Introduction to Laser-Wakefield Acceleration

Matthias Fuchs

Outline

- Electron interacting with a strong electro-magnetic field
- Introduction to plasma physics
- Plasma waves
- Laser-plasma acceleration

Laser-Wakefield Acceleration

- Invented by Tajima & Dawson, 1979

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Laser Electron Accelerator

T. Tajima and J. M. Dawson
Department of Physics, University of California, Los Angeles, California 90024
(Received 9 March 1979)

An intense electromagnetic pulse can create a weak of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. Existing glass lasers of power density 10^{18}W/cm^2 shone on plasmas of densities 10^{18} cm^{-3} can yield gigaelectronvolts of electron energy per centimeter of acceleration distance. This acceleration mechanism is demonstrated through computer simulation. Applications to accelerators and pulsers are examined.

- Short-pulse high-power lasers no where near existence
- used computer simulations that today a cell phone could do in a fraction of the time

Electron in strong electromagnetic field I

- Plane wave, linearly polarized along y direction

$$\overrightarrow{E_l}(x,t) = \hat{y}E_0\cos(k_l x - \omega_l t)$$
$$\overrightarrow{B_l}(x,t) = \hat{z}B_0\cos(k_l x - \omega_l t)$$

$k_l = \dfrac{2\pi}{\lambda_l}$: wavenumber

ω_l : angular frequency

Electron in strong electromagnetic field II

- e in plane e-m wave (non-relativistic)

$$\frac{d\vec{p}_e}{dt} = \overrightarrow{F}_L$$

Lorentz force

$$\frac{d}{dt}(m\vec{v}_e) = -e(\overrightarrow{E}_l + \vec{v}_e \times \overrightarrow{B}_l)$$

$$\overrightarrow{E}_l(x,t) = \hat{y}E_0\cos(k_l x - \omega_l t)$$

$$\overrightarrow{B}_l(x,t) = \hat{z}B_0\cos(k_l x - \omega_l t)$$

$B_0 = \frac{E_0}{c}$ -> for v_e <<c : v x B term negligible

$$\vec{v}_e(t) = \hat{y}\frac{eE_0}{m\omega_l}\sin(k_l x - \omega_l t)$$

$$y(t) = \frac{eE_0}{m\omega_l^2}\cos(k_l x - \omega_l t)$$

Electron in strong electromagnetic field III

- Maximum velocity

$$\vec{v}_e(t) = \hat{y}\frac{eE_0}{m\omega_l}\sin(k_l x - \omega_l t)$$

$$v_{max} = \frac{eE_0}{m\omega_l}$$

– relativistic for $v_{max} \to c$:

$\frac{v_{max}}{c} \to 1$: $\underbrace{\frac{eE_0}{\omega_l mc}}_{\stackrel{!}{=}\, a_0} \to 1$

a_0 : normalized laser field

$\boxed{a_0 = \frac{eE_0}{\omega_l mc}} \gtrsim 1$ → relativistic interaction

→ also need to consider $\vec{v} \times \vec{B}$ term

Interpretation of a_0

other way to look at it:

$\omega = ck = c\,\frac{2\pi}{\lambda}$

$$a_0 = \frac{eE_0}{\omega_l\, mc} = \frac{\overbrace{eE_0\lambda_l}^{\text{work done by } E_0 \text{ over } \lambda_l}}{2\pi\, \underbrace{mc^2}_{\bar{e}\text{ rest mass energy (Einstein)}}}$$

-> for $a_0 \gtrsim 1$: $\bar{e}$ gains energy $\approx mc^2$ over a distance λ_l

Example: normalized laser field 100 TW

engineering formula: $a_0 \approx \lambda[\mu m] \cdot \sqrt{\frac{I_0 [W/cm^2]}{1.4 \cdot 10^{18}}}$

100 TW laser @KIT: $P = \frac{2.5\ J}{25\ \underbrace{fs}_{10^{-15}\,s}} = 100 \cdot 10^{12}\ W$

- focused to 20 μm spot:

$$I = \frac{P}{d^2} = \frac{100 \cdot 10^{12}\ W}{(20 \cdot 10^{-4})^2\ cm^2} = \underbrace{\frac{100}{400}}_{1/4} \cdot \underbrace{10^{12} \cdot 10^{8}}_{10^{20}} = 2.5 \cdot 10^{19}\ \frac{W}{cm^2}$$

$\lambda = 1\,\mu m$:

$$a_0 \approx 1 \cdot \sqrt{\frac{2.5 \cdot 10^{19}}{\underbrace{1.4}_{\approx 2} \cdot \underbrace{10^{18}}_{10}}} \approx \sqrt{20} \approx \underline{\underline{4.5}}$$

Outline

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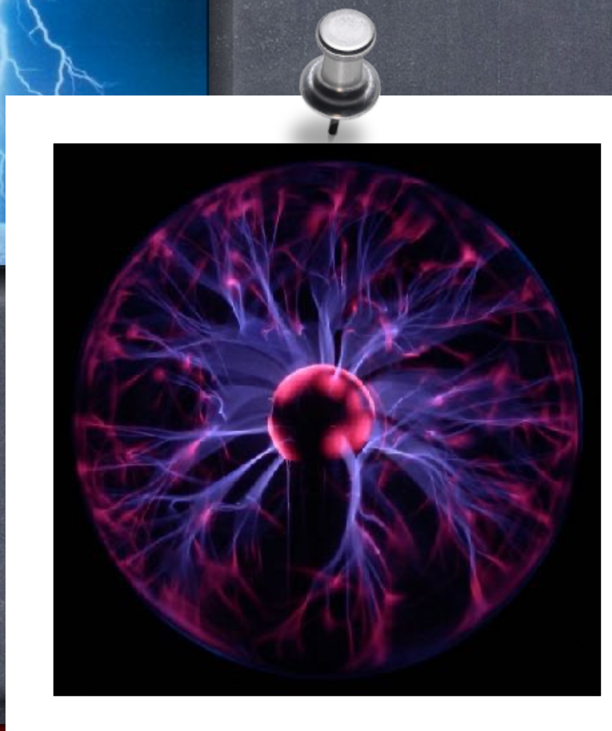
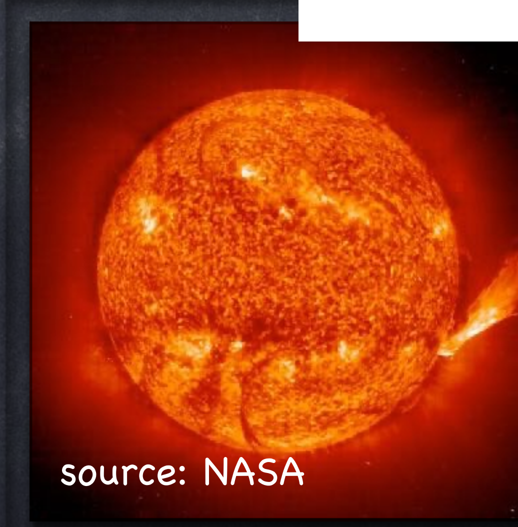
- Electron interacting with a strong electro-magnetic field
- Introduction to plasma physics
  - Plasma frequency
  - Light propagation in plasmas
- Plasma waves
- Laser-plasma acceleration

Plasma

- "fourth state of matter"
- consists of separated positive and negative charges (e.g. ionized gas)
- electrically neutral
- by separating electrons from the ions, enormous electric fields can be generated
- Unlike electrons, ions are static on the timescales of the interaction (ion movement on ~0.1 ns - scale) due to higher mass



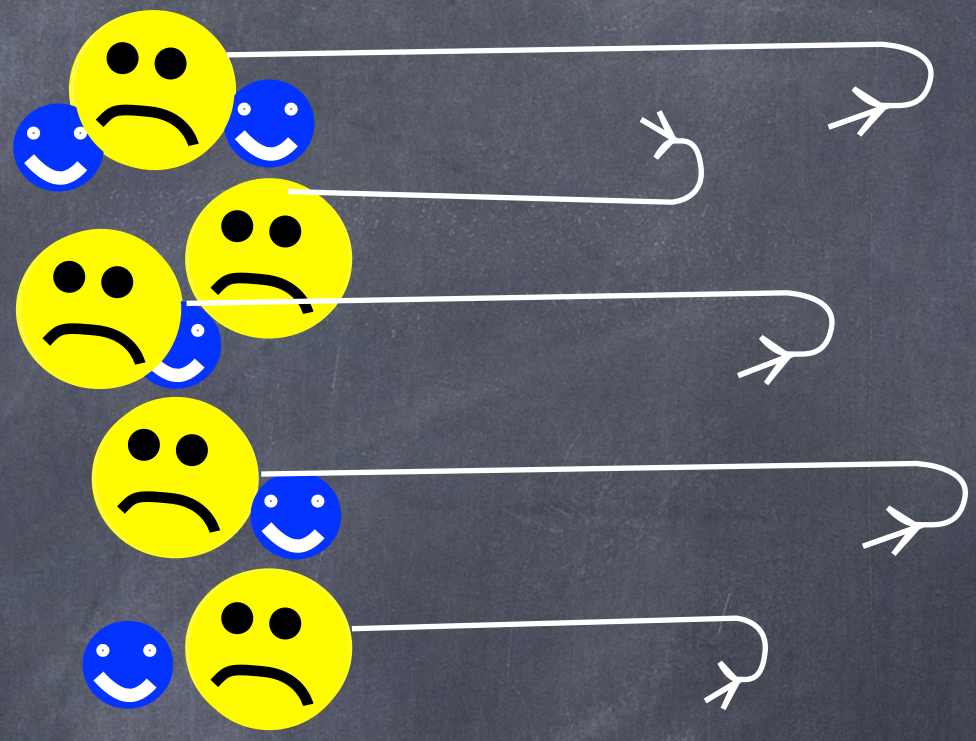


source: NASA

Plasma properties: Plasma oscillations

- displacement of electrons
- creates regions of positive and negative charges
- sets up restoring electrical field
- electrons are accelerated back, overshoot
- harmonic oscillation with "plasma frequency":

$$\omega_{p,e} = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}}$$



Laser propagation in plasmas I

relevant Maxwell-eqns:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

plane waves: $\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)}$ -> $\vec{\nabla} \times \vec{E} = i\vec{k} \times \vec{E}$:

$$i\vec{k} \times \vec{E} = i\omega \vec{B} \qquad (*)$$

$$i\vec{k} \times \vec{B} = \mu_0 \vec{j} + i\frac{\omega}{c^2}\vec{E} \qquad (**)$$

k x (*) & use $A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$

(**)

$$k^2 \vec{E} - \vec{k} \cdot (\vec{k} \cdot \vec{E}) = \frac{\omega^2}{c^2}\left(i\frac{\vec{j}}{\epsilon_0 \omega} + \vec{E} \right)$$

-> find expression for current density j

Laser propagation in plasmas II

current due to movement of electrons
(ions remain stationary for high frequencies)

-> current density: $\vec{j} = -n_e e \vec{v}_e$ n_e: electron density

dependence of j on E:

-> use Lorentz force: $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

-> to get first order eqn. of motion

$$m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -e\vec{E}$$

$$\left(\vec{E} = \vec{E}_0 \mathrm{e}^{\mathrm{i}(\vec{k}\vec{r}-\omega t)} \right)$$

=> $$\vec{j} = -n_e e \vec{v} = \frac{n_e e^2}{m_e}\frac{1}{\mathrm{i}\omega}\vec{E}$$

remember: $$k^2\vec{E} - \vec{k}\cdot(\vec{k}\cdot\vec{E}) = \frac{\omega^2}{c^2}\left(\frac{\vec{j}}{\mathrm{i}\epsilon_0\omega} + \vec{E}\right)$$

for e-m waves: $\vec{k} \perp \vec{E}$ => $$(c^2k^2 - \omega^2)\vec{E} = -\frac{n_e e^2}{m_e\epsilon_0}\vec{E}$$

Laser propagation in plasmas III

$$(c^2k^2-\omega^2)\vec{E} = -\underbrace{\frac{n_e e^2}{m_e\epsilon_0}}_{\omega_p^2}\vec{E}$$

-> dispersion relation for an e-m in plasma:

$$\omega^2 = \omega_p^2 + c^2k^2$$

-> phase velocity of wave:

$$v_\phi = \frac{\omega}{|\vec{k}|}$$

-> refractive index of plasma:

$$\eta = \frac{c}{v_\phi} = \sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2}$$

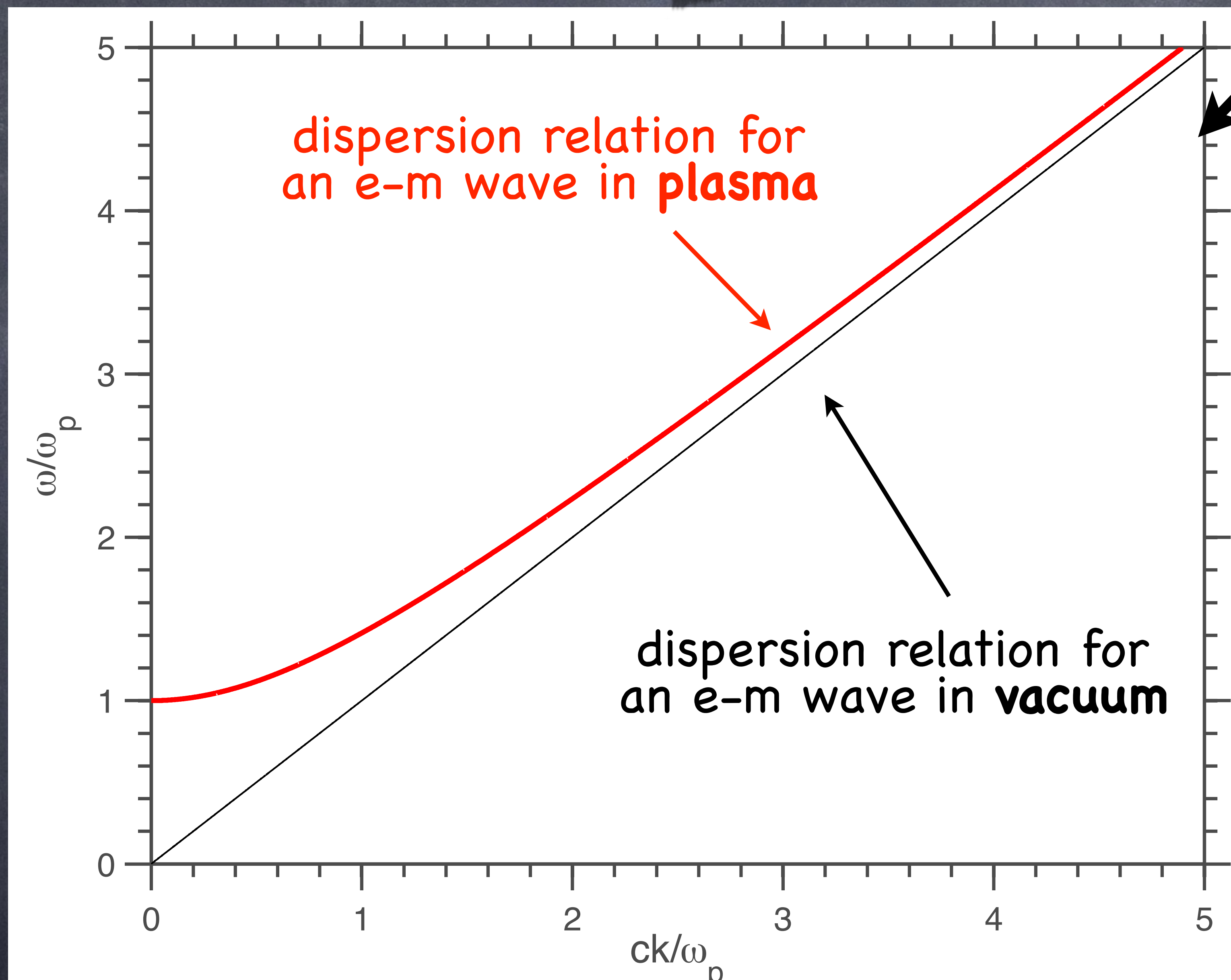
-> group velocity of wave:

$$v_{\text{group}}^{\text{laser}} = \frac{d\omega_L}{dk_L} = c\sqrt{1-\omega_p^2/\omega_L^2}$$

Dispersion relation I

-> dispersion relation: $$\omega^2 = \omega_p^2 + c^2 k^2$$

ω/ω_p



for $\omega \gg \omega_p$:

disp. rel. of plasma approaches that of vacuum

-> response of electrons (ω_p) too slow to respond to high frequencies

-> light field doesn't "feel" (couple to) electrons

for $\omega < \omega_p$:

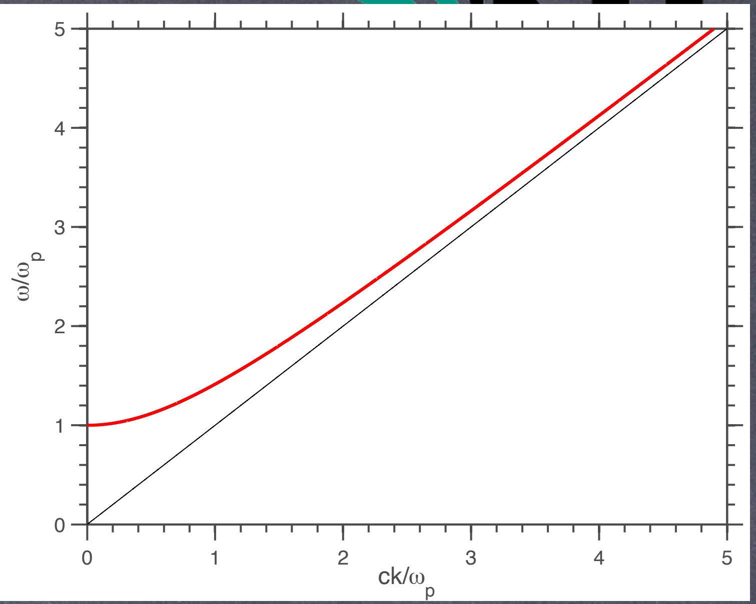
wave can't propagate in plasma

-> gets reflected or damped

-> plasma electrons shield fields that oscillate at a frequency $< \omega_p$

Dispersion relation II

$$\left(\omega^2 = \omega_p^2 + c^2 k^2\right) \;\rightarrow\; k = \frac{\sqrt{\omega^2 - \omega_p^2}}{c}$$



for $\omega < \omega_p$: -> k is imaginary

=> wave decays as: $$\exp\left(-x\sqrt{\frac{\omega_p^2 - \omega^2}{c}}\right)$$

with skin depth: $$\delta = |k|^{-1} = \frac{c}{(\omega_p^2 - \omega^2)^{1/2}}$$

distance over which wave amplitude is decreased by factor 1/e

critical plasma density: $(\omega = \omega_p)$

remember: $$\omega_{p,e} = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}}$$

critical plasma density: $$n_c = \frac{m_e \epsilon_0 \omega^2}{e^2}$$

overcritical plasma

Outline

- Electron interacting with a strong electro-magnetic field
- Introduction to plasma physics
- Plasma waves
  - Ponderomotive potential
  - Ponderomotive plasma wave excitation
  - Properties of plasma wave
- Laser-plasma acceleration

Laser-matter interaction I: the ponderomotive force

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eqns of motion revisited:

from Lorentz force: $m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -e\vec{E}$

for a wave with varying amplitude: $\vec{E} = \vec{E}_0(\vec{x},t)\cos(\vec{k}\vec{x} - \omega t + \Phi)$

=> $$\vec{v}_e(\vec{x},t) = -\frac{e}{m_e}\int \vec{E}_0(\vec{x},t)\cos\left(\vec{k}\vec{x} - \omega t + \Phi\right)\mathrm{d}t + \vec{v}_0$$

initial electron velocity == 0

-> quiver motion of electron

-> averaging over one oscillation period of the quiver energy ($E_\mathrm{q} = \frac{1}{2}m_\mathrm{e}|\vec{v}|^2$) :

=> ponderomotive potential:

$$U_\mathrm{P} = \langle E_\mathrm{q}\rangle = \frac{e^2}{4m_\mathrm{e}\omega^2}|\vec{E}_0|^2$$

$$\langle E_\mathrm{kin}\rangle = U_p = \frac{a_0^2}{2}m_e c^2$$

-> for a0>1: electron gains energy comparable with rest mass

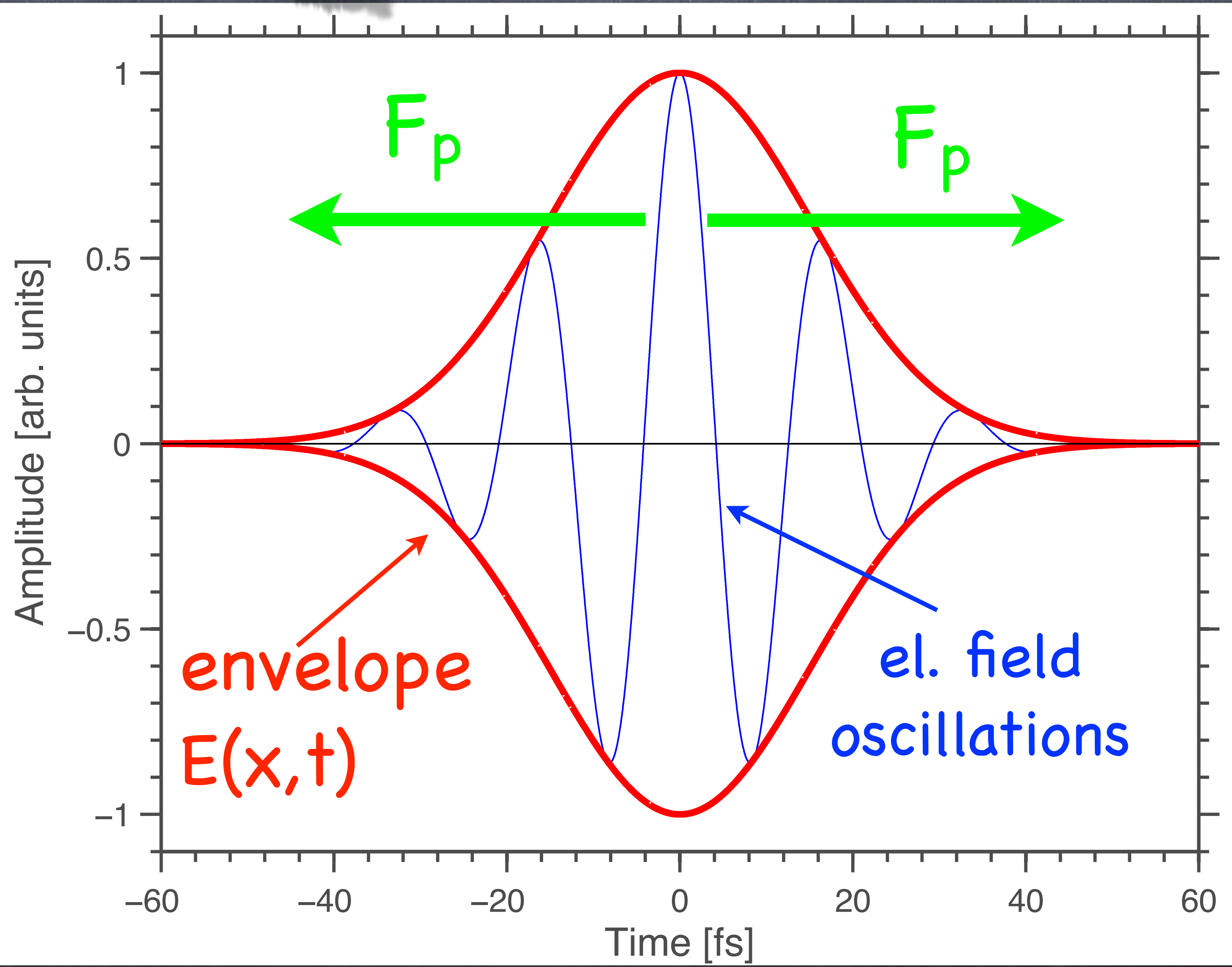
Laser-matter interaction II: the ponderomotive force

ponderomotive potential: $U_\mathrm{P} = \langle E_\mathrm{q} \rangle = \dfrac{e^2}{4 m_\mathrm{e} \omega^2} |\vec{E}_0|^2$

ponderomotive force: $\vec{F}_\mathrm{P} = -\vec{\nabla} U_\mathrm{P}$

-> is directed along the gradient of a laser-pulse envelope

=> pushes electrons towards regions of lower laser intensity

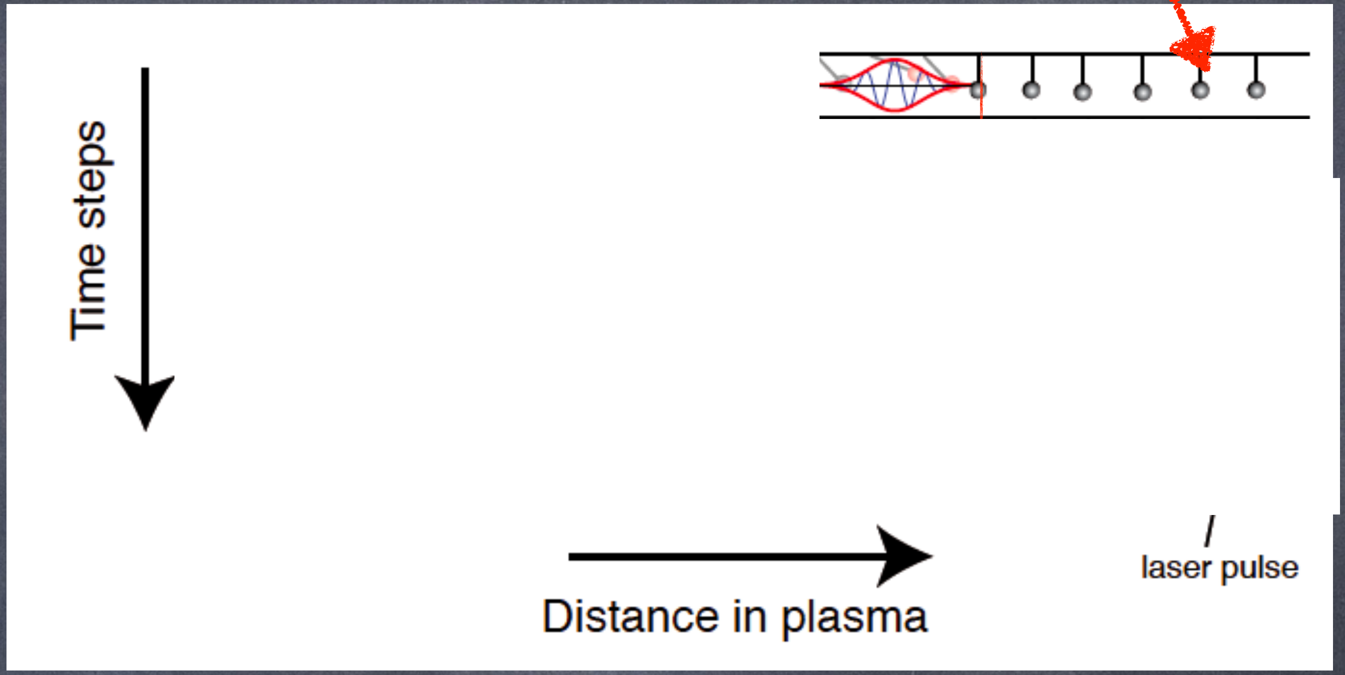


Amplitude of a Gaussian laser pulse, 25 fs duration (FWHM)

Plasma Waves

- laser excites a plasma wave
- ultrashort laser pulse "kicks" electrons
- electrons are pulled back by stationary ions
- electrons oscillate with plasma frequency: $\omega_{p,0} = \sqrt{\frac{e^2 n_0}{m_e \epsilon_0}}$
- collective motion forms a plasma wave that is propagating at laser group velocity
- no charge transport: just oscillations

plasma electrons

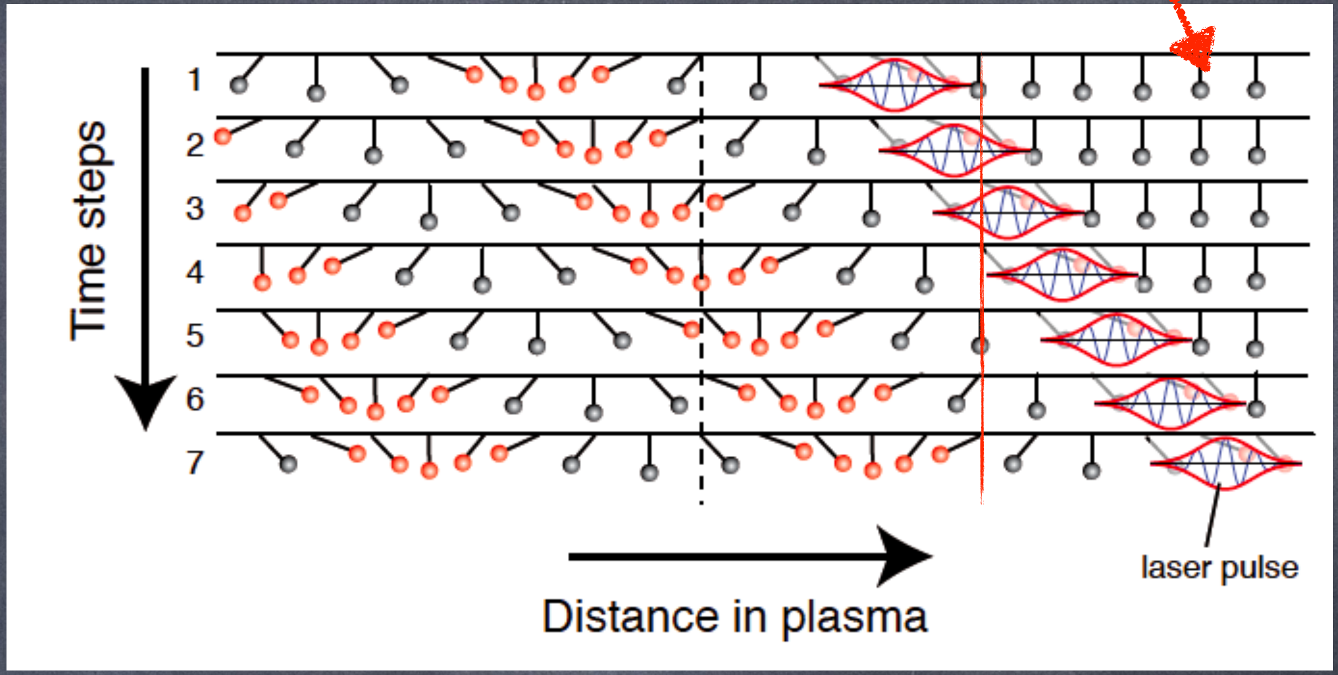


Dissertation M. F., after Dawson, Sci. American (1989)

Plasma Waves

- charge separation in the plasma wave
  - set up longitudinal electrical field: laser-wakefield
- particles injected into wakefield get accelerated!

plasma electrons



Dissertation M. F., after Dawson, Sci. American (1989)

Derivation of Plasma Wave 1

- "Delta" force moving with v_l : transfers momentum $m_e u_0$ at laser front

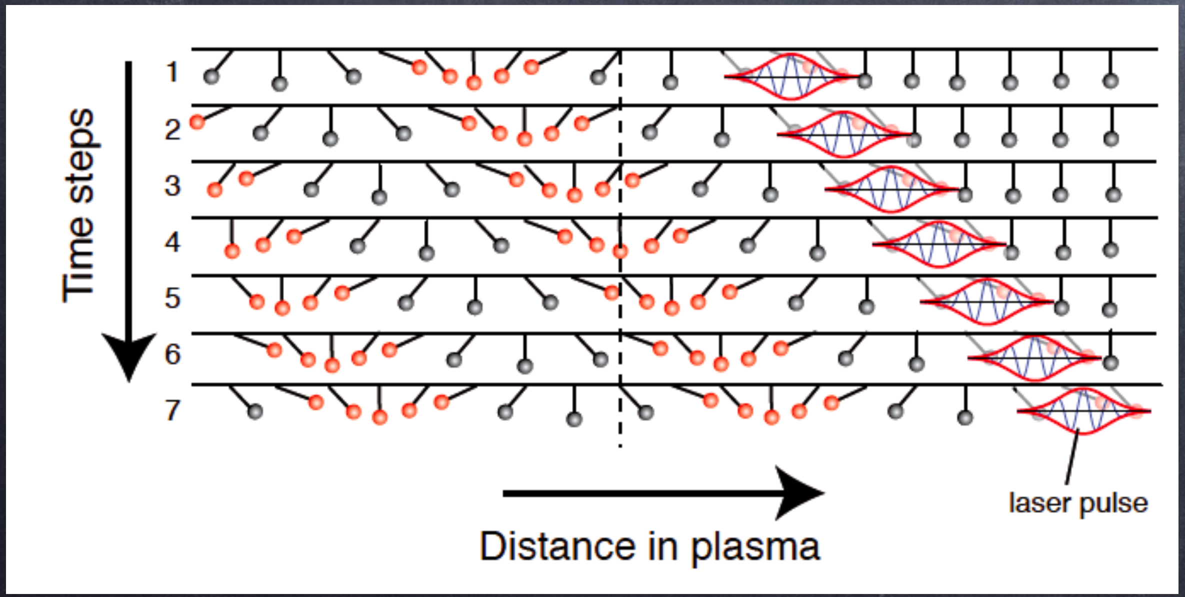
$$f \simeq m_e u_0 \delta(t - x/v_l)$$

- Electrons at laser front oscillate with plasma frequency. Electron Velocity:

$$u_x = u_0 \cos(\omega_p \tau)\ \Theta(\tau)$$

$$\tau = t - x/v_l$$

$$\Theta(\tau) = \begin{cases} 0 & \text{for } \tau \geq 0 \\ 1 & \text{for } \tau < 0 \end{cases}$$



Plasma Response

- linearized fluid equations: assume background with a small perturbation $\delta n_e = n_e - n_0$ $\quad n_e \ll n_0$

"cold fluid" equations:

- continuity equation: $$\frac{\partial}{\partial t}\delta n + n_0 \vec{\nabla}\cdot\vec{u} \simeq 0$$
- momentum equation: $$\frac{\partial \vec{u}}{\partial t} \simeq \nabla\phi - \nabla a_0^2/2,$$
- Poisson equation: $$\nabla^2\phi \simeq k_p^2 \frac{\delta n}{n_0}$$

n: electron density

u: fluid velocity

a_0: relativistic laser potential (~ laser intensity)

k_p: plasma wavenumber

ϕ : electrostatic potential

$$\frac{\partial}{\partial t} n_e \simeq - n_0 \frac{\partial}{\partial x} u_x$$

(a_0 = 0)

$$\frac{\partial}{\partial t} u_x \simeq \frac{eE_x}{m_e}$$

Derivation of Plasma Wave 2

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linearized fluid equations:

$$\frac{\partial}{\partial t} n_e \simeq -n_0 \frac{\partial}{\partial x} u_x$$

$$\frac{\partial}{\partial t} u_x \simeq \frac{eE_x}{m_e}$$

Electron oscillation

$$u_x = u_0 \cos(\omega_p \tau)\ \Theta(\tau)$$

Electron density:

$$\delta n_e \simeq n_0 \frac{u_0}{v_l} \cos(\omega_p \tau)\ \Theta(\tau)$$

$$\delta n_e = n_e - n_0$$

$$\Theta(\tau) = \begin{cases} 0 & \text{for } \tau \geq 0 \\ 1 & \text{for } \tau < 0 \end{cases}$$

(longitudinal) electric field:

$$E_x \simeq \frac{m_e \omega_p u_0}{e} \sin(\omega_p \tau)\ \Theta(\tau)$$

$$\tau = t - x/v_l$$

Plasma Wave Properties

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- describe plasma electrons as fluid (use continuity-, momentum- and Poisson equation)

solution for ultrashort Gaussian laser pulse with frequency >> ω_p

-> sinusoidal wave with period λ_p

- <u>electron density:</u>

$$\frac{\delta n}{n_0} \sim \frac{a_0^2}{2} \sin(k_p \xi)$$

a_0^2 ~ laser intensity

- <u>electric field</u>

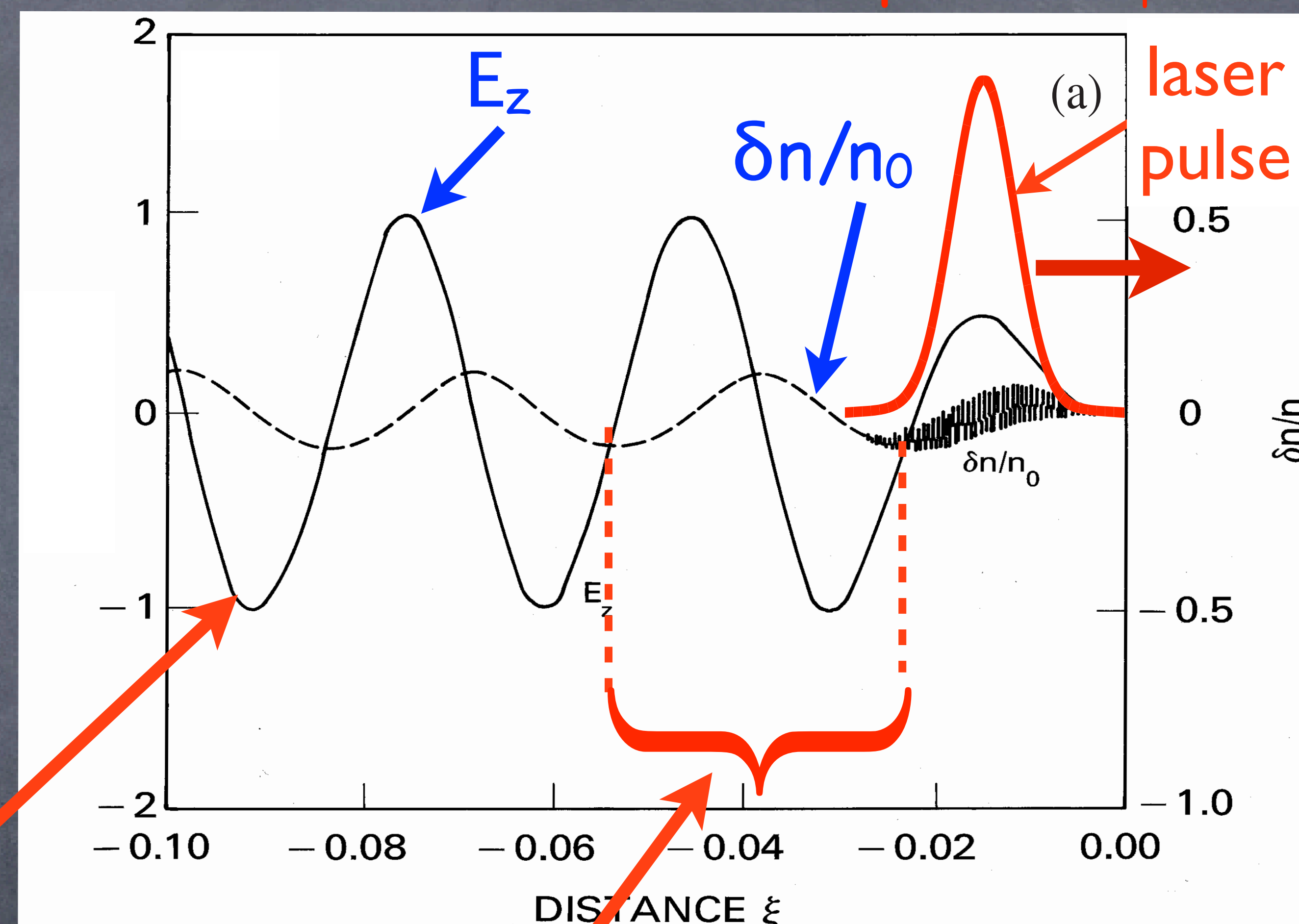
$$E_z \sim \underbrace{\frac{m_e c \omega_p}{e}} \frac{a_0^2}{2} \cos(k_p \xi)$$

$$E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]}$$

-> typical density: $n_0 = 10^{18}$ cm^{-3}

=> $E_0 \approx 100$ GV/m !!

(RF: 20 MV/m)



graph: E. Esarey, IEEE Tr[ans.] Plasma Sci. 24, 252-288

λ_p = 30 μm (100 fs)

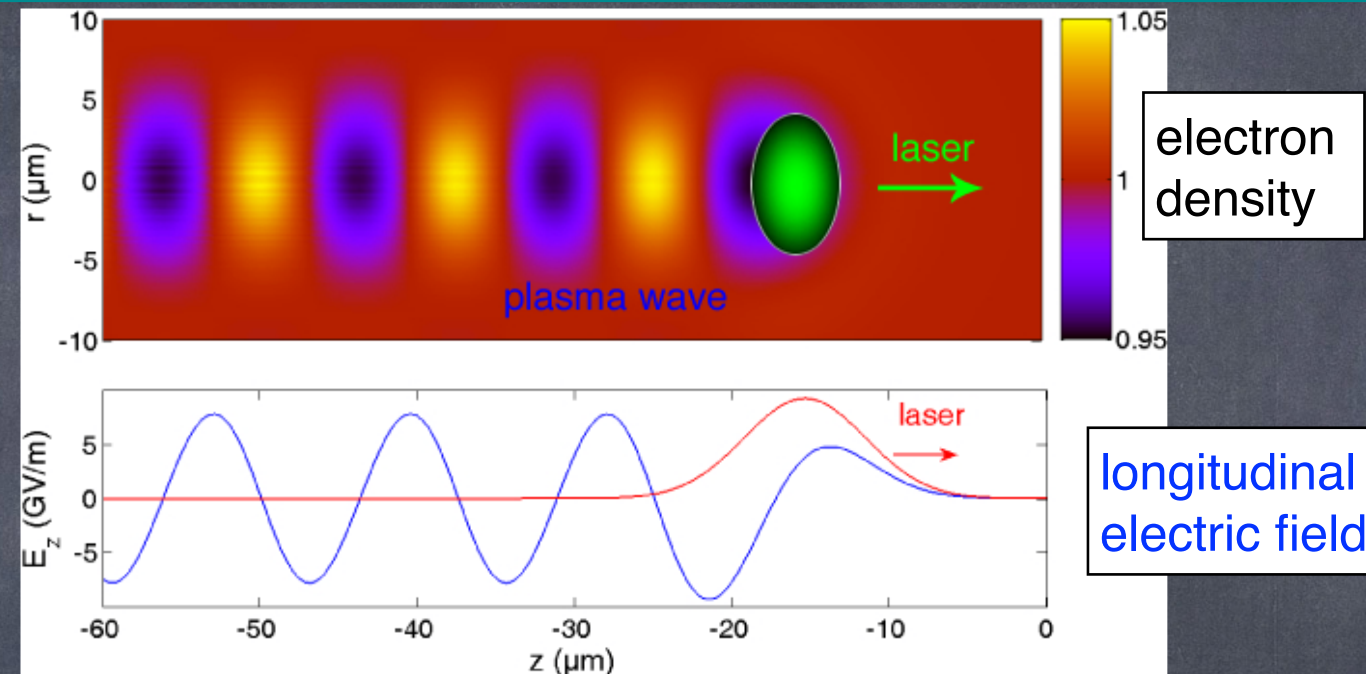
Outline

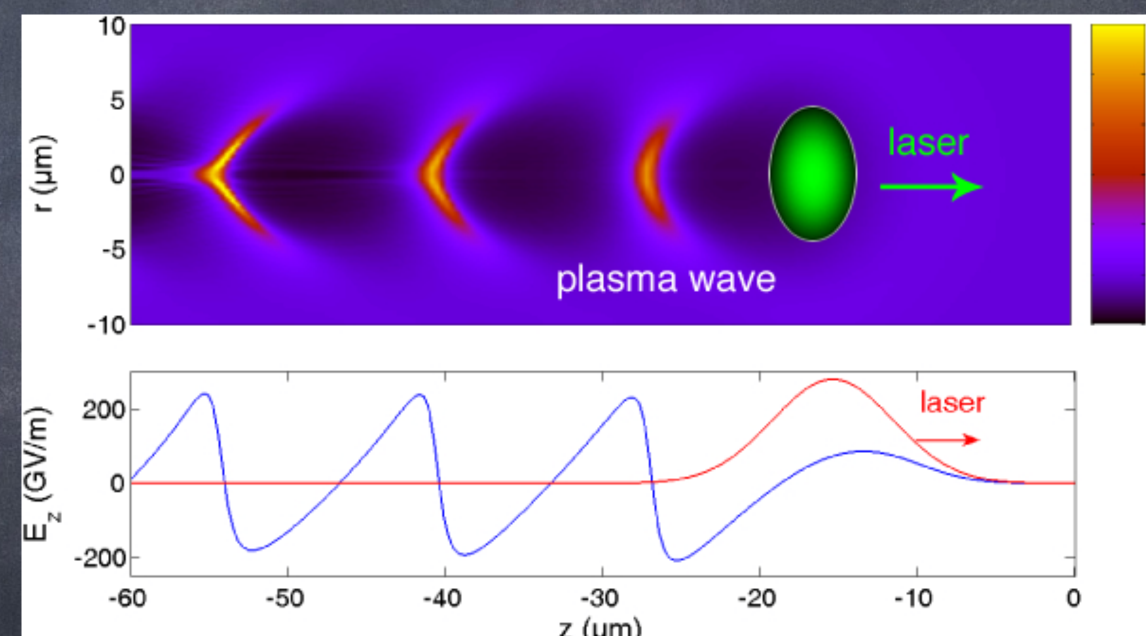
- Electron interacting with a strong electro-magnetic field
- Introduction to plasma physics
- Plasma waves
- Laser-plasma acceleration
    - Discussion: Laser-wakefield acceleration
    - Maximum energy gain
    - Limits of laser-wakefield acceleration

Wakefield

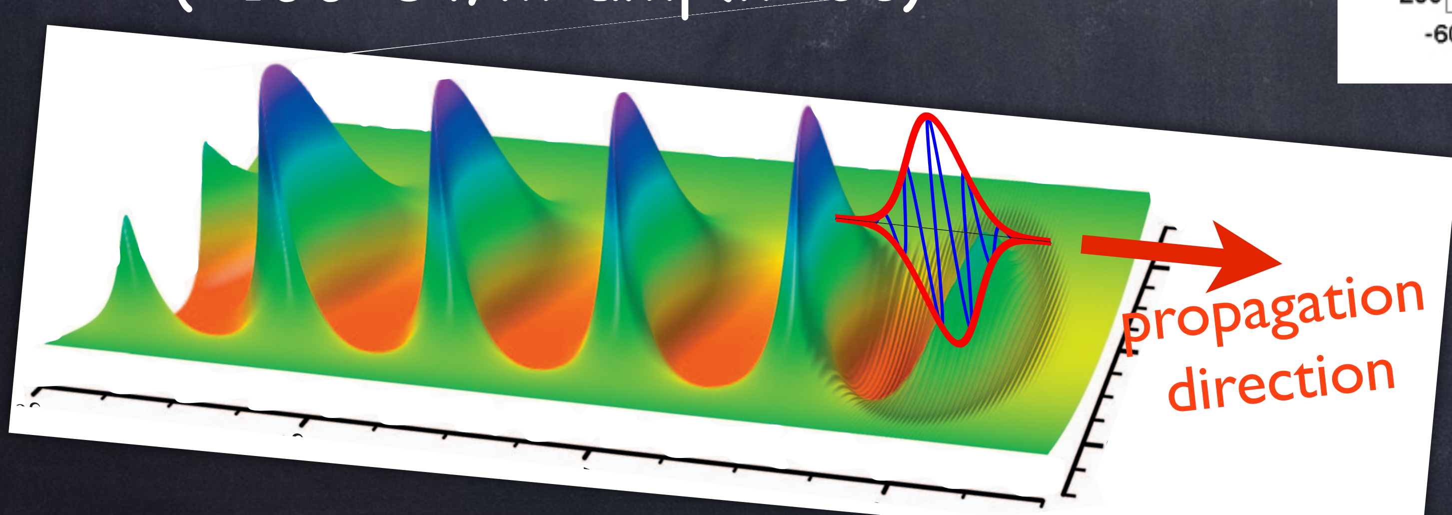
- <u>Linear Wakefield</u>
  laser: $a_0 = 0.35$
  -> sinusoidal density perturbation
  -> ~GV/m sinusoidal longitudinal el. field

- <u>Nonlinear Wakefield</u>
  laser: $a_0 \sim 2$
  -> can only be solved numerically
  -> density spikes
  -> sawtooth el. field
  (~100 GV/m amplitude)





V. Malka, in Proc. of the CERN Accelerator School (2016)



source: Shadwick, UNL

Wake Properties

For linear wakes:

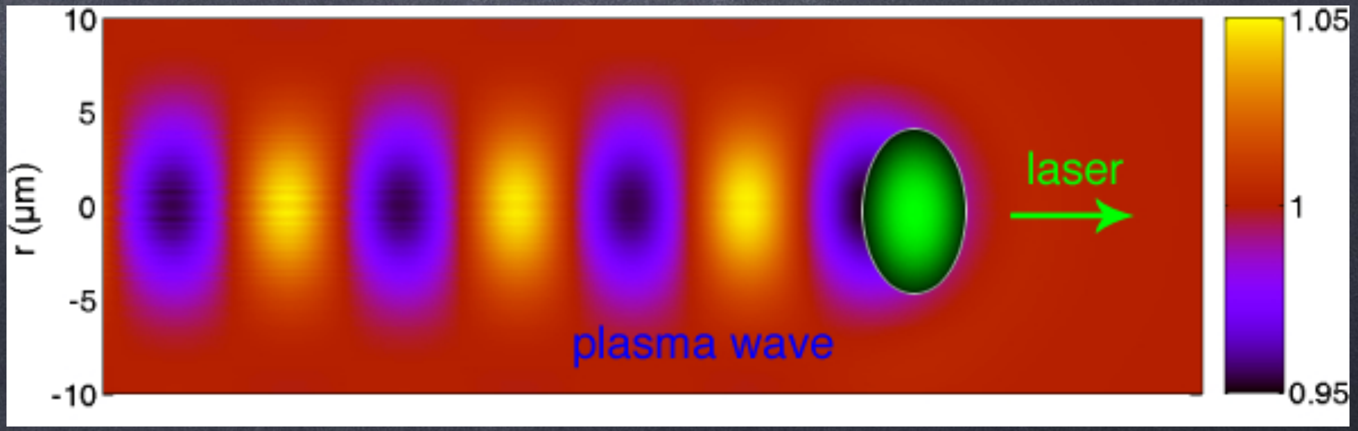
- estimated accelerating field (cold non-relativistic wavebreaking limit)

$$E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]}$$

- for typical densities n_0 = 1x10^{18} cm^{-3}: E_0 = 100 GV/m !! (= 10 V/Å; close to atomic unit electric field !!)

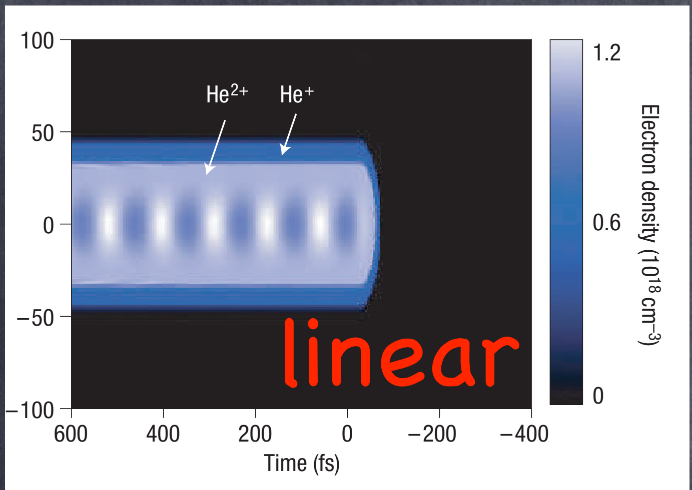
- size of accelerating structure: (~plasma wavelength)

$$\lambda_{p,0}[\mu\mathrm{m}] = \frac{2\pi c}{\omega_{p,0}} = 3.33 \times 10^{10} \left(n_0[\mathrm{cm}^{-3}]\right)^{-1/2}$$

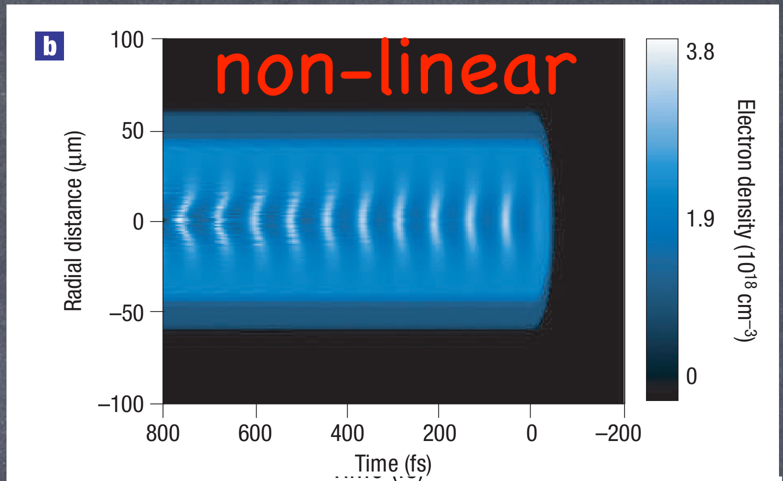


- for n_0 = 1x10^{18} cm^{-3}: λ_p = 30 μm !! (=> or 100 fs)

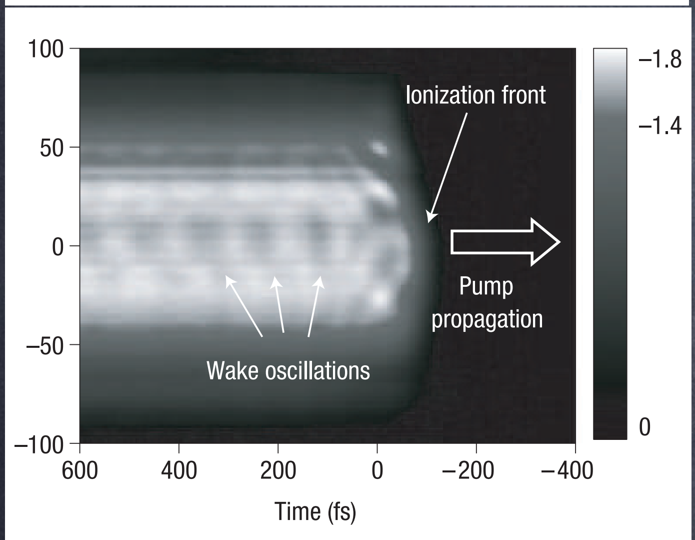
Experimental Observation



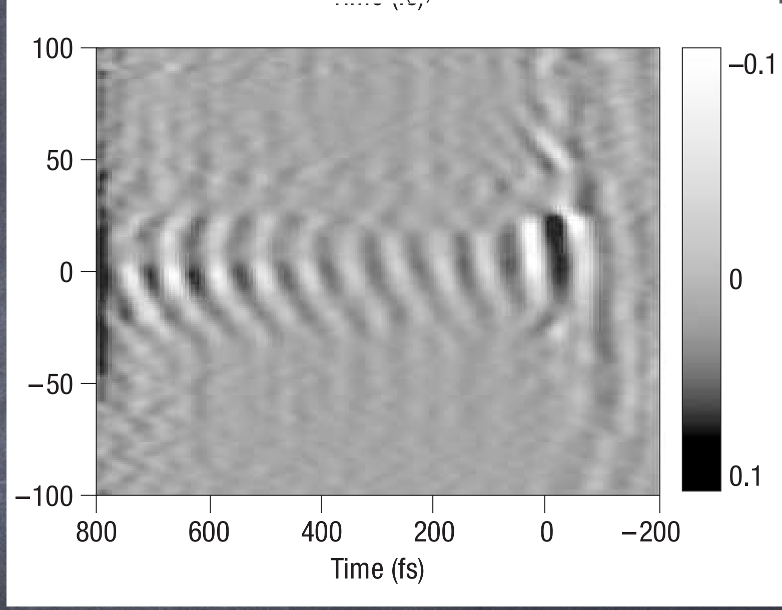
simulation



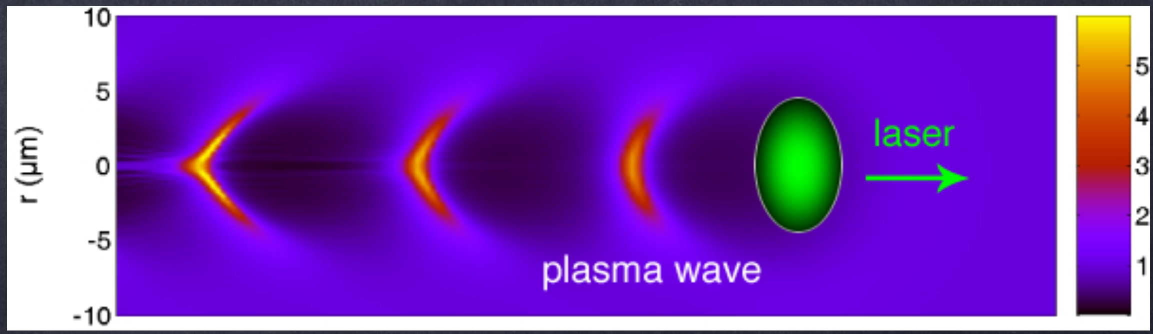
N. Matlis, et al.
Nature Phys., 2, 749 (2006)



experiment
using frequency-domain
holography



non-linear: later buckets
become "horseshoe" shaped



Electron Acceleration

Hamiltonian of electron motion

laser intensity

electrostatic potential of wake

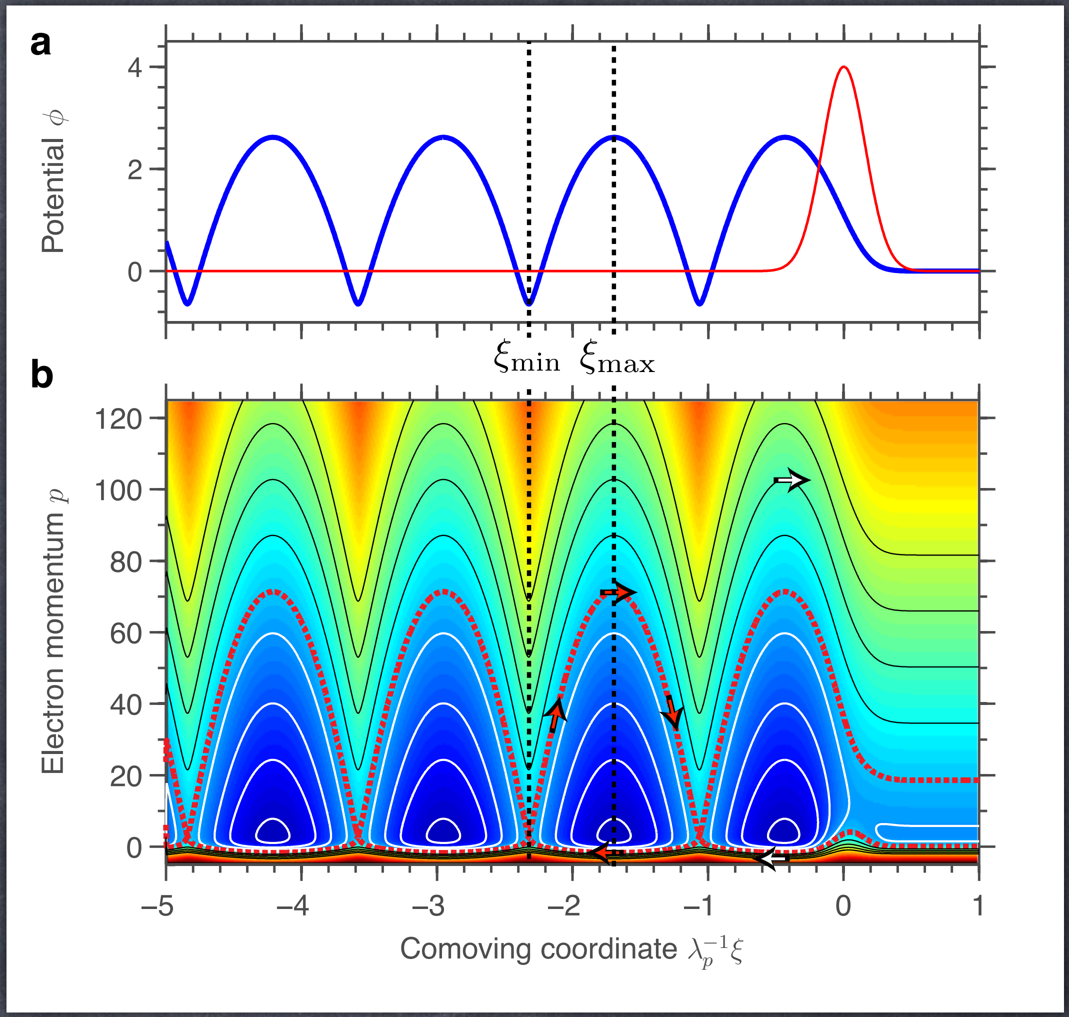
$$H(p_z, \xi) = \sqrt{p_z^2 + 1 + a^2} - \beta_p p_z - \phi(\xi)$$

long. é momentum

velocity of plasma wave

$$\xi = z - v_g t$$

co-moving frame -> distance from laser pulse



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- electrons inside separatrix are accelerated (red)
- electrons outside (white): background plasma electrons
  (flow backwards wrt laser or slowly overtake wake)

Wakefield Properties I

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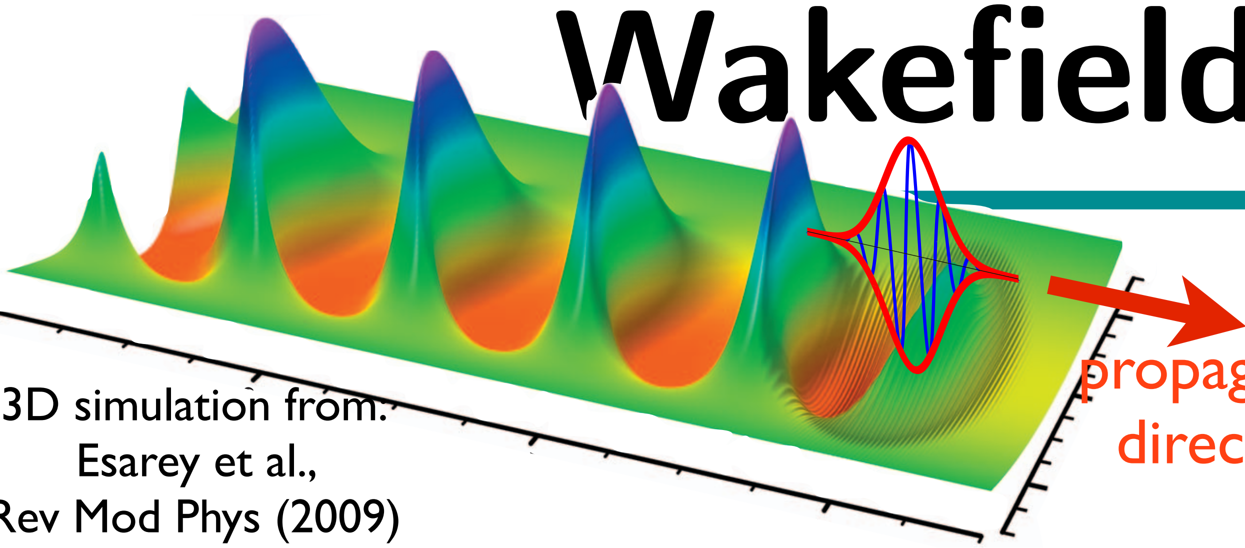
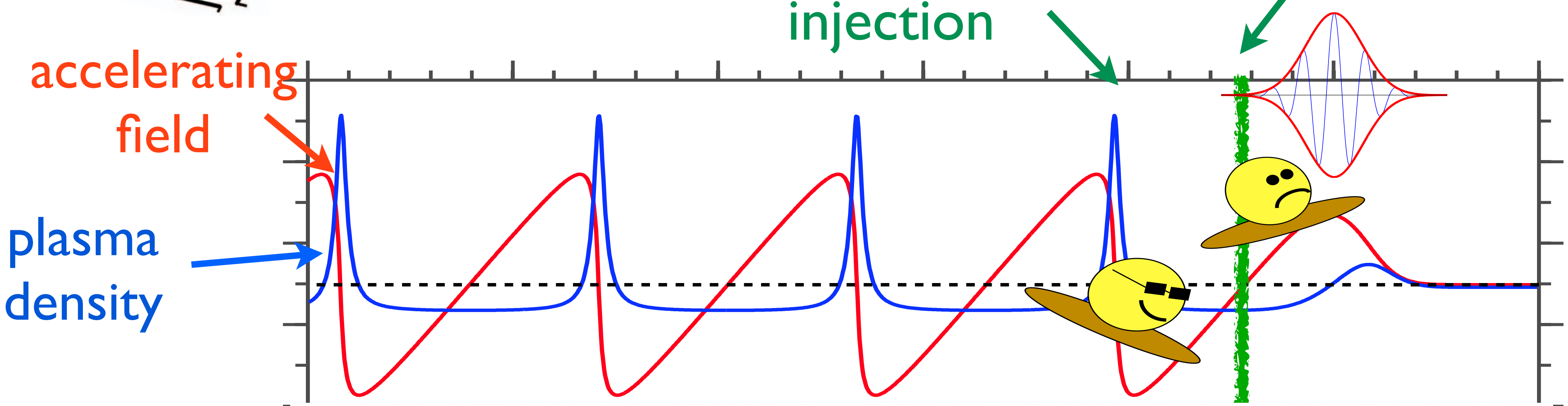
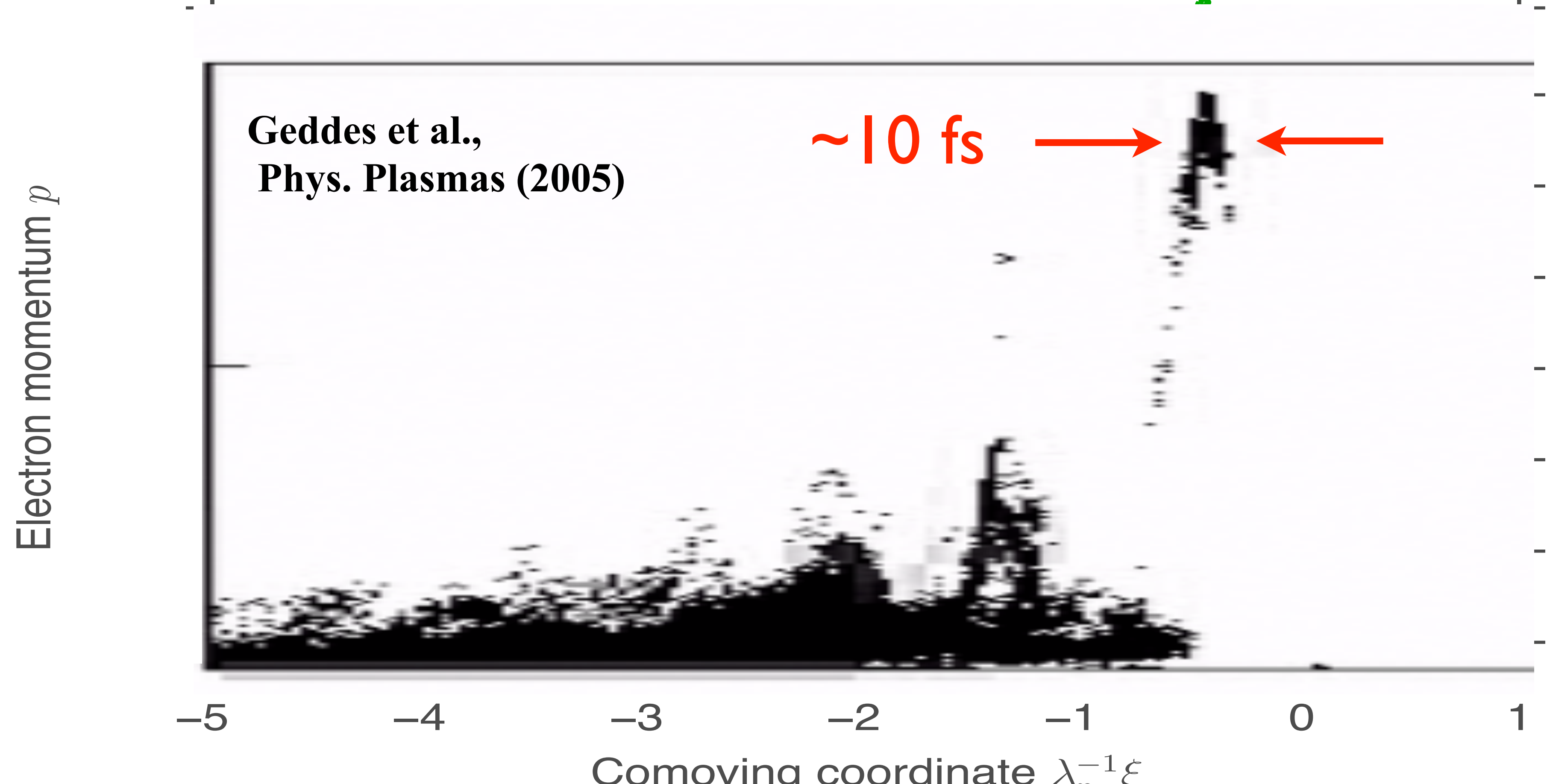


3D simulation from: Esarey et al., Rev Mod Phys (2009)





- electron bunch duration

quasi-monoenergetic peak indicates: fraction of plasma wavelength: λ_p = 15 µm (50 fs)

Wavebreaking

quick summary:

- oscillating plasma electrons form plasma wave
- plasma wave propagates ~with laser group velocity: typically: $\gamma_p \approx$ 10-100 (including 3D effects)

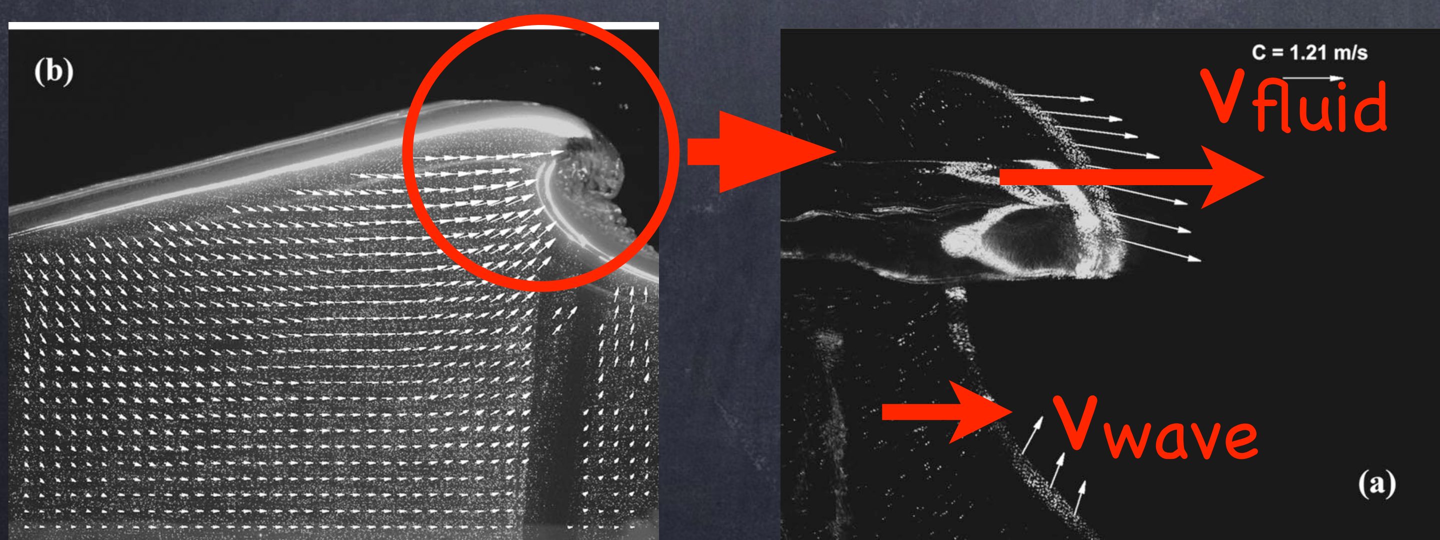
$$\gamma_p = \gamma_{\mathrm{laser,g}} \simeq \frac{\omega}{\omega_p}$$

- electron fluid velocity:
  -> depends on laser intensity

$$\gamma_e = \frac{(1+a^2)+(1+\phi)^2}{2(1+\phi)}$$

- "self" injection through wavebreaking

-> at sufficiently high laser intensity, the electron fluid velocity is higher than plasma wave phase velocity $\gamma_e > \gamma_p$ => wave breaking

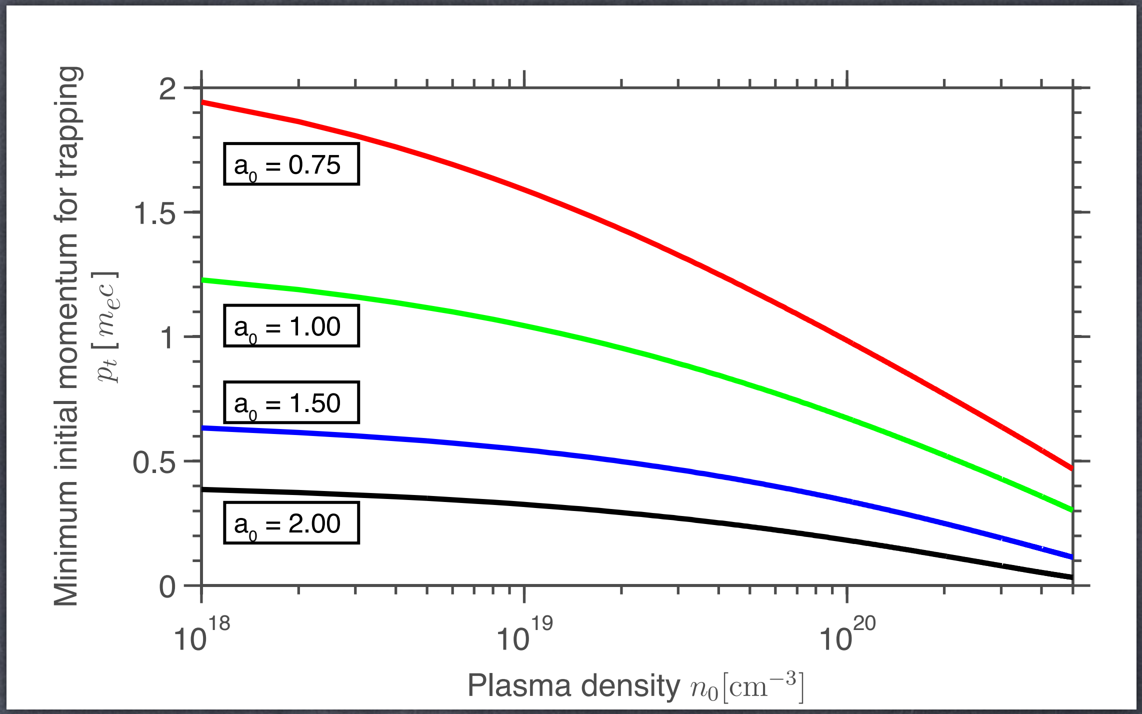


Chang & Liu, Phys. Fluids 10 (1) 1998

Electron Injection

- Electron Trapping
  - electrons get trapped (cross into separatrix) at the back of the bucket (at the potential minimum)
  - minimum momentum required for trapping

$$p_t = \beta_p \gamma_p (1 - \gamma_p \phi_{\min}) - \gamma_p \sqrt{(1 - \gamma_p \phi_{\min})^2 - 1}$$



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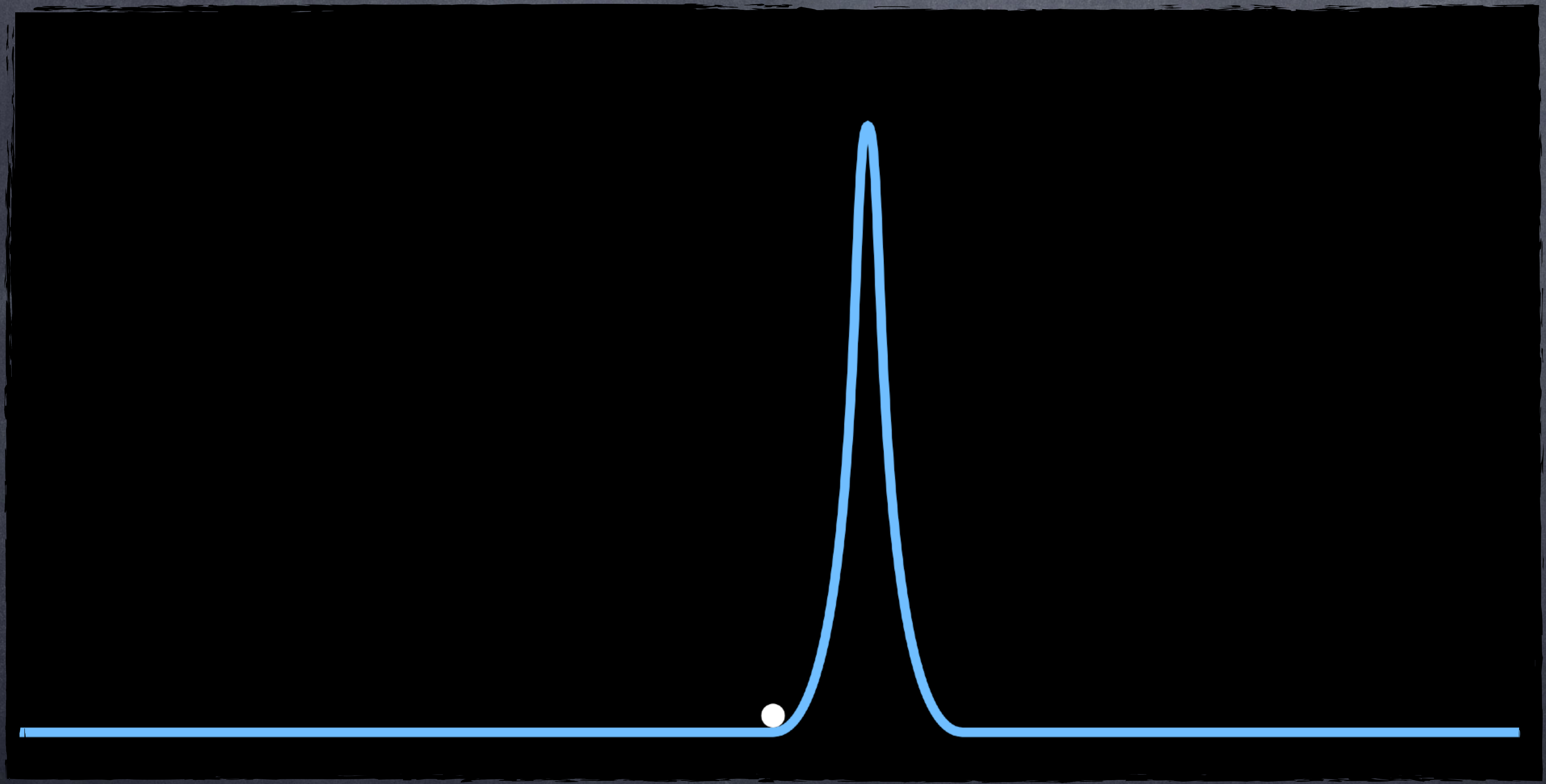
  - implicitly depends on laser intensity a_0 and plasma density n_0

bigger separatrix amplitude

  - lower momentum required for higher laser intensities and higher plasma densities

slower wake velocity

Untrapped electron



Electron velocity too small for trapping

Trapped & accelerated electron

Electron velocity sufficiently high

Accelerator Length, Maximum Energy

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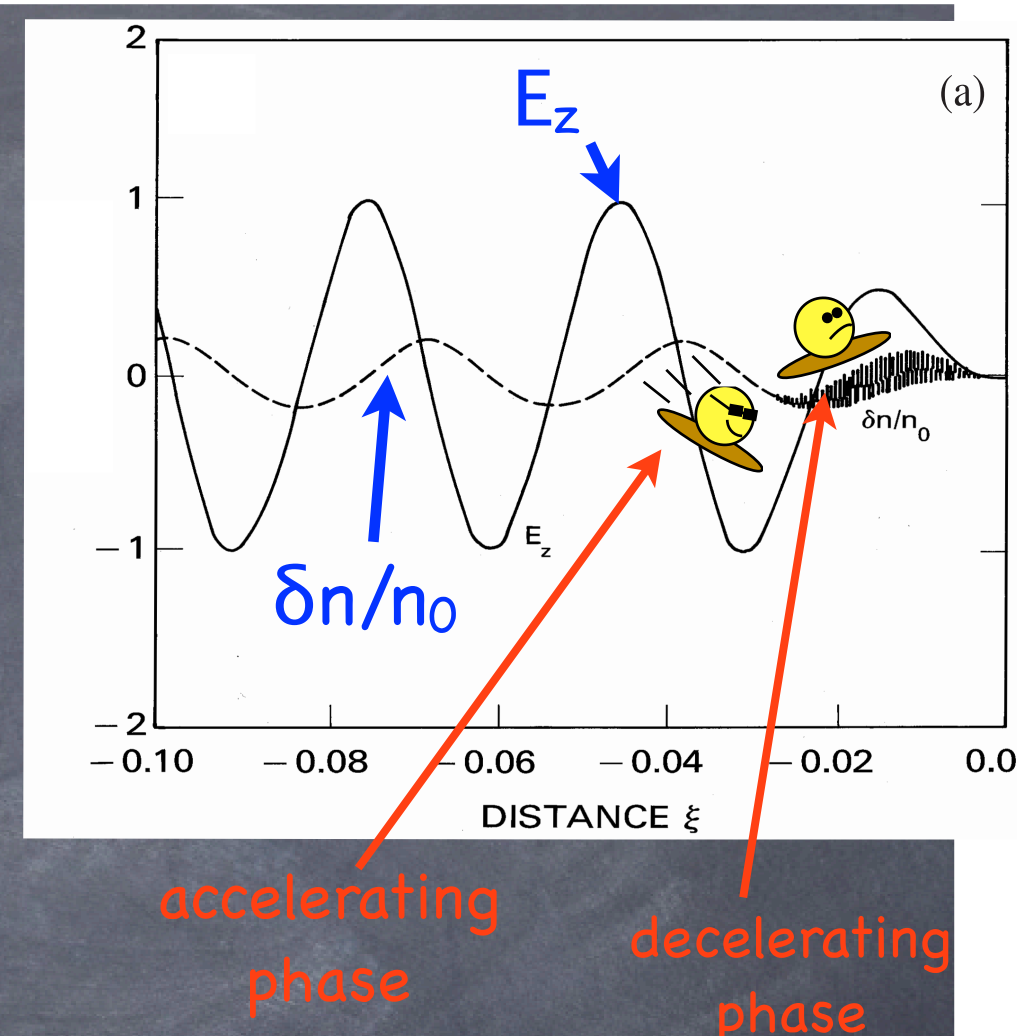
- dispersion relation : $\omega^2 = \omega_p^2 + c^2 k^2$
  (for laser in plasma)
- plasma wave moves
  w laser group velocity: $v_g = \frac{d\omega}{dk} \underset{\cdots}{=} \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} \cdot c$
  slightly slower than c



- dephasing length L_d:

distance until electrons (moving with c) outrun accelerating phase

$$t_{wave} = \frac{L_d}{v_g}$$

$$t_e = \frac{L_d + \frac{\lambda_p}{2}}{c}$$

$$t_{wave} = t_e$$

Accelerator Length, Maximum Energy

- for $\omega_p \ll \omega$: $v_g = \sqrt{1-\frac{\omega_p^2}{\omega^2}}\, c \approx \left(1 - \frac{\omega_p^2/\omega^2}{2}\right) c$

$\frac{\omega_p^2}{\omega^2} \stackrel{!}{=} x^2$

$t_{wave} = \frac{L_d}{v_g}$

$t_e = \frac{L_d + \frac{\lambda_p}{2}}{c}$

$t_{wave} = t_e$: $\frac{L_d}{\left(1-\frac{x^2}{2}\right)\cancel{c}} = \frac{L_d + \lambda_p/2}{\cancel{c}}$

$$L_d \underbrace{\left(\frac{1}{1-\frac{x^2}{2}} - 1\right)}_{} = \frac{\lambda_p}{2}$$

$$\frac{1-\left(1-\frac{x^2}{2}\right)}{1-\frac{x^2}{2}} = \frac{x^2}{2\left(1-\frac{x^2}{2}\right)} = \frac{x^2}{2-x^2}$$

$$L_d = \frac{2-x^2}{x^2}\,\frac{\lambda_p}{2}$$

$\left(\frac{\omega_p}{\omega}\right)^2 \ll 1$

$$= \frac{\cancel{2}}{x^2}\,\frac{\lambda_p}{\cancel{2}} = \underbrace{\frac{\omega_p^2}{\omega^2}}_{\frac{\lambda_p^2}{\lambda^2}} \lambda_p = \frac{\lambda_p^3}{\lambda^2}$$

- **dephasing length L_d:** $L_d \simeq \frac{\lambda_p^3}{\lambda^2} \sim n_0^{-3/2}$

distance until electrons (moving with c) outrun accelerating phase

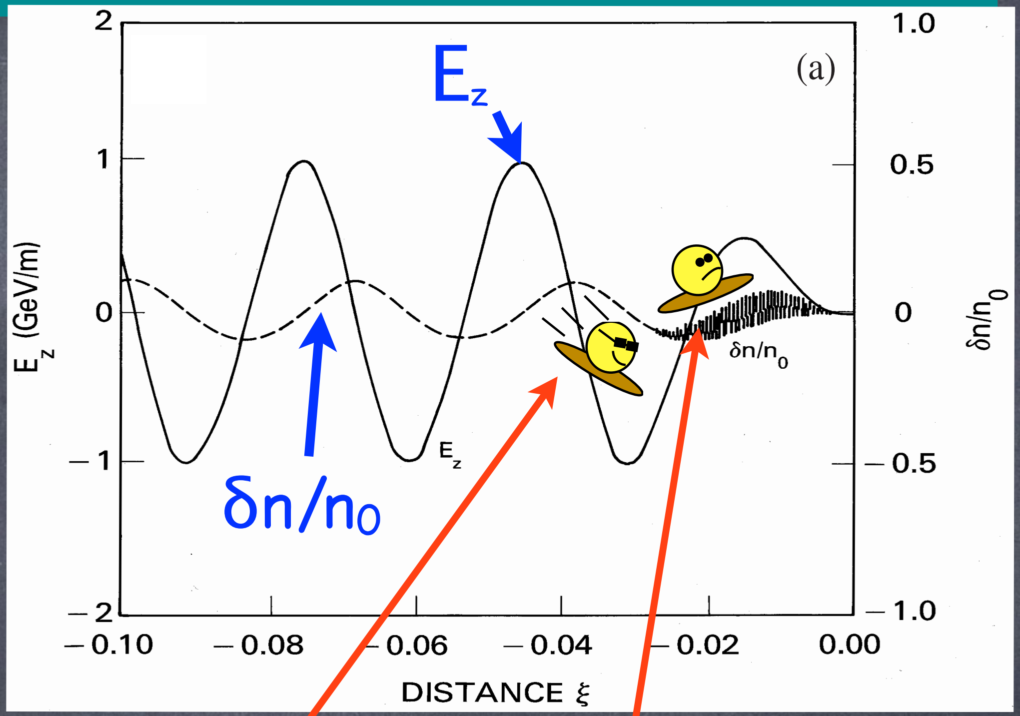
Accelerator Length, Maximum Energy

- dephasing length:

distance until electrons (moving with c) outrun accelerating phase

$$L_d \simeq \frac{\lambda_p^3}{\lambda^2} \sim n_0^{-3/2}$$

$\lambda_p \sim \frac{1}{\sqrt{n_0}}$



accelerating phase

decelerating phase

- max electron energy:

assume const. E_0

$$W_{\max} = eE_0 L_d \sim \frac{1}{n_0}$$

$$E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]}$$

$\Rightarrow$ overall higher energy gain for lower plasma densities

Accelerator Length, Maximum Energy

- dephasing length:

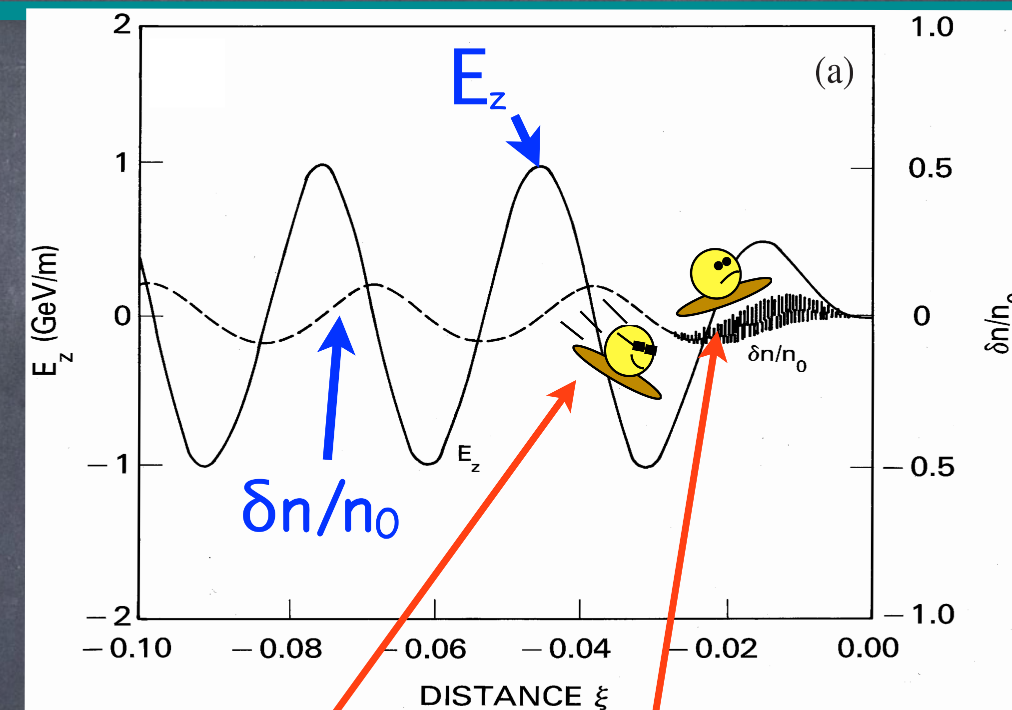
$$L_d \simeq \frac{\lambda_p^3}{\lambda^2} \sim n_0^{-3/2}$$

for $n_0 = 3 \cdot 10^{17}\,\mathrm{cm}^{-3}$:

$$\lambda_{p,0}[\mu m] = \frac{2\pi c}{\omega_{p,0}} = 3.33 \times 10^{10} \left(n_0[\mathrm{cm}^{-3}]\right)^{-1/2} = 60\,\mu m$$

$$L_d = \frac{(60\,\mu m)^3}{(0.8\,\mu m)^2} = 0.3\ m$$

laser → $(0.8\,\mu m)$



accelerating phase

decelerating phase

- max electron energy:

$$W_{max} = eE_0 L_d \sim \frac{1}{n_0}$$

assume const. E_0

$$E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]} \approx 52\ \frac{GV}{m}$$

$\Rightarrow W_{max} = 15.6\ GeV$

- Experimental results:

PHYSICAL REVIEW LETTERS **122**, 084801 (2019)

Editors' Suggestion Featured in Physics

Petawatt Laser Guiding and Electron Beam Acceleration to 8 GeV in a Laser-Heated Capillary Discharge Waveguide

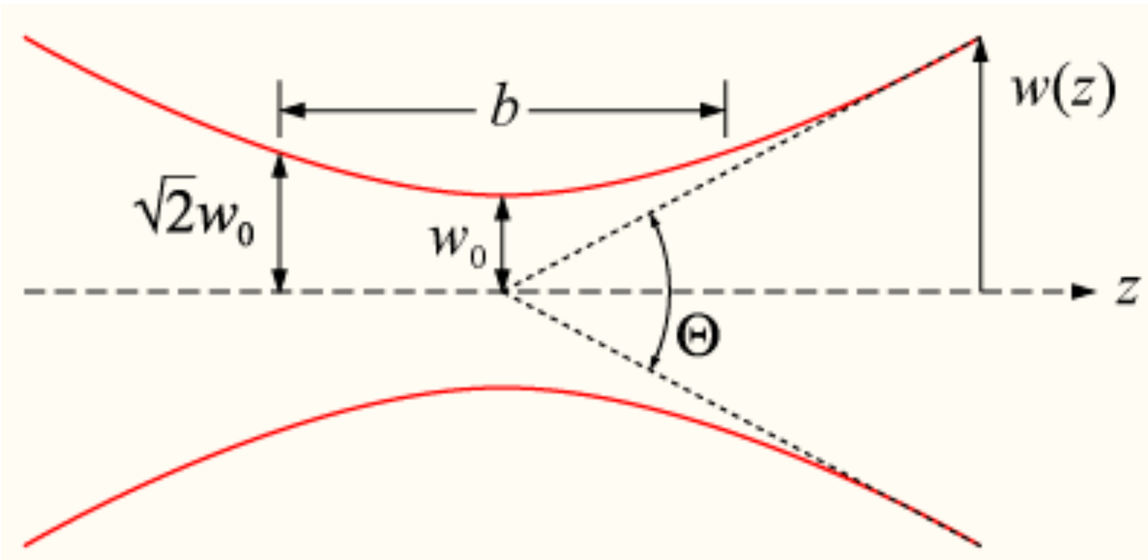
A. J. Gonsalves,[1,*] K. Nakamura,[1] J. Daniels,[1] C. Benedetti,[1] C. Pieronek,[1,2] T. C. H. de Raadt,[1] S. Steinke,[1] J. H. Bin,[1] S. S. Bulanov,[1] J. van Tilborg,[1] C. G. R. Geddes,[1] C. B. Schroeder,[1,2] Cs. Tóth,[1] E. Esarey,[1] K. Swanson,[1,2] L. Fan-Chiang,[1,2] G. Bagdasarov,[3,4] N. Bobrova,[3,5] V. Gasilov,[3,4] G. Korn,[6] P. Sasorov,[3,6] and W. P. Leemans[1,2,†]

along the capillary axis [20], and that this structure can extend the LPA length to 20 cm (15 diffraction lengths) at low ($\approx 3.0 \times 10^{17}\ \mathrm{cm}^{-3}$) density. This enabled the generation of electron beams with quasimonoenergetic peaks in energy up to 7.8 GeV using a peak laser power of 850 TW.

For L = 20 cm:

W_{max} = 52 GV/m *0.2m = 10.4 GeV

Other Acceleration Limits: Laser Diffraction



$$Z_R = \pi r_0^2/\lambda$$

for 25 μm focus: Zr = 0.5 mm

Guiding

plasma index of refraction:

$$\eta = \sqrt{1-\left(\frac{\omega_p}{\omega_{las}}\right)^2} = \sqrt{1-\frac{4\pi e^2 n_e}{m_e}}$$

phase velocity:

$$v_{ph} = \frac{c}{\eta}$$

How to guide a laser?

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$$\eta = \sqrt{1-\left(\frac{\omega_p}{\omega_{las}}\right)^2} = \sqrt{1-\frac{4\pi e^2 n_e}{m_e}}$$

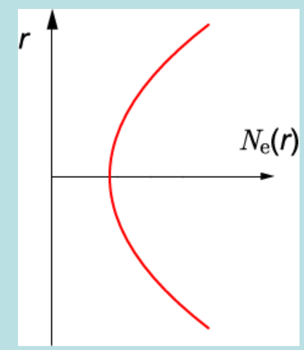
guiding using density gradient: "Plasma fiber"

$$v_{ph} = \frac{c}{\eta}$$

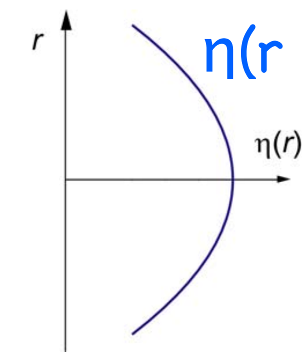
relativistic self focussing
-> self guiding

engineering transverse plasma profile (low density on axis)

Plasma channel
transverse variation of ***density*** yields refractive index profile

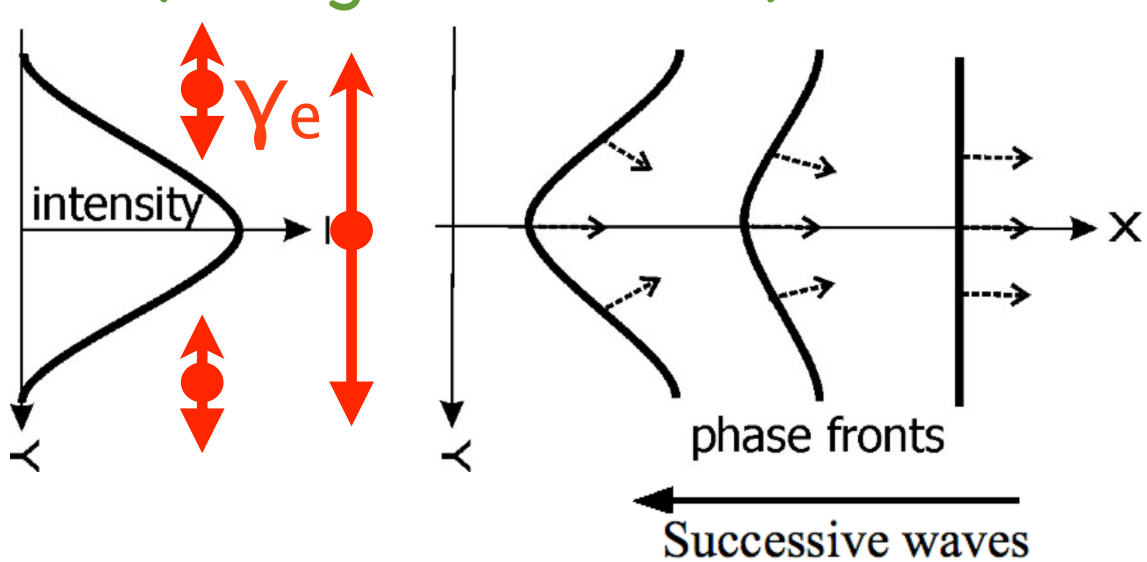


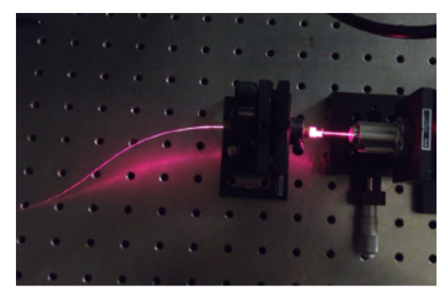
$$\eta \approx 1 - \frac{1}{2}\frac{n_e(r)e^2}{\gamma m_e \varepsilon_0 \omega^2}$$

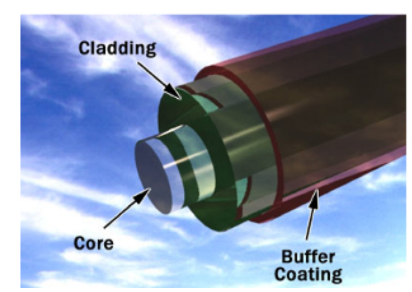


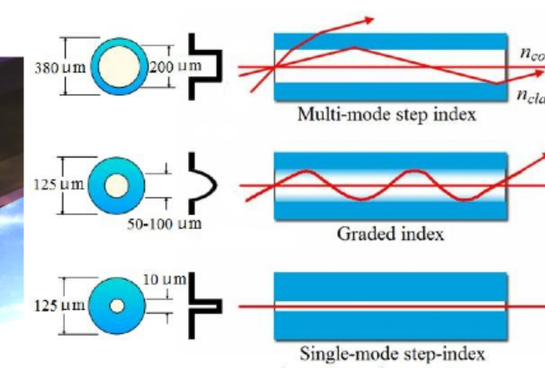
Self-focussing: $v_{ph} = c/n_R$

increase in electron mass on axis (at high intensities)

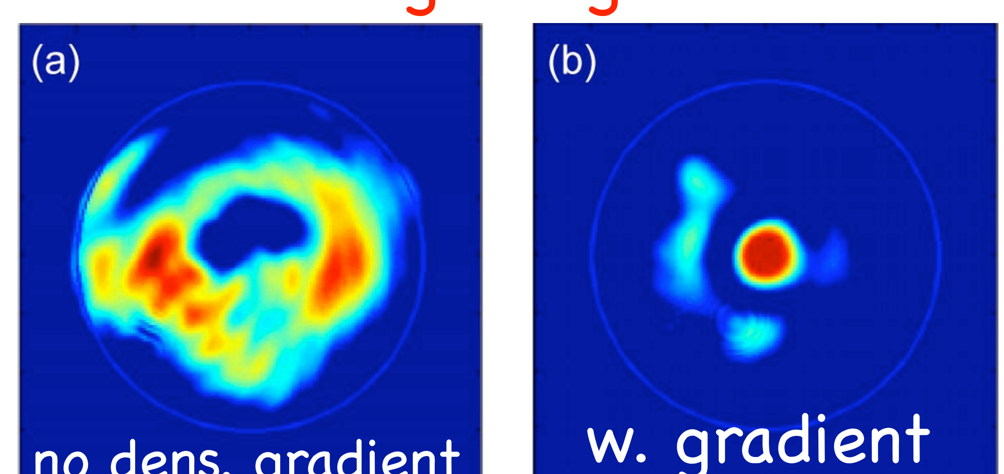






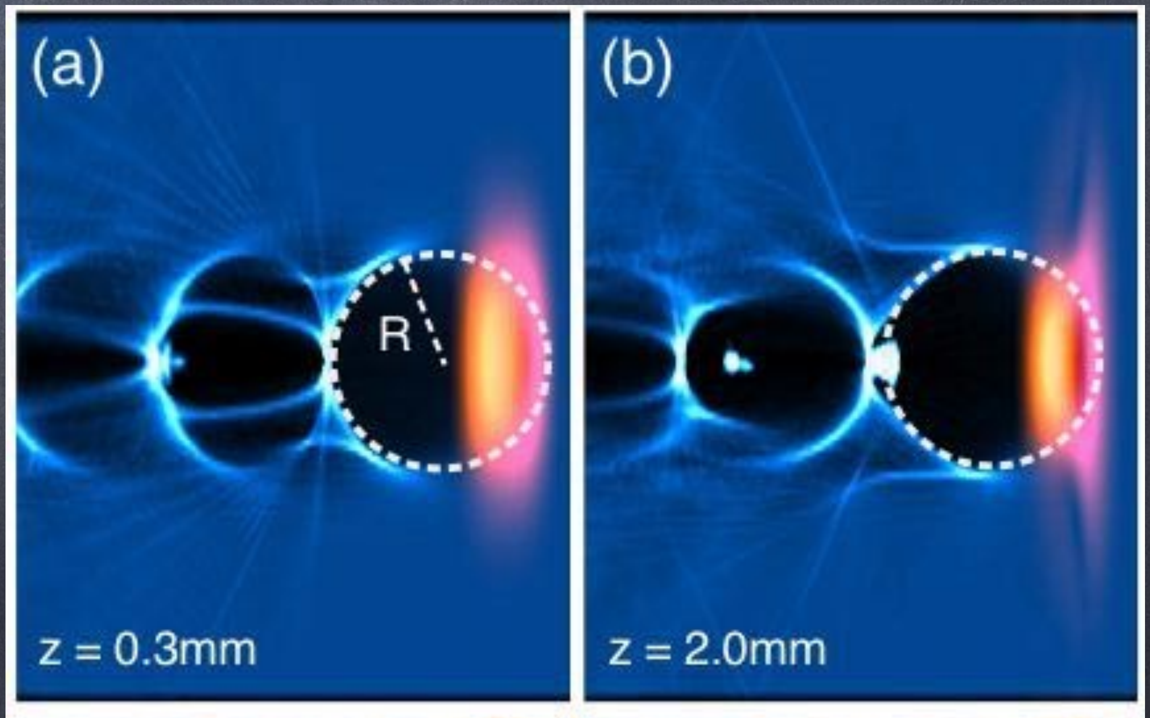


laser guiding



"Bubble"-Regime

- for $a_0 >> 1$
- electrons also transversally expelled
- completely cavitated spherical ion "bubble" trailing laser pulse
- radius of bubble depends on laser and plasma properties



Lu et al., PRSTAB (2007)

The Bubble

- Assume spherical cavity: fields can be derived using Gauss' Law

$$E_z(\xi) \simeq \frac{\xi}{2} k_p E_0$$

E_0: nonrel wave breaking limit

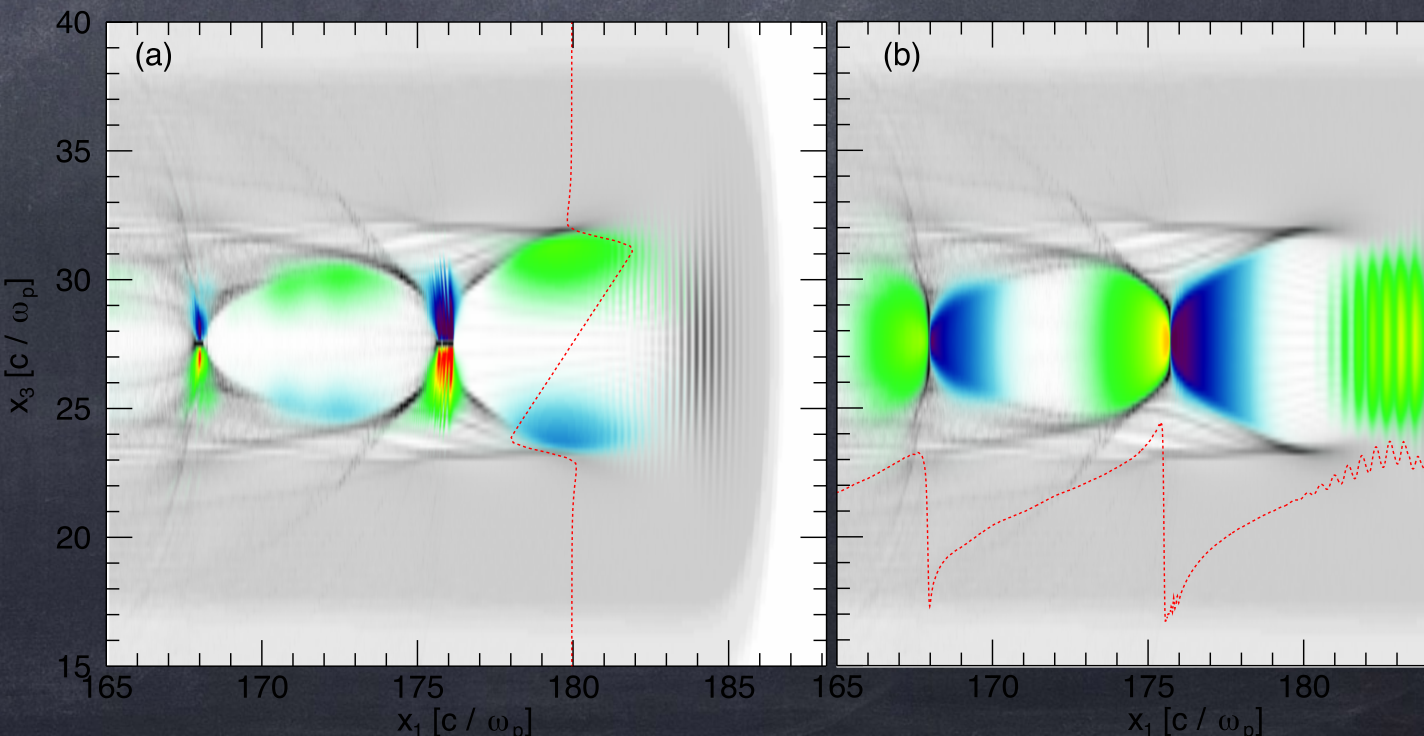
- Accelerating field: (linear with distance)
  - same field strength across different transv. positions
  - max field at ξ=R:

$$E_{\max} = \sqrt{a_0} E_0$$

- Transverse (restoring) field

$$E_r(r_\perp) - B_\Theta(r_\perp) = \frac{r_\perp}{2} k_p E_0$$

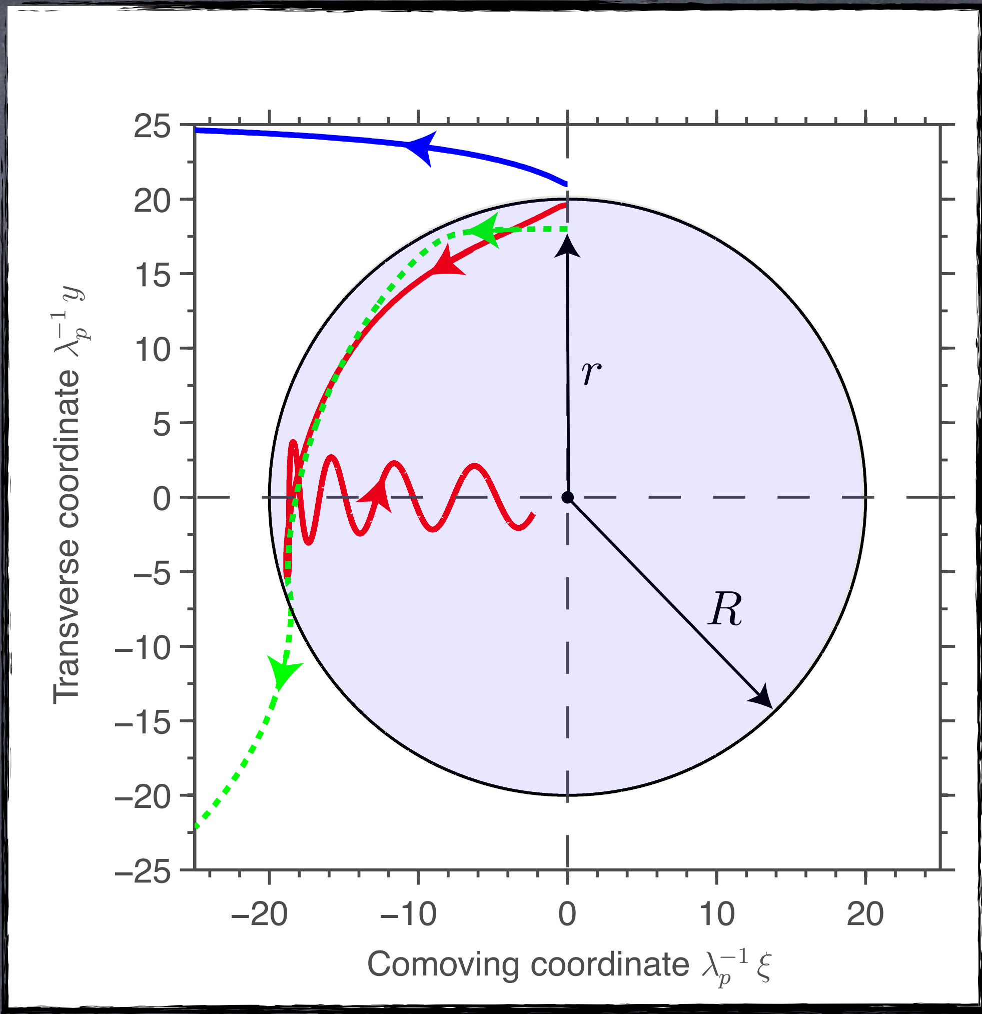
Vieira, in Proc. of the CERN Accelerator School (2016)



Injection into Bubble

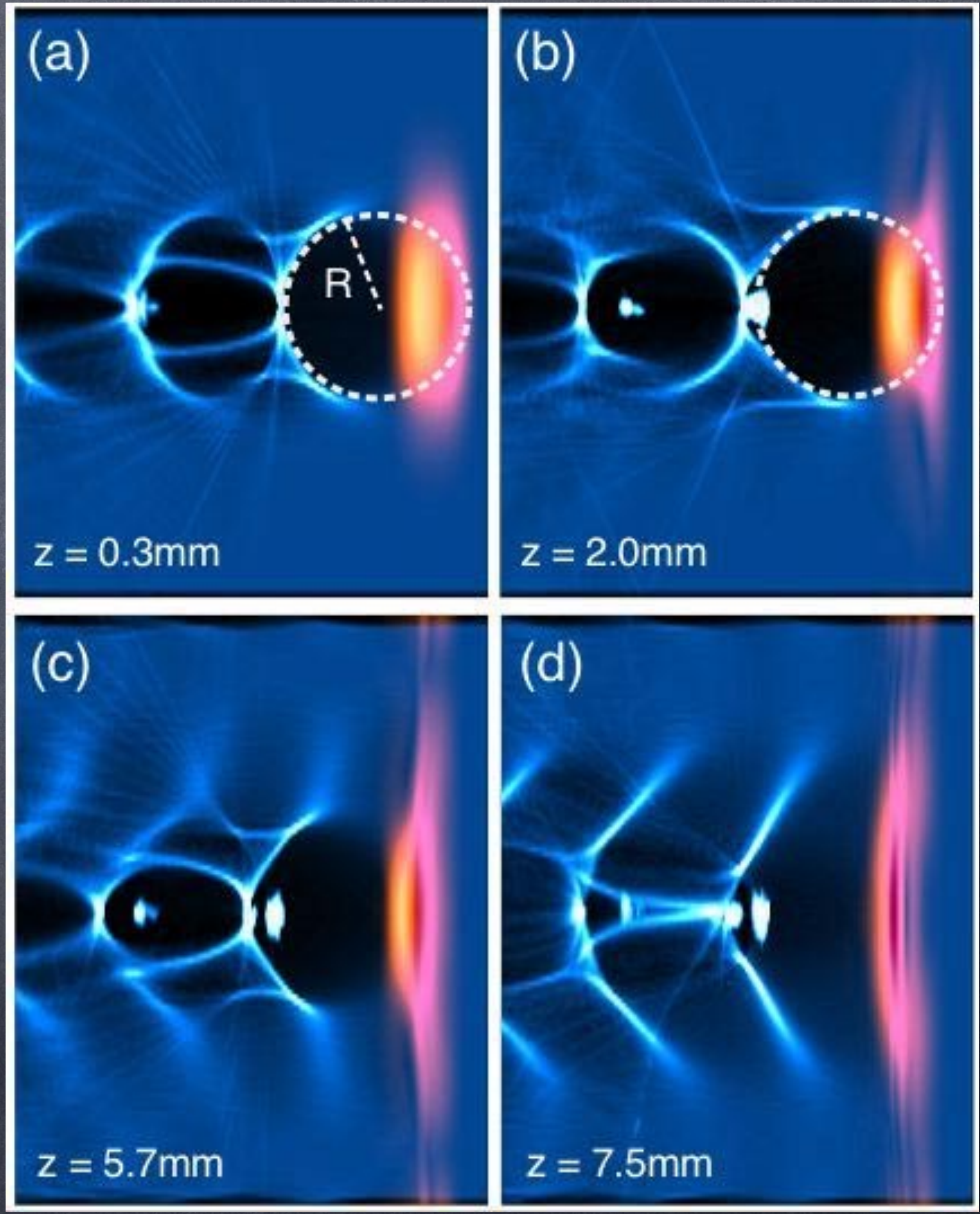
"bubble" or blowout regime:

- transversally expelled electrons -> pulled back by space charge field of ions
- electrons with suitable initial conditions (red) undergo sufficient longitudinal acceleration while bubble passes by
- injection at back of bubble
- electron have finite transverse momentum
- perform transv. (betatron) oscillations -> move on sinusoidal trajectory during acceleration
- amplitude of transv. motion decreases with increasing electron energy as: $(r_\beta \propto \gamma^{-1/4})$



Bubble dynamics

- dynamics and evolution highly nonlinear
- detailed description including laser evolution, space charge effects of injected electrons, … and their feedback on bubble structure need large-scale 3D particle-in-cell (PIC) simulations



Lu et al., PRSTAB (2007)