Since $0 < \beta < 1$, we get a worst-case singularity of $(1-x/z)^{-1}$:

$$\frac{(1-x/z)^{\beta-1}}{(1-x)^{\beta-1}} = \frac{1}{z-x} (1-x/z)^{\beta} (1-x)^{1-\beta}.$$

$\implies 1/(z-x)$ is a strict overestimate of the PDF ratio in this limit. Take an extra factor $k \approx 10$ for safety away from the limit.

What about numerical limits in the shower?

3 regions:



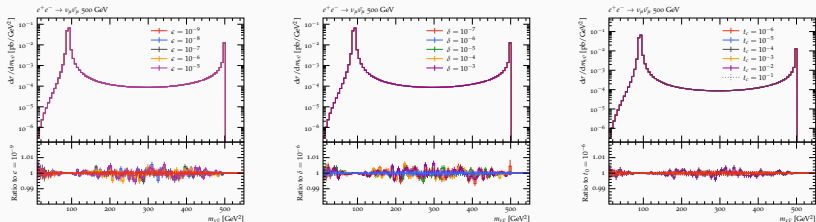
Parton shower evolution $x \rightarrow x/z$ is

- *allowed* within $[0, 1 - \delta]$ and into $[1 - \delta, 1 - \epsilon]$
- and *forbidden* to start from $[1 - \delta, 1]$

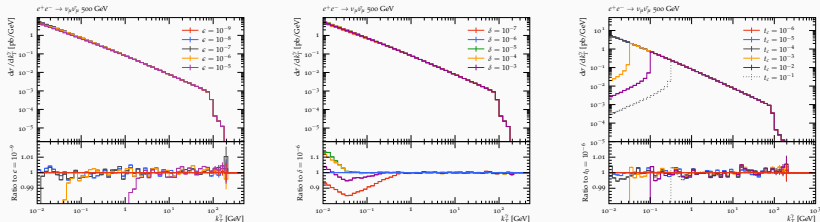
Validation of initial-state MC@NLO

Independence of technical parameters

Born-level observables: independent at per-mille level

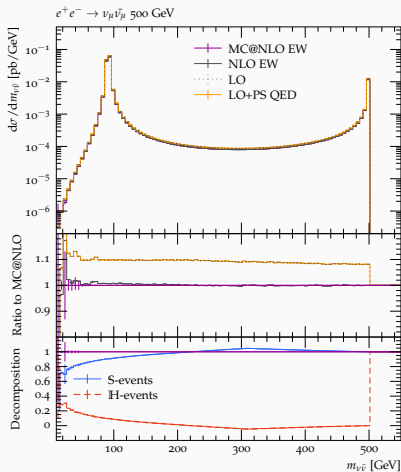


Radiative observables: predictable differences in soft radiation



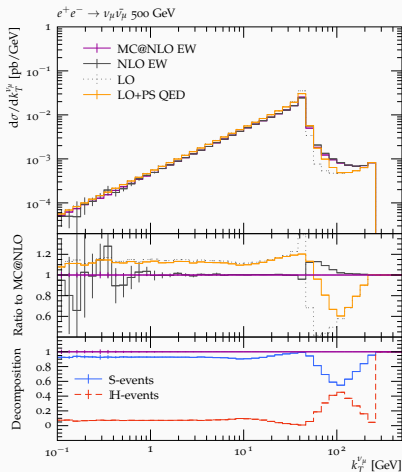
→ Choose $\epsilon = 10^{-8}$, $\delta = 10^{-4}$, $t_c = 10^{-6}$ GeV²

$e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ at 500 GeV – final state invariant mass



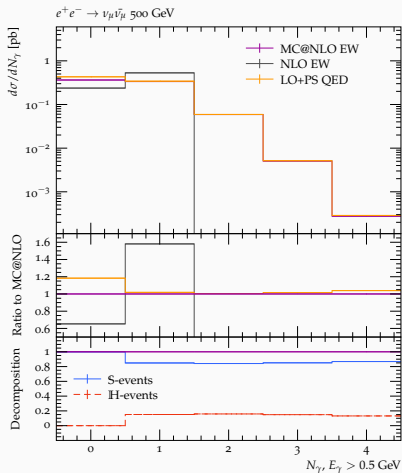
- Invariant mass is unchanged by initial-state shower emissions
- NLO V and R corrections are flat in $m_{\nu\bar{\nu}}$
- Lower ratio plot shows makeup in terms of $\mathbb{S}$ and $\mathbb{H}$

$e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ at 500 GeV – neutrino transverse momentum



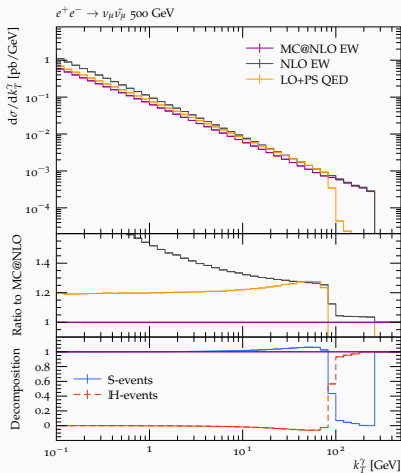
- $k_T^{\nu_\mu}$ affected by shower at LO but much smaller effect at NLO
- Very good agreement with fixed-order NLO in both tails

$e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ at 500 GeV – photon multiplicity



- $\mathbb{H}$ events contribute positively to extra photon production
- LO+PS, MC@NLO and $\mathbb{S}$ -event ratio flat beyond first emission
- Cut used here is 0.5 GeV

$e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ at 500 GeV – hardest photon k_T

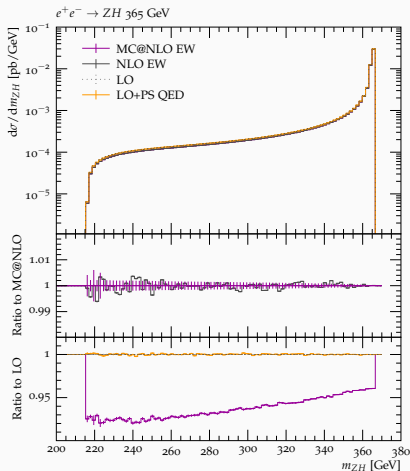


- Good agreement with NLO at high k_T^γ where H-events dominate
- Shower-like behaviour at low k_T^γ : less divergent than NLO
- Unlike QCD, Sudakov peak $\ll$ detectable photon energy

Results: ZH production at 365 GeV

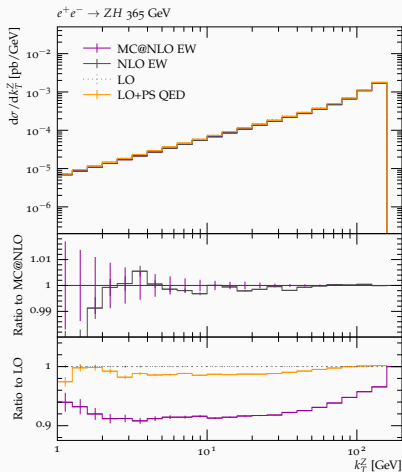
Results: ZH production at 365 GeV – final state invariant mass

Set $t_c = 10^{-3} \text{ GeV}^2$ for this run



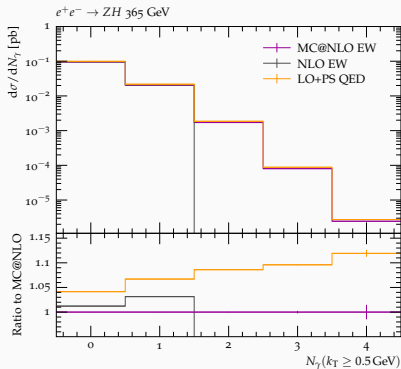
- IS shower does not change prediction as expected – excellent agreement with fixed-order
- NLO correction large and negative
- Local K -factor up to 8%

Results: ZH production at 365 GeV – Z transverse momentum



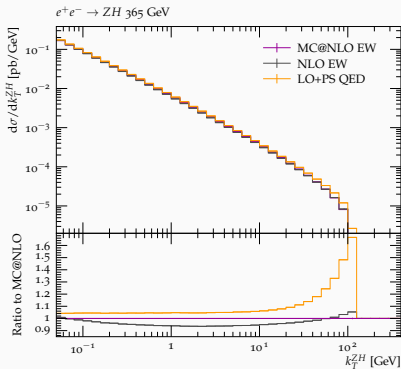
- Z k_T mostly unchanged by parton shower even at LO
- NLO correction is largest for small k_T^Z as expected
- MC@NLO errors are larger but far fewer events generated since it samples the tail better

Results: ZH production at 365 GeV – photon multiplicity



- MC@NLO corrects overproduction of photons by LO+PS
- Ratio is no longer flat beyond 1 photon – more ZH -events with $t_{\text{start}} > \mu_Q^2$

Results: ZH production at 365 GeV – final state transverse momentum



- Shows recoil of the final state against ISR
- Unlike $\nu\bar{\nu}$, no exact agreement between MC@NLO and NLO at high k_T
- This is due to different resonance structure & scales
- MC@NLO flattens out quicker than LO+PS at low k_T

Conclusions

What we did:

- ‘Tamed’ the electron structure function for use in an NLO-matched parton shower
- Created a method that is largely process-independent and automated (applicable to future colliders *and* past/current low-energy colliders)
- Produced the first EW NLO+PS prediction for $e^+e^- \rightarrow ZH$

Next steps:

- Merge into public release of SHERPA
- Apply to low-energy $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \pi^+\pi^-$
- Compare with BABAYAGA, YFS, fixed-order NLO
- Improve description of initial-final interference in parton shower

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Thanks for listening!

Backup

Backup: Veto algorithm, briefly

For a splitting function f we need to sample from, but cannot find F^{-1} :

1. Find a simple overestimate function g
2. Find the indefinite integral G and its inverse G^{-1}
3. Sample t, z, ϕ from G using the usual method, e.g.
 $z = G^{-1}[G(z_{\min}) + r(G(z_{\max}) - G(z_{\min}))]$, where r is a uniform random number $r \in (0, 1)$
4. Accept the splitting with probability $f(t, z, \phi)/g(t, z, \phi)$, otherwise veto it

Aside

There is a weighted generalisation of the veto algorithm which we use to handle the negative splitting functions which come from same-sign charge dipoles in QED.

The initial-state splitting function for $f \rightarrow f\gamma$ is

$$f = 2\alpha \left(\frac{2(z+y)}{1-z} + 1 - z - y \right) \frac{\frac{x}{z} f_e(\frac{x}{z}, t)}{x f_e(x, t)}$$

which we will overestimate with

$$g = 2\alpha \left(\frac{2}{1-z} \right) \frac{k}{z-x}$$

and choose $k = 10$. Integrating this analytically, we get

$$G = 4k\alpha \frac{1}{1-x} \log \left(\frac{(1-z_{\min})(z_{\max}-x)}{(1-z_{\max})(z_{\min}-x)} \right)$$

New z generation is a little more complicated...

$$z = \frac{x+R}{1+R}, \text{ where } R = \frac{z_{\min}-x}{1-z_{\min}} \left(\frac{(1-z_{\min})(z_{\max}-x)}{(1-z_{\max})(z_{\min}-x)} \right)^r$$

Backup: Dipole efficiency

- For efficiency in processes where there are many charged particles around, each splitter is assigned a single opposite sign same flavour (OSSF) spectator
- Phase space considerations mean that we should assign pairs that were produced together – we estimate these by minimising dipole mass
- Similar schemes used in PYTHIA simple shower [Bierlich et al. '22](#) and VINCIA [Skands, Verheyen '20](#)
- Can separately turn off initial-final interference

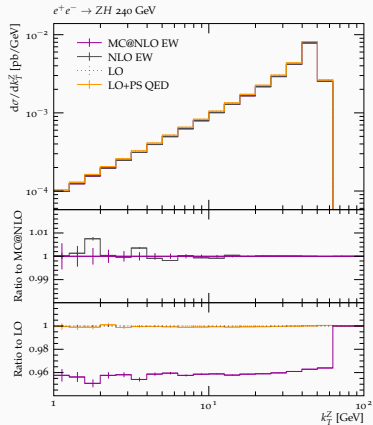
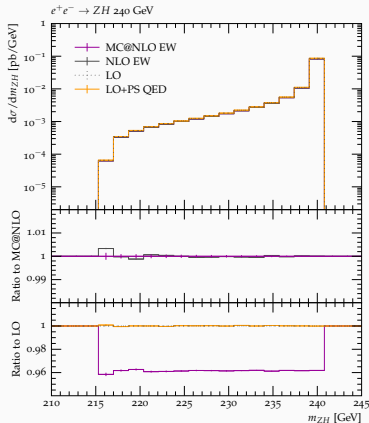
Backup: Scale identification

- 'Core scale' for Born process identified as shower starting scale μ_Q
- In $e^+e^- \rightarrow \text{neutral}$, choose neutral invariant mass
- In $e^+e^- \rightarrow \text{charged}$, not 100% clear, but in our implementation we choose $\sqrt{s'} = (p_{e^+} + p_{e^-})$
- Use MEPS-type clustering for identification of hard scale when $n > 4$ (e.g. in $\mathbb{H}$ -events)
- Two possible cases:
 - $t_H < \mu_Q^2$ The shower should start from t_H to avoid an unordered configuration and double counting
 - $t_H > \mu_Q^2$ Not clear-cut, t_H is now the hardest scale, but do not want to extend parton shower beyond its region of validity
- We choose to start the shower from t_H in both cases

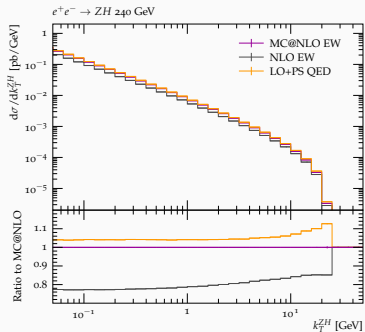
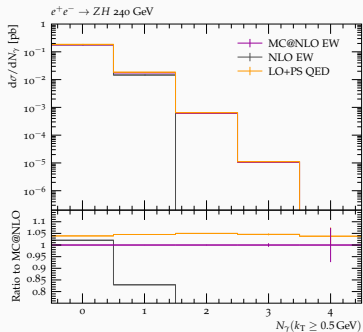
Backup: Mixed electroweak scheme

- For the hard scattering, helpful to use an EW scheme which resums higher-order corrections, e.g. G_μ scheme, $\alpha(m_Z)$ scheme
- Real (long-distance) photons should always couple with Thomson limit $\alpha(0)$ since their wavefunction renormalisation cancels logarithms in α renormalisation
- To reconcile this, define mixed scheme [Denner, Dittmaier '19](#)
 - All NLO parts of the calculation are defined in the $\alpha(0)$ scheme
 - All Born parts are defined in G_μ scheme

Backup: $e^+e^- \rightarrow ZH$ at 240 GeV



Backup: $e^+e^- \rightarrow ZH$ at 240 GeV



Backup: Validation of MC@NLO one-step shower

