

Numerical Methods for Electroweak Corrections and Applications



GHENT
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FACULTY
OF SCIENCES

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Contents

- **WHY?**
 - Electroweak corrections;
 - Drell-Yan process.
- **HOW?**
 - Analytical VS Numerical methods;
 - Series expansion method.
- **WHAT?**
 - Applications to the Drell-Yan process:
 - QCDxEW corrections;
 - NNLO QED corrections;
 - Towards NNLO EW, fermionic sector.

Personal Selection!

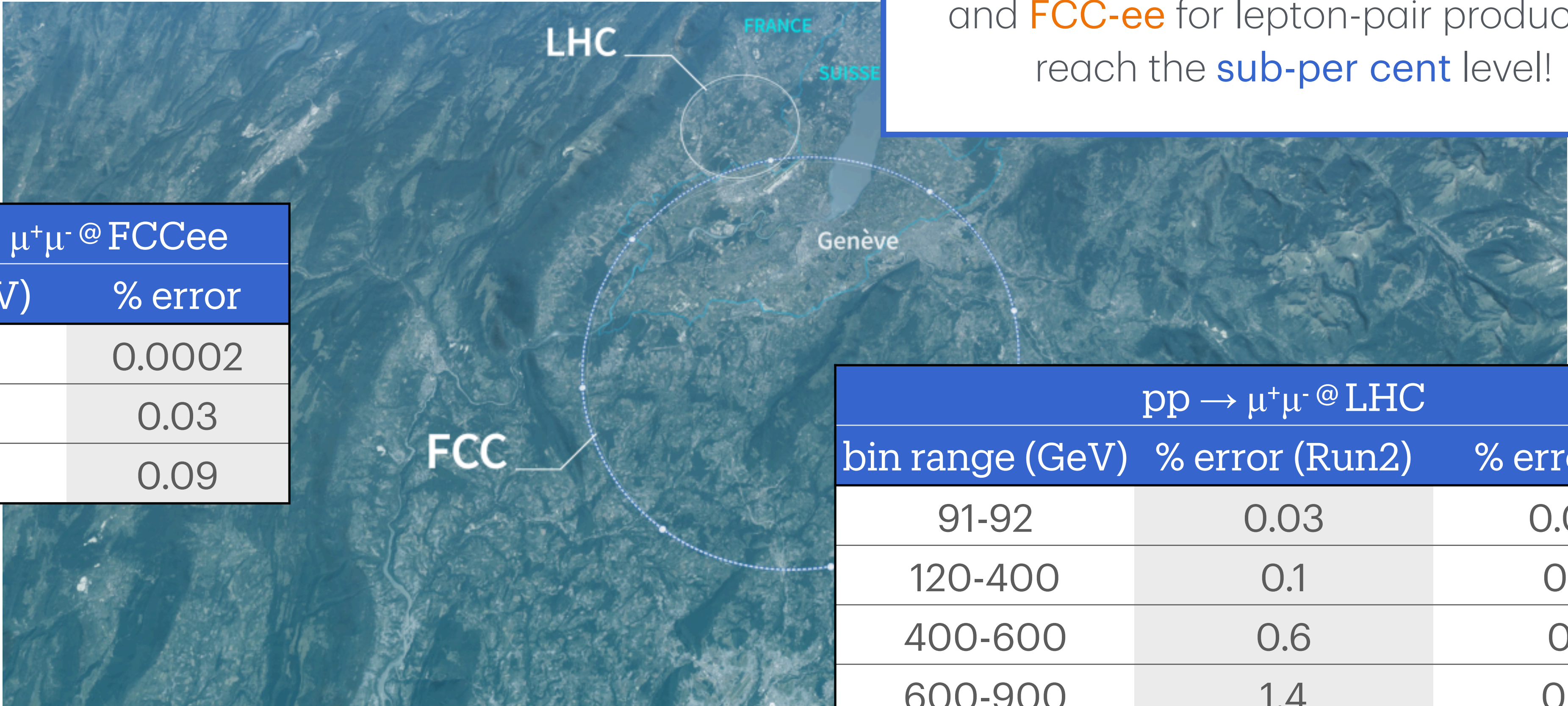


Introduction

Introduction

Motivations

The projected statistical errors at **HL-LHC** and **FCC-ee** for lepton-pair production reach the **sub-per cent** level!

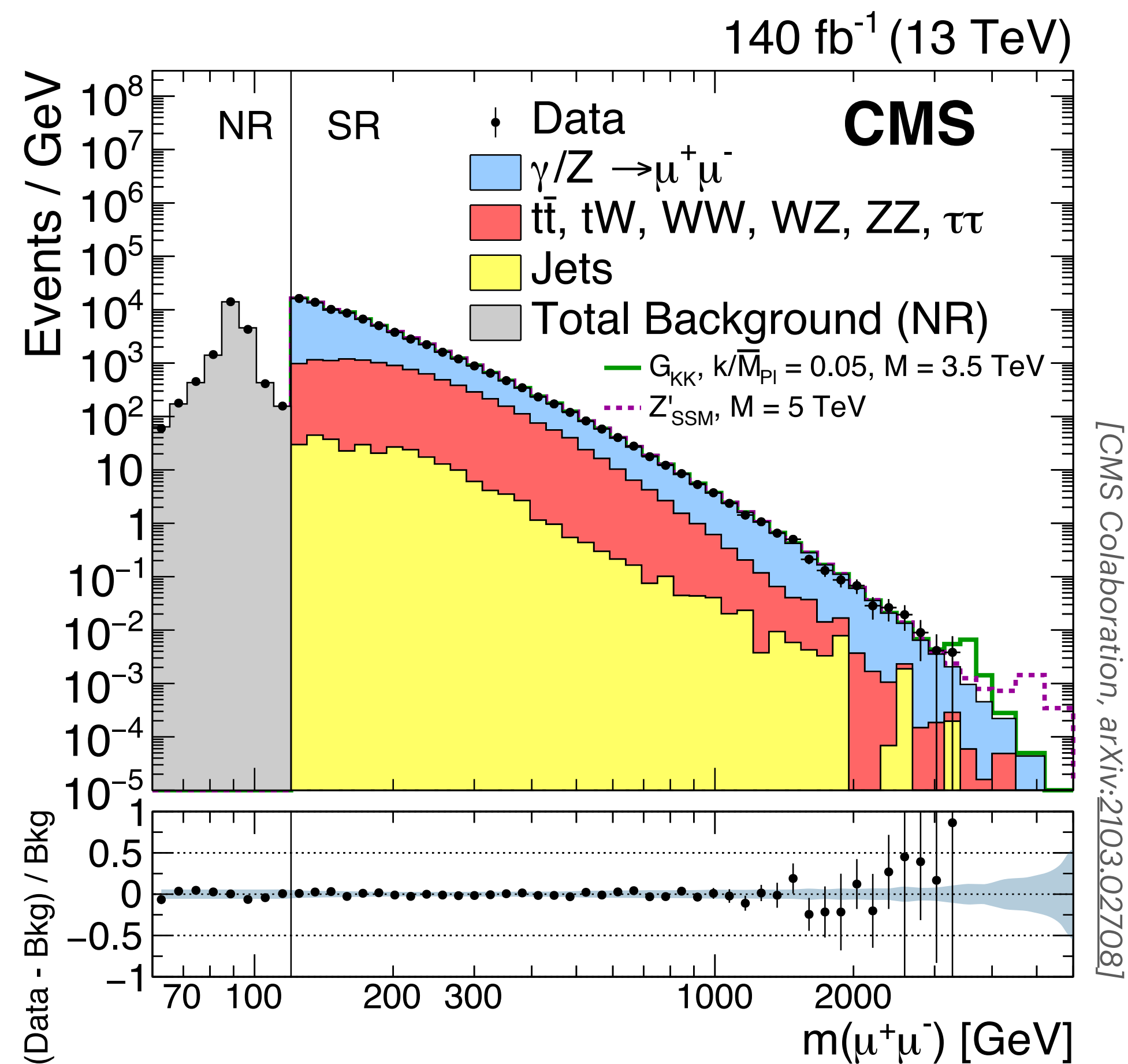


pp → μ ⁺ μ ⁻ @ FCCee	
√s (GeV)	% error
91	0.0002
240	0.03
365	0.09

pp → μ ⁺ μ ⁻ @ LHC		
bin range (GeV)	% error (Run2)	% error (HL)
91-92	0.03	0.006
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

Introduction

The Drell-Yan Process: $q+\bar{q} \rightarrow l+\bar{l}$

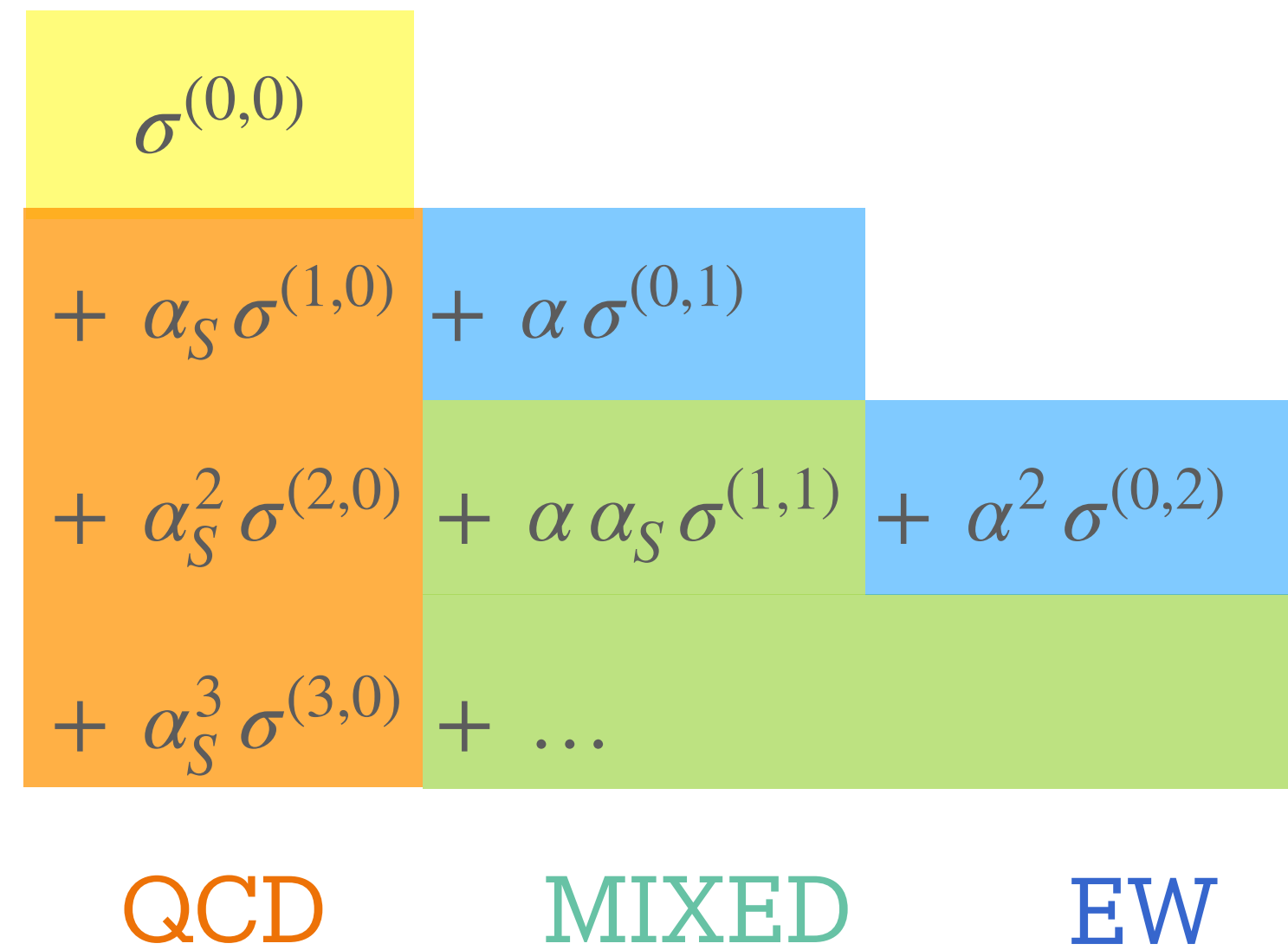


Precision Observables:

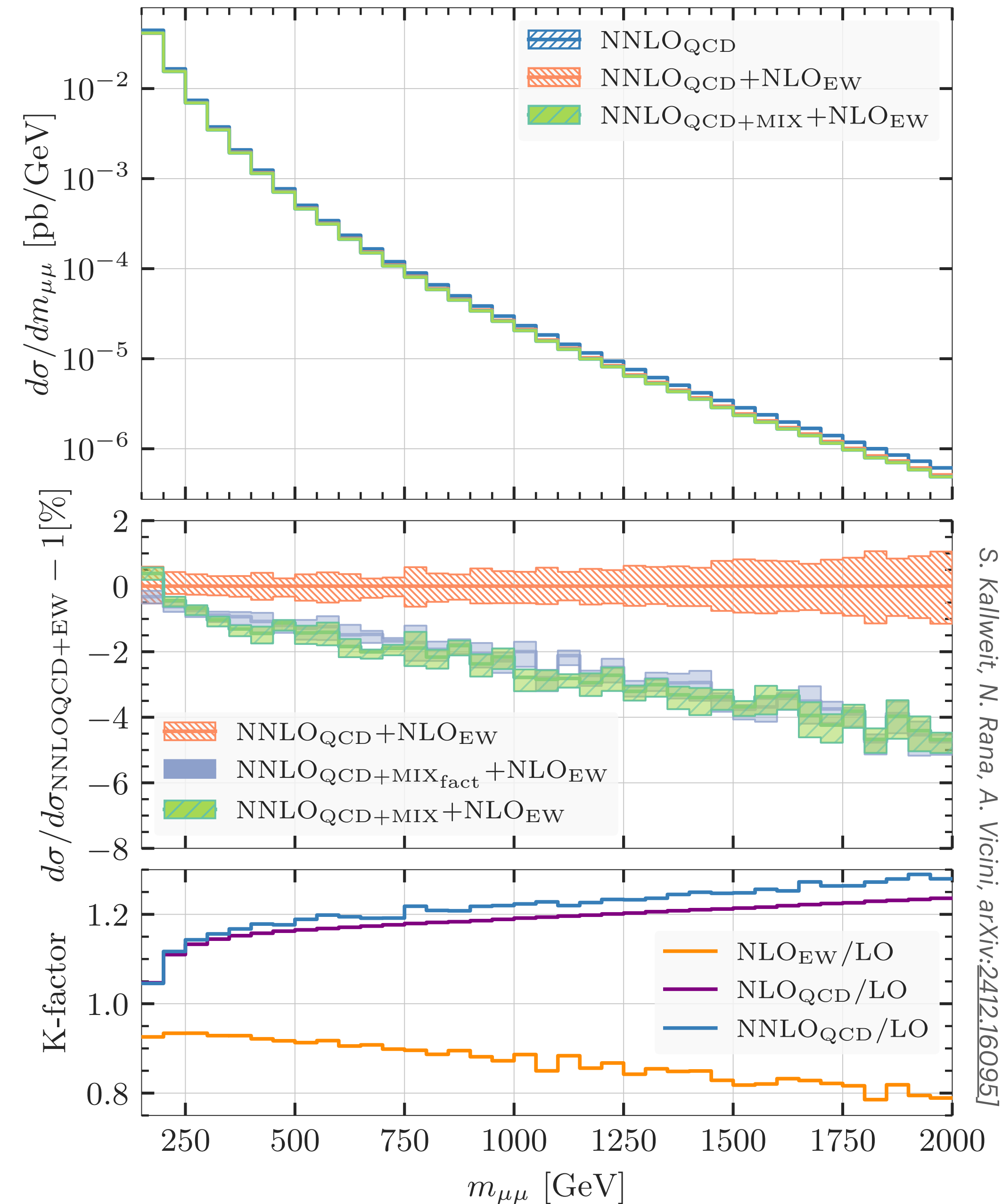
- Lepton-pair invariant mass:
 - at Z resonance: **Z** properties;
 - AFB: $\sin^2 \theta_{eff}^\ell$;
- Lepton-pair transverse mass:
 - W resonance: m_W ;
- Charged lepton transverse momentum:
 - m_Z, m_W ;
- Lepton-pair (pseudo-)rapidity:
 - **PDF** constraints;
- Lepton-pair transverse momentum:
 - low transverse momentum - α_S ;
 - large transverse momentum - **PDF** constraints;
- Large invariant mass tail:
 - **BSM** Signal, **PDF** constraint.

Introduction

Size of EW effects



- The **mixed corrections** provide a relevant contribution, **increasing at higher invariant mass** and larger than the statistical error expected by the end of **HL-LHC**.



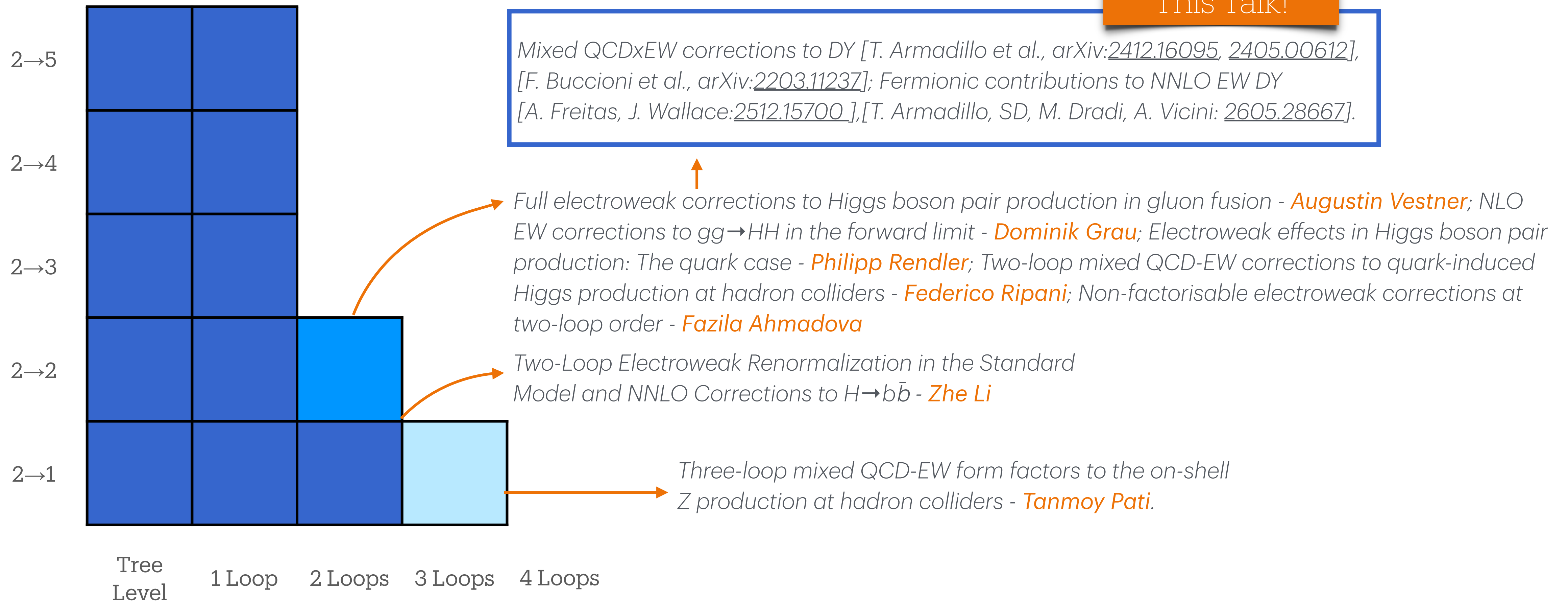
[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]

State of the Art

EW Corrections

Loop amplitudes are one of the main bottlenecks of higher-order predictions.

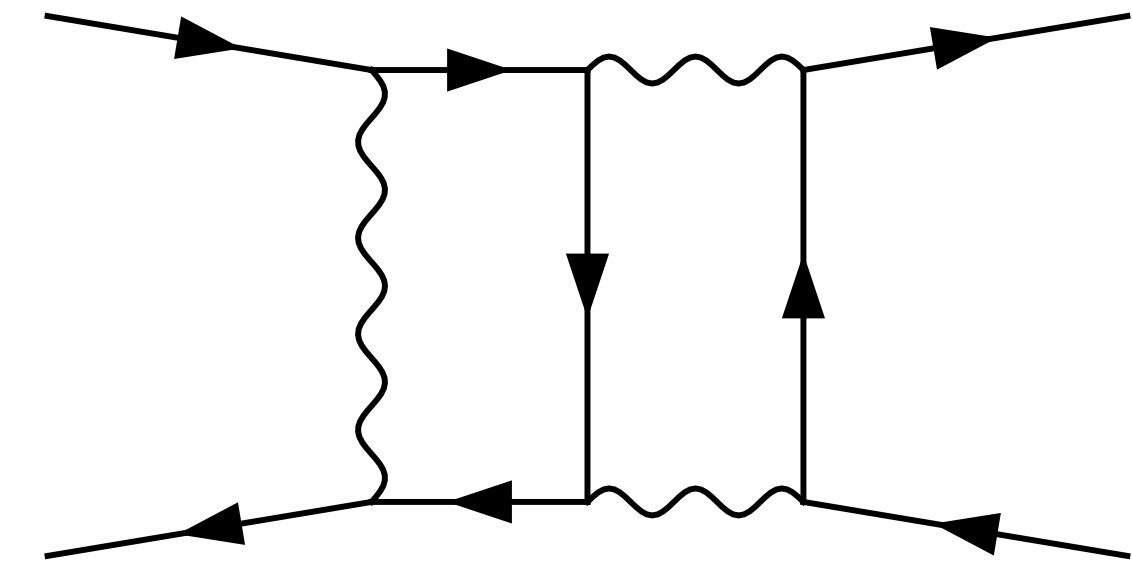
This Talk!



Numerical Methods

The Challenges

What makes EW corrections challenging?

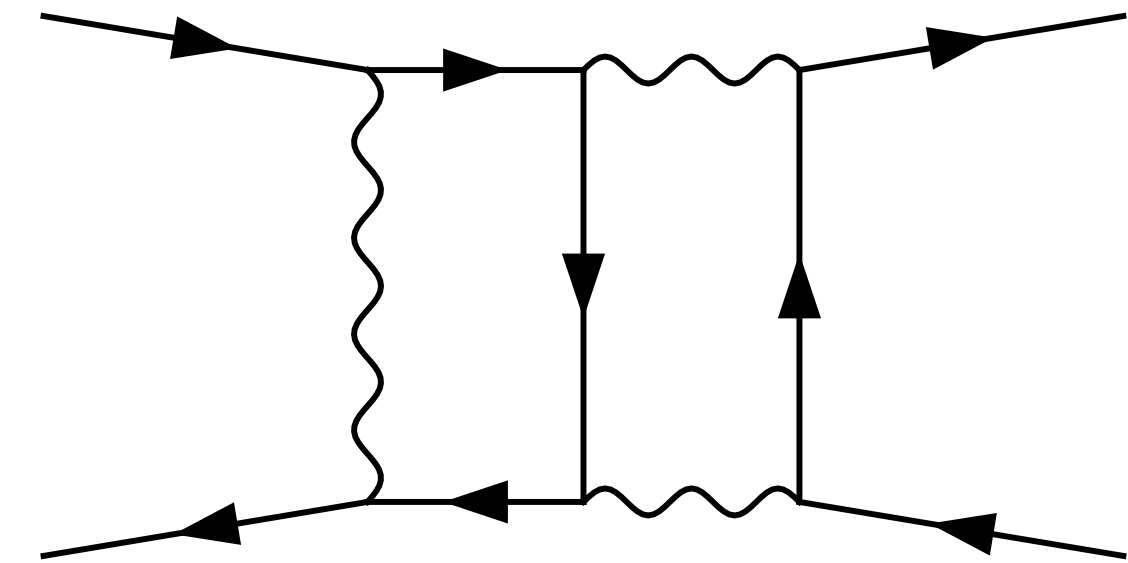


- **Large** computations ($\mathcal{O}(10^4)$ diagrams):
systematic **bookkeeping** is a priority;
- Treatment of γ_5 :
how can γ_5 be consistently used in **dimensional regularisation**?
- **Renormalisation program**:
how to renormalise the EW theory at **NNLO**?
- Additional internal **massive lines**:
additional scales in the problem ($m_Z, m_W, m_H \dots$) bring additional complications, in particular **elliptic integrals**;
- Need for the **complex mass scheme**:
requires **analytically continuing** the master integrals on the complex plane!

$$\mathcal{A} = \sum c_i I_i$$

The Challenges

What makes EW corrections challenging?



• Large computations ($\mathcal{O}(10^4)$ diagrams): systematic bookkeeping is a priority; • Treatment of γ_5: how can γ_5 be consistently used in dimensional regularisation?	Automation?
• Renormalisation program: how to renormalise the EW theory at NNLO?	QFT
• Additional internal massive lines: additional scales in the problem ($m_Z, m_W, m_H...$) bring additional complications, in particular elliptic integrals; • Need for the complex mass scheme: requires analytically continuing the master integrals on the complex plane!	Numerical Methods?

$$\mathcal{A} = \sum c_i I_i$$

Computing an amplitude

Evaluating the integrals

$$\mathcal{A} = \sum c_i I_i$$

- The integrals can be computed by following different **strategies**:

Analytic methods

- Master Integrals are closed under differentiation: they satisfy a **system of differential equations**;
- Canonical basis**: differential equation in dlog form, solution in terms of **special functions**;
- Internal masses** complicate the picture: **elliptic functions**, **Calabi-Yau**, ...

See the talk from Vasily

Numerical methods

- Numerical integration via sector decomposition: **PYSECDEC** [G. Heinrich et al. [2305.19768](#)] ;
- Numerical integration of a **parametric representation** (Feynman parameter, Baikov variables, ...);
- Numerical solution of the differential equation via **series expansion**.

See the talk from Stephen

This Talk!

How to compute integrals

Strengths and weaknesses of the methods

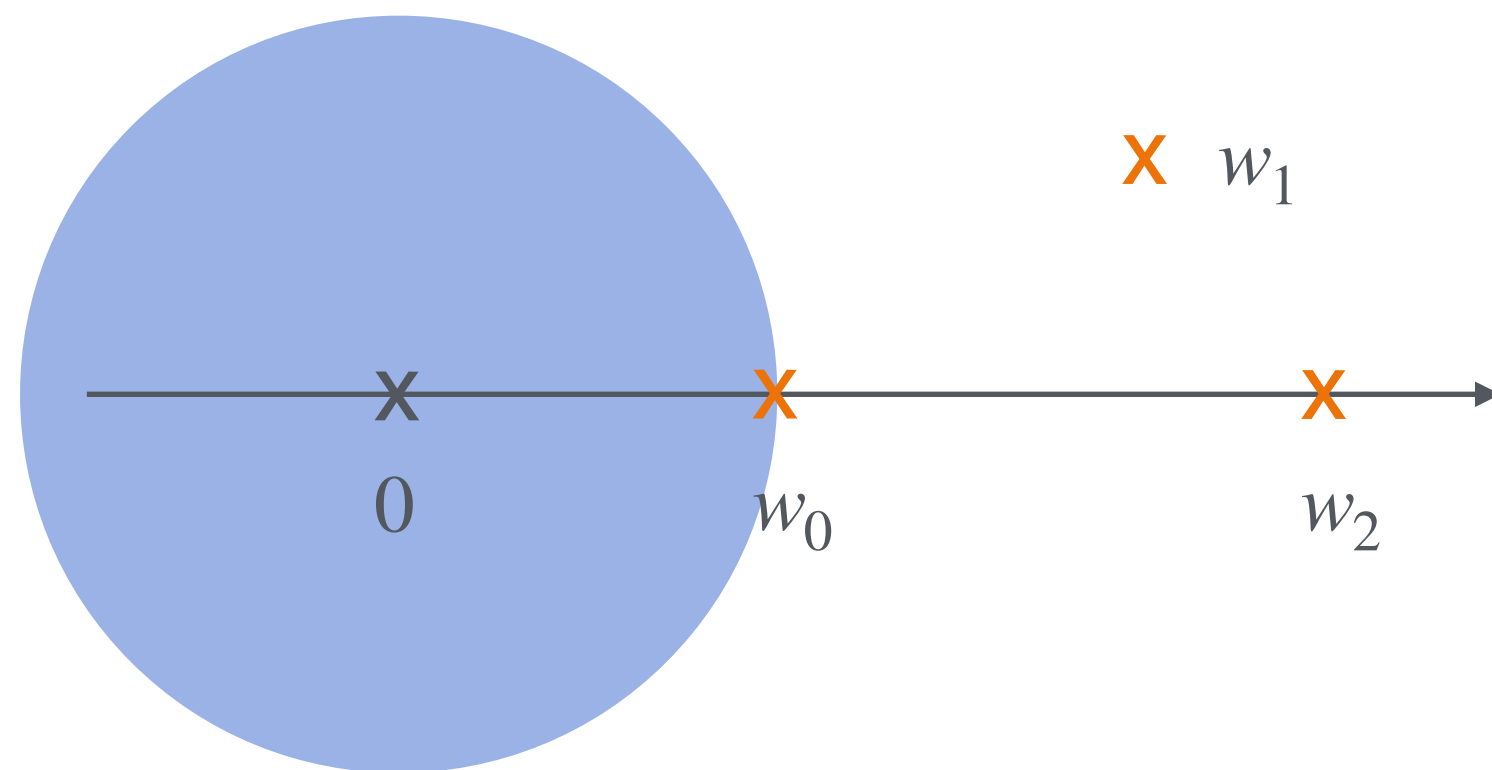
	ANALYTICAL METHODS	NUMERICAL METHODS	SEMI-ANALYTICAL METHODS
Functional relations	✓	✗	✗
Analytic properties (poles location)	✓ <i>Mathematical study of function properties.</i>	✗ <i>In pySecDec, the pole structure is exploited at the integrand level.</i>	✓ <i>Pole structure in the coefficients of the differential equations.</i>
Numerical evaluation	<i>Series expansion of analytical functions.</i>	• Monte Carlo integration; • Numerical quadrature of parametric representations (Feynman, Baikov, ...).	<i>Result expressed as series expansion.</i>
Error esitnation	Estimation through the truncation of the series.	• Monte Carlo: $1/\sqrt{n}$; • Numerical quadrature: comparison between different number of nodes.	Estimation through the truncation of the series.

Series Expansion

Taylor and logarithmic expansion

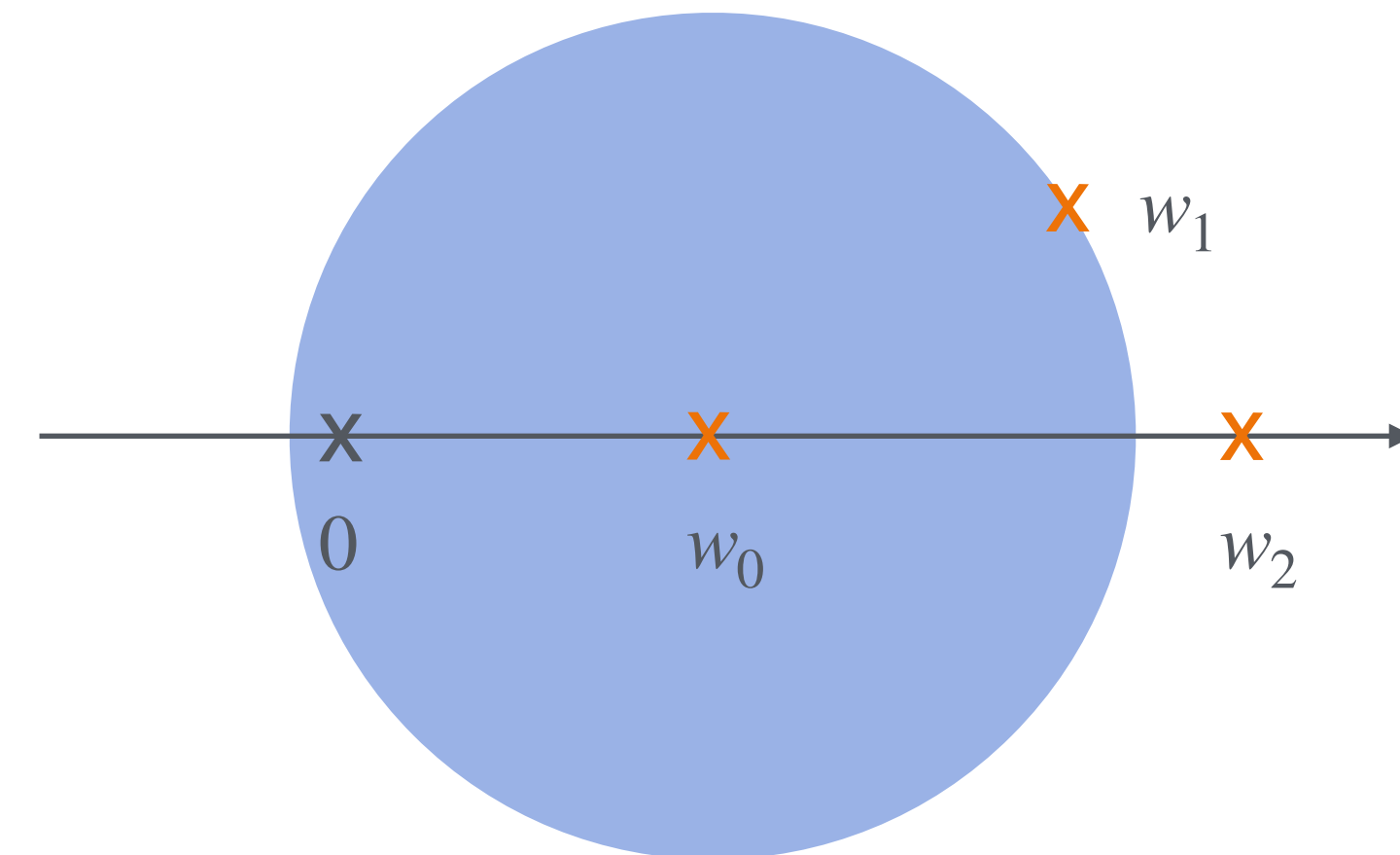
Taylor Ansatz:

$$f(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$



Logarithmic ansatz:

$$f(x) = x^r \sum_{k,j=0}^{\infty} c_{k,j} (x - w_0)^k \log^j(x - w_0)$$



- Original implementation [F.Moriello, arXiv:1907.13234] in **DIFFEXP**, [M.Hidding, arXiv:2006.05510];
- Now, multiple codes are available (**SEASYDE** [T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2205.03345], **AMFLOW** [R. Huang, X. Liu, Y. Ma, arXiv: 2607.08477], **LINE** [R. Prisco, J. Ronca, F. Tramontano, arXiv: 2501.01943], ...).

See the talk from Renato

AMFlow

[R. Huang, X. Liu, Y. Ma , arXiv: [2607.08477](#)]

[X. Liu and Y.-Q. Ma, arXiv: [2201.11669](#)]

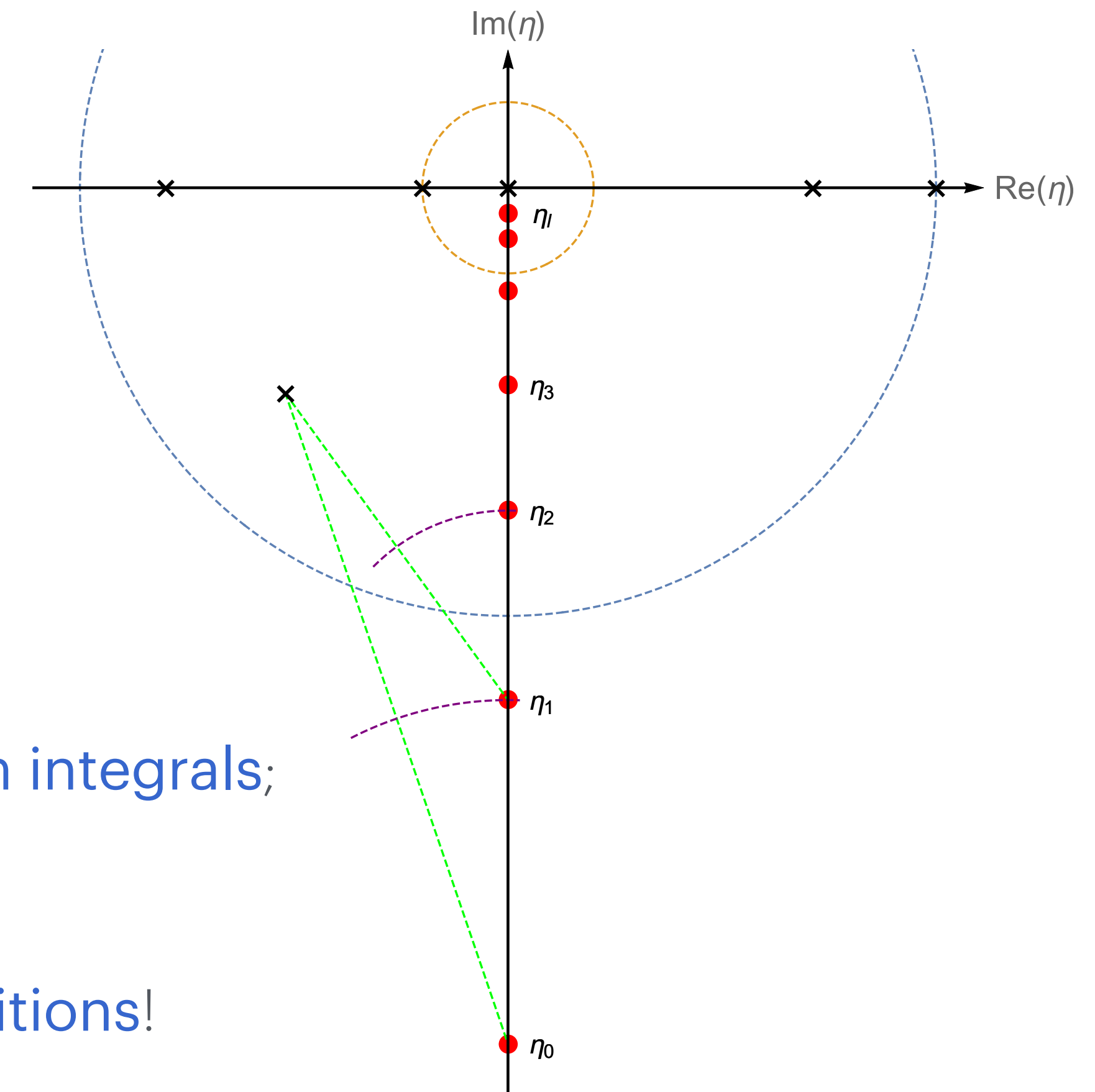
No need to compute boundaries

- Introduce an **auxiliary mass** parameter η :

$$I(\vec{\nu}, \vec{s}, \epsilon) = \int \prod_{i=1}^L \frac{d^D \ell_i}{i\pi^{D/2}} \frac{\mathcal{D}_{K+1}^{-\nu_{K+1}} \dots \mathcal{D}_N^{-\nu_N}}{(\mathcal{D}_1 + i0^+)^{\nu_1} \dots (\mathcal{D}_K + i0^+)^{\nu_K}}$$

$$I_{\text{aux}}(\vec{\nu}, \vec{s}, \epsilon, \eta) = \int \prod_{i=1}^L \frac{d^D \ell_i}{i\pi^{D/2}} \frac{\mathcal{D}_{K+1}^{-\nu_{K+1}} \dots \mathcal{D}_N^{-\nu_N}}{(\mathcal{D}_1 - \eta)^{\nu_1} \dots (\mathcal{D}_K - \eta)^{\nu_K}}$$

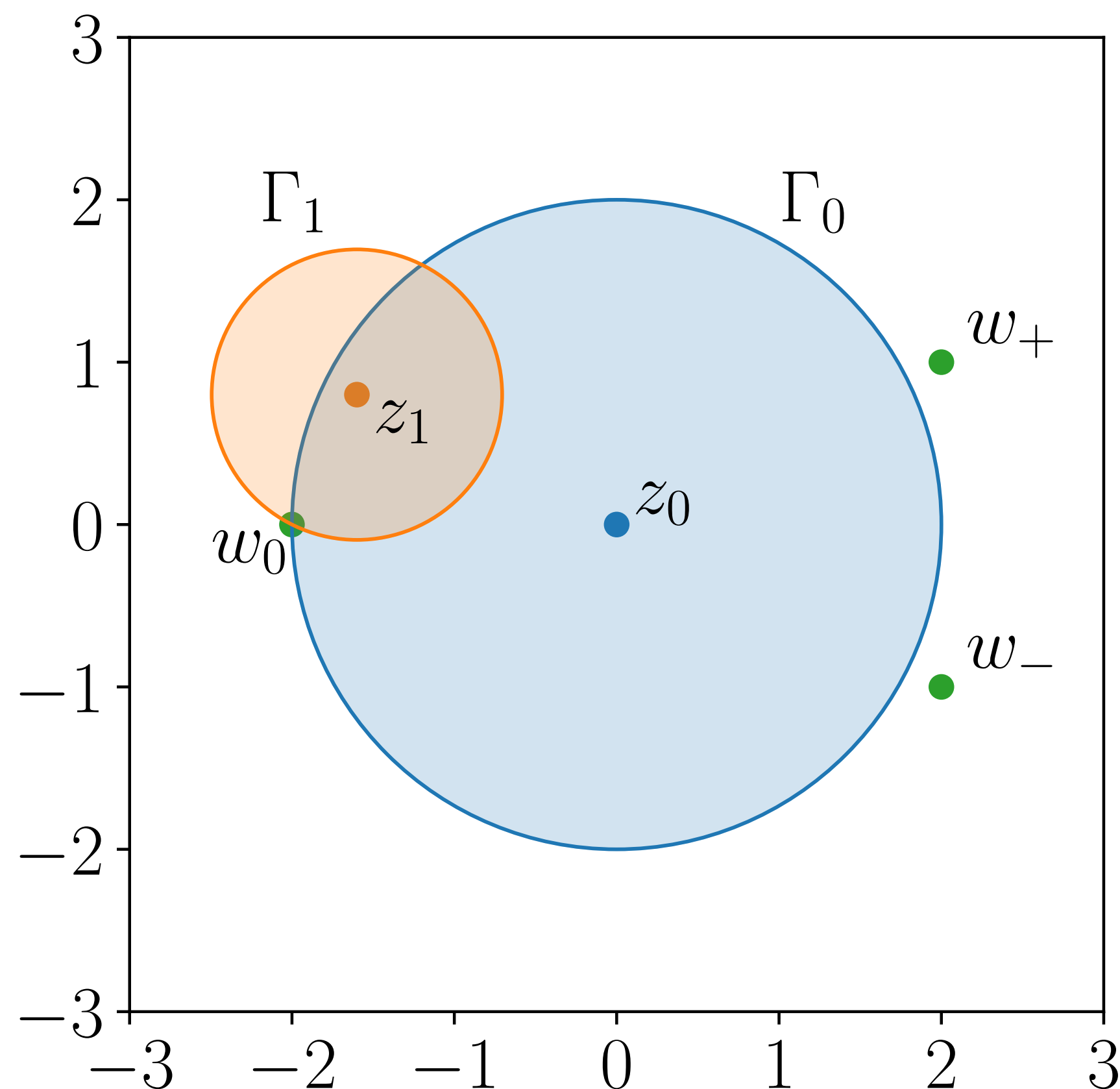
- The limit $\eta \rightarrow \infty$ can be computed **analytically** in terms of **vacuum integrals**;
- The limit $\eta \rightarrow 0$ provides the **target** integrals;
- The auxiliary mass flow approach does **not** require **boundary conditions**!



NEW: AMFlow2

Alternate recursion mode, High-performance differential equation solver, improved IBP interfaces, ...

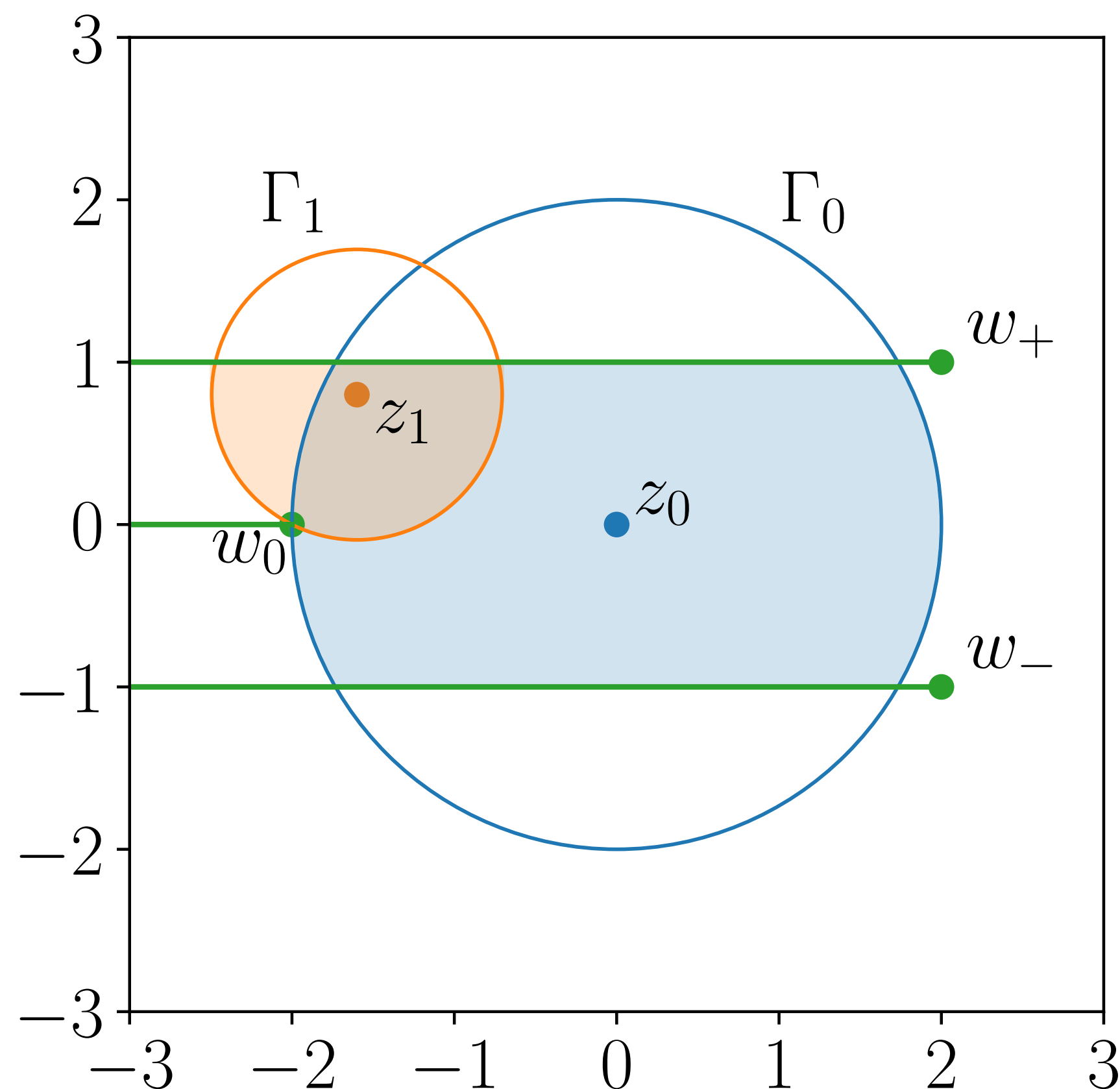
Moving into the complex plane



- A **gauge-invariant** definition of the mass can be obtained through the **complex mass scheme**;
- Complex kinematical variables require analytically continuing the solution over the complex plane;
- **SEASYDE** is a **MATHEMATICA** package implementing the series expansion approach on **the complex plane**.
- The radius of convergence of the series is limited by the presence of **poles**;
- “Transport” of the boundary conditions needs to consider **branch cuts**.



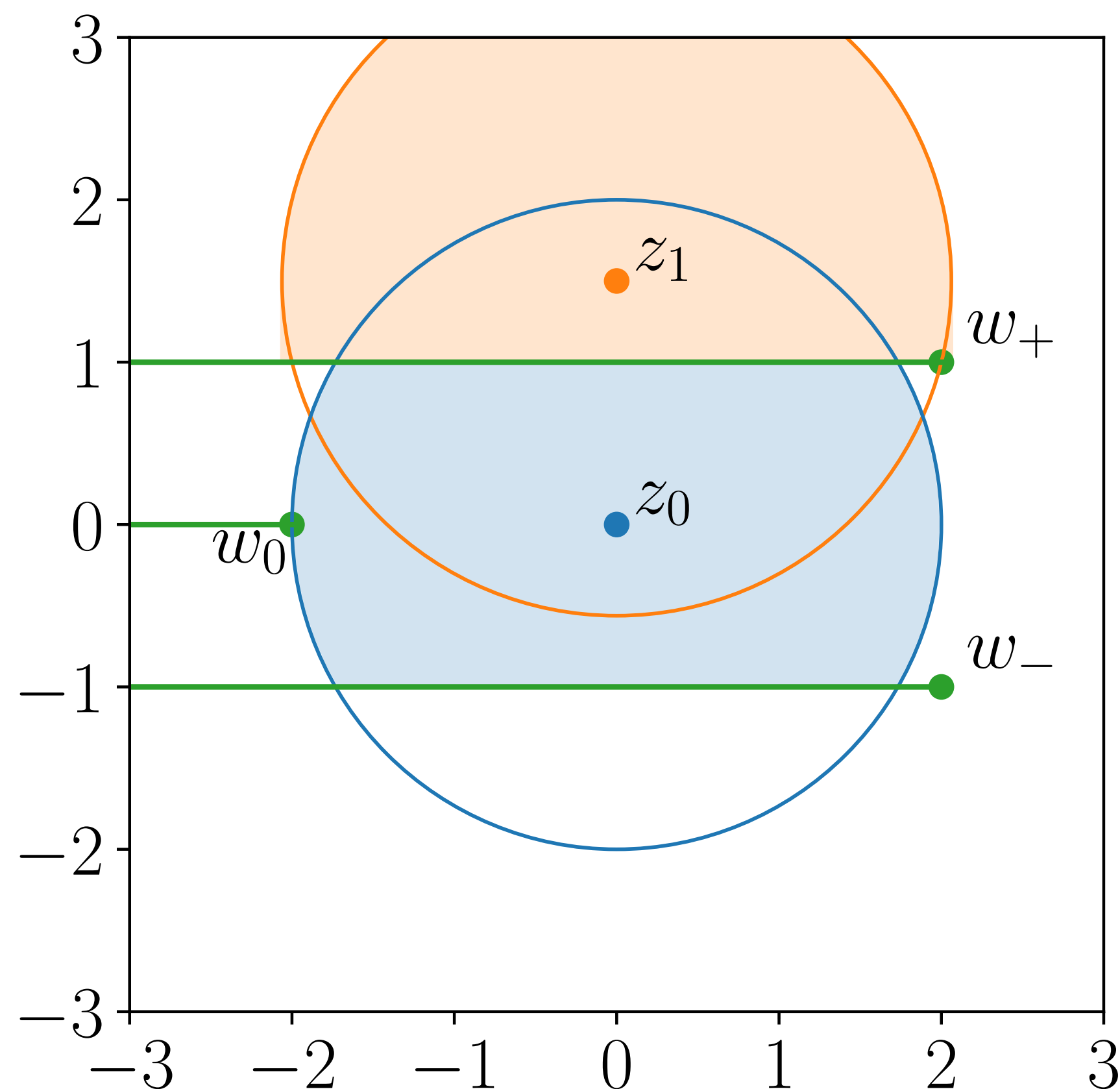
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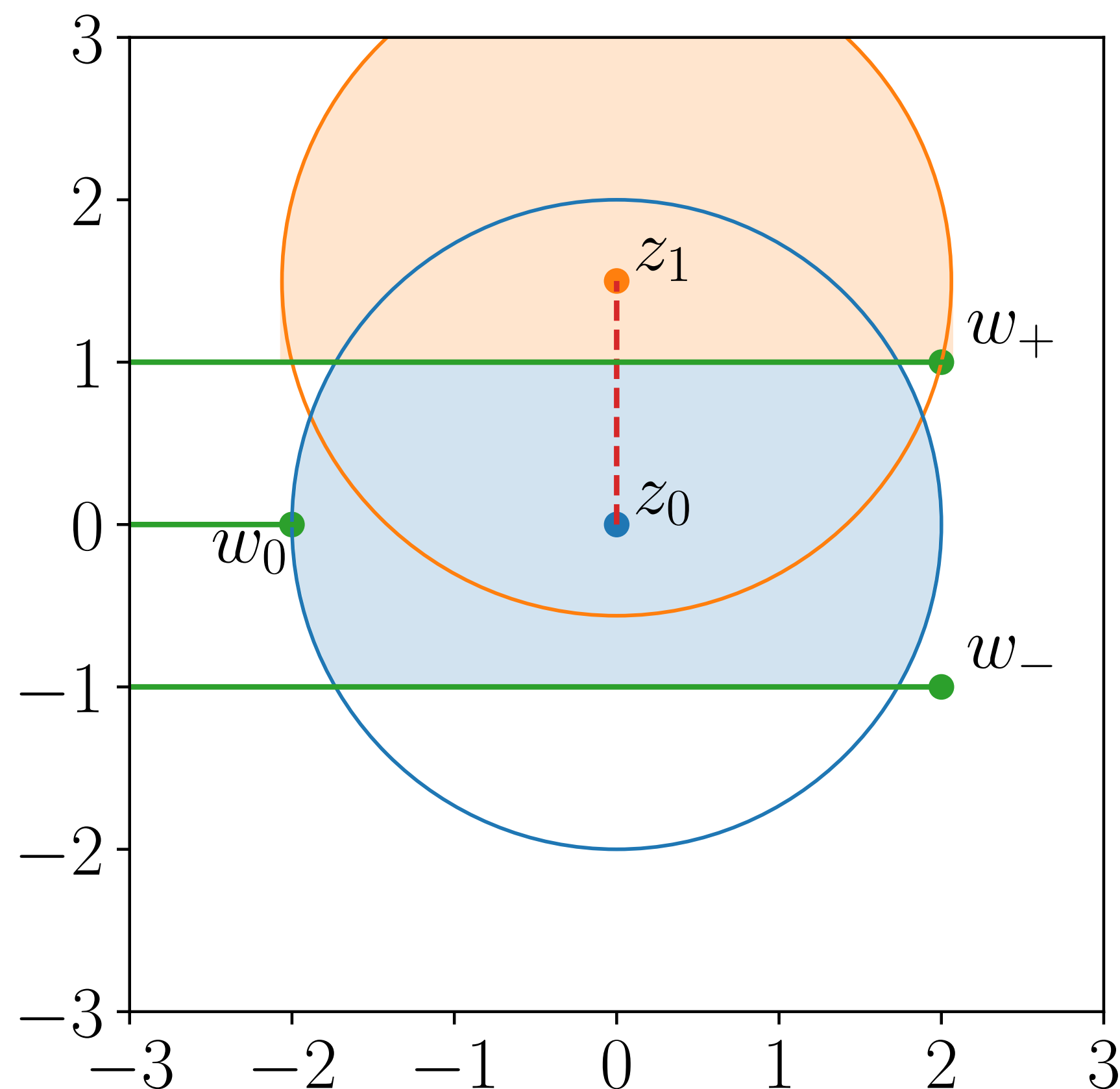
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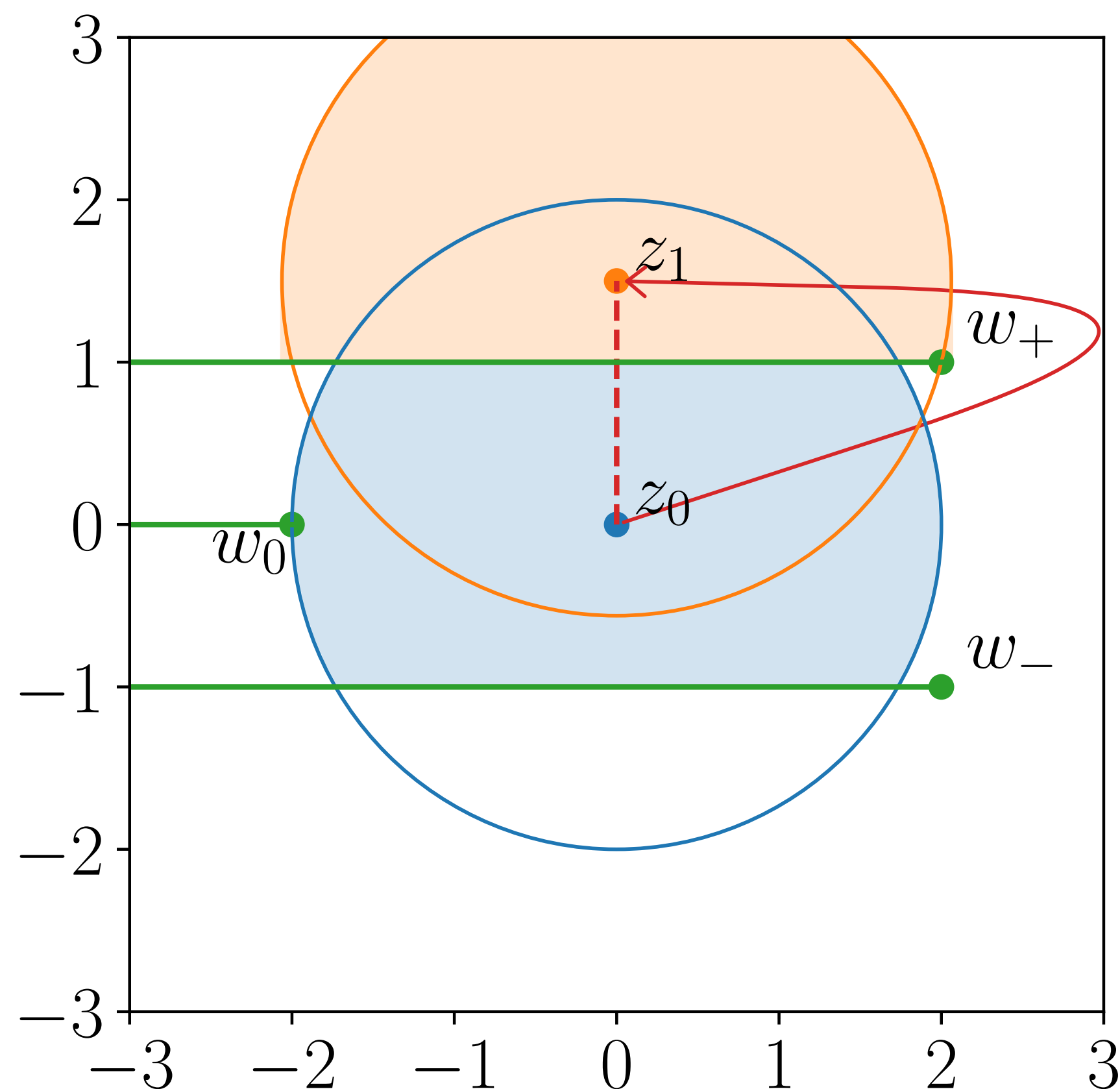
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Applications

Mixed QCD-EW Corrections

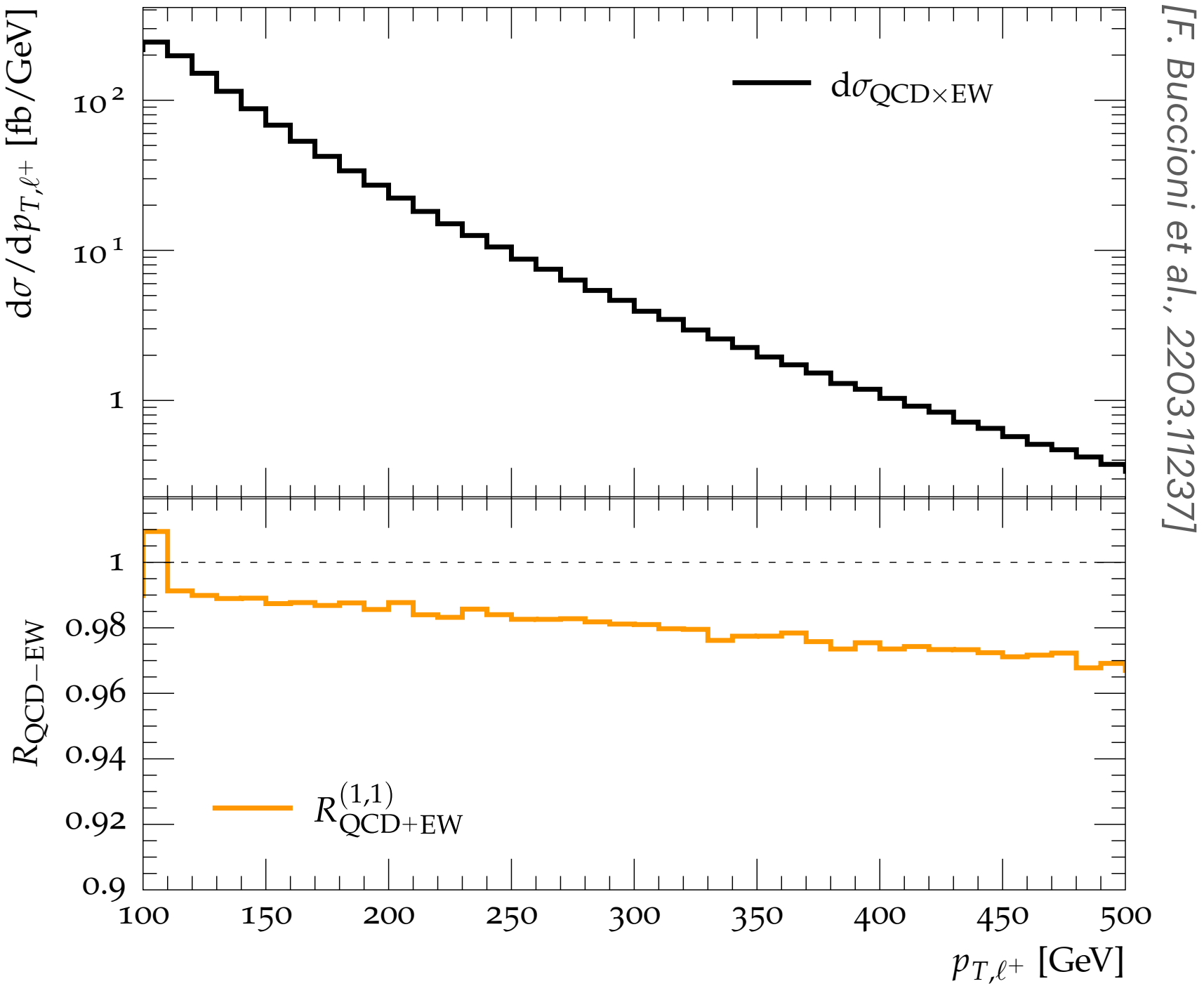
[T. Armadillo, et al, arXiv:2412.16095]

[F. Buccioni et al., arXiv:2203.11237]

Neutral Current Drell-Yan

- Two fully **independent** calculations: massive bare leptons and **semi-analytic amplitudes** [T. Armadillo, et al., arXiv:2412.16095]; massless dressed leptons and **analytic amplitudes** [F. Buccioni et al., arXiv:2203.11237].
- Dressing the massive leptons allowed for **comparison**, finding excellent agreement: differences are due to the lepton mass and are negligible for fiducial cross sections.

	T. Armadillo et al.	F. Buccioni et al.
Final State Leptons	Massive	Massless
Subtraction Method	q_T subtraction	Nested Subtraction
2-loop Amplitudes	Semi-Analytic	Analytic
$\sigma^{(1,1)}(qq)$ [pb]	-15.7(7)	-16.80
$\sigma^{(1,1)}(qg)$ [pb]	2.15(7)	2.07
$\sigma^{(1,1)}(q\gamma)$ [pb]	-0.231(9)	-0.207
$\sigma^{(1,1)}(g\gamma)$ [pb]	0.266(1)	0.2668

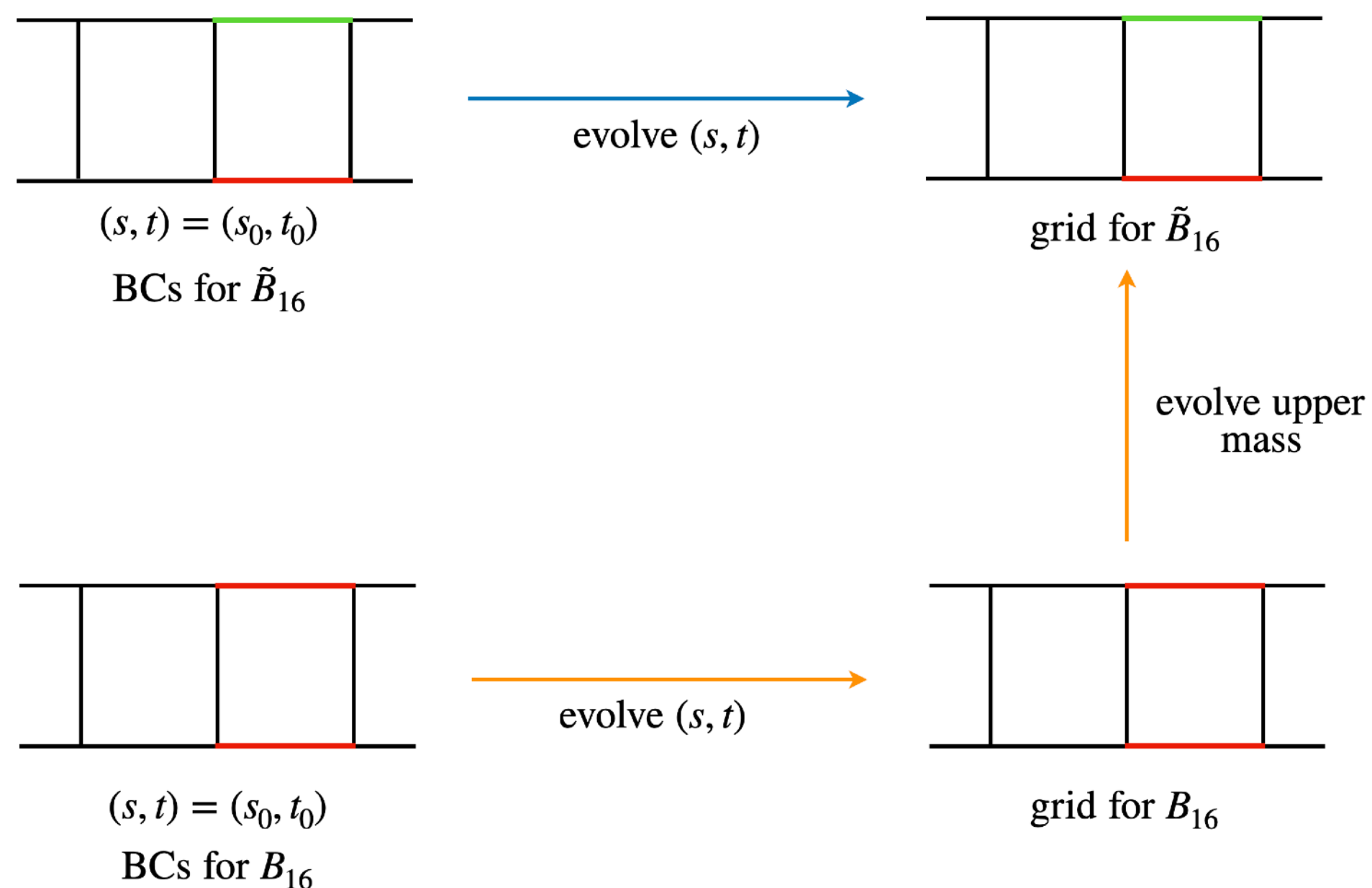
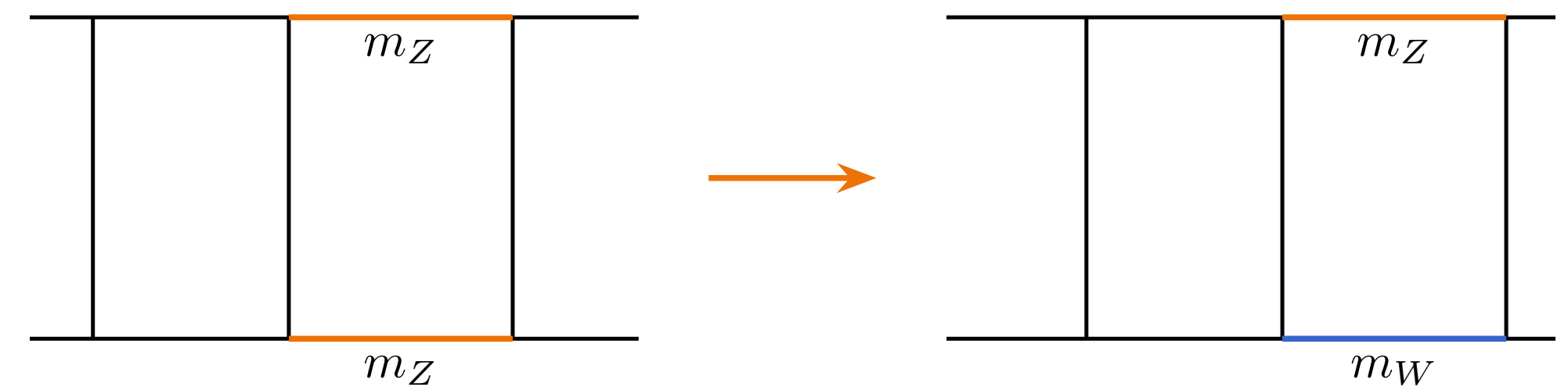


Mixed QCD-EW Corrections

[T. Armadillo, R. Bonciani, SD, N.Rana,
A.Vicini, arXiv:2405.00612]

Charged Current Drell-Yan

- Computationally **similar** to neutral-current Drell-Yan;
- Extra complexity comes from **new diagrams** with two **different** internal massive lines;
- At the moment, only **semi-analytical** results are available.



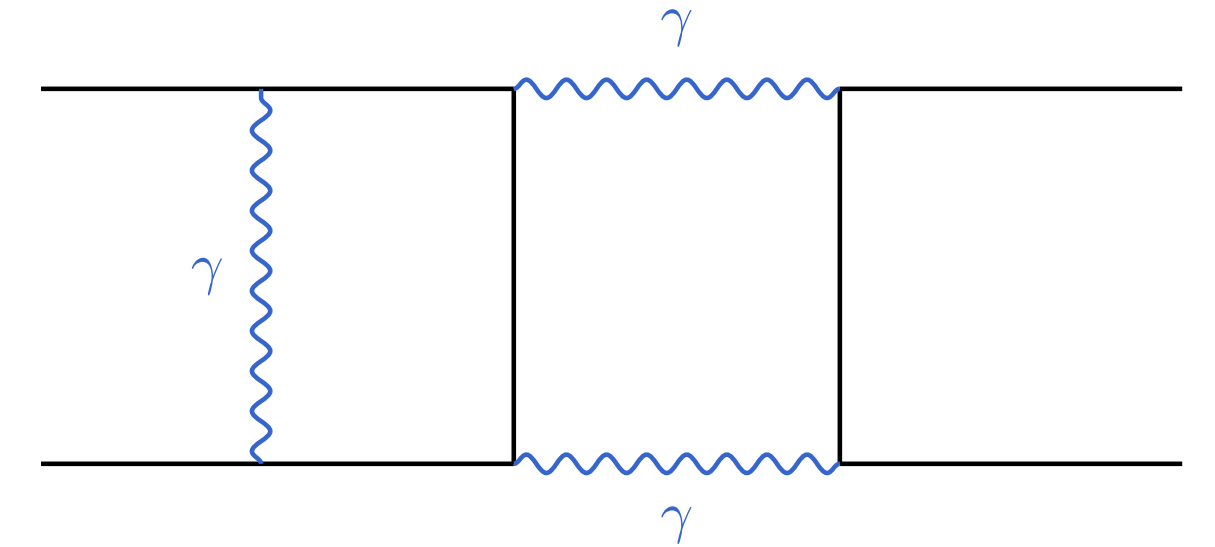
- Numerical grid for the amplitude computed with **SEASYDE**;
- Possible approaches for the **evolution**:
 - Compute the boundary conditions with AMFlow; use the **differential equations in s and t** to build the grid;
 - Use the neutral-current Drell-Yan grid as a boundary condition; use the **differential equation in one of the masses**;
- The differential equation in the mass allows keeping a **dependence** on the W boson mass.

Towards NNLO EW

[R. Bonciani et al., arXiv:2106.13179]

NNLO QED

- **QED**: easiest subset of EW corrections: no γ_5 , no massive gauge bosons;
- **Analytic** results have been computed in terms of special functions;
- IR structure **shared** with the full EW calculation;
- **IR subtraction operator** can be obtained via **abelianisation** of the corresponding QCD operator:



$$C_F \rightarrow Q_{qle}^2$$

$$C_A \rightarrow 0$$

$$T_R n_{lh} \rightarrow \sum_{lh} N_{C,i} Q_i^2$$

Expansion in the couplings:

$$|\hat{\mathcal{M}}\rangle = 4\pi\hat{\alpha} \sum_{i,j} \left(\frac{\hat{\alpha}_S}{4\pi}\right)^i \left(\frac{\hat{\alpha}}{4\pi}\right)^j |\hat{\mathcal{M}}^{(i,j)}\rangle$$

$$|\mathcal{M}^{(0,0),fin}\rangle = |\mathcal{M}^{(0,0)}\rangle$$

$$|\mathcal{M}^{(0,1),fin}\rangle = |\mathcal{M}^{(0,1)}\rangle - \mathcal{F}^{(0,1)} |\mathcal{M}^{(0,0)}\rangle$$

$$|\mathcal{M}^{(0,2),fin}\rangle = |\mathcal{M}^{(0,2)}\rangle - \mathcal{F}^{(0,2)} |\mathcal{M}^{(0,0)}\rangle - \mathcal{F}^{(0,1)} |\mathcal{M}^{(0,1),fin}\rangle$$

Towards NNLO EW

[T. Armadillo, SD, M. Dradi,
A.Vicini, arXiv:2511.20365]

NNLO QED

s=10000 GeV ² ; t=-2500 GeV ² ; μ _R =m _Z .		
ε ⁻⁴	0.43895747601285225040892043215096	0
	-0.43895747601285225040892043215096	
	0	
ε ⁻³	-50.507936715422813794673578353212	0
	50.507936715422813794673578353212	
	0	
ε ⁻²	729.67339746440682380540184060883	0
	-1334.17941387822635237175428142592	
	604.50601641381952856635244081709	
ε ⁻¹	26528.6623446604895668120585896735	0
	-463.09023418725937944723920907369	
	-26065.57211047323018736481938059986	
ε ⁰	401847.4473451308127296540393826352	97243.8715776209203558943130413070
	4376.8127679941628242540225839402	
	-308980.388535504055198013748925268	

- **Semi-analytic** calculation to prove automation in the computational framework **ABISS** + **SEASYDE**;
- Main methodological choice: **arbitrary numerical precision** at each step of the calculation;
- Control of the cancellation of the poles with **35-digit** precision;
- Successful **comparison** with analytic results.

$$\langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,2)} \rangle - \mathcal{I}^{(0,1)} \langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,1)} \rangle - \mathcal{I}^{(0,2)} \langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,0)} \rangle$$

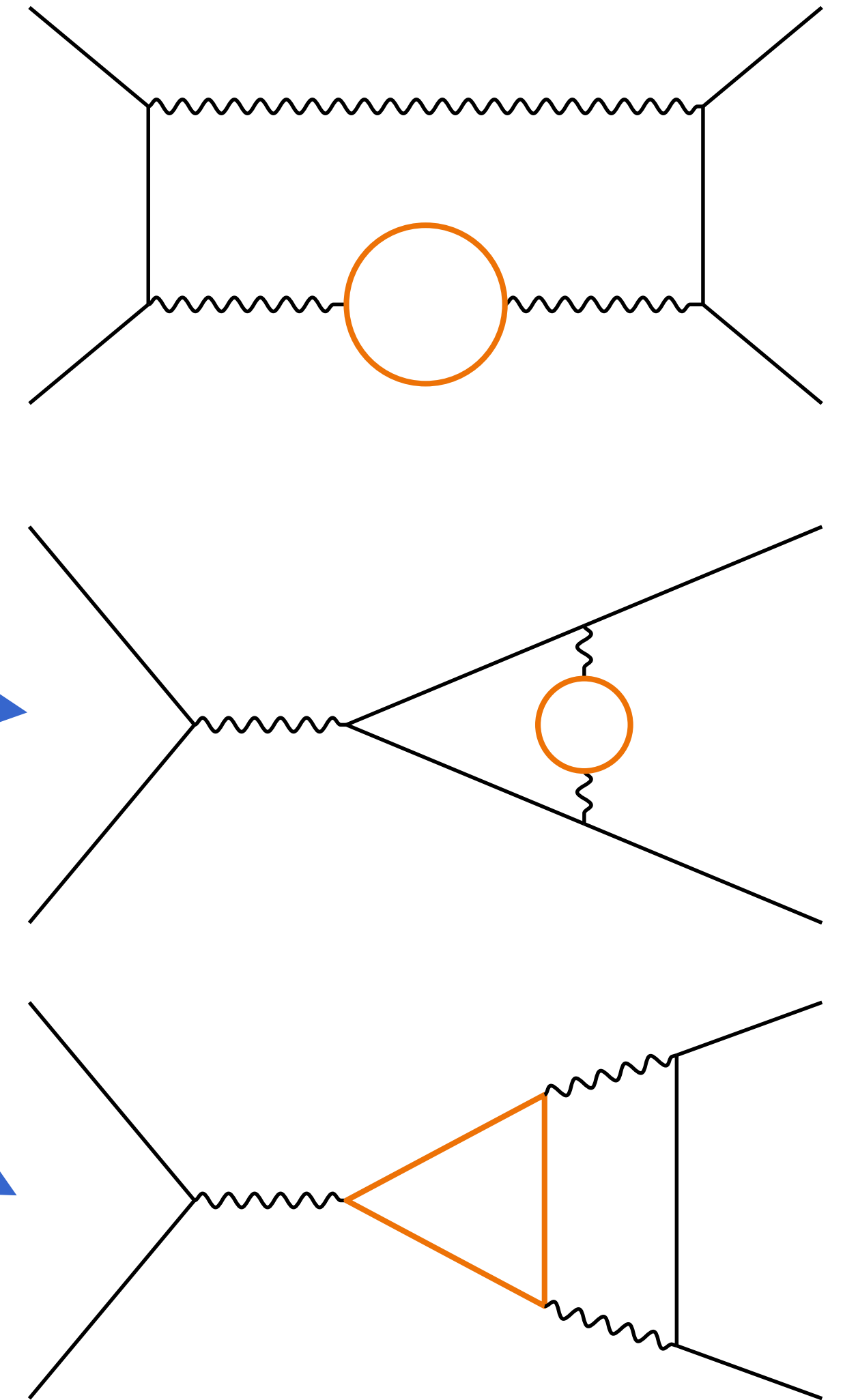


Towards NNLO EW

NNLO EW: Fermionic Contribution

- **Fermionic contribution**: at least one closed fermion sub-loop;
- **Gauge-invariant subset** which is expected to be relatively large due to the top quark mass;
- Direct **numerical integration** of each contribution (**no IBP**):
 - **Vertex and box diagrams with a sub-loop fermion self-energy**: dispersion relation for the self-energy (numerical integration over σ);
 - **Vertex diagrams with a triangle fermion sub-loop**: Feynman parametrisation followed by dispersion relation (numerical integration over x and σ)
- Numerical integration performed in **C++** (GNU Scientific Library, QUADPACK).

[A. Freitas, J. Wallace: [2512.15700](#)]



Towards NNLO EW

[T. Armadillo, SD, M. Dradi,
A. Vicini: 2605.28667]

NNLO EW: Fermionic Contribution



$$\langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,2)} \rangle - \mathcal{F}^{(0,1)} \langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,1)} \rangle - \mathcal{F}^{(0,2)} \langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,0)} \rangle$$

s=10000 GeV ² ; t=-2500 GeV ² ; $\mu_R=m_Z$.		
ε^{-3}	-35.4668	0
	35.4672	
	0	
ε^{-2}	1856.03 + 374.962 i	0
	-1207.81 + 241.417 i	
	-648.21 - 616.378 i	
ε^{-1}	-35083.52 - 4774.08	0
	9463.56 - 5495.62 i	
	25620.07 + 10269.72 i	
ε^0	-523472.8 + 82010.2 i	-454654.8 + 118286.9 i
	32550.2 - 16686.4 i	
	36267.7 + 52963.0 i	

- Alternative calculation within **ABISS** + **SEASYDE**;
- Calculation performed with the **muon mass** regulating collinear divergences;
- Cancellation of the poles up to m_μ^2 power corrections ($10^{-6} - 10^{-8}$ relative effect);
- Decreasing by a factor of $100 m_\mu$, the cancellation of the poles improves from 10^{-8} to 10^{-12} in the case of the γ/Z mediated box corrections.

Summary

- **Higher-order EW corrections** are needed to fully exploit the scientific potential of HL-LHC and FCC-ee;
- Multiloop EW corrections pose phenomenal **challenges**;
- Need for **automation**;
- **Numerical** and **semi-analytical** methods have brought impressive progress in the evaluation of amplitudes;
- On our way to meeting the requirements of HL-LHC?

	ANALYTICAL	NUMERICAL	SEMI-ANALYTICAL
NCDY NNLO QCDxEW	2022		2022
CCDY NNLO QCDxEW			2024
NCDY NNLO QED	2021		2025
NCDY NNLO QED (Fermionic)		2026	2026

Summary

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Thank you!

	ANALYTICAL	NUMERICAL	SEMI-ANALYTICAL
NCDY NNLO QCDxEW	2022		2022
CCDY NNLO QCDxEW			2024
NCDY NNLO QED	2021		2025
NCDY NNLO QED (Fermionic)		2026	2026

Backup slides

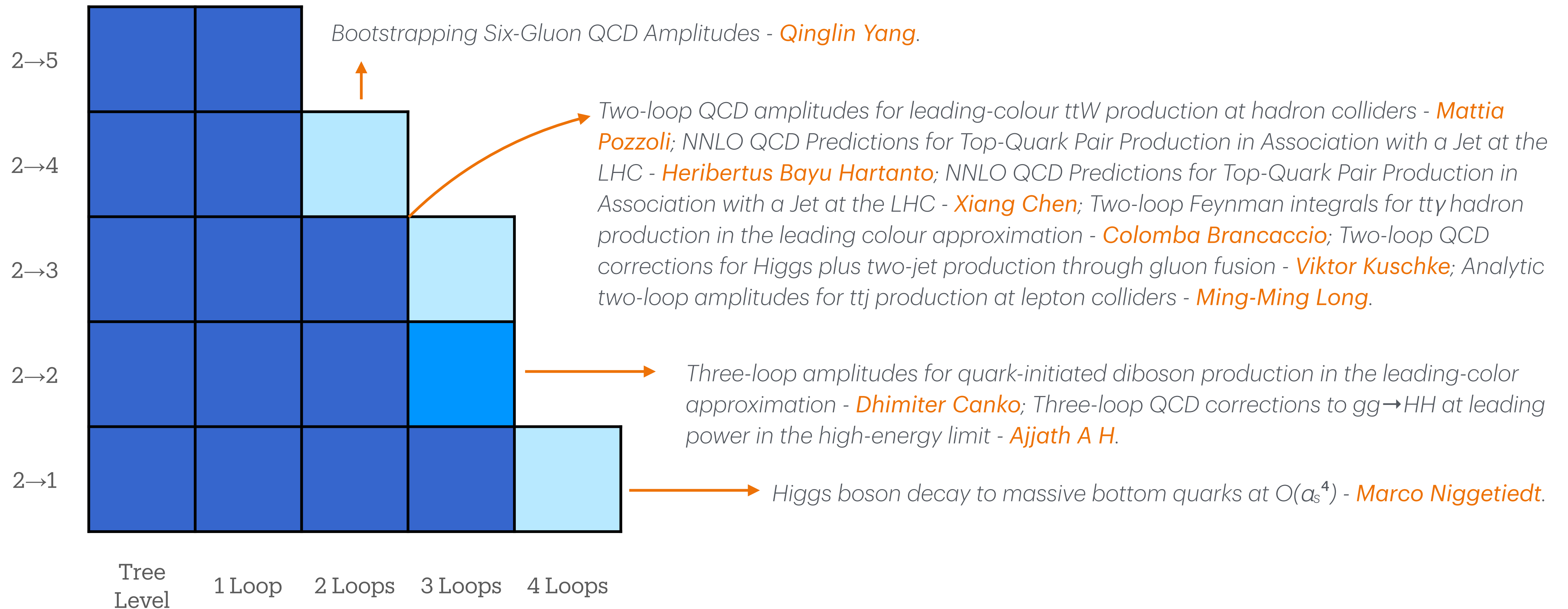
State of the Art

Loop amplitudes are one of the main bottlenecks of higher-order predictions.

State of the Art

QCD Corrections

Loop amplitudes are one of the main bottlenecks of higher-order predictions.



Series Expansion

A simple example

$$\begin{cases} f'(x) + \frac{1}{x^2 - 4x + 5} f(x) = \frac{1}{x + 2} \\ f(0) = 1 \end{cases}$$

Ansatz:

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f'_{hom}(x) = \sum_{k=0}^{\infty} (k + r) c_k x^{(k+r-1)}$$

$$\begin{cases} rc_0 = 0 \\ \frac{1}{5}c_0 + c_1(r + 1) = 0 \\ \frac{4}{25}c_0 + \frac{1}{5}c_1 + c_2(2 + r) = 0 \\ \dots \end{cases}$$

$$f_{hom}(x) = 5 - x - \frac{3}{10}x^2 + \frac{11}{150}x^3 + \dots$$

Expanded around $x' = 0$

$$\begin{aligned} f_{part}(x) &= f_{hom}(x) \int_0^x dx' \frac{1}{(x' + 2)} f_{hom}^{-1}(x') \\ &= \frac{1}{2}x - \frac{7}{40}x^2 + \frac{2}{75}x^3 + \dots \end{aligned}$$

$$f(x) = f_{part}(x) + C f_{hom}(x)$$

$$f(0) = 1 \rightarrow C = \frac{1}{5}$$

HANDLING γ_5

Prescriptions in dimensional regularisation

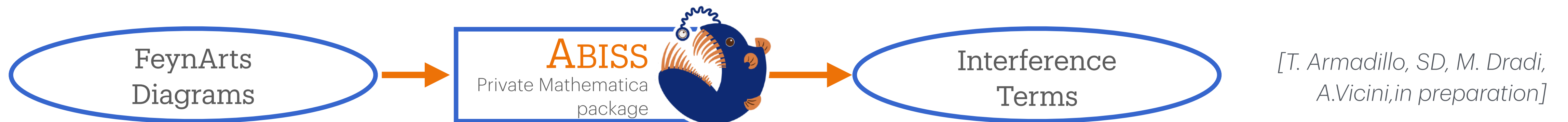
	'T HOOFT AND VELTMANN NUCL. PHYS. B 44 (1972) 189–213	KREIMER ET AL. PHYS. LETT. B 237 (1990) 59–62
Anticommutation	✗	✓
Cyclicity	✓	✗
Implementation	• $\bar{\gamma}_\mu \rightarrow \mu = 0, \dots, 3$ • $\hat{\gamma}_\mu \rightarrow (-2\epsilon)$ dimensional • $\{\bar{\gamma}_\mu, \gamma_5\} = 0$; $[\hat{\gamma}_\mu, \gamma_5] = 0$;	• Define a consistent reading point as the “start” of each dirac trace: fixed point prescription;
Comments	• The most mathematically sound approach; • Increased computational load; • Chiral symmetry needs to be restored with ad hoc counterterms; • Non-trivial implementation at higher orders.	• More streamlined computational implementation; • Single traces give different results for different choices of the fixed point; • Sanity check: verify that the final result does not depend on the choice of the fixed point.

HANDLING γ_5

Prescriptions in dimensional regularisation

- For neutral-current Drell-Yan, proven [M. Heller, A. von Manteuffel, R. M. Schabinger and H. Spiesberger, arXiv:hep-ph/2012.05918] that at two loops the two prescriptions yield:
 - different** bare scattering amplitudes;
 - same** finite UV-renormalised and IR-subtracted amplitudes.

- ABISS** is a MATHEMATICA package to perform Dirac and Lorentz algebra, **automating** Kreimer's prescription;



- A consistent implementation of Kreimer's scheme **requires**:
 - treating $\epsilon_{\mu\nu\rho\sigma}$ as a pure **4-dimensional** object;
 - computing **higher-dimensional** integrals.
- Successful automated treatment of **2→2** processes;
- Next Steps:** Generalisation to arbitrary processes? Implementation of 't Hooft-Veltmann prescription?

$$\begin{aligned}k^\mu &= \bar{k}^\mu + \hat{k}^\mu \\g^{\mu\nu} &= \bar{g}^{\mu\nu} + \hat{g}^{\mu\nu} \\ \bar{g}^{\mu\nu} \bar{g}_{\mu\nu} &= 4\end{aligned}$$