

Particle Accelerators

KSETA topical course — 20 & 21 October 2025
Bastian Härer & Matthias Fuchs (IBPT)



Overview

■ Session 1 — Monday 20 October 2025

- Applications of accelerators
- Accelerator types
- Basics of beam optics and dynamics

■ Session 2 — Tuesday 21 October 2025

- Current and future projects
- Laser Plasma Acceleration
- KIT accelerator research



Indico page

<https://indico.kit.edu/event/5296/>

- Schedule
- Slides

Accelerators — why?

■ The experiment as the driving force

The "classical" experiment for the investigation of small structures involves bombarding the sample with a probe (radiation/particle) of defined direction, energy, momentum, intensity. Scattered radiation/particles and/or secondary radiation/particles are then observed.

■ Particles with high energies first for nuclear physics, later also for elementary particle physics

Radiation wavelength < structural dimension

$$\lambda < 10^{-15} \text{ m}$$

The energy of radiation or particles (de Broglie) is thus

$$E_{\text{rad}} = 2\pi \frac{\hbar c}{\lambda} \approx 2 \times 10^{-10} \text{ J}$$

The electronvolt

■ More practical unit: definition of 1 eV

If the electron with its charge

$$e = 1.602 \times 10^{-19} \text{ C}$$

passes through the voltage

$$U = 1 \text{ V}$$

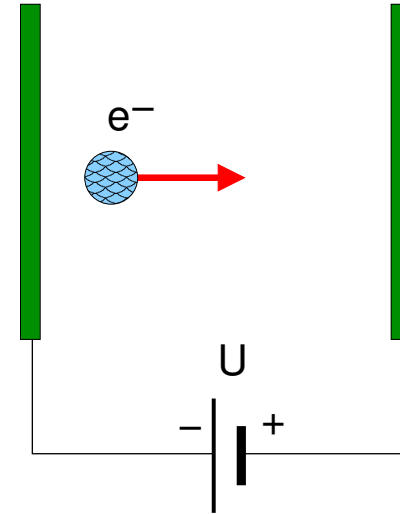
it has gained the following energy:

$$E = eU = 1.602 \times 10^{-19} \text{ J} \Rightarrow E = 1 \text{ eV}$$

■ Common values are:

$$1 \text{ keV} = 10^3 \text{ eV} \quad 1 \text{ MeV} = 10^6 \text{ eV}$$

$$1 \text{ GeV} = 10^9 \text{ eV} \quad 1 \text{ TeV} = 10^{12} \text{ eV}$$



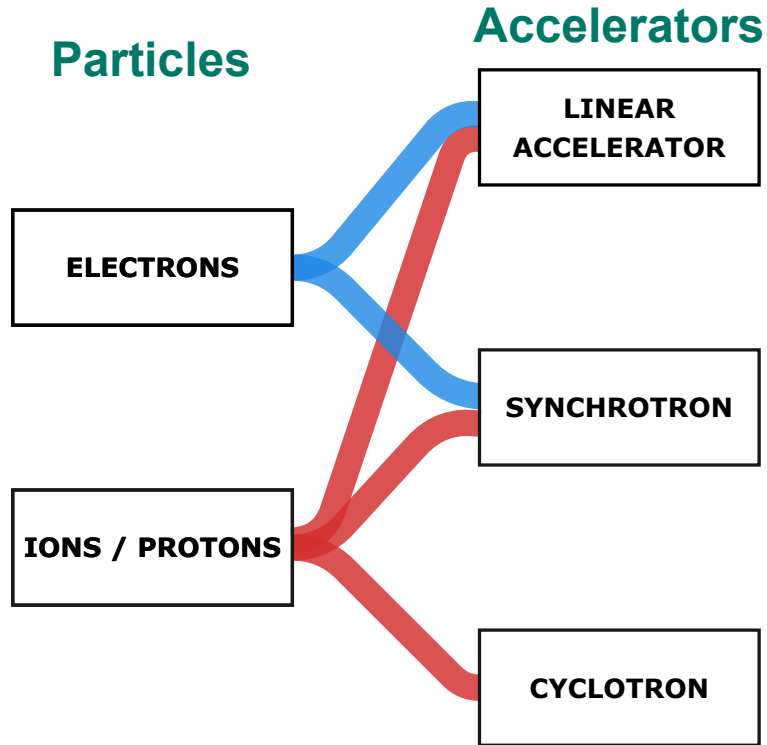
What do we accelerate?

Charged particles

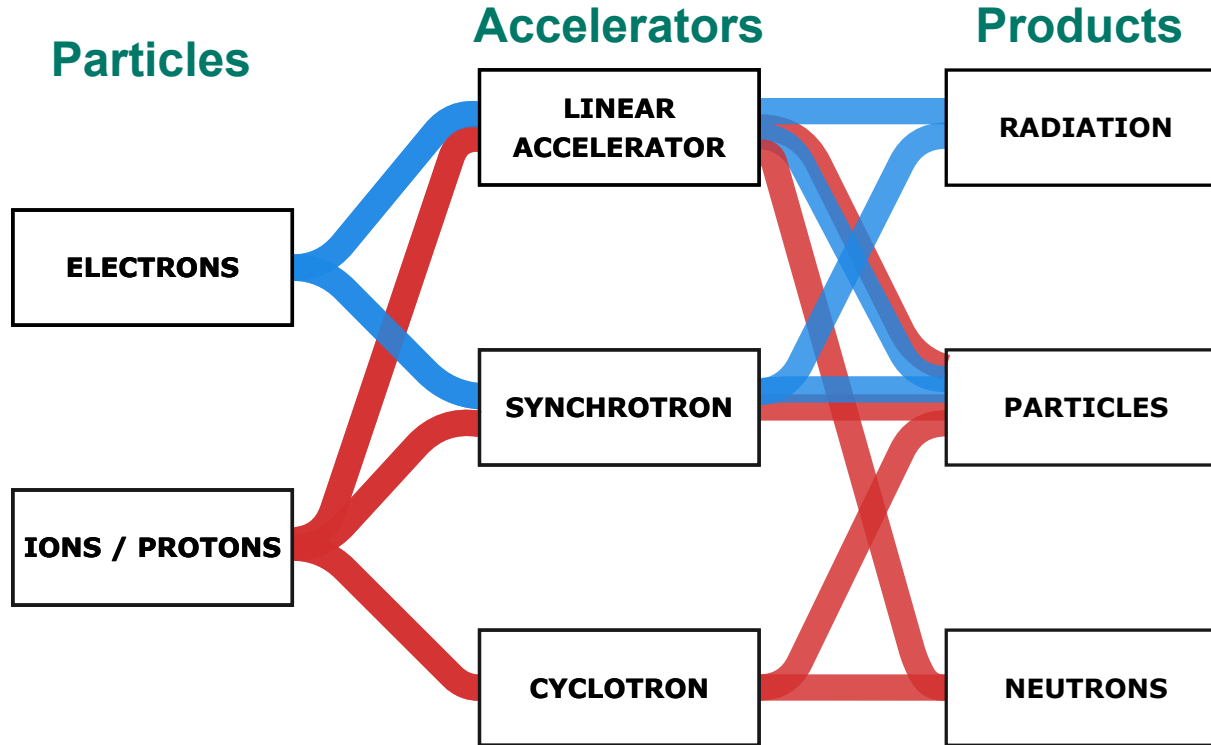
ELECTRONS

IONS / PROTONS

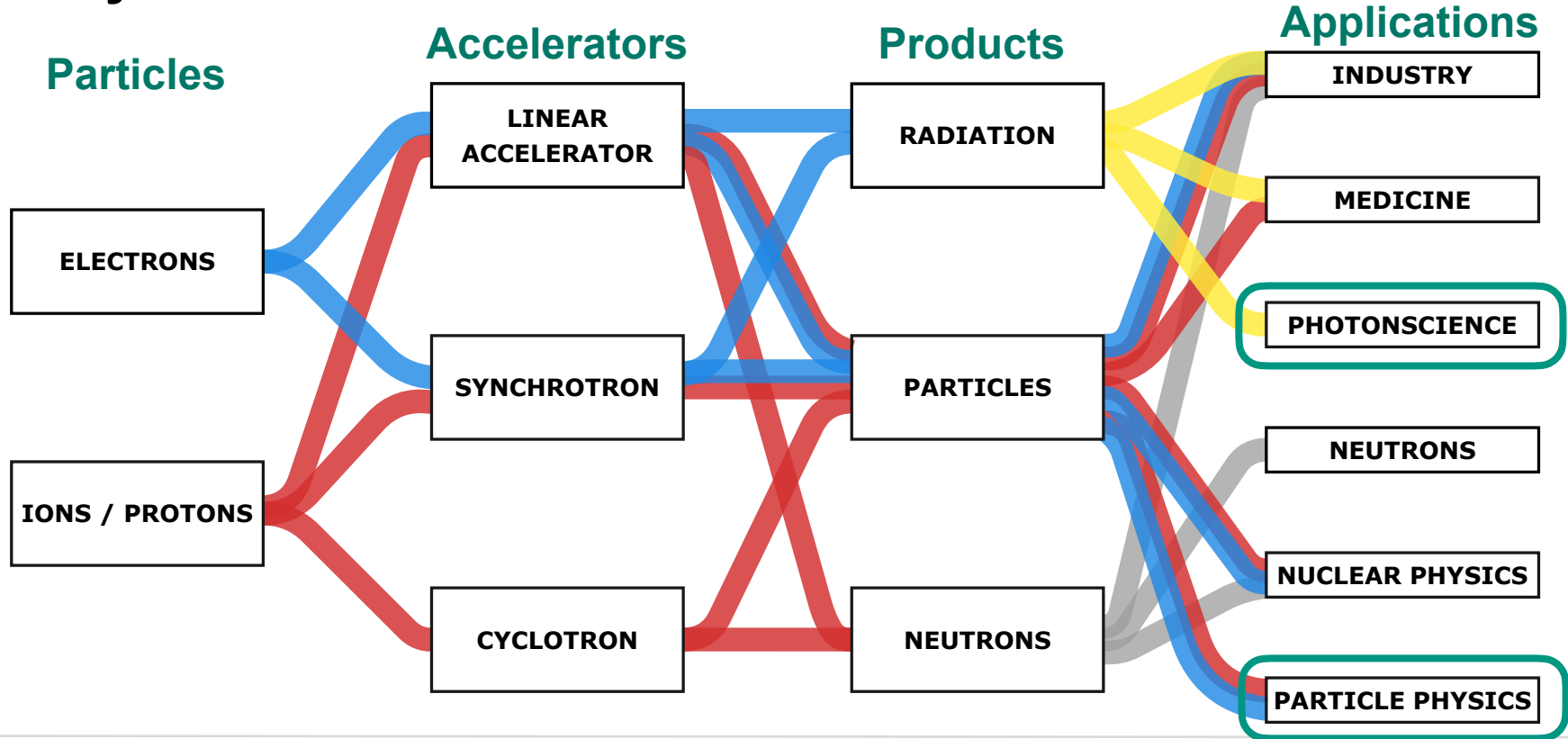
What types of accelerators are there?



Why do we accelerate?



Why do we accelerate?



Einsatzgebiete von Beschleunigern

Forschung

Teilchenphysik



Tumorthherapie



Medizin

Tiernahrung

Lebensmittel-
bestrahlung



Kosmische
Strahlen

Bildgebung



T. van de Kamp et al.,
DOI: 10.1038/
s41467-018-05654-y

Lithographie



Strahlenvernetzung &
farbl. Veränderung



Rob Lavinsky, iRocks.com
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<https://commons.wikimedia.org/w/index.php?curid=10127470>

Schmuck

Kabel / Elektro-
bauteile

Fahrzeug-
industrie

Synchrotron-
strahlung

Applications of particle accelerators

Table 13.1 Particle accelerators—world-wide (after Dearnaley 1987)

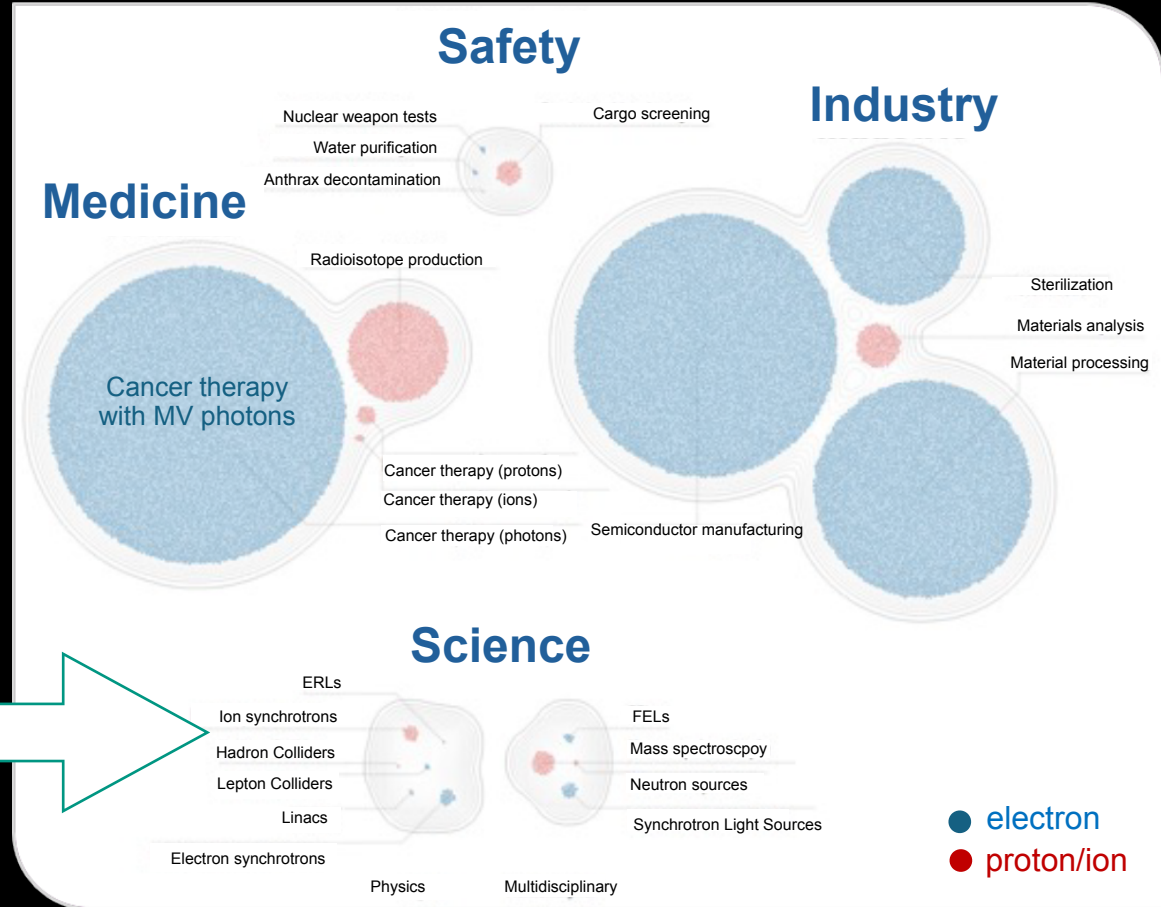
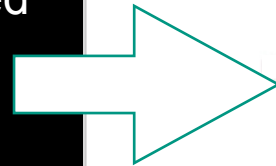
Category of accelerators	Number in use
(1) High-energy accelerators of more than 1 GeV	112
Biomedical accelerators	
(2) Radiotherapy	>4000
(3) Research including biomedical research	800
(4) Medical radioisotope production	~200
(5) Accelerators in industry	~1500
(6) Ion implanters	>2000
(7) Surface modification centres and research	~1000
(8) Synchrotron radiation sources	~50
Total in 1994	10 000

Reference: E. Wilson, An Introduction to particle Accelerators, Oxford University Press, 2001

Accelerators of the world

Only a small fraction of the world's accelerators serve solely science.

Most of the systems are used in industry and medicine.



Simplest accelerator

- Well-known: Cathode ray tube

- In old television sets:

Acceleration to about 20 keV

- Start: $\vec{F} = e(\vec{v} \times \vec{B} + \vec{E})$

Energy change with movement in the EM field:

$$\Delta E = e \int (\vec{v} \times \vec{B} + \vec{E}) d\vec{r}$$

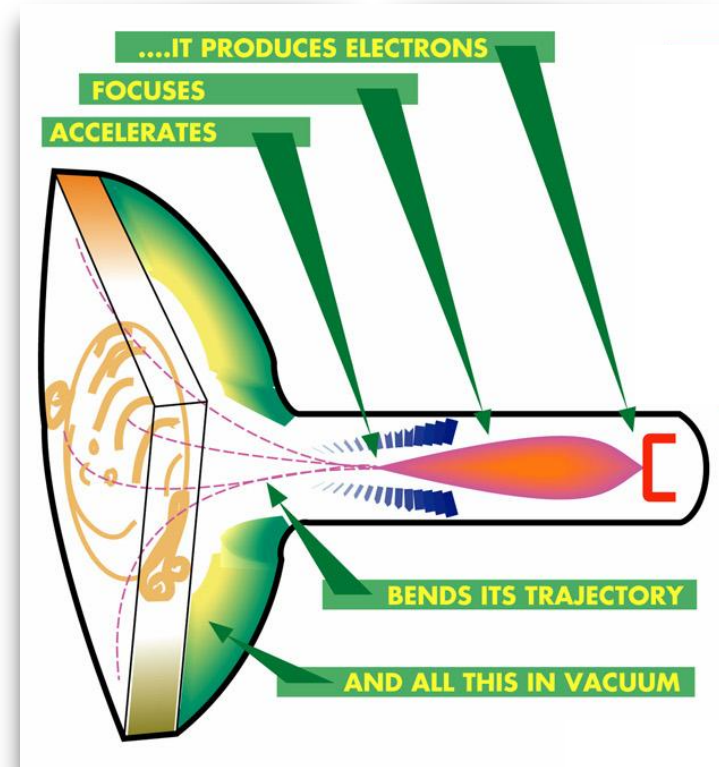
No energy change through $\vec{B}$

Energy gain only through electric fields:

$$\Delta E = e \int \vec{E} d\vec{r} = eU$$

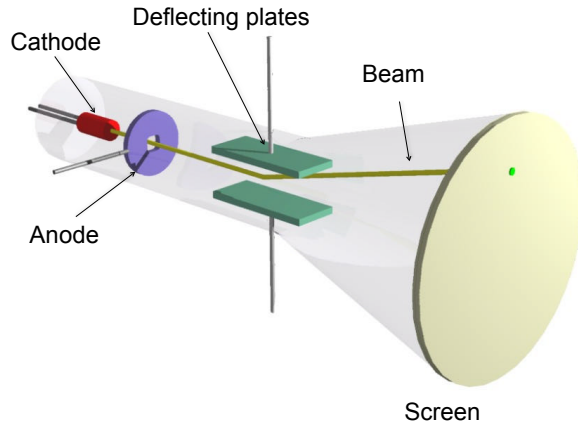
- Application:

Thermionic e guns, X-ray tubes



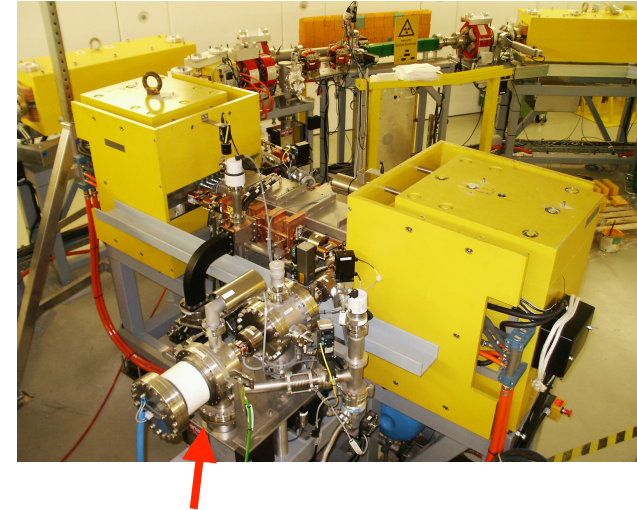
Creation of free electrons

- Glow filament is heated up, electrons emerge at the surface (cathode)
- Voltage at the anode accelerates electrons



Cathode ray tube of an electron gun

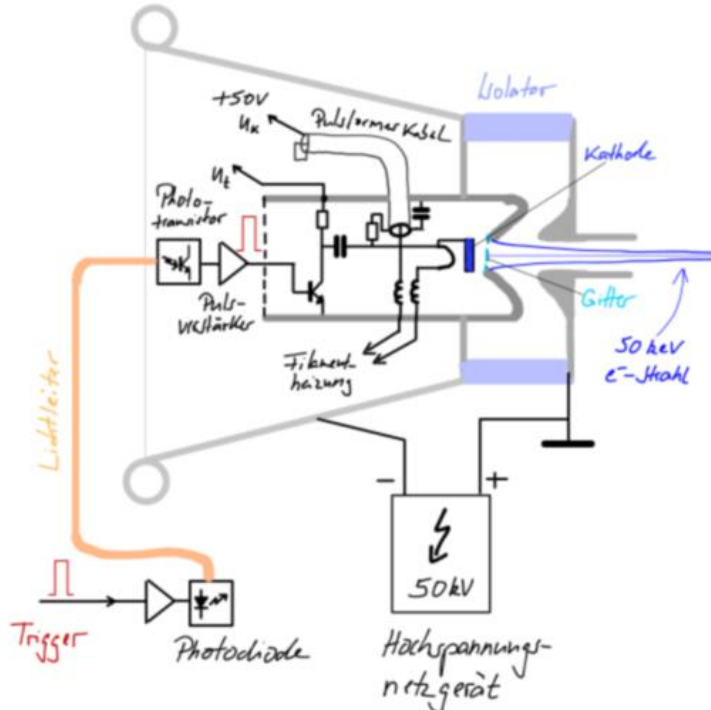
Demonstration during accelerator lab course!



Electron gun at KARA (90'000 V)

Electron sources: e guns

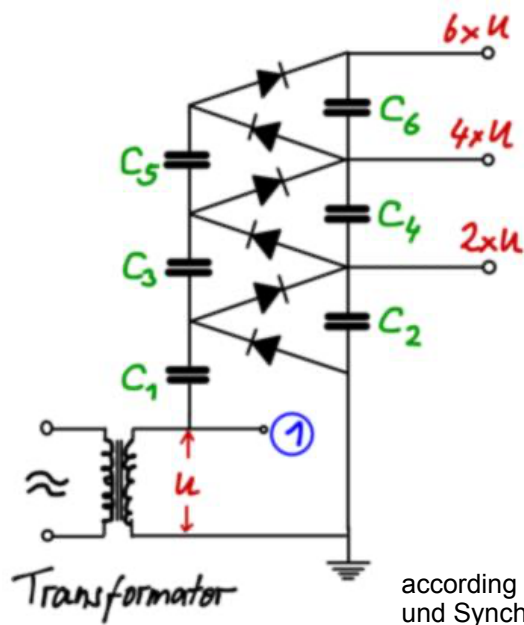
- $U = 5 \text{ eV to } 100 \text{ keV}$
- Currents ranging from nA to mA, pulsed up to few 10 mA
- UHV environment



according to: K. Wille, Physik der Teilchenbeschleuniger und Synchrotronstrahlungsquellen, Teubner Studienbücher, 2. Auflage, 1996.

Electrostatic accelerators

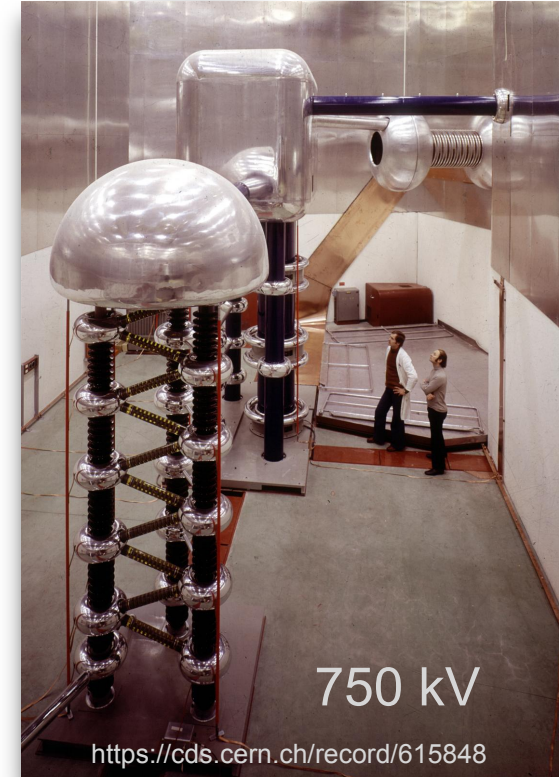
■ Cockcroft-Walton accelerator (1930s)



“Greinacher rectifier”:

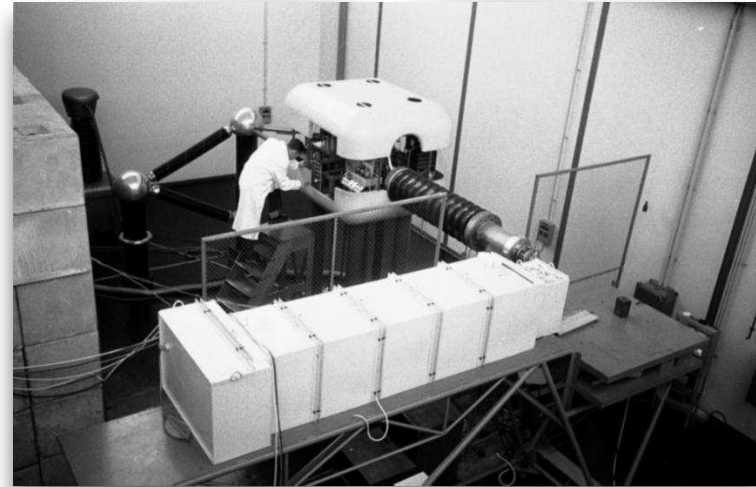
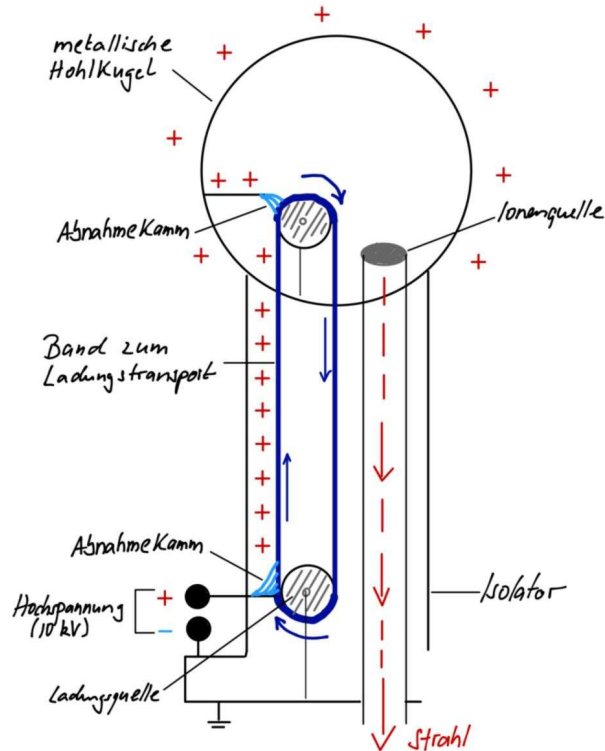
- voltages up to 4 MV
- for μs pulses
- currents up to 100 mA

according to: K. Wille, Physik der Teilchenbeschleuniger und Synchrotronstrahlungsquellen



<https://cds.cern.ch/record/615848>

Van-de-Graaff

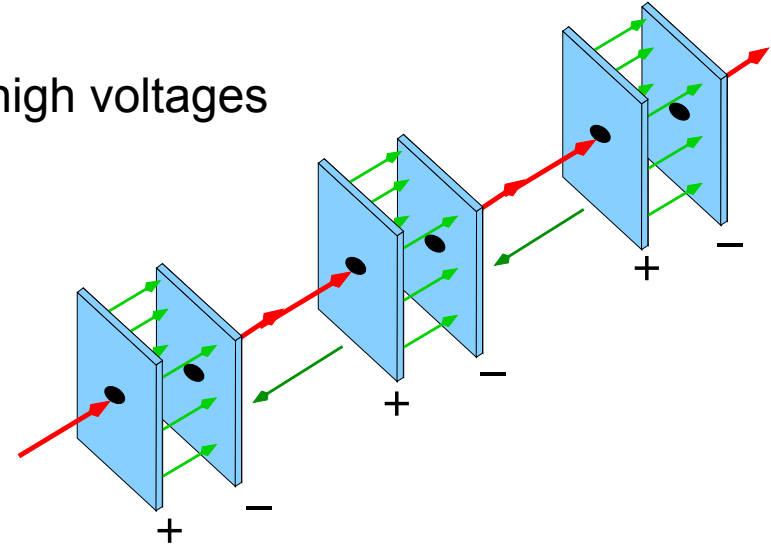
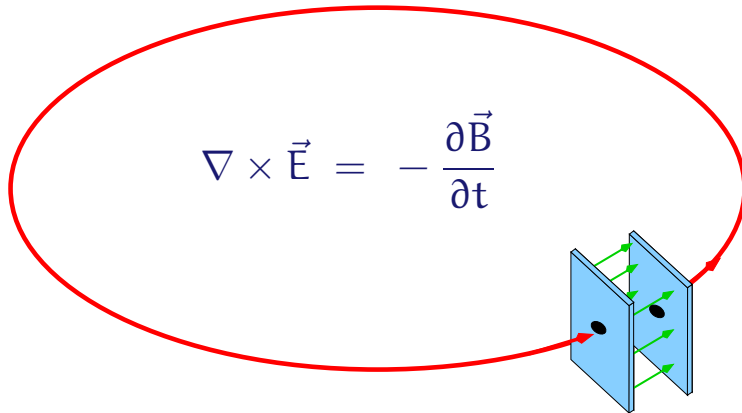


- Start of development 1930
- Focusing effect of the ring-shaped electrodes
- Voltages up to 2 MV
- With insulating gas (e.g. SF₆) under high pressure even up to 10 MV

Limits

■ Limits of electrostatic accelerators:

- Electrical breakdown and discharge at too high voltages
- Accelerators became larger and larger
- Impossible to build several accelerating stages one after the other with static fields



■ Way out:

Transition to time-varying EM fields

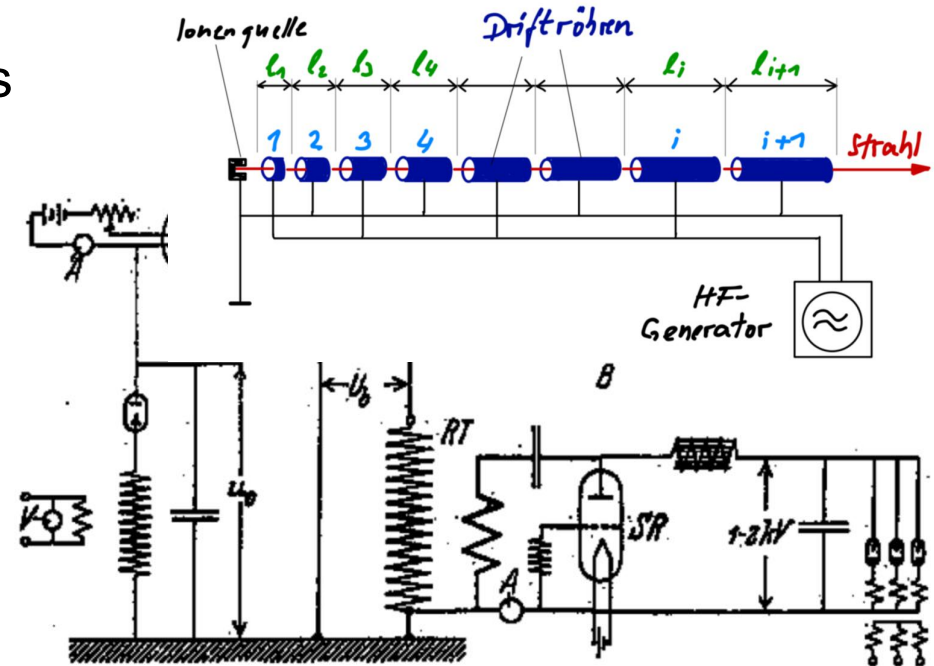
Linear accelerators — Linacs

- Acceleration with electric RF fields

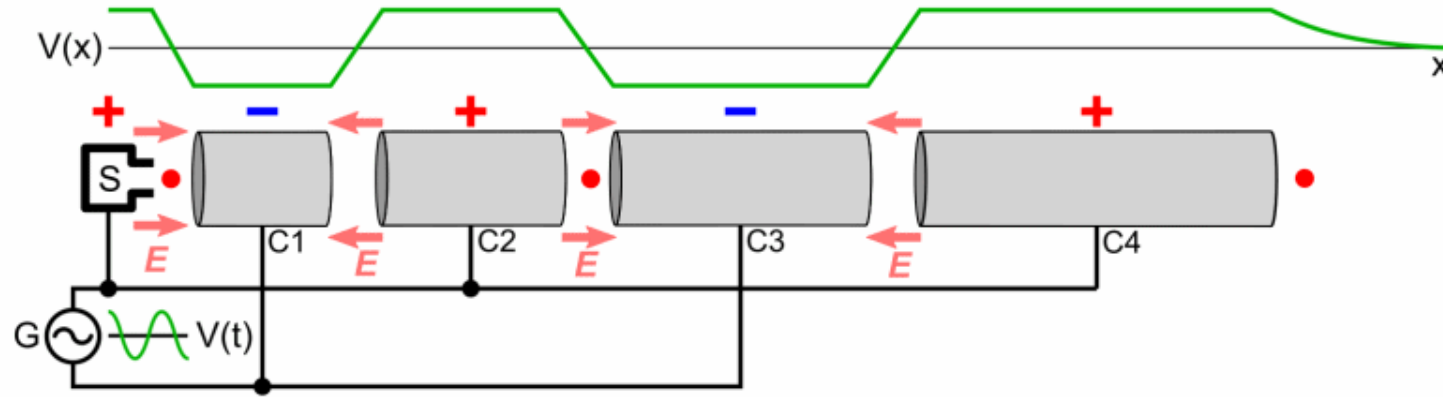
$$U(t) = U_0 \sin \omega t$$

- Drift tubes of increasing length

- 1925: proposed by Ising
- 1928: proof of principle by Wideröe



Wideröe linac — illustration



Reference: https://en.wikipedia.org/wiki/Cyclotron#/media/File:Linear_accelerator_animation_16frames_1.6sec.gif

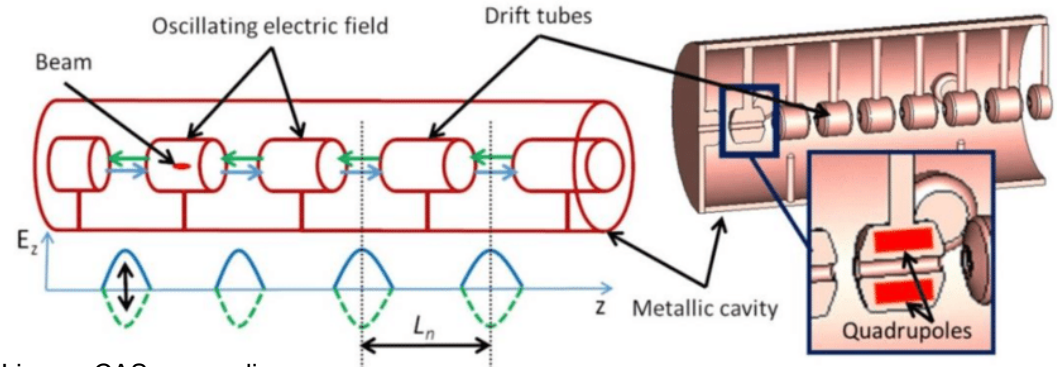
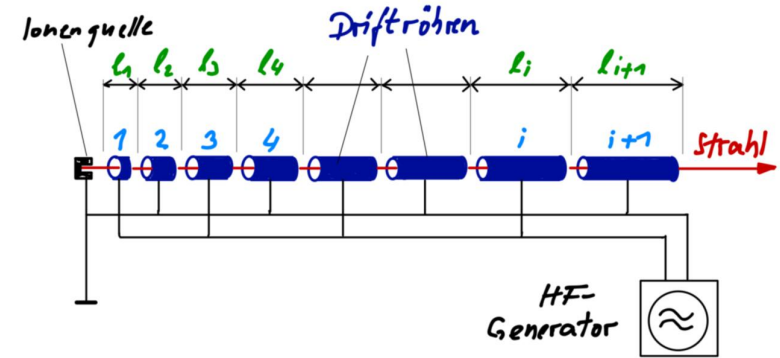
Linear accelerators — Linacs

- Acceleration with electric RF fields

$$U(t) = U_0 \sin \omega t$$

- Drift tubes of increasing length

- 1925: proposed by Ising
- 1928: proof of principle by Wideröe
- 1946: Alvarez structure



Reference: D. Alesini: Linacs, CAS proceedings

Drift tube linacs

- Energy after drift tube n for particle with charge q and $v_n \ll c$

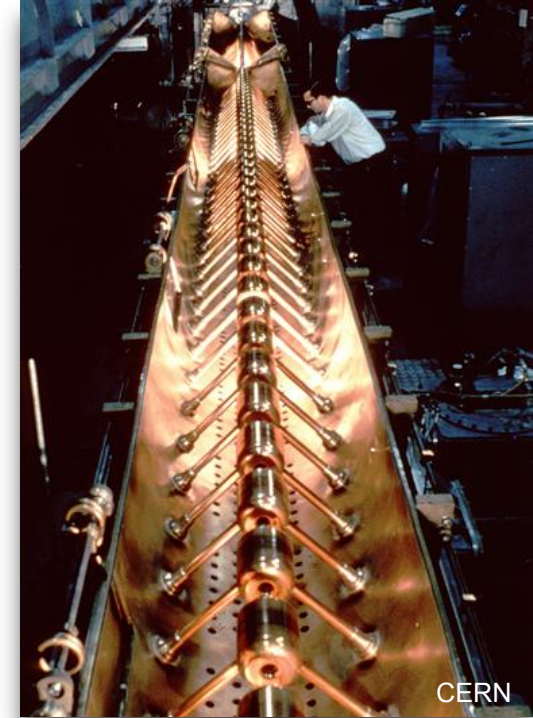
$$E_n = nqU_0 \sin \Psi_s \quad \text{with kinetic energy} \quad E_{n,\text{kin}} = \frac{1}{2}mv_n^2$$

- Running through a drift space lasts exactly half an RF period length $T_{\text{RF}}/2$. The distance between n^{th} and $(n+1)^{\text{th}}$ gap is then

$$l_n = \frac{v_n T_{\text{RF}}}{2} = \frac{v_n \lambda_{\text{RF}}}{2c} = \beta_n \frac{\lambda_{\text{RF}}}{2}$$

- Assuming a constant RF frequency $f_{\text{RF}} = c/\lambda_{\text{RF}}$ yields

$$l_n = \frac{1}{f_{\text{RF}}} \sqrt{\frac{nqU_0 \sin \Psi_s}{2m}} \propto \sqrt{n}$$



CERN

Reference: <https://cds.cern.ch/record/39283>

UNILAC at GSI

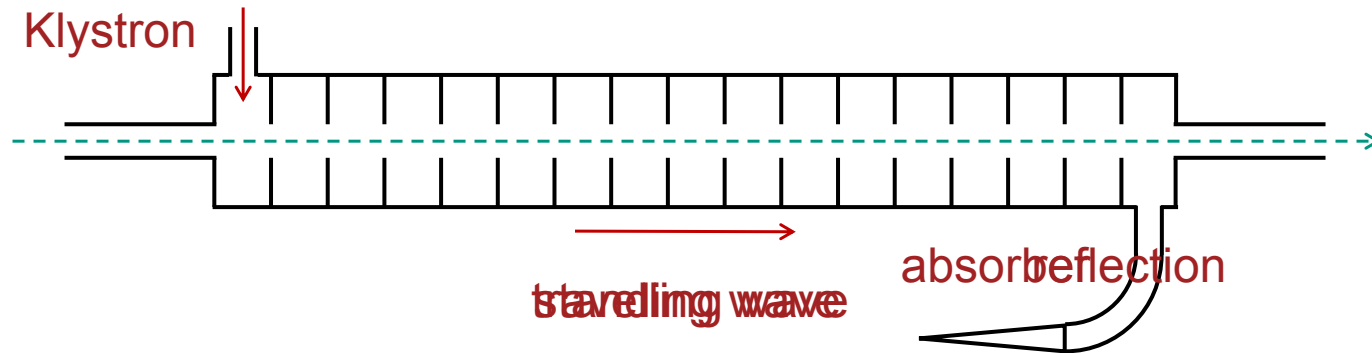


G. Otto, GSI Helmholtzzentrum für Schwerionenforschung

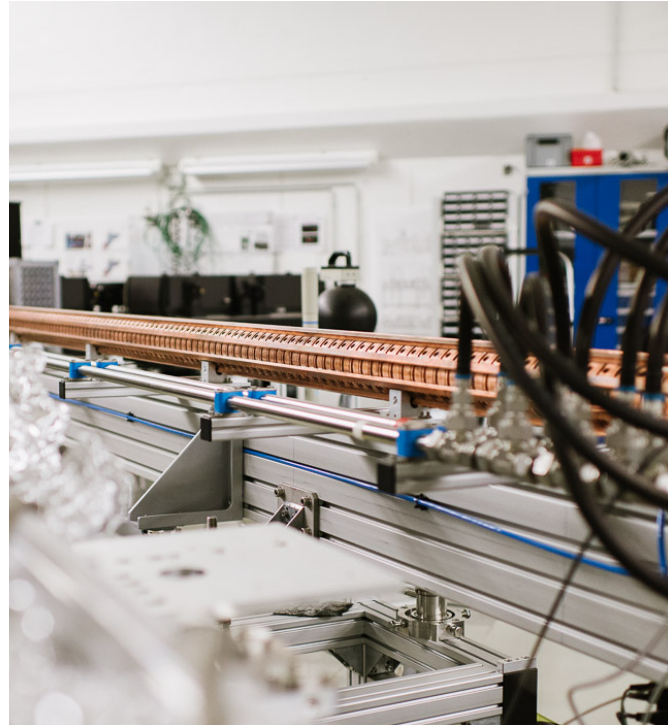
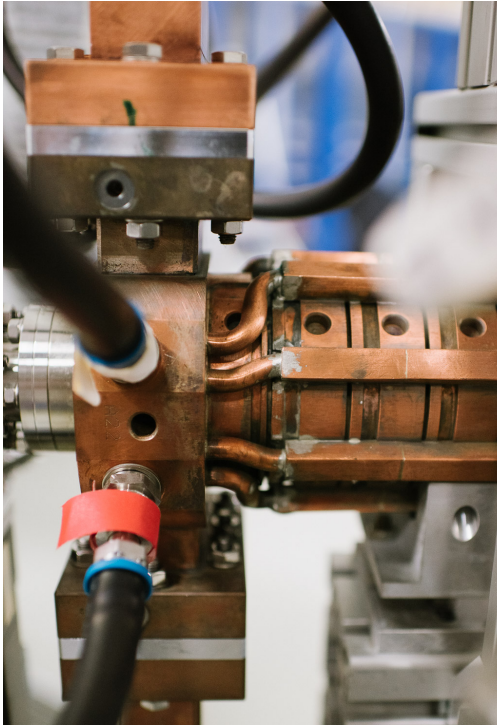
J. Hosan, FAIR GmbH

Electron linacs: waveguide structures

- Electrons very quickly reach relativistic velocities. $\Rightarrow v \approx c$
- Since their velocity does not increase, the length of drift tubes does not have to increase.
$$l_n = \frac{v_n T_{\text{RF}}}{2} = \frac{v_n \lambda_{\text{RF}}}{2c} = \beta_n \frac{\lambda_{\text{RF}}}{2} \approx \frac{\lambda_{\text{RF}}}{2}$$
- Many electron linacs use **waveguide structures** instead of drift tubes.

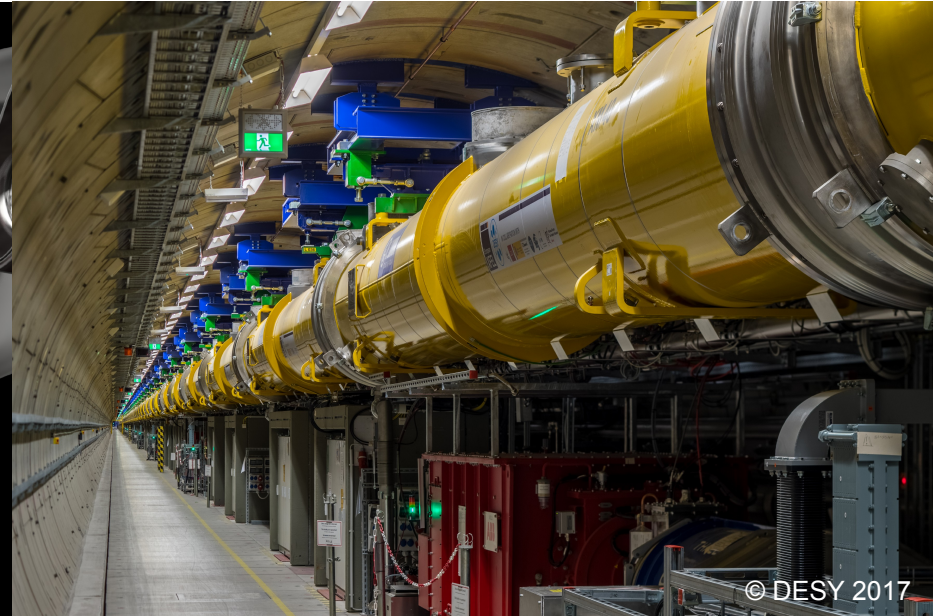
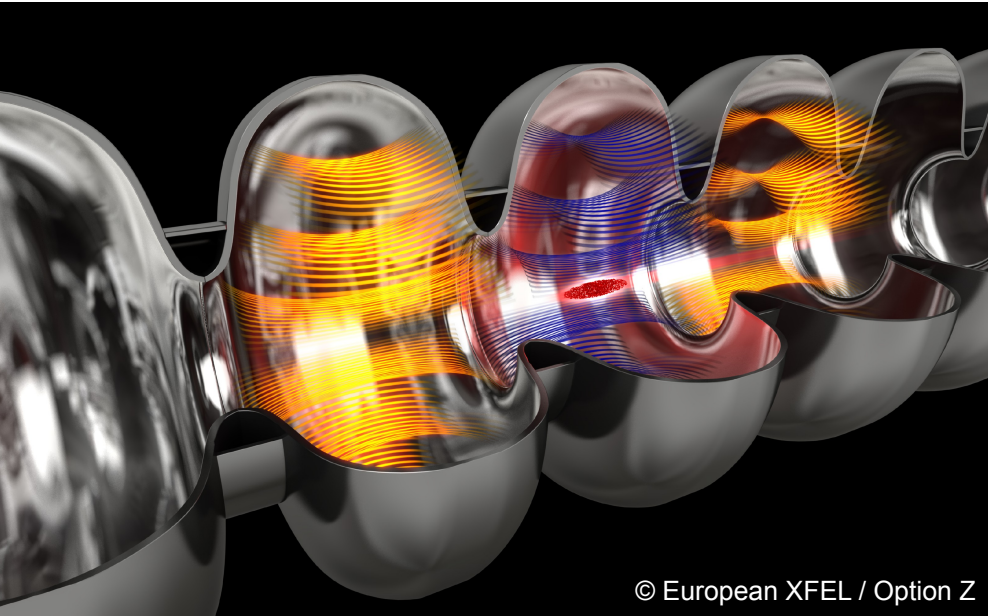


Example: The FLUTE linac



Reference: K. Heil

Example: European XFEL



- 1.7 km long linac, 17.5 GeV beam energy
- 96 superconducting “Tesla” cavities: Nb, $T < 2\text{ K}$

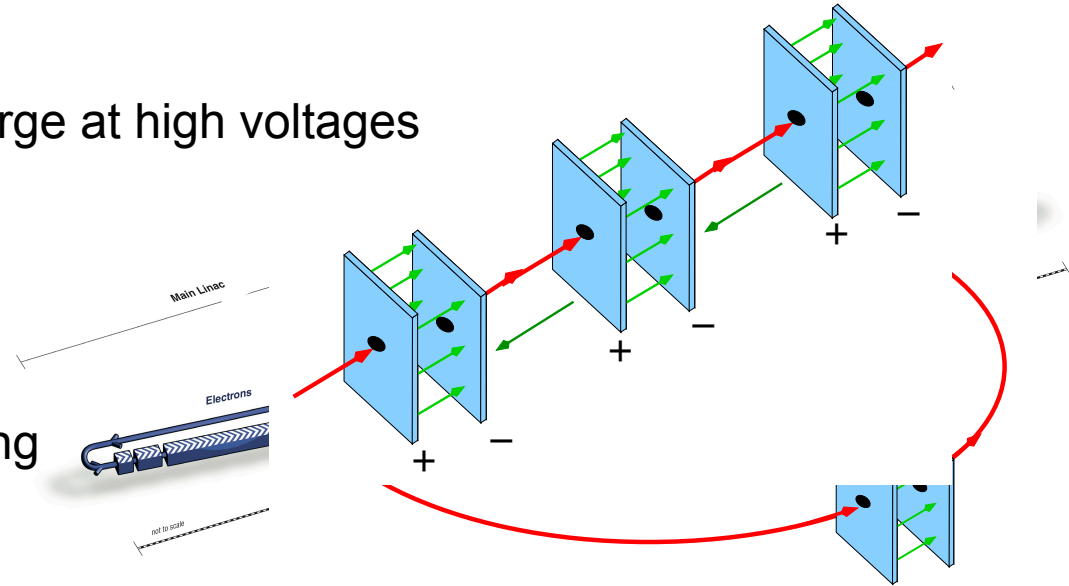
Transition to circular accelerators

■ Electrostatic accelerators:

- Corona formation and discharge at high voltages
- Staging not possible

■ Linacs:

- $E \propto L$
- Accelerator becomes very long for high energies

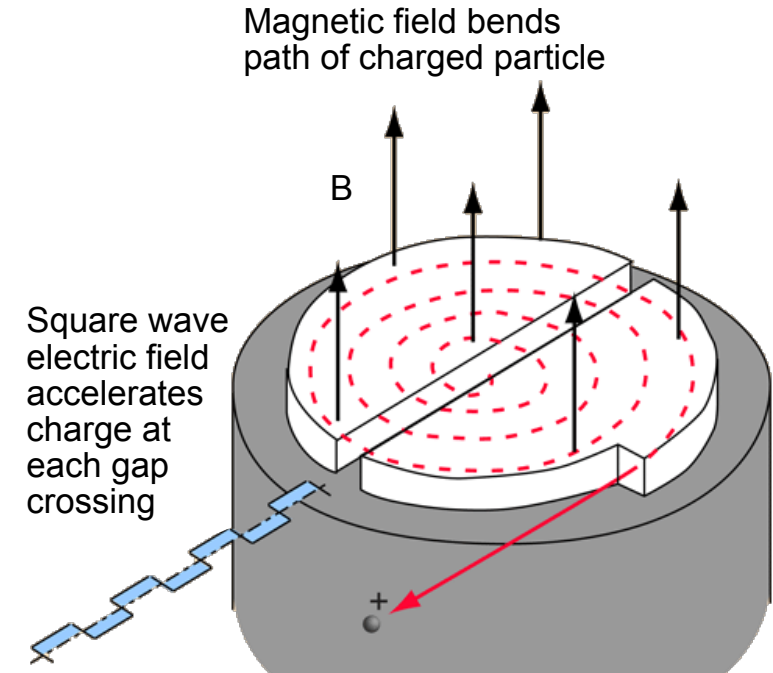


■ Solution:

- Switch to a **circular trajectory** and use **same acceleration stage** several times

Cyclotron

- Acceleration in the RF field between the so-called “dees”
- Circular orbit with increasing radius due to vertical magnetic field
- Proposed by Lawrence 1930
- First cyclotron in operation in 1932, built with Livingston

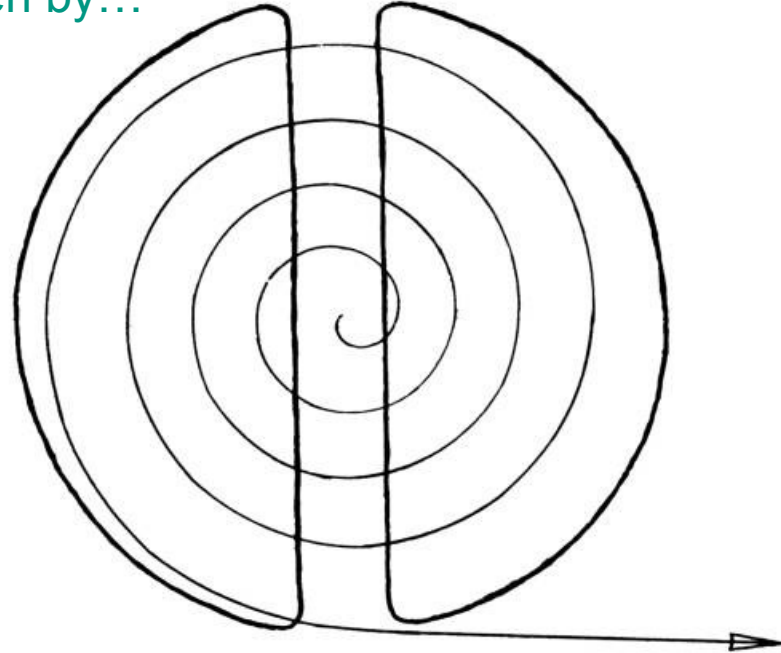


Reference:

<http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/cyclot.html>

The cyclotron humoristically

The cyclotron, as seen by...

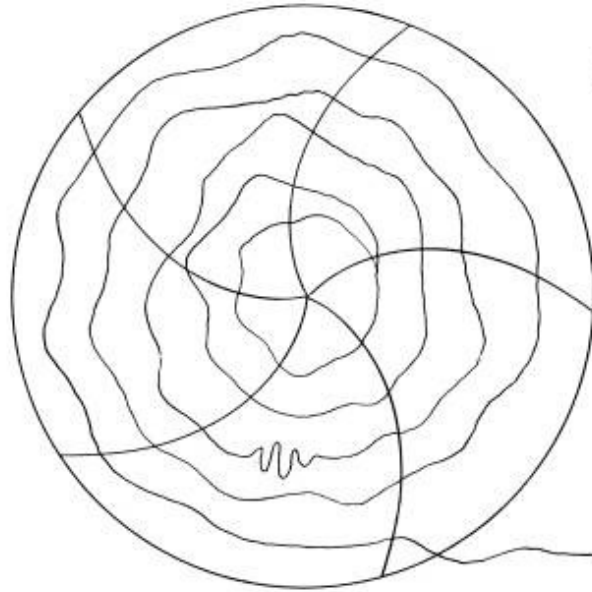


... the inventor.

David L. Judd and Ron MacKenzie: https://people.nsl.msui.edu/~lund/msu/phy905_2018/lec_lund/judd_cartoon.pdf

The cyclotron humoristically

The cyclotron, as seen by...



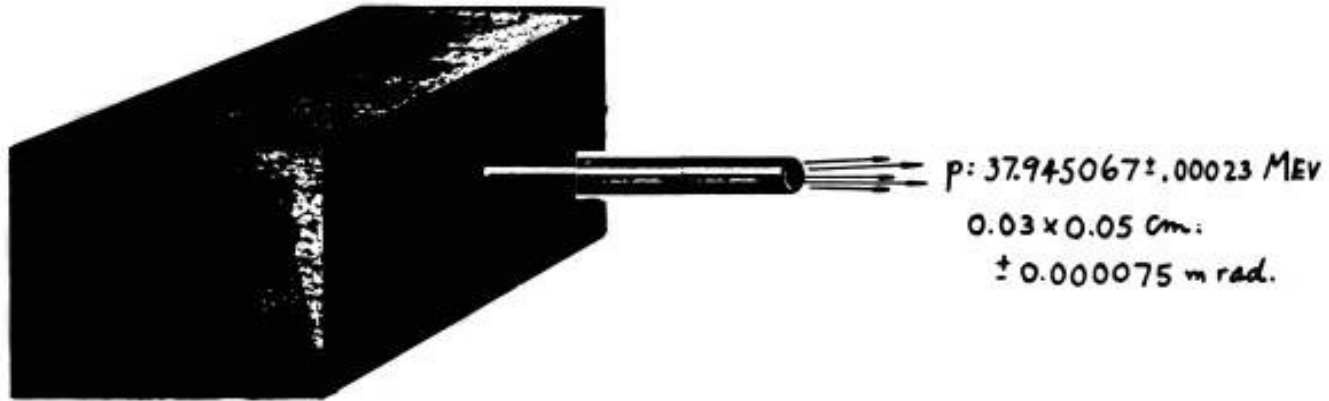
$$r = r_0 \left[1 + \left(\frac{f r \omega}{c} \right) \cos(3\theta + \delta_0 + \delta_1 r) + \right. \\ \left. \left(\frac{f r \omega}{c} \right)^2 \cos(5\theta + \delta_2 - \delta_3 r^2) + \right. \\ \left. \left(\frac{f r \omega}{c} \right)^3 \cos(7\theta + \delta_4 - \delta_5 r^3) + \right. \\ \left. \dots \right] \times \left\{ \frac{e^{3/5} r^2 \ln Z}{1 + \left(\frac{a}{r} \right)^{7/4}} \right\}$$

$$\frac{d\phi}{dt} = \left[\sin(\omega t - k\phi) - \sin k\phi - \frac{3}{5} f f_1 f_2 f_3' \right] \frac{e V_0}{2\pi \omega}$$

... the theoretical physicist.

The cyclotron humoristically

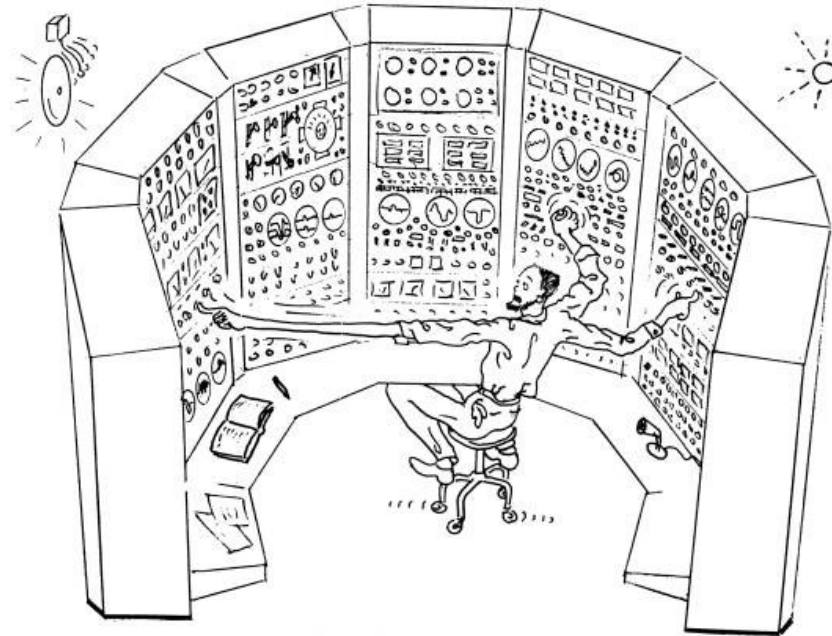
The cyclotron, as seen by...



... the experimental physicist.

The cyclotron humoristically

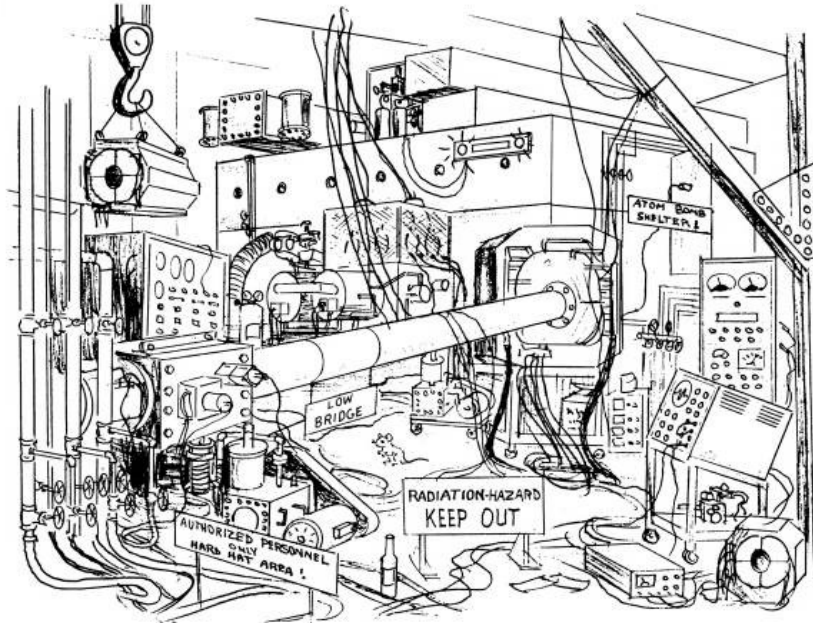
The cyclotron, as seen by...



... the operator.

The cyclotron humoristically

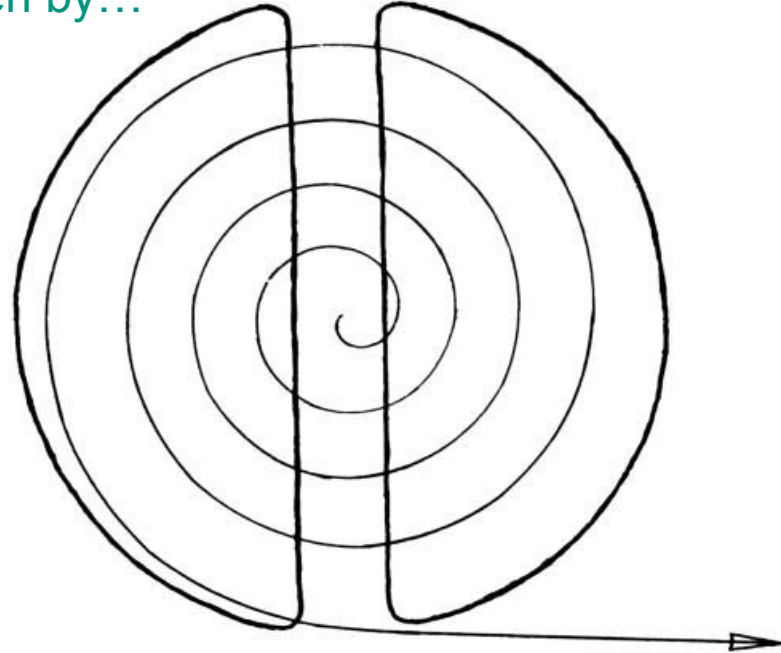
The cyclotron, as seen by...



... the visitor.

The cyclotron humoristically

The cyclotron, as seen by...



... the student.

Cyclotron: principle

- Vertical homogeneous magnetic field: $\vec{B} = (0,0,B_z)$
- A particle with velocity $\vec{v}$ and charge q experiences the force

$$\vec{F} = \dot{\vec{p}} = \frac{d}{dt}(m\vec{v}) = q\vec{v} \times \vec{B}.$$

- Since the particles motion is confined to the horizontal plane ($v_z = 0$), we obtain

$$\dot{p}_x = m\dot{v}_x = qv_y B_z \quad \text{and} \quad \dot{p}_y = m\dot{v}_y = -qv_x B_z.$$

- Second differentiation and combination yields

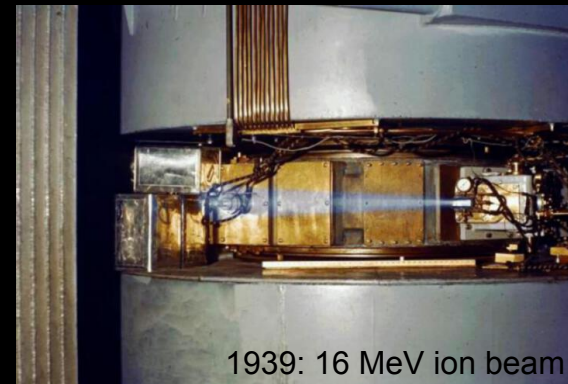
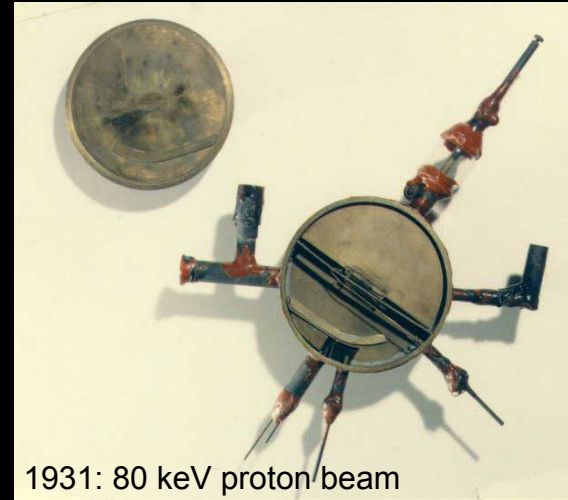
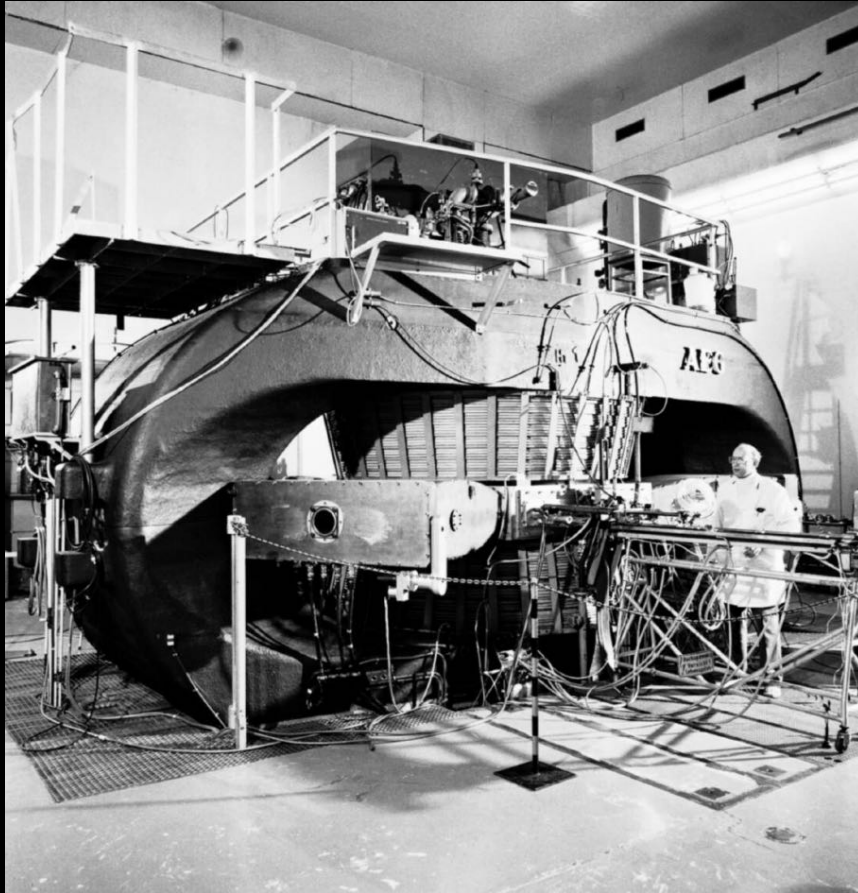
$$\ddot{v}_x + \frac{q^2}{m^2} B_z^2 v_x = 0 \quad \text{and} \quad \ddot{v}_y + \frac{q^2}{m^2} B_z^2 v_y = 0.$$

- Solutions of this equations of motion:

$$v_x(t) = v_0 \cos \omega_z t \quad \text{and} \quad v_y(t) = v_0 \sin \omega_z t$$

- The particles follow a circular path in the x-y-plane with the constant **cyclotron frequency**

$$\omega_z = \frac{q}{m} B_z$$



Applications today: proton therapy



PROTON
THERAPY

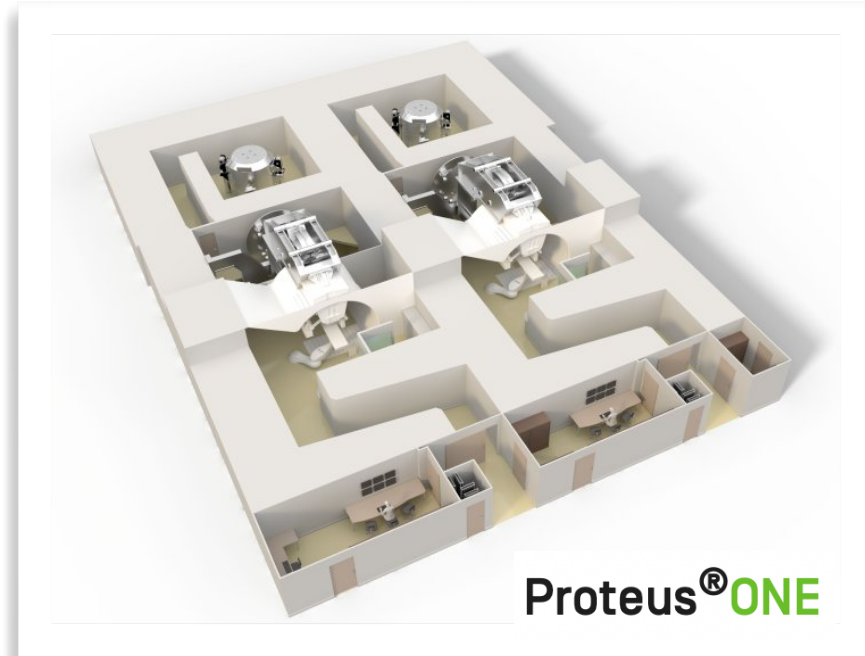


70 MeV proton cyclotron (2008)

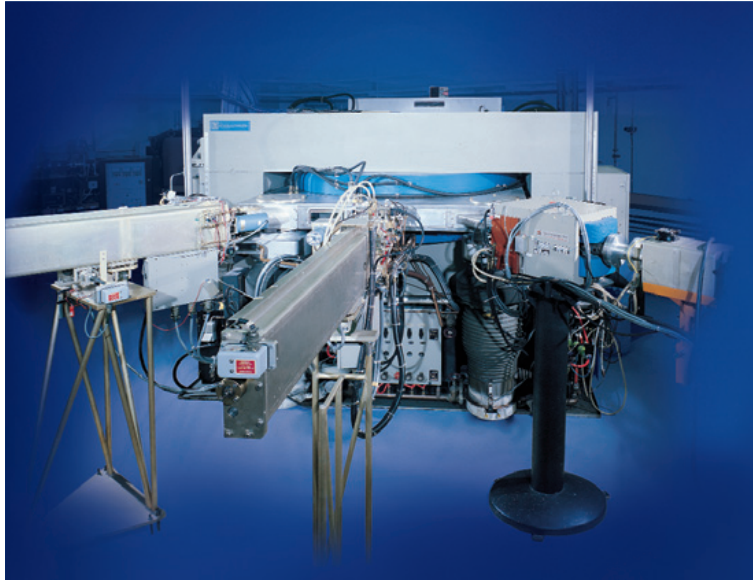
References:

https://de.wikipedia.org/wiki/Zyklotron#/media/Datei:Cyclotron_ARRONAX.jpg

<https://www.iba-protontherapy.com/>



Zyklotron AG at Campus North



- Production of radioisotopes for medicine and technology
- KIT spin-off (2000)

Reference: <https://www.zyklotron-ag.de/downloads>

Cyclotron: modifications

■ Cyclotron parameters:

- Acceleration with constant RF frequency $\omega_{\text{RF}} = \omega_z$
- Accelerated particle species: mainly protons, deuterons, α particles up to 22 MeV per electron charge
- Typical RF frequencies of about 10 MHz

■ Problem for high energies: $\beta \rightarrow 1$

- Cyclotron frequency decreases with energy according to

$$\omega_z = \frac{qB_z}{\gamma m_0}$$

■ Solutions:

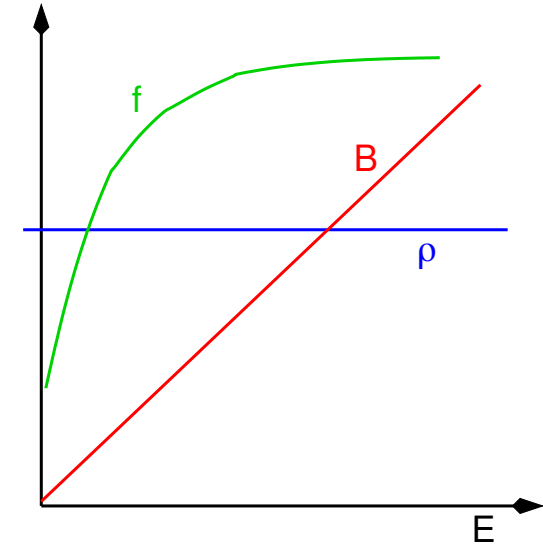
- Synchronous adjustment of the RF frequency (Synchro-Cyclotron)
- Variable magnetic field in radial direction (Isocyclotron) with

$$\omega_z = \frac{qB_z(r)}{\gamma m_0} = \text{const.}$$



Synchrotron

- Reminder: For constant magnetic field the orbit radius increases with particle energy: $\rho = E / (ecB)$
- For 1 GeV particles following magnetic fields and radii are technically feasible:
Iron dominated magnet: $B = 1.5 \text{ T}$, $\rho = 2.22 \text{ m}$
Superconducting magnet: $B = 5 \text{ T}$, $\rho = 0.67 \text{ m}$
→ Quickly becomes unpractical for high energies.
- In 1943 Mark Oliphant combines three old ideas — acceleration in resonator gaps, variation of RF frequency and pulsed magnetic fields — to a new concept: [the synchrotron](#).
- **Breakthrough:** Discovery of the [phase focusing](#) principle in the synchrotron (Veksler 1944, McMillan 1945).
- In 1947 the first synchrotron was built for 700 MeV electrons.
- Today: Particle energies up to several TeV

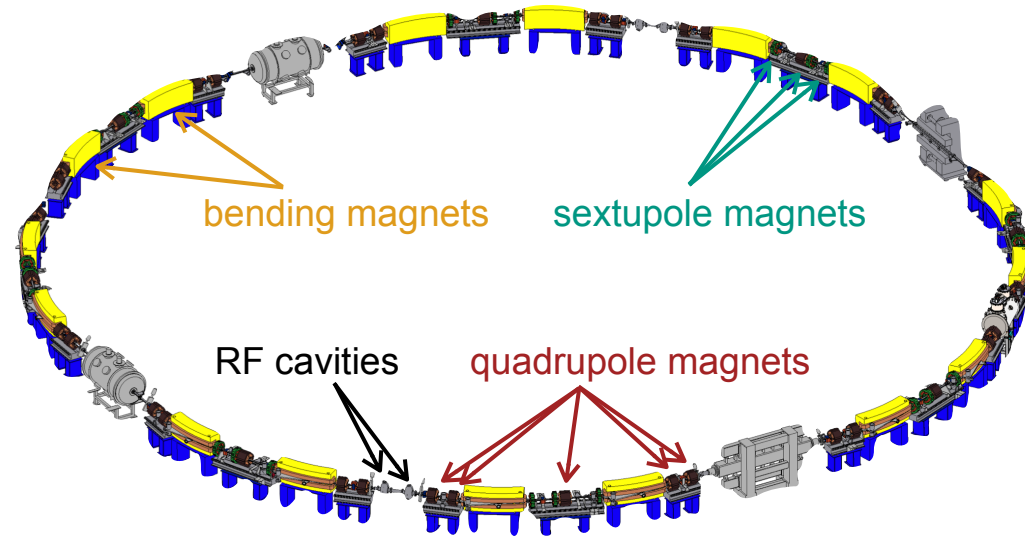


Synchrotron

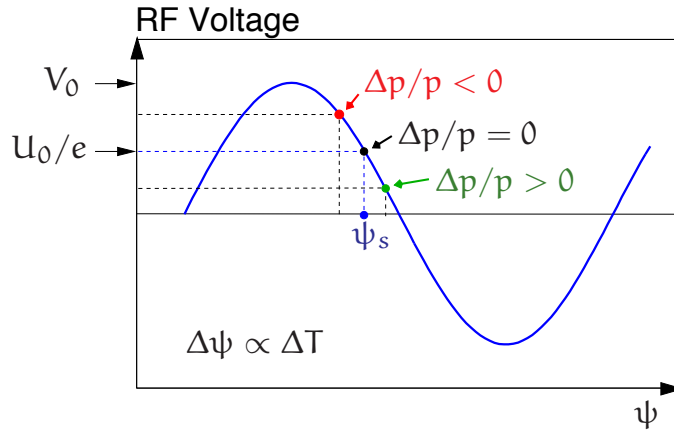
- Stationary orbit, therefore light magnetic structure (“lattice”)
- Acceleration in RF waveguides (“cavities”)
- Relation between circumference and revolution frequency f_{rev} and the RF frequency:

$$f_{\text{RF}} = h f_{\text{rev}}$$

h : harmonic number

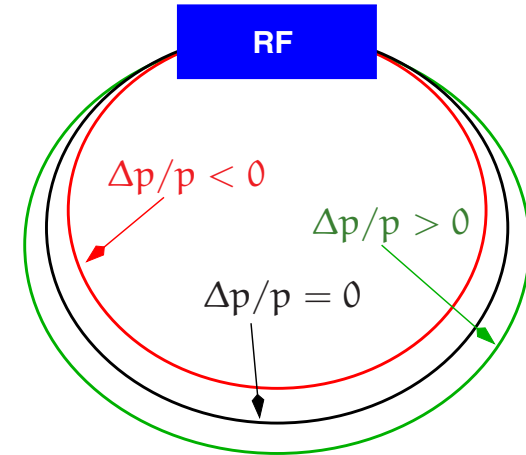


Phase focusing



- Acceleration in RF cavity by sinusoidal altering RF field
 - RF oscillation is synchronous with the revolution of the “reference particle”
- $$p = p_0 \rightarrow \Delta p/p = 0$$

- Arrival time determines acceleration voltage
- The particles oscillate around the stable phase Ψ_s with synchrotron frequency Ω_s
 $\Rightarrow$ Particles form bunches



Storage rings

- **Until now:** Accelerated particle beam is extracted focused on a target/sample.
 - Limited duration in the synchrotron: effects like scattering at residual gas molecules and intra-beam scattering not relevant
- **Storage rings:** The beam is stored at fixed energy for many hours after acceleration
 - Application of synchrotrons as light sources and colliders
 - Ultra-high vacuum needed ($<10^{-10}$ mbar)
 - Small beam sizes ($\rightarrow$ interaction of particles within the beam) for sufficient “**luminosity**” (collider) or “**brilliance**” (synchrotron light source)
 - Additionally in the collider: interactions between the particles of the two beams



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Emission of synchrotron radiation

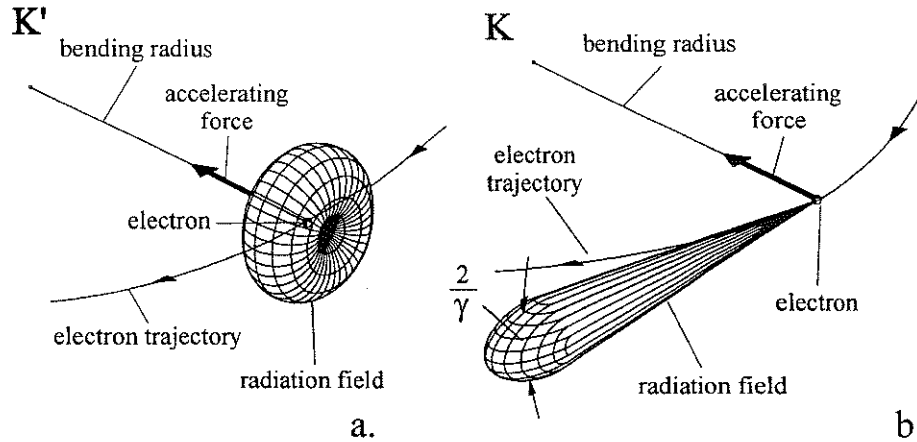


Fig. 2.3 Transformation of the axially symmetric radiation distribution in the centre of mass frame (a) into the sharply forward peaked distribution in the laboratory frame K (b).

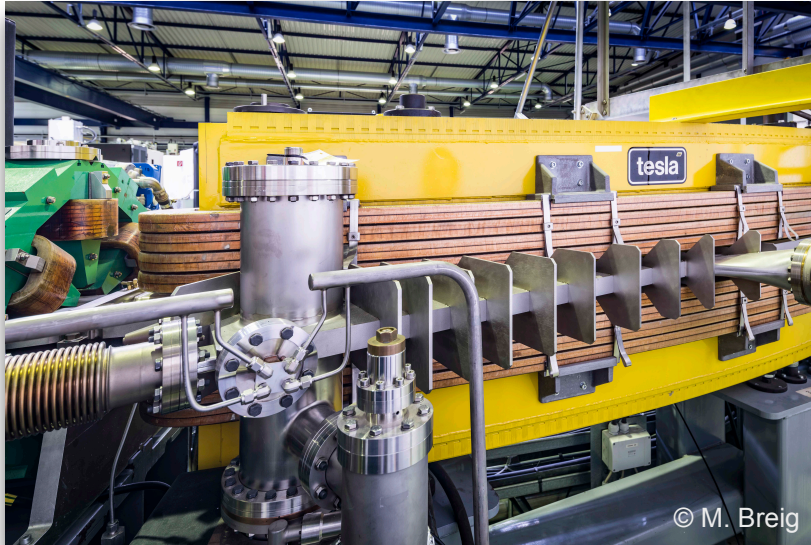
K. Wille, The Physics of Particle Accelerators

$$P_\gamma = \frac{c C_\gamma}{2\pi} \frac{E^4}{\rho^2}$$

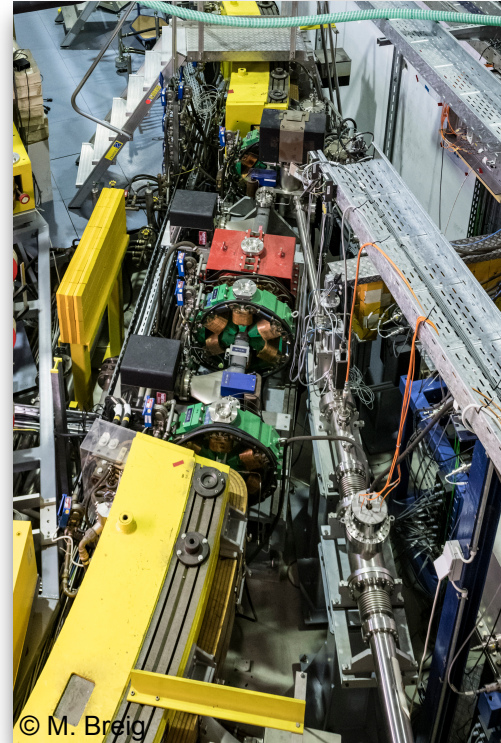
$$C_\gamma = \frac{e^2}{3\epsilon_0} \frac{1}{(m_e c^2)^4} = 8.8460 \times 10^{-5} \frac{\text{m}}{\text{GeV}}$$

- Broad spectrum
- High flux
- High spectral brightness
- Polarization can be controlled
- Pulsed time structure

Synchrotron light sources



Light port: synchrotron radiation is extracted...

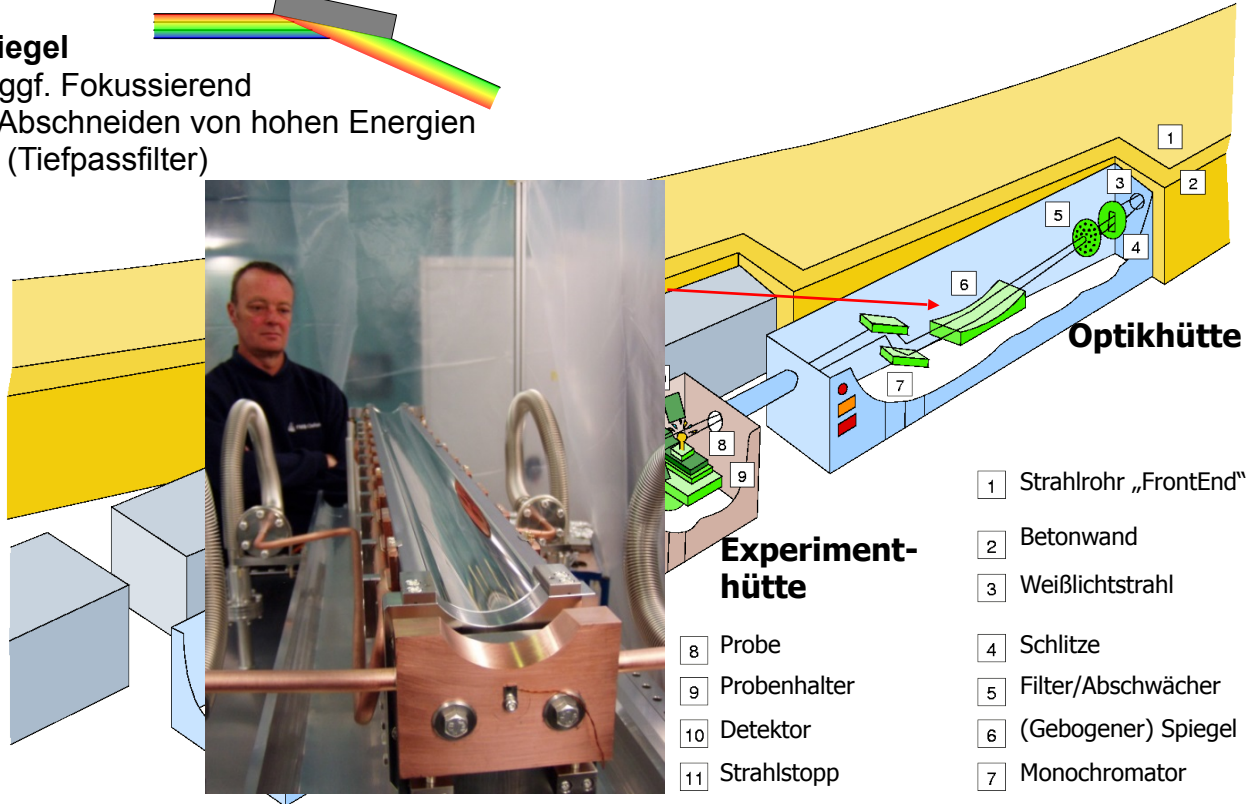
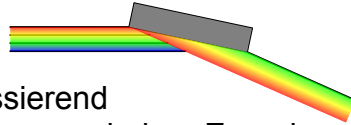


... and transferred to the beamlines.

Strahlrohr Aufbau

Spiegel

- ggf. Fokussierend
- Abschneiden von hohen Energien (Tiefpassfilter)

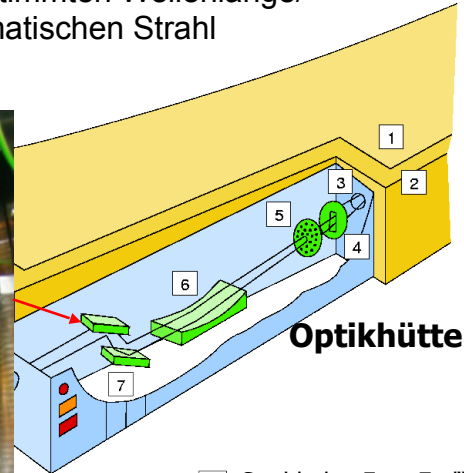
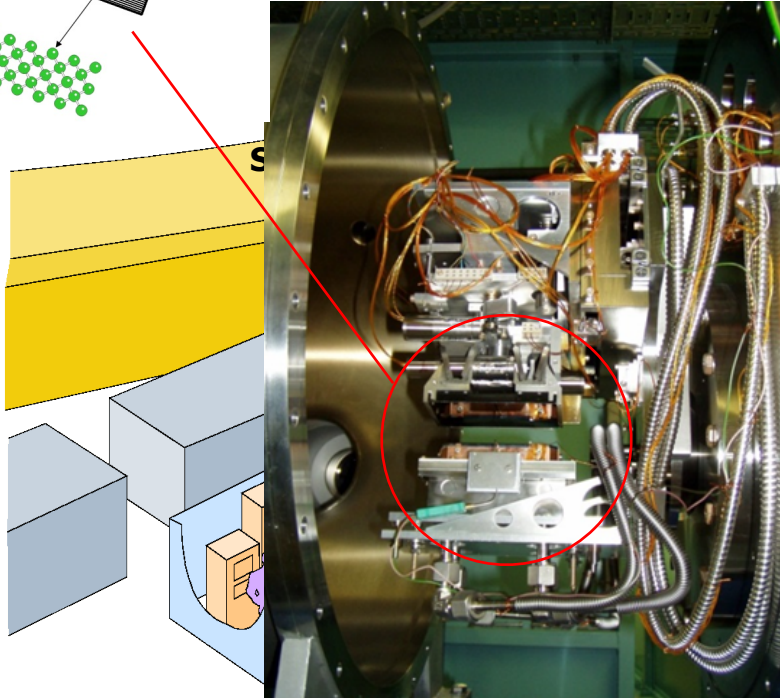
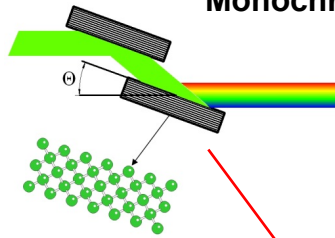


- 1 Strahlrohr „FrontEnd“
- 2 Betonwand
- 3 Weißlichtstrahl
- 4 Schlitz
- 5 Filter/Abschwächer
- 6 (Gebogener) Spiegel
- 7 Monochromator

- 8 Probe
- 9 Probenhalter
- 10 Detektor
- 11 Strahlstopp

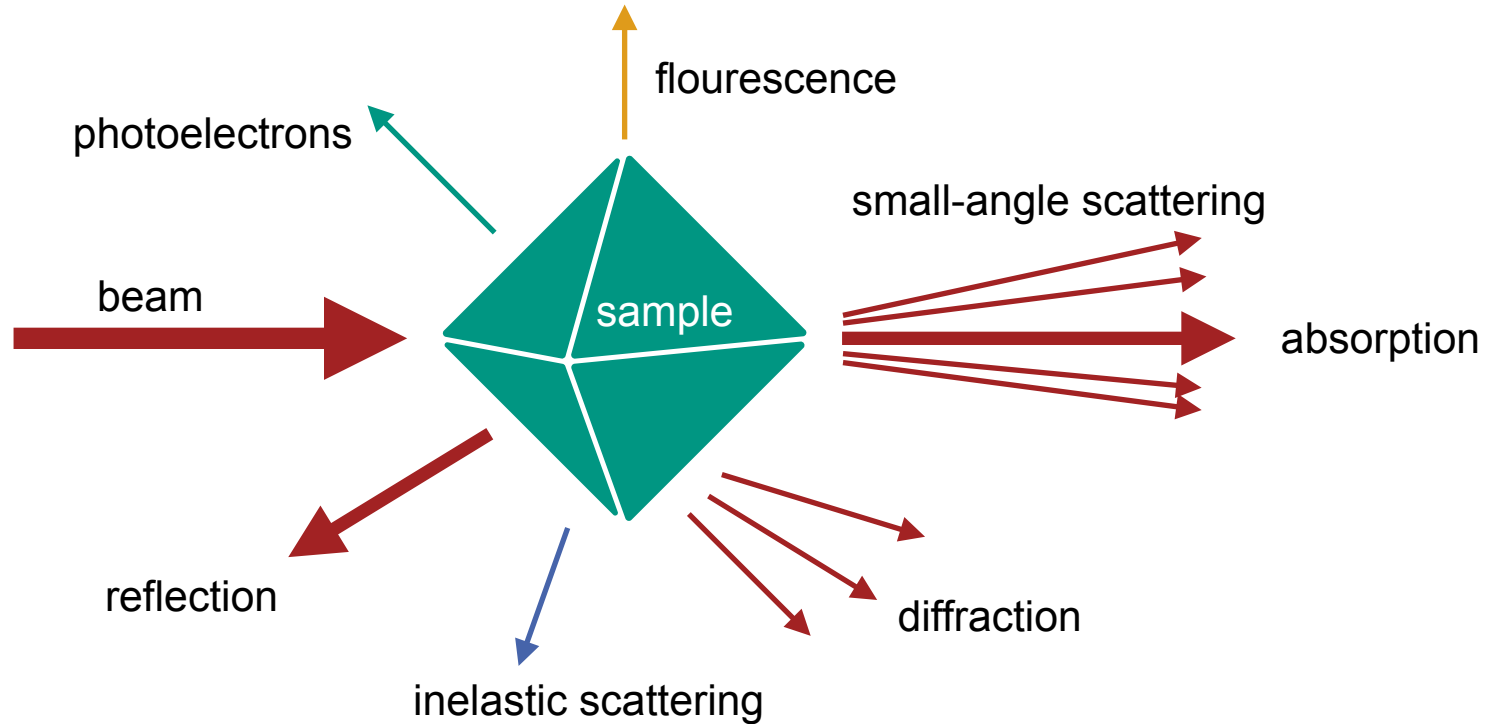
Strahlrohr Aufbau

Monochromator: dient zur Auswahl einer bestimmten Wellenlänge/
Energie aus dem polychromatischen Strahl



Optikhütte

- 1 Strahlrohr „FrontEnd“
- 2 Betonwand
- 3 Weißlichtstrahl
- 4 Schlitze
- 5 Filter/Abschwächer
- 6 (Gebogener) Spiegel
- 7 Monochromator



More than 50 light sources around the world



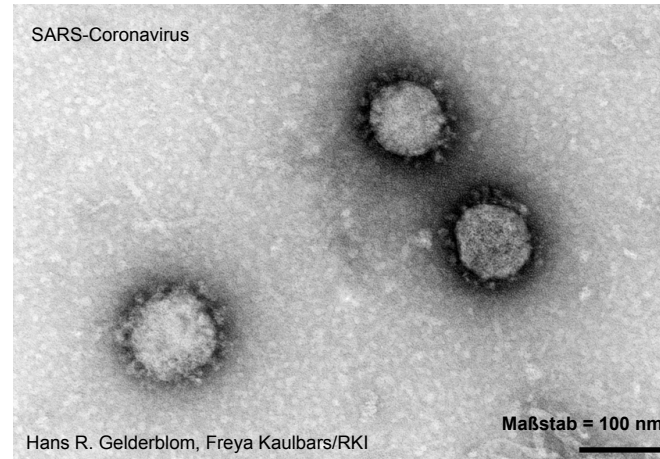
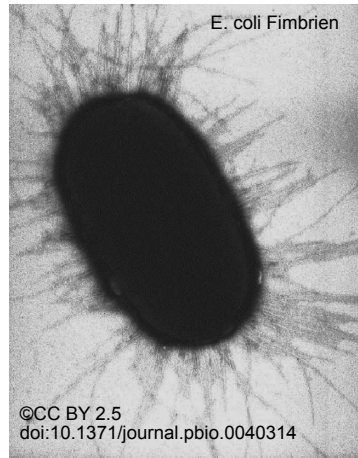
An accelerator “just” to make light?

- Synchrotron radiation is special light with unique properties
- One important point: structures smaller than half the wavelength become blurred



An accelerator “just” to make light?

- Optical light microscopes can resolve bacteria (about 1000 nm) but not viruses (about 10 nm)
- Bacteria detected 1675 (light microscope), viruses 1940 (electron microscope).



$$\lambda = 10 \text{ nm} :$$

$$E_{\text{rad}} = \frac{hc}{\lambda} = 126 \text{ eV}$$

$$\Rightarrow f = 3 \times 10^{16} \text{ Hz}$$

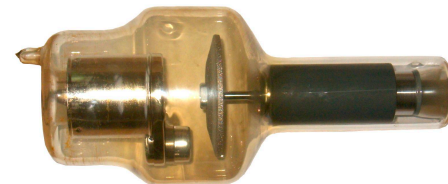
- X-rays needed for many scientific investigations

Methods to generate X-rays

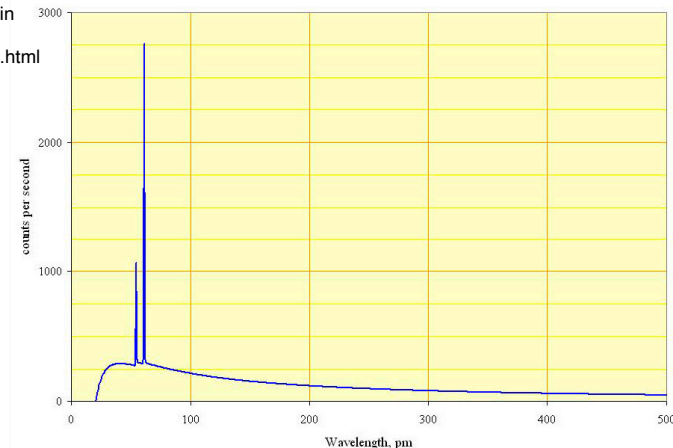
- Only few known methods
- **Radioactive decay**
 - can not be controlled
 - single frequency
 - emission in the entire solid angle range
- **X-ray tube**
 - Continuous radiation
 - Non-adjustable line on top of broadband spectrum
 - Emission in the entire solid angle range
 - Low brilliance
- **Synchrotron radiation**
 - Expensive
 - Large-scale facility



By Nobel foundation - Public Domain
http://nobelprize.org/nobel_prizes/physics/laureates/1901/rontgen-bio.html



Von Rschiedon, CC BY-SA 3.0
<https://commons.wikimedia.org/w/index.php?curid=1993131>



<https://commons.wikimedia.org/wiki/File:TubeSpectrum.jpg>

Brilliance

low brilliance

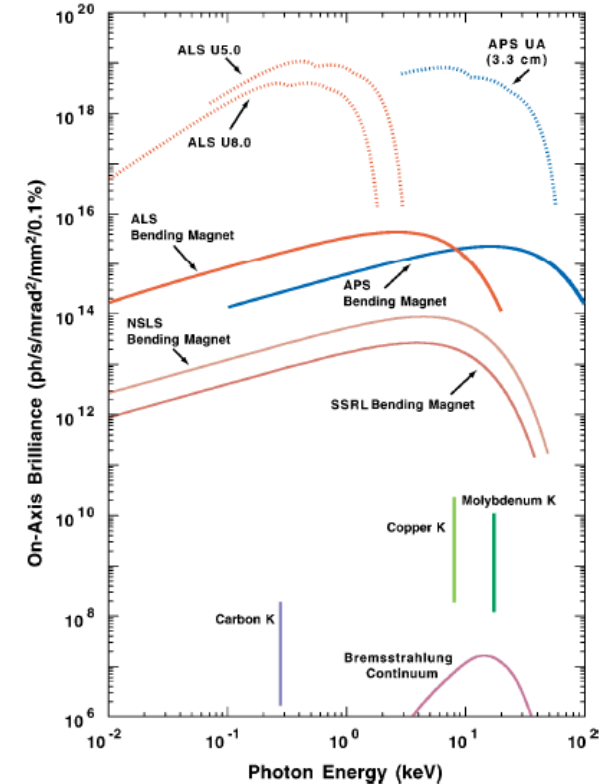


high brilliance

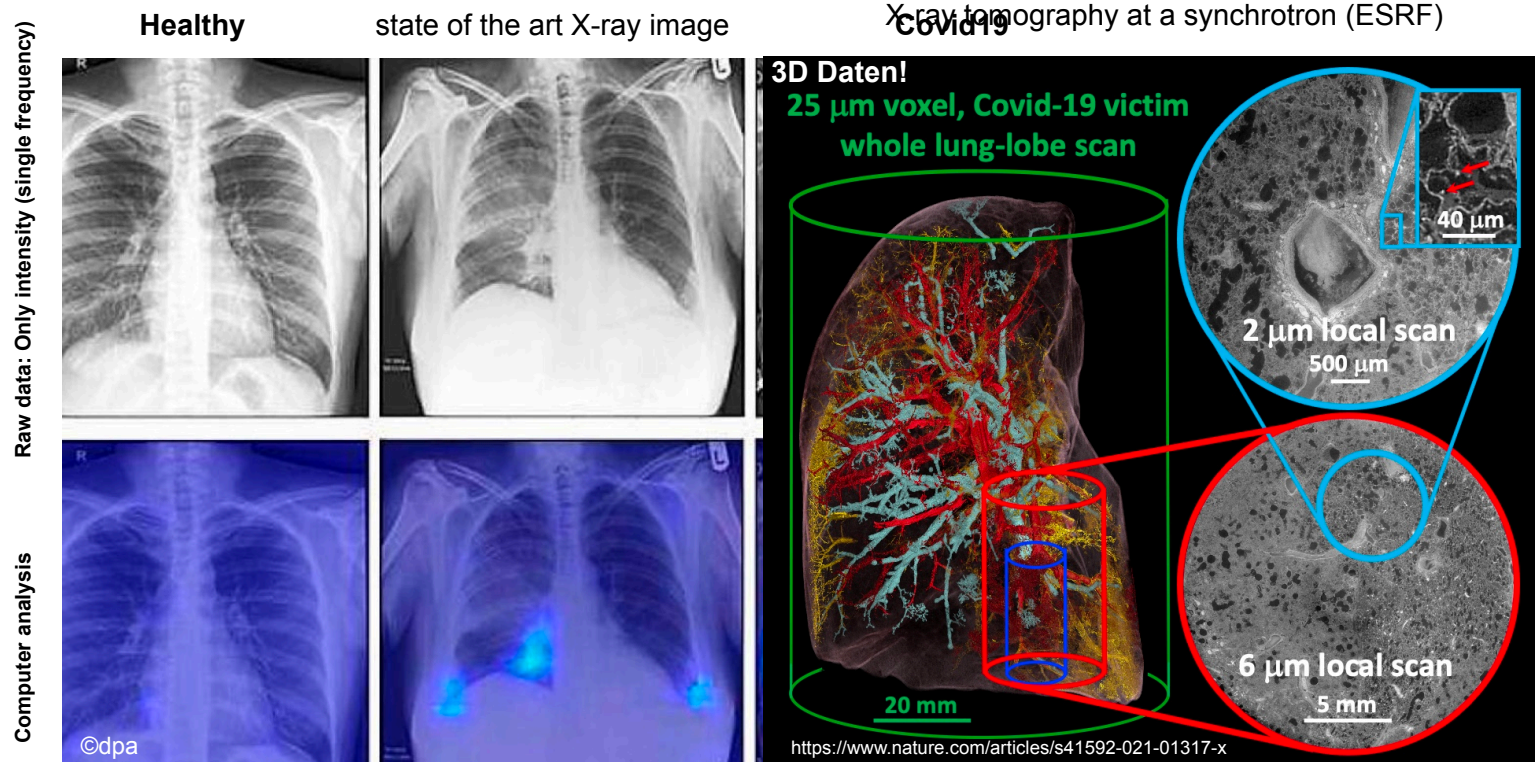


- Property of the source
- Measure of light quality
- Light intensity per time, area, solid angle, bandwidth

$$B = \frac{\Delta N}{t \cdot A \cdot \Delta\Omega \cdot \frac{\Delta\lambda}{\lambda}}$$



Impact of brilliance



Particle physics

Fixed target experiments

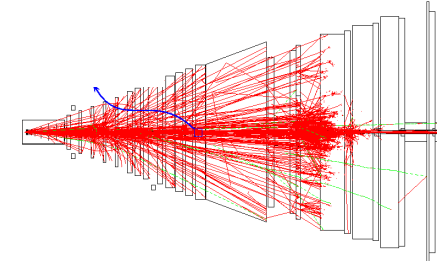
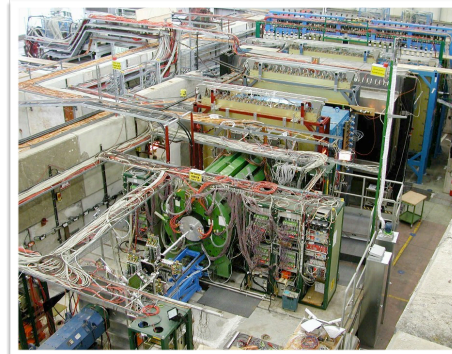
- extracted beam hits a target
- high event rate
- limited energy reach

$$E_{\text{cm}} \propto \sqrt{E_{\text{beam}}}$$

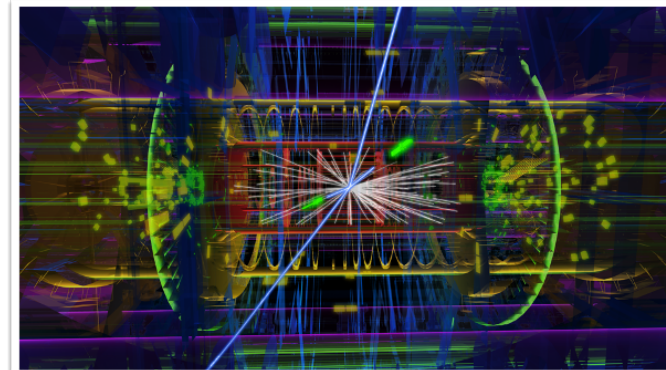
Beam-beam collisions

- low event rate (luminosity)
- high energy reach

$$E_{\text{cm}} = E_{\text{beam 1}} + E_{\text{beam 2}}$$



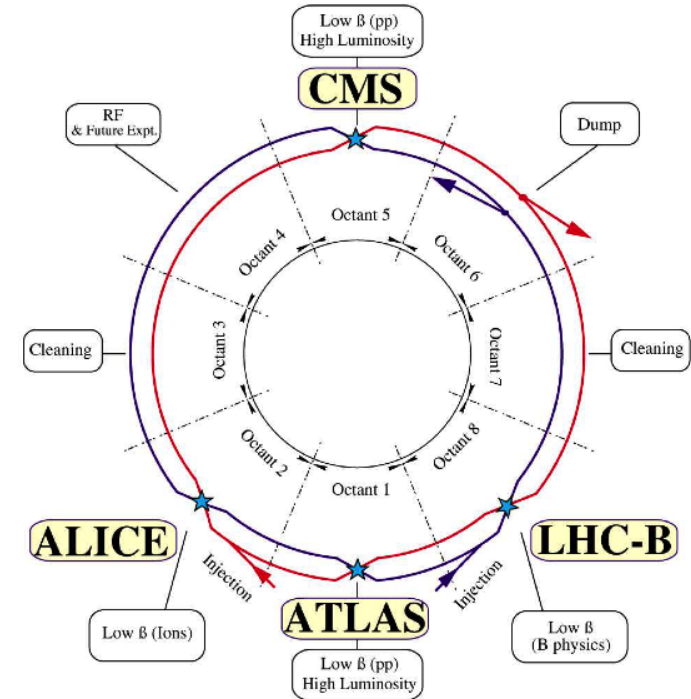
fixed target event $p + W \rightarrow \text{xxxxx}$



ATLAS event display:
 $H \rightarrow e^+ + e^- + \mu^+ + \mu^-$

High-energy particle colliders

- Two particle beams are collided in special interaction regions, where the beams are squeezed to minimum sizes.
- **Advantage** of storage rings compared to linear colliders: **more than one interaction region possible**
- **Largest colliders so far: LEP/LHC at CERN in Geneva with 27 km circumference**
LEP (e^+e^-): 100 GeV beam energy (single ring)
LHC (pp): 7 TeV beam energy (double-ring)



Luminosity

■ (Most) important parameter of colliders

■ Both beams are on same trajectory with $\vec{v}_1 = -\vec{v}_2$ (“head-on collision”)

■ Consideration: 1 particle collides with beam of size A

■ Probability for collision: $N (\sigma_p/A)$

■ For N particles/beam: $N^2 (\sigma_p/A)$

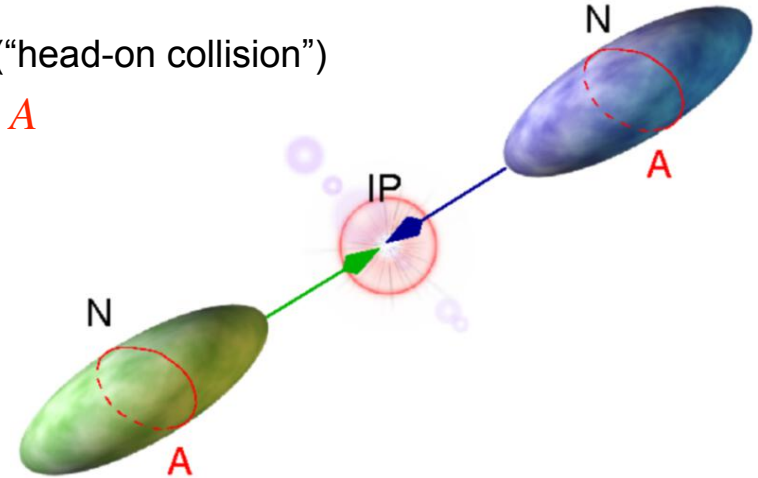
■ Bunches cross with revolution frequency f_{rev}

Event rate: $\dot{N}_p = (N^2 f_{\text{rev}}/A) \sigma_p = \mathcal{L} \sigma_p$

$$\mathcal{L} = \frac{I_+ I_-}{4\pi e^2 f_{\text{rev}} n_b \sigma_x \sigma_y}$$

⇒ Beam current $I = eNf_{\text{rev}}$, number of bunches n_b , beam size at interaction point (IP) $\sigma_{x,y}$

⇒ Typical values: $\mathcal{L} \approx 10^{30}$ bis $10^{34} \text{ cm}^{-2}\text{s}^{-1}$

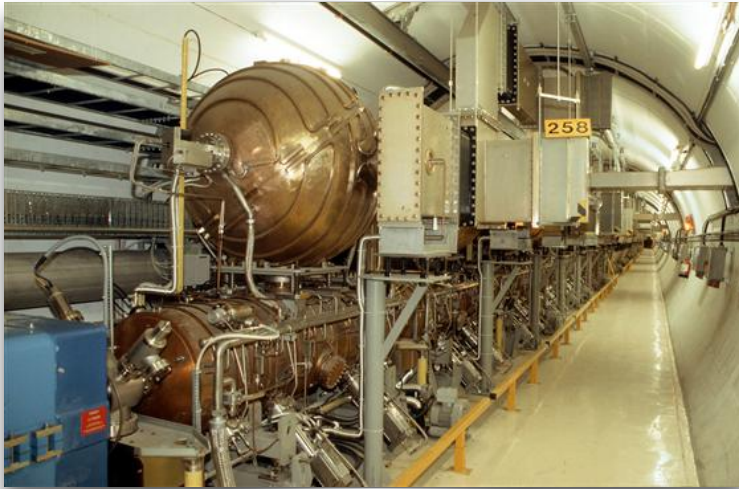


Large-scale synchrotrons: LEP



Reference: <http://cds.cern.ch/record/39027>

LEP components



- Circumference 26.7 km
- 3336 dipole magnets (length: 5.75 m, weight: 4.6 t, magnetic field up to 0.1 T)
- more than 800 quadrupole magnets



- 272 superconducting (sc) Nb-Cu cavities
- 16 sc Nb cavities
- 48 (+8) Cu cavities
- Acceleration voltage of up to 3570 MV

References: <https://cds.cern.ch/record/39293> and .../41673

Large-scale synchrotrons: LHC



Reference: <https://supernova.eso.org/exhibition/images/cern-aerial-cc/>

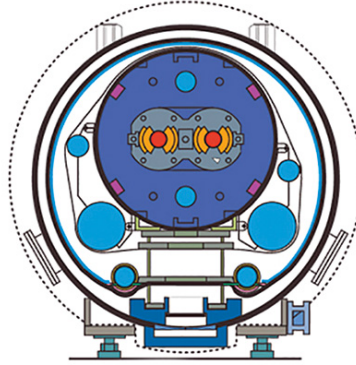


Large Hadron Collider

- Protons and heavy ions
- Double-ring collider
 - ⇒ 2808 bunches per ring (LEP: 16)
- 1232 sc bending magnets
 - ⇒ cooled with super-fluid helium 1.9 K
 - ⇒ length 14.3 m, field 8.33 T (LEP: 0.1 T)
- 8 cavities per beam: 16 MV (LEP 3570 MV)

- $\mathcal{L} \approx 1.0 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

(LEP: $\mathcal{L} \approx 1.6 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$)



CERN

Choice of particle species

Hadrons

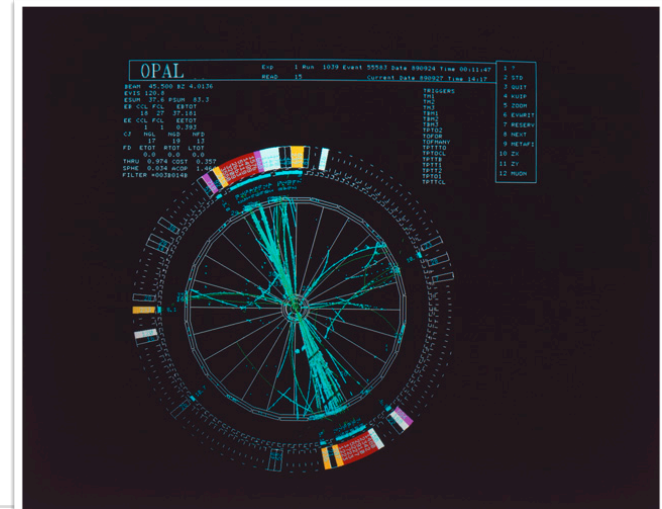
- Heavier, easier to reach high energies
→ discovery machines (“frontier of physics”)
- Don’t radiate (much)
- Collision of quarks → not all nucleon energy available in collision
→ huge background

Electrons & positrons

- Beam dynamics driven by emission of synchrotron radiation
- Elementary particles
- Well-defined CM energy → precision measurements
- Polarisation possible

$$\begin{aligned}
 m_p &= 938 \text{ MeV}/c^2 & E &= 10 \text{ GeV} & \rightarrow \gamma_p &= 11 \\
 m_e &= 0.511 \text{ MeV}/c^2 & & & \rightarrow \gamma_e &= 19570
 \end{aligned}$$

Event display of OPAL at LEP



Basics of beam dynamics

- A bunch typically consists of some 10^9 - 10^{11} particles.
- Different processes (synchrotron radiation, scattering at residual gas molecules, ...) lead to a distribution of beam energy around the nominal energy.
- Without focusing, the vacuum chambers would have to be large requiring extremely strong magnets.
- Electric and magnetic fields are used to steer and focus the particle beams:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

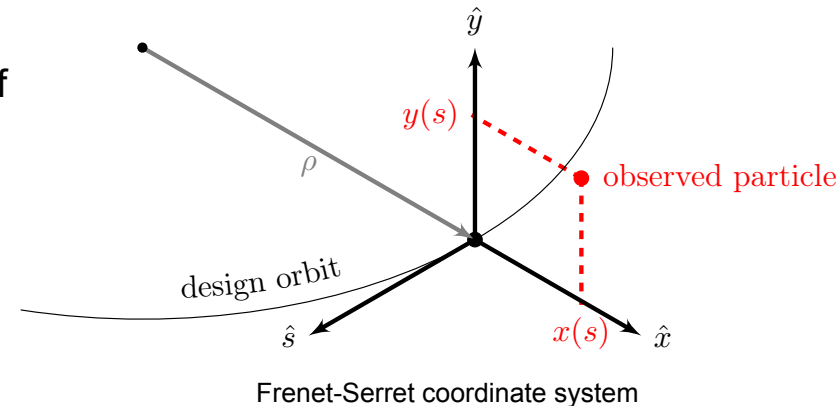
- At relativistic velocities E and B have the same effect if

$$E = cB$$

$$\rightarrow B = 1 \text{ T} \quad \checkmark \quad \Leftrightarrow \quad E = 3 \times 10^8 \text{ V/m} \quad \times$$

- Right-handed orthogonal coordinate system

Simplification: $\vec{v} = (0, 0, v_s)$ and $\vec{B} = (B_x, B_y, 0)$



Magnetic fields along the beam trajectory

- For highly relativistic particles follows from equality of Lorentz and Centripetal Force

$$\frac{1}{\rho(x, y, s)} = \left| \frac{e}{p} B_y(x, y, s) \right| = \left| \frac{e c}{\beta E} B_y(x, y, s) \right|$$

- Taylor expansion of the magnetic field in the vicinity of the nominal trajectory ($x = 0$):

$$B_y(x) = B_{y0} + \frac{dB_y}{dx} x + \frac{1}{2!} \frac{d^2 B_y}{dx^2} x^2 + \frac{1}{3!} \frac{d^3 B_y}{dx^3} x^3 + \dots$$

- This immediately leads to

$$\begin{aligned} \frac{e}{p} B_y(x) &= \frac{e}{p} B_{y0} & + \frac{e}{p} \frac{dB_y}{dx} x & + \frac{1}{2!} \frac{e}{p} \frac{d^2 B_y}{dx^2} x^2 & + \frac{1}{3!} \frac{e}{p} \frac{d^3 B_y}{dx^3} x^3 + \dots \\ &= \frac{1}{\rho} & + k x & + \frac{1}{2!} m x^2 & + \frac{1}{3!} o x^3 + \dots \\ &\Rightarrow \text{dipole} & + \text{quadrupole} & + \text{sextupole} & + \text{octupole} + \dots \end{aligned}$$

- Structures that only consist of **dipole and quadrupole fields** are called “**linear lattices**”. The linear lattice determines trajectory (“orbit”) and focusing properties, higher-order field components are used for correction and error compensation.

Dipole fields define geometry

Condition for circular orbit

■ Lorentz force

$$F_L = evB$$

■ Centripetal force

$$F_{\text{centr}} = \frac{\gamma m_0 v^2}{\rho}$$



$$\frac{p}{e} = B \rho$$

“Beam rigidity”

The strength of the dipole magnets and the size of the machine define the maximum momentum (or energy) of the particles that can be carried in the machine.

Field strength defined by

coil current
gap height



$$B = \frac{\mu_0 n I}{h}$$

→ **keep the beam
dimensions *small* !!!**

Bending angle and particle momentum

- The integrated dipole strength (along “s”) defines the momentum of the particle beam.

$$d\theta = \frac{ds}{\rho} \approx \frac{dl}{\rho} = \frac{B dl}{B\rho} = \frac{e}{p_0} B dl \quad \Rightarrow \quad \int B dl = 2\pi \frac{p_0}{e}$$

Example: LHC 7 TeV proton storage ring

- $B = 8.3 \text{ T}$

- $N = 1232$

- $l = 14.3 \text{ m}$

$$\int B dl \approx N l B = 2\pi \frac{p_0}{e}$$

$$p_0 = \frac{N l B e}{2\pi} = 7 \frac{\text{TeV}}{c}$$



Quadrupole

- The quadrupole field increases linearly with transverse distance from the magnet center:

$$B_x(y) = - g \cdot y$$

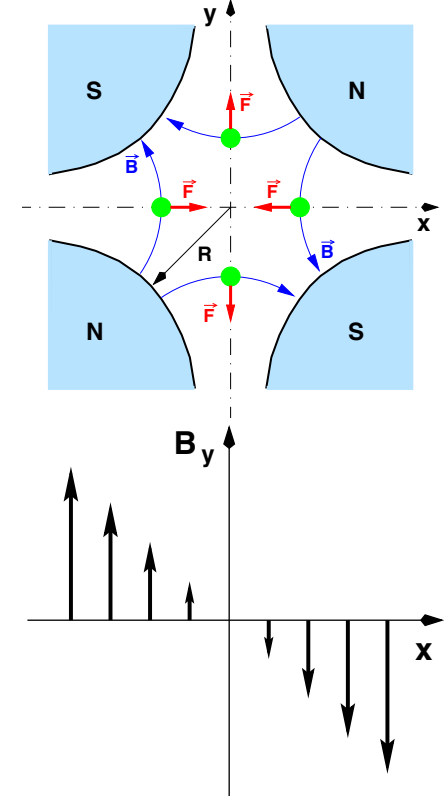
$$B_y(x) = - g \cdot x$$

- Is I the current in n windings of the coils and R the distance of the poles to the magnet center, a perfectly formed pole has the gradient

$$g = \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = \frac{2\mu_0 n I}{R^2}.$$

- Definition of a normalized quadrupole strength in analogy of the deflection strength $1/\rho$ of a dipole magnet:

$$k = \frac{e}{p} g$$



Quadrupole “focal length”

- In analogy to the dipole magnet, the normalized quadrupole gradient can be written as

$$k[\text{m}^{-2}] = 0.2998 \frac{g[\text{T/m}]}{p[\text{GeV}/c]}$$

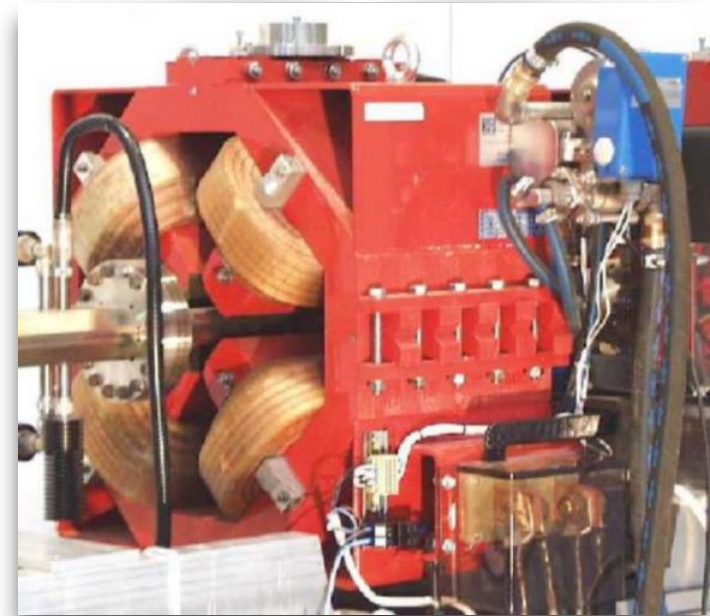
- The deflection (“kick”) of a particle passing through a quadrupole of length l with offset x is

$$\Delta x' = \Delta \left(\frac{dx}{ds} \right) = k l x.$$

- The focal length f of a quadrupole of length l is given by

$$\frac{1}{f} = -k l.$$

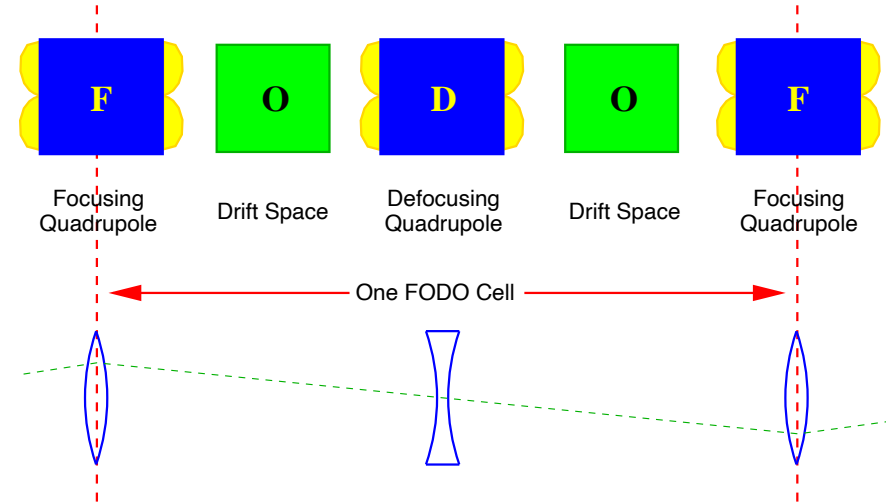
- In the case of $f \gg l$, we speak of **thin lenses**, no matter how large l actually is.



KARA quadrupole

The FODO lattice

- The alternating arrangement of (horizontally) focusing and (horizontally) defocusing quadrupoles allows to build compact systems. A frequently used arrangement is the periodic so-called **“FODO structure”**.

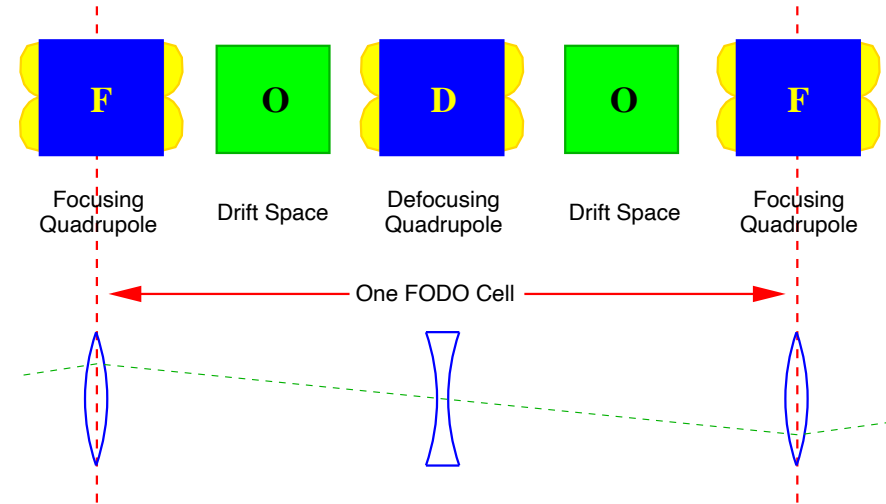


- F and D quadrupoles have comparable strength and the distance between two equal lenses is $\lesssim 2f$.



The FODO lattice

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- F and D quadrupoles have comparable strength and the distance between two equal lenses is $\lesssim 2f$.

Equations of motion

- For a (horizontally) focusing quadrupole is $k < 0$.
- A particle with vertical offset y in a thin quadrupole of length ds and strength k undergoes a vertical deflection of

$$dy' = -k y ds.$$

- With this, a differential equation for the motion can be written immediately, called “Hill's differential equation”, with the periodic coefficient $k(s)$:

$$y'' + k(s) y = 0$$

- In general, for the coordinate $u(s)$ and the length of the periodic structure l we write

$$u'' + K(s) u = 0$$

$$\text{with } K(s + l) = K(s)$$

$$K(s) = \begin{cases} -k(s) + \frac{1}{\rho(s)^2} & \text{horizontal} \\ +k(s) & \text{vertical} \end{cases}$$

Solutions of Hill's differential equation

- Structure of a simple harmonic oscillator — but with variable reset constant $K(s)$
- Focussing of quadrupoles creates transverse oscillation around the design orbit, the “betatron oscillation”.

- The independent solutions are

$$u(s) = a\sqrt{\beta(s)} e^{\pm i(\Phi(s) + \Phi_0)}$$

$$\text{with } \Phi'(s) = \frac{1}{\beta(s)} \text{ and } a = \text{const.}$$

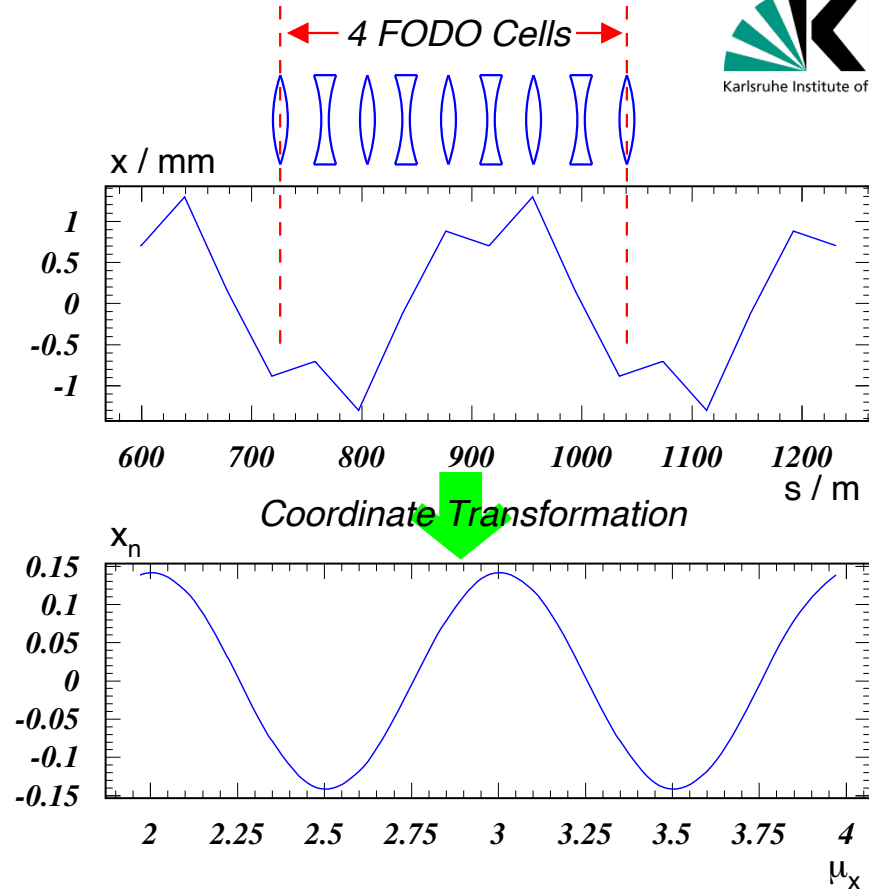
- The phase function $\Phi(s)$ increases non-linearly with time or longitudinal position s .

Particle trajectory

Illustration:

- Particle trajectory in regular FODO cells
- Periodicity of 4 FODO cells
- Normalised representation:

$$X(s) = x(s) / \sqrt{\beta(s)}$$

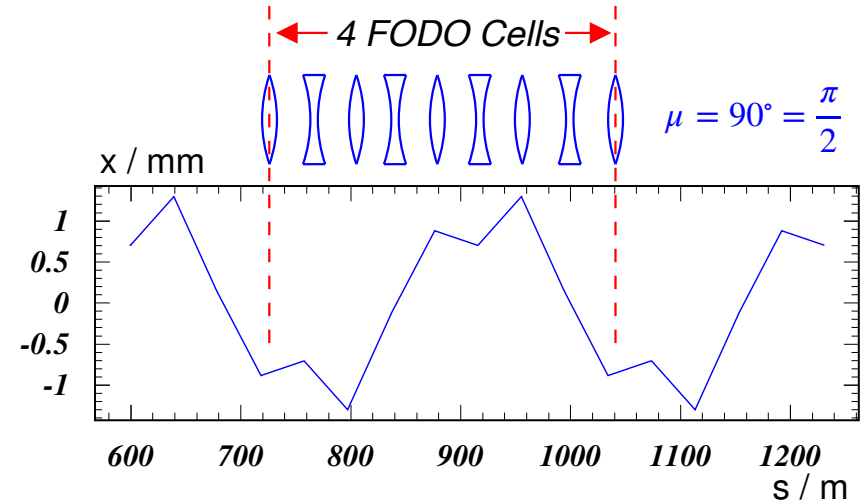


Phase advance and tune

- The difference of the phase functions is called the **phase advance** and is related to the betafunction:

$$\mu = \Phi(s_2) - \Phi(s_1) = \int_{s_1}^{s_2} \frac{1}{\beta(s)} ds$$

- The **phase advance of one revolution** is called the “**tune**” and gives the number of transverse oscillations per turn



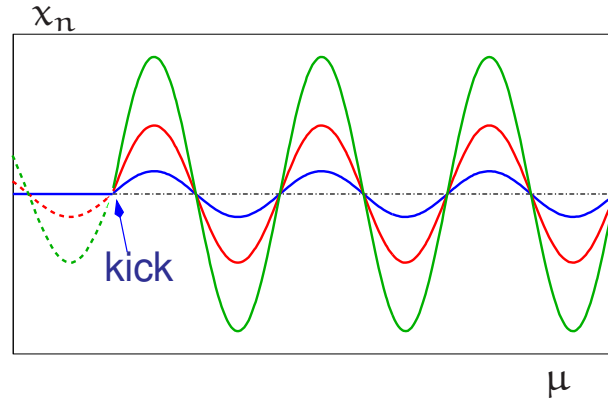
$$Q = \frac{1}{2\pi} \oint \frac{1}{\beta(s)} ds$$

Accelerators are precision instruments

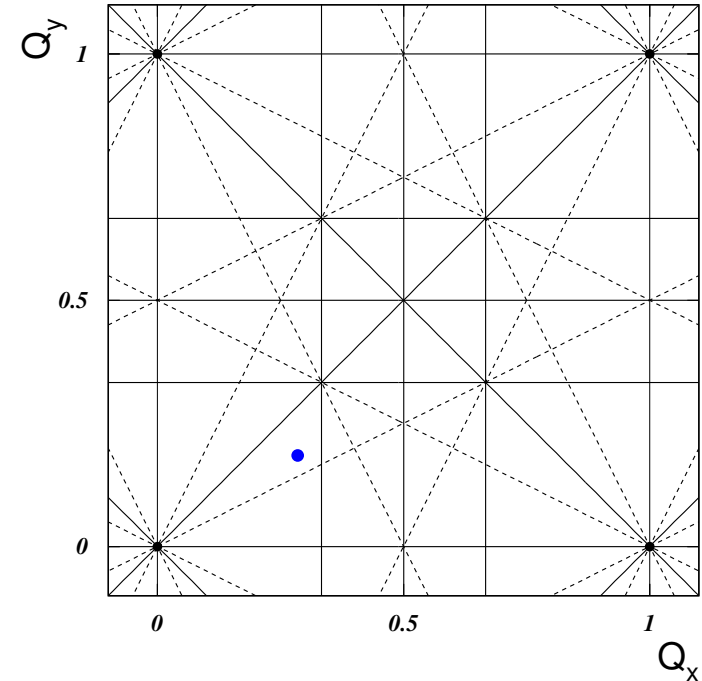


They are tuned

Tunes and resonances



- If the tune reaches certain rational numbers, the movement becomes unstable.
- Condition for optical resonances:
 $mQ_x + nQ_y = p$ mit $m, n, p \in \mathbb{Z}$
- Order of the resonance: $|m| + |n|$



The tunes define the “working point”.

In phase space

- The equations of the particle trajectory (solution of Hill's differential equation) and its derivative with respect to s form a parametric representation of an ellipse in u - u' space at location s :

$$u(s) = a\sqrt{\beta(s)} \cos(\Phi(s) - \delta)$$

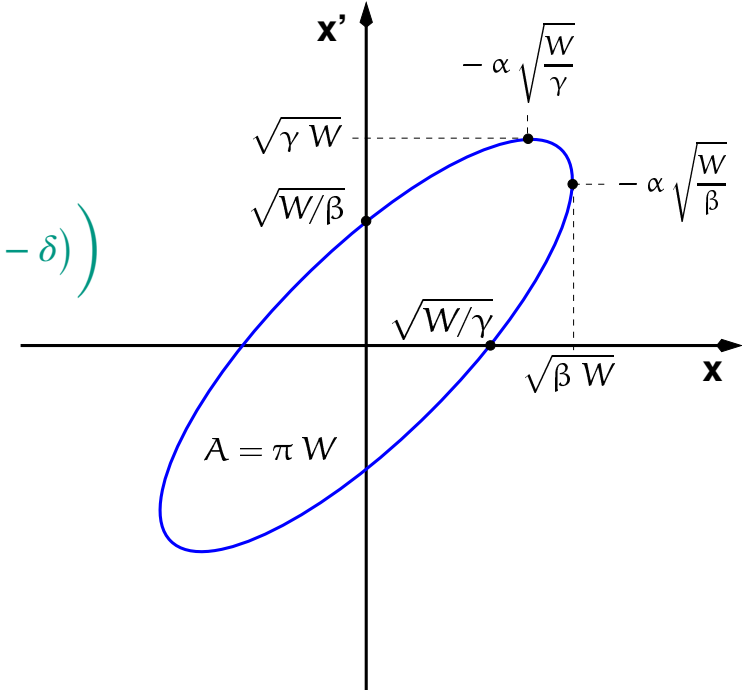
$$u'(s) = -\frac{a}{\sqrt{\beta(s)}} \left(\sin(\Phi(s) - \delta) - \frac{1}{2} \frac{d}{ds} \beta(s) \cos(\Phi(s) - \delta) \right)$$

- In absence of dissipative forces, the surface of this ellipse, πa^2 , is independent of s (Liouville's theorem).
- The parameter a^2 is referred to as the “Courant Snyder invariant”.

$$\gamma u^2 + 2\alpha uu' + \beta u'^2 = W_u$$

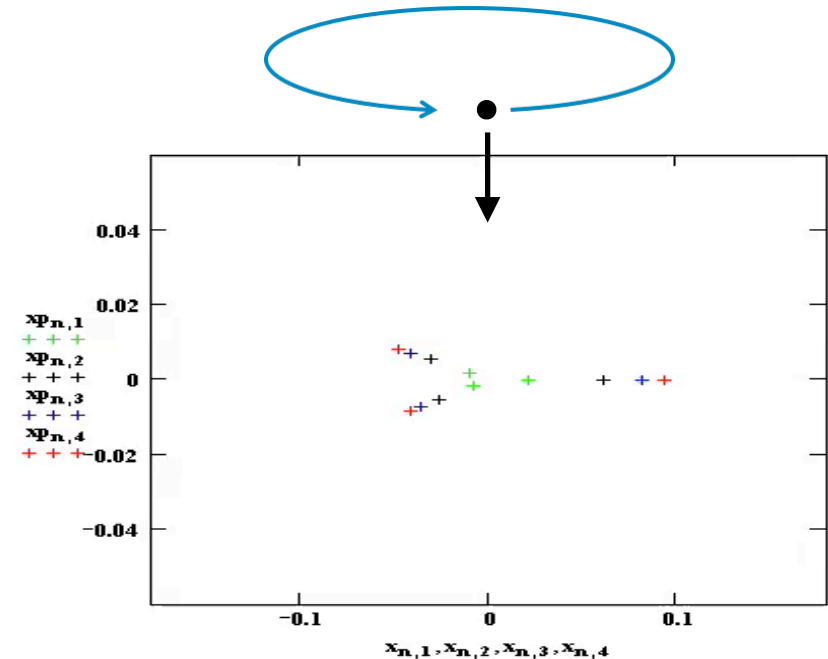
$$\alpha = -\frac{1}{2} \beta' \quad \text{and} \quad \gamma = \frac{1 + \alpha^2}{\beta}$$

- The location dependent functions $\alpha(s)$, $\beta(s)$ and $\gamma(s)$ are called the “optical functions” (formerly often known as “Twiss parameters”).

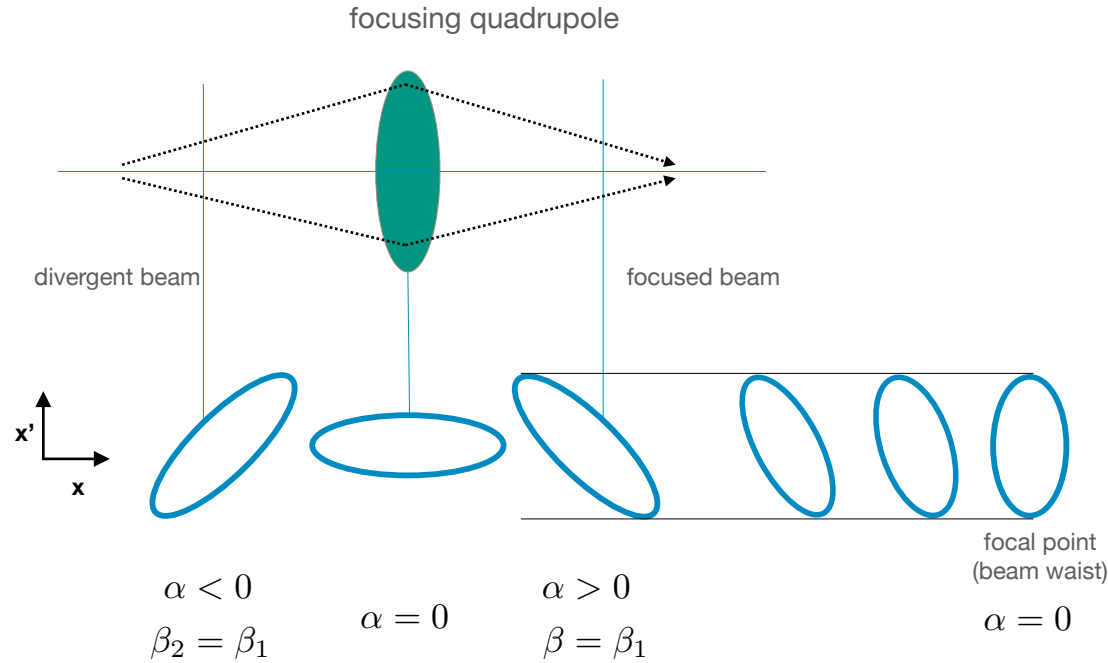


Phase space ellipse II

- In the case of a ring or matched cell, the periodicity imposes equality on the input and output α and β values.
- This means that the particle returns after each turn to the same ellipse but at phases
 $\mu_0 = b, \mu_1 = b + 2\pi Q, \mu_2 = b + 4\pi Q,$
....., $\mu_n = b + n \cdot 2\pi Q.$



Evolution of phase space ellipse along the lattice

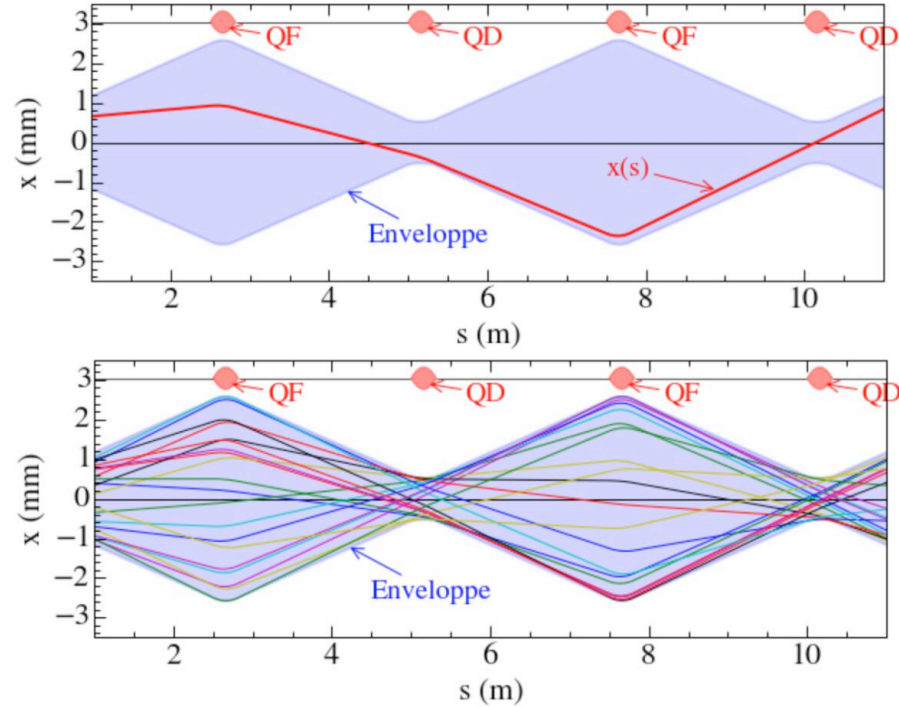


Beam envelope function

- The trajectories $x_i(s)$ are individual solutions of the equation of motion and have different initial conditions.
- The beta function and the constant a describe the envelope function of all possible particle trajectories:

$$E(s) = a\sqrt{\beta(s)}$$

- Since β is determined by the magnetic lattice, the constant a remains to take influence on the beam size besides that.



Emittance

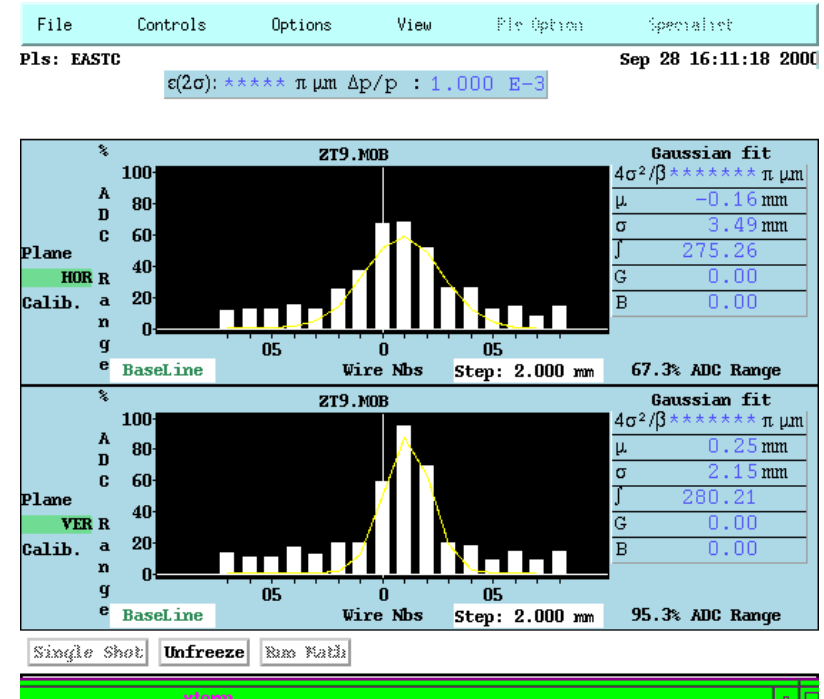
- As a consequence of **Liouville's theorem** the W is locationally invariant. W corresponds to a “**single particle emittance**”.
- In good approximation, the transverse charge density distribution in a particle beam is Gaussian.
- The beam size is defined as the standard deviation of the charge density distribution:

$$\sigma_u(s) = \sqrt{\varepsilon \beta(s)}$$

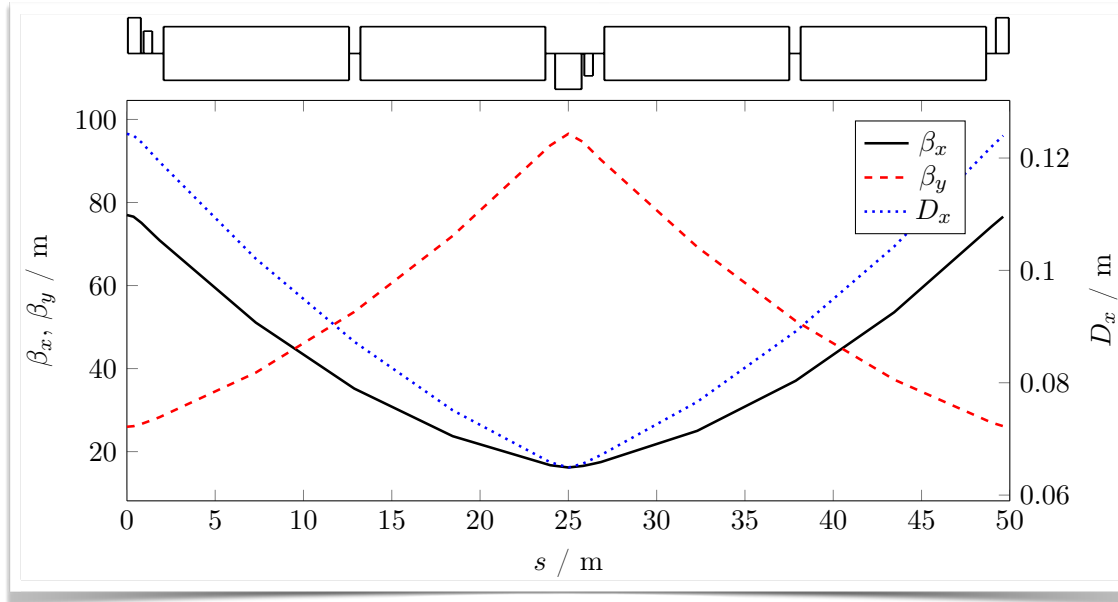
- The **emittance** is thus given by

$$\varepsilon = \frac{\sigma_u^2(s)}{\beta(s)}.$$

- The maximum possible emittance limited by the aperture is called the **acceptance**.



Beam optics example



$$\sigma_u(s) = \sqrt{\epsilon \beta(s)} e^{\pm i(\Phi(s) + \Phi_0)}$$

- Beam optics functions of a FODO cell designed for the FCC-ee
- β is determined only by the focusing properties of the magnetic lattice
- ϵ is the beam emittance

Momentum offset: dispersion orbit

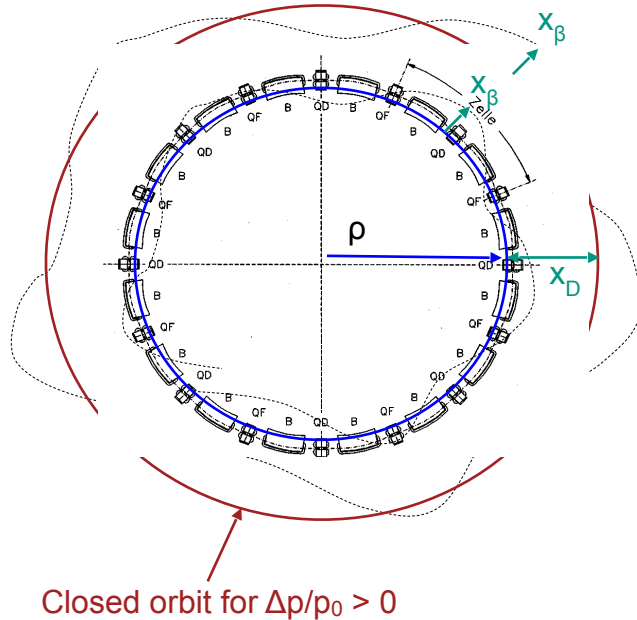
- The deflection angle in the dipole magnets depends on the particle energy.
- In a storage ring, dipoles deflect horizontally, therefore mainly horizontal dispersion.
- Vertical dispersion is caused by field errors or misalignment for example.
- As a consequence, the equation of motion becomes an inhomogeneous differential equation with an additional term on the right hand side:

$$x'' + K(s) x = \frac{\Delta p}{p_0} \frac{1}{\rho}$$

- The horizontal particle position at location s relative to the nominal orbit consists of two components:

$$x(s) = x_\beta(s) + x_D(s)$$

Dispersion orbit with homogeneous dipole field



- x_D can be understood as a special orbit for the particle with momentum offset.
- Normalizing this orbit with respect to $\Delta p/p_0$ yields the dispersion function

$$D_x(s) = \frac{\Delta x}{\Delta p/p_0}$$

that describes the change of transverse position Δx due to a momentum deviation $\Delta p/p_0$.

- The total transverse offset for a particle with $\Delta p/p_0 \neq 0$ is then

$$x(s) = x_\beta(s) + x_D(s) = x_\beta(s) + D_x(s) \frac{\Delta p}{p_0}.$$

- The beam size increases due to off-momentum particles in dispersive sections:

$$\sigma = \sqrt{\epsilon\beta + D^2(\Delta p/p_0)^2}$$

High-energy storage rings

Hadron storage rings

- Heavy particles require strong B fields
- Push for highest B fields up to technical limit
- Energy limit given by maximum acceptable circumference

$$2\pi \frac{p_0}{e} = \int B \, ds$$

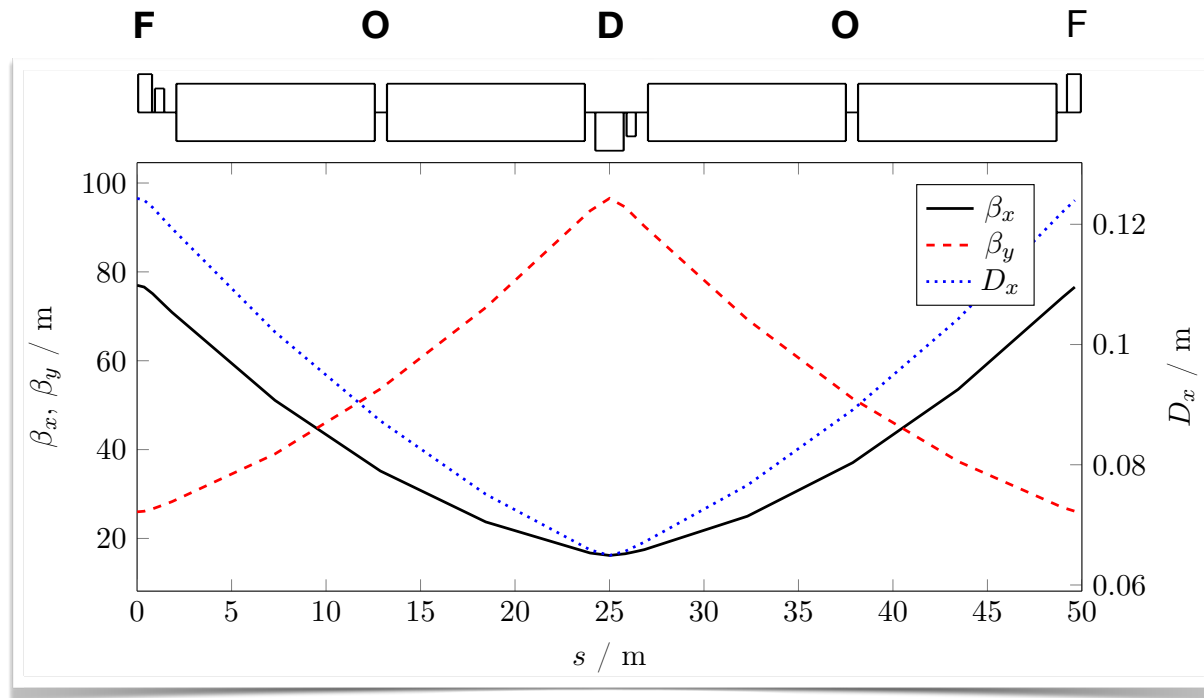
Electron storage rings

- Synchrotron light dominated
- Push for small B fields thus large bending radius

$$P_\gamma = \propto \frac{\gamma^4}{\rho^2}$$

Common feature: For high beam energies
→ **Push for highest possible dipole filling factor**

FODO structure



Arc FODO cell designed for FCC-ee

e⁺e⁻ colliders vs. synchrotron light sources

Collider

- High dipole filling factor → FODO structure
- High energy → large circumference
→ Naturally small emittance

Synchrotron light source

- Small footprint desired
- Low emittance beams for high brilliance

$$B(\lambda) = \frac{F(\lambda)}{(2\pi)^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'}} \propto \frac{1}{\epsilon_x \epsilon_y} \quad \text{with photon flux } F(\lambda)$$

$$\mathcal{L} = \frac{N_1 N_2 n_b f}{4\pi \sigma_x^* \sigma_y^*}$$

N particles per bunch
 n_b number of bunches
 f revolution frequency



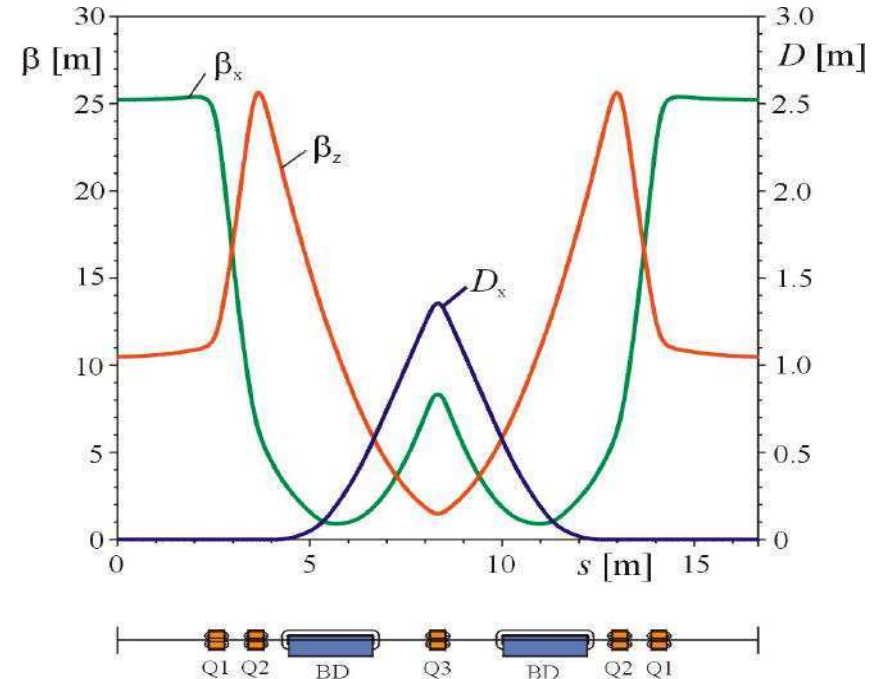
Jean-Luc Revol: ESRF - The European Synchrotron, JUAS 2020

Double Bend Achromat Struktur

Chasman-Green Lattice

- Achromat: that is, dispersion vanishes at the beginning and at the end.
- Dispersion is generated locally by one dipole and immediately corrected again by the central quadrupole and the second dipole.
- Many drift spaces with vanishing dispersion allow for
 - operation of insertion devices and
 - small emittances.

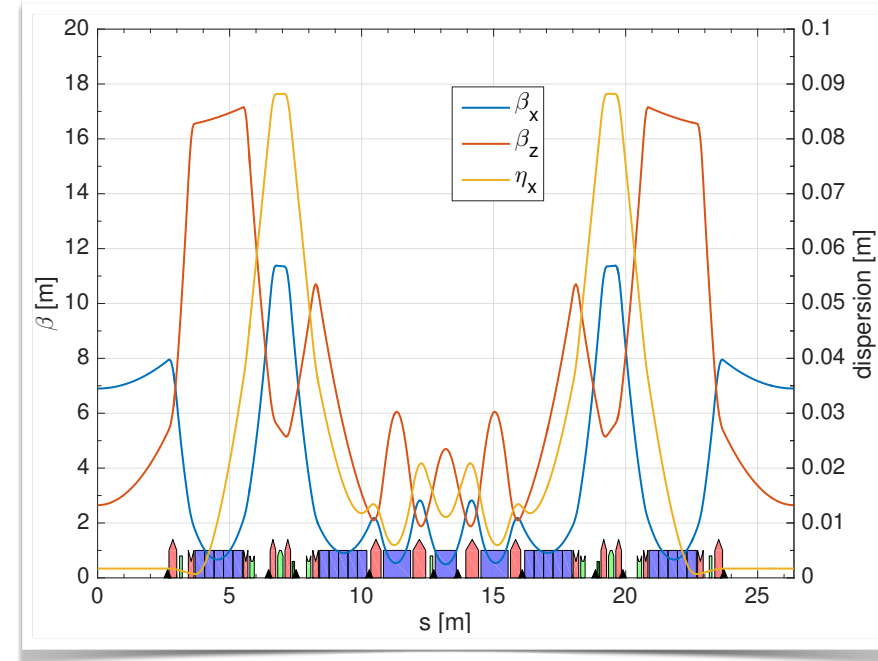
■ Typical magnetic structure for 3rd generation synchrotron light sources



Multi Bend Achromat structures (MBA)

- Highly specialised lattice, thus less flexibility
- Smaller magnets, higher number
- Special magnet technology
 - Combined function magnets
 - Permanent/hybrid magnets
 - Modular magnets
- Full energy injection, "top-up", no ramping
- **Goal: 24/7 operation at lowest emittances**

Energie	E	6 GeV
Umfang	C	844 m
Emittanz	ϵ_x	133 pm rad



7 Bend Achromat, ESRF-EBS
L. Farvacque 2015

Take-home messages

- Particle accelerators are versatile and have many applications in science, medicine and industry
- There are different types of accelerators that are used depending on the application.
- In research, accelerators are most prominently used as light sources and colliders — in most cases synchrotrons.
- The magnetic structure („lattice“) is different for colliders and light sources.
- The optical beam parameters are defined by the lattice.

