

# Modelling In-Ice Secondary Cosmic-Ray Cascades

ARENA

10/06/26

RADAR ECHO TELESCOPE

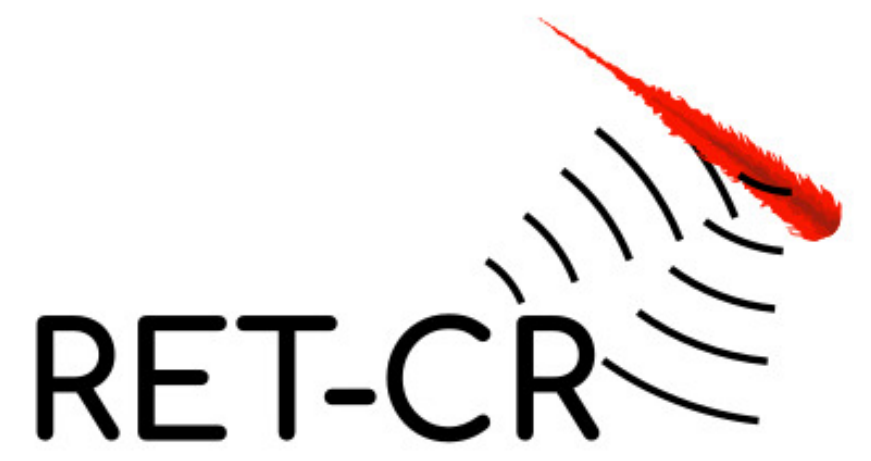


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Krijn de Vries, Enrique Huesca Santiago,  
Jannes Loonen, Katie Mulrey, Krishna Nivedita,  
Jethro Stoffels, Simona Toscano*



UNIVERSITÉ  
LIBRE  
DE BRUXELLES

# Motivation



▶ Searching for in-ice CR cascades using in-ice radar system

▶ Investigating possibility to also apply method to in-ice Askaryan from CRs

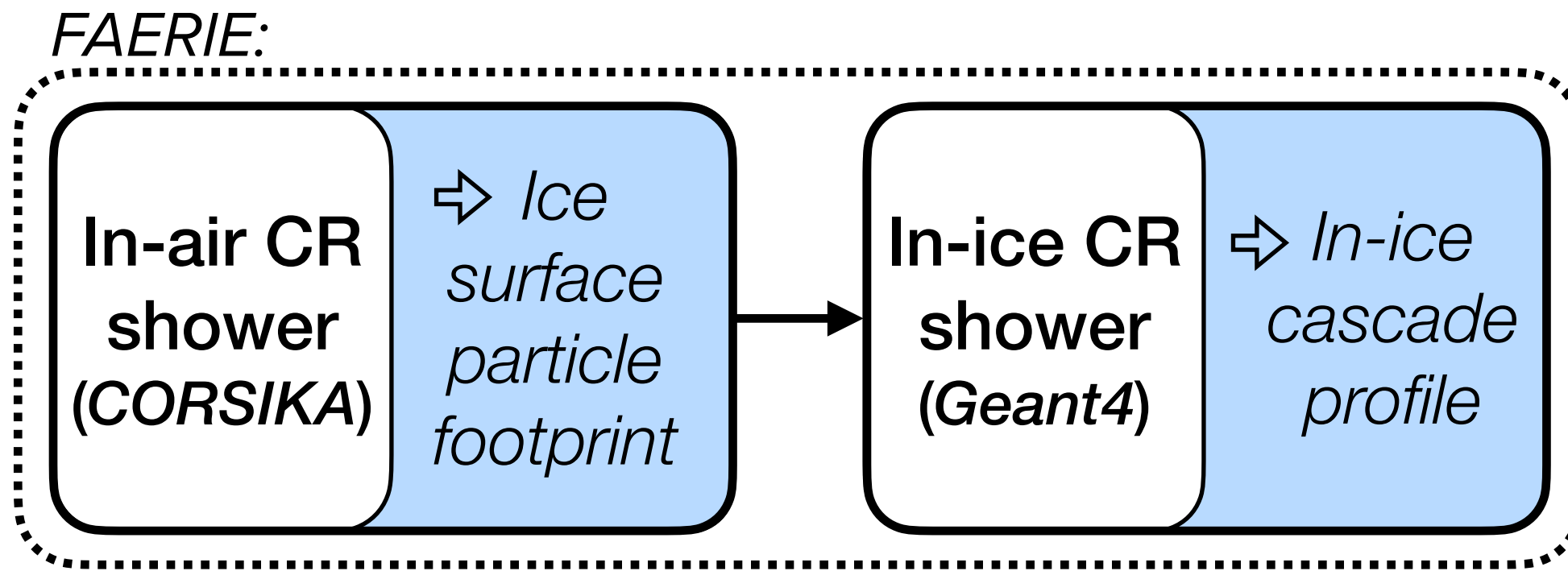
↓  
Need to be able to accurately and quickly simulate this cascade type

Background for in-ice radio neutrino detectors e.g. ↓  
*ARA collaboration, 2026 (arXiv:2510.21104)*

# Simulation pipeline

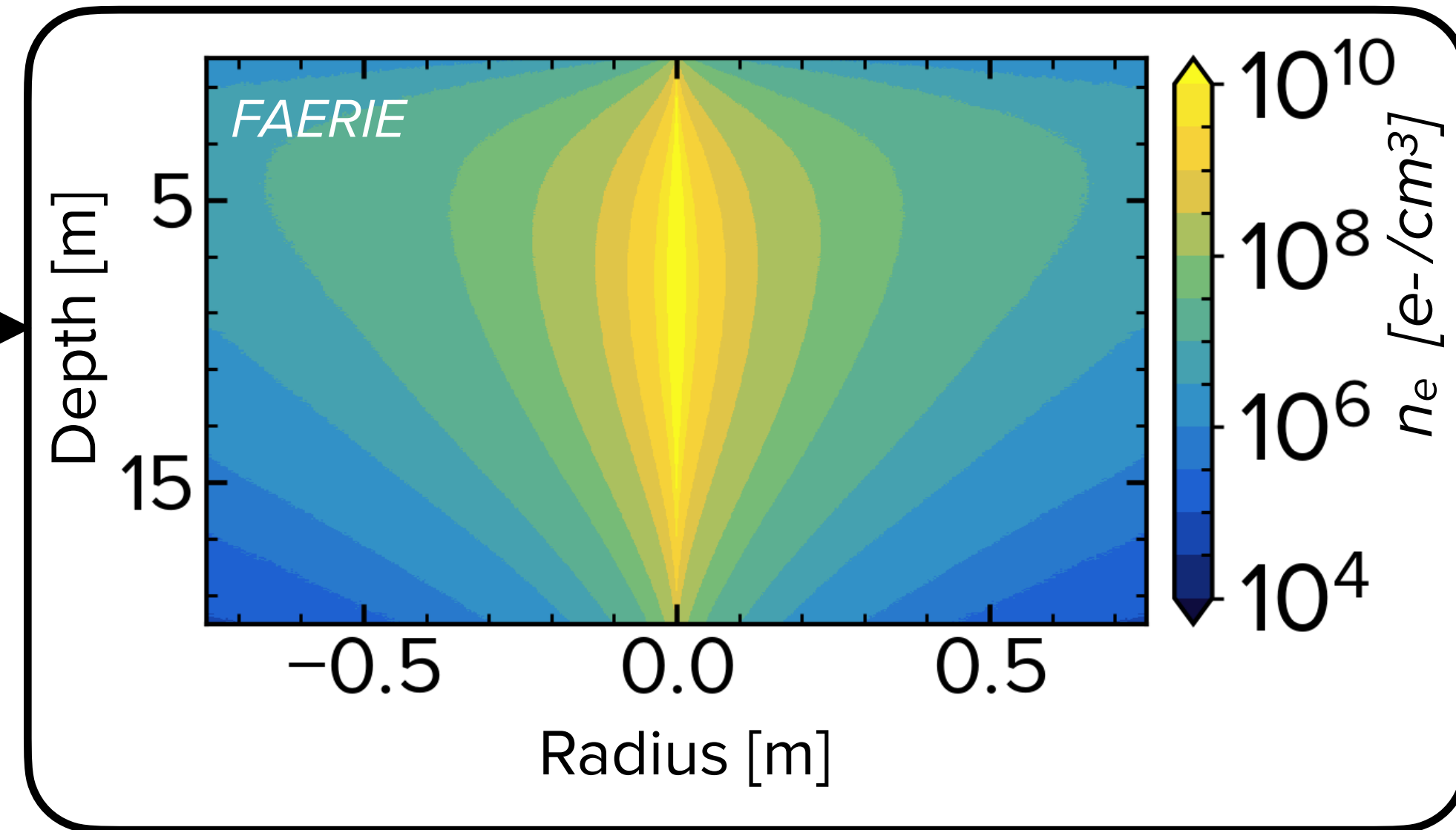


- ▶ Can use **FAERIE** code package to obtain an accurate particle-level simulation



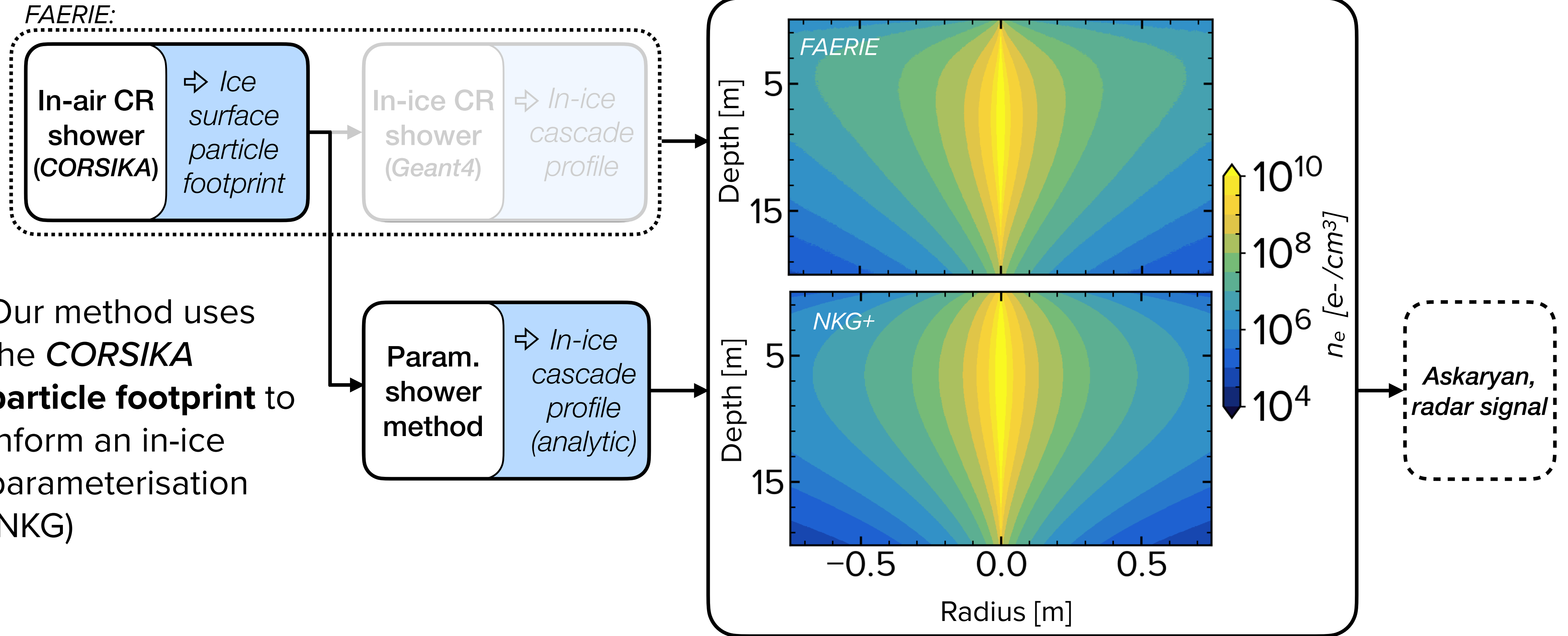
- ▶ Final output is both the **in-ice profile** and the radio signal

$10^{17.0}$  eV shower, in-ice electron density



# Simulation pipeline

- ▶ Can use **FAERIE** code package to obtain an accurate particle-level simulation

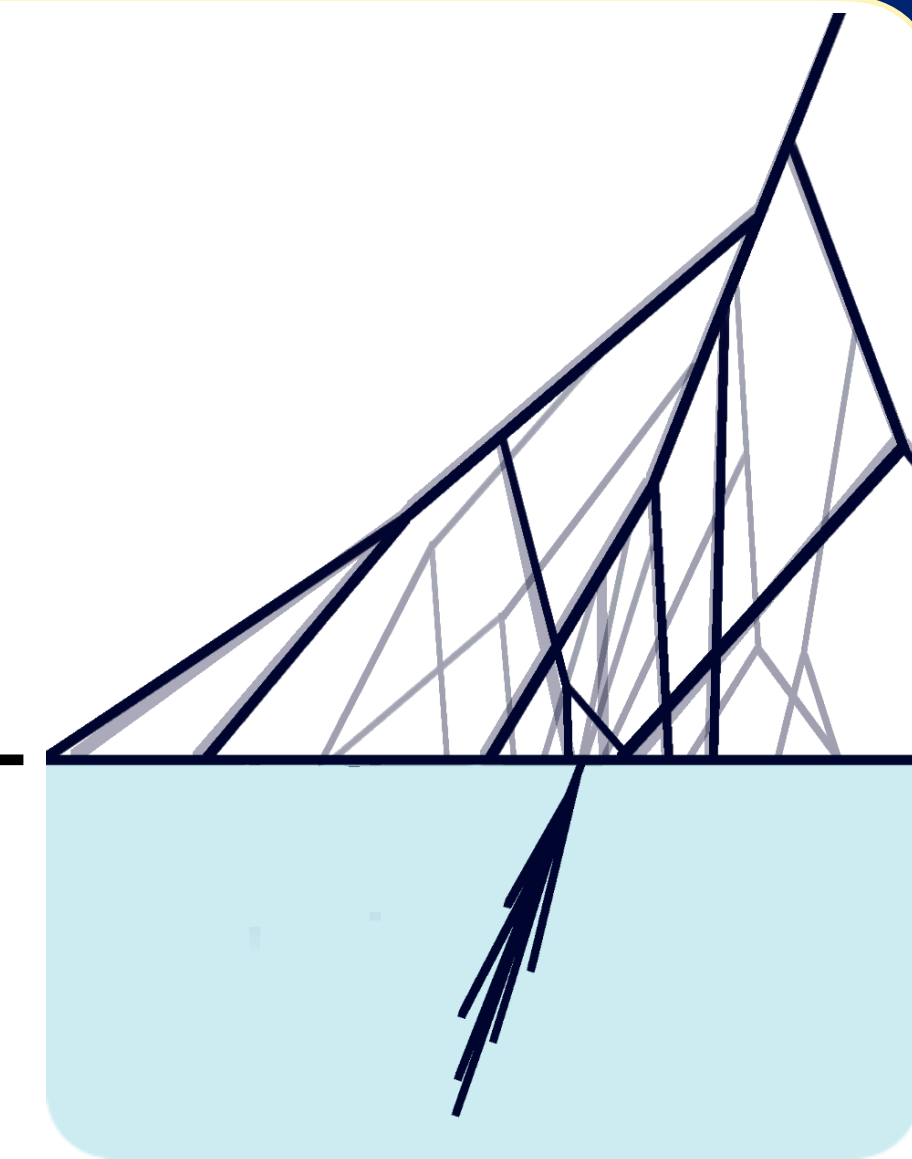
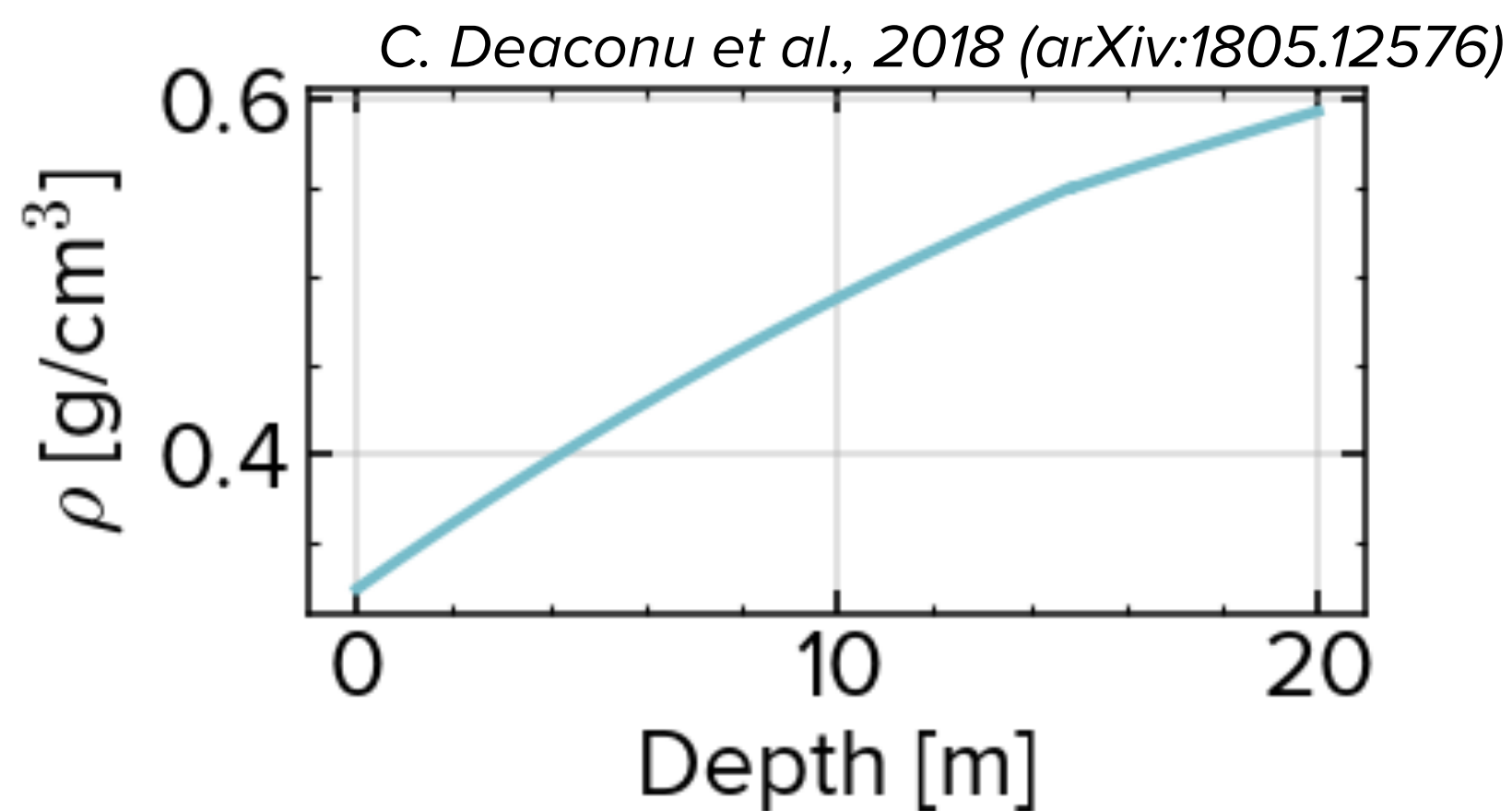


- ▶ Our method uses the **CORSIKA particle footprint** to inform an in-ice parameterisation (NKG)

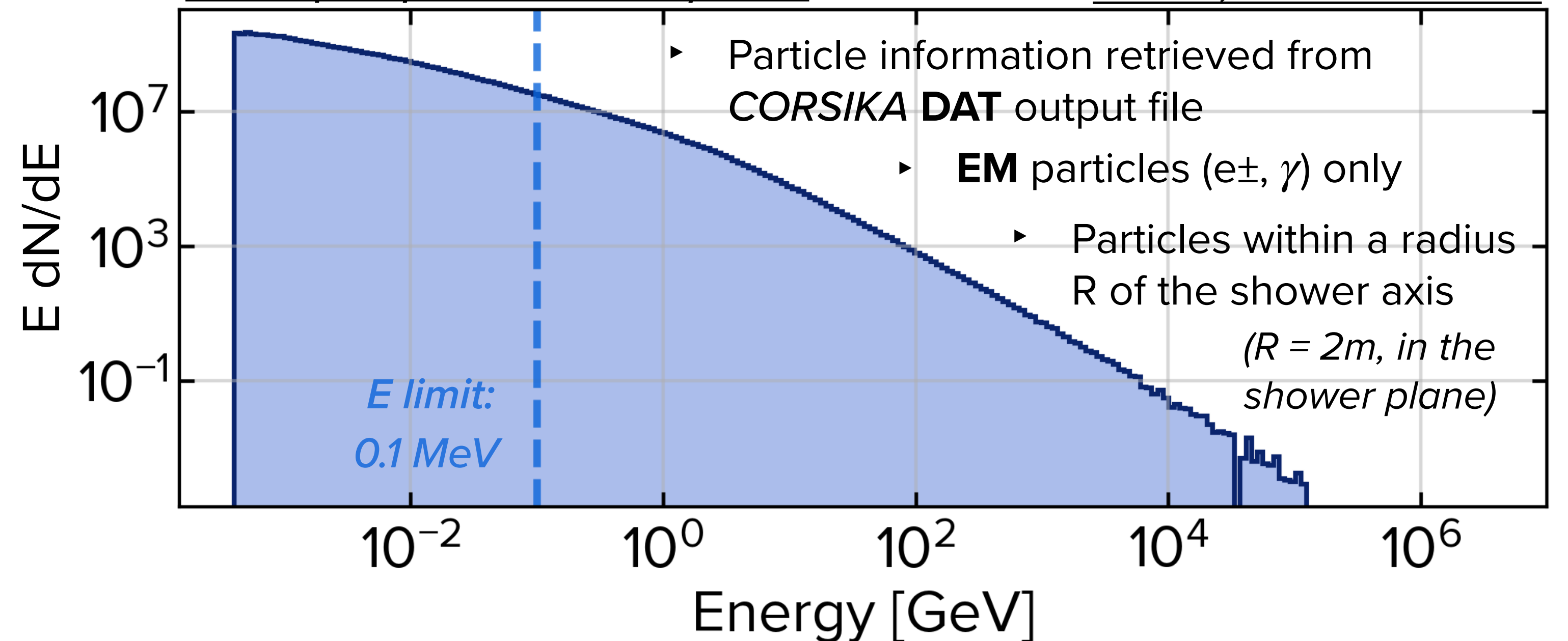
$10^{17.0}$  eV shower, in-ice electron density

# Simulations

- ▶ Compare in-ice cascade output from *FAERIE* to NKG+ method:
- ▶ *CORISKA* simulations:  $10^{16}$  -  $10^{17.5}$  eV energies,  $0 - 45^\circ$  zeniths, proton primary
- ▶ *Geant4*: Using a Greenland ice density profile (double exponential) ↓



Example particle footprint:



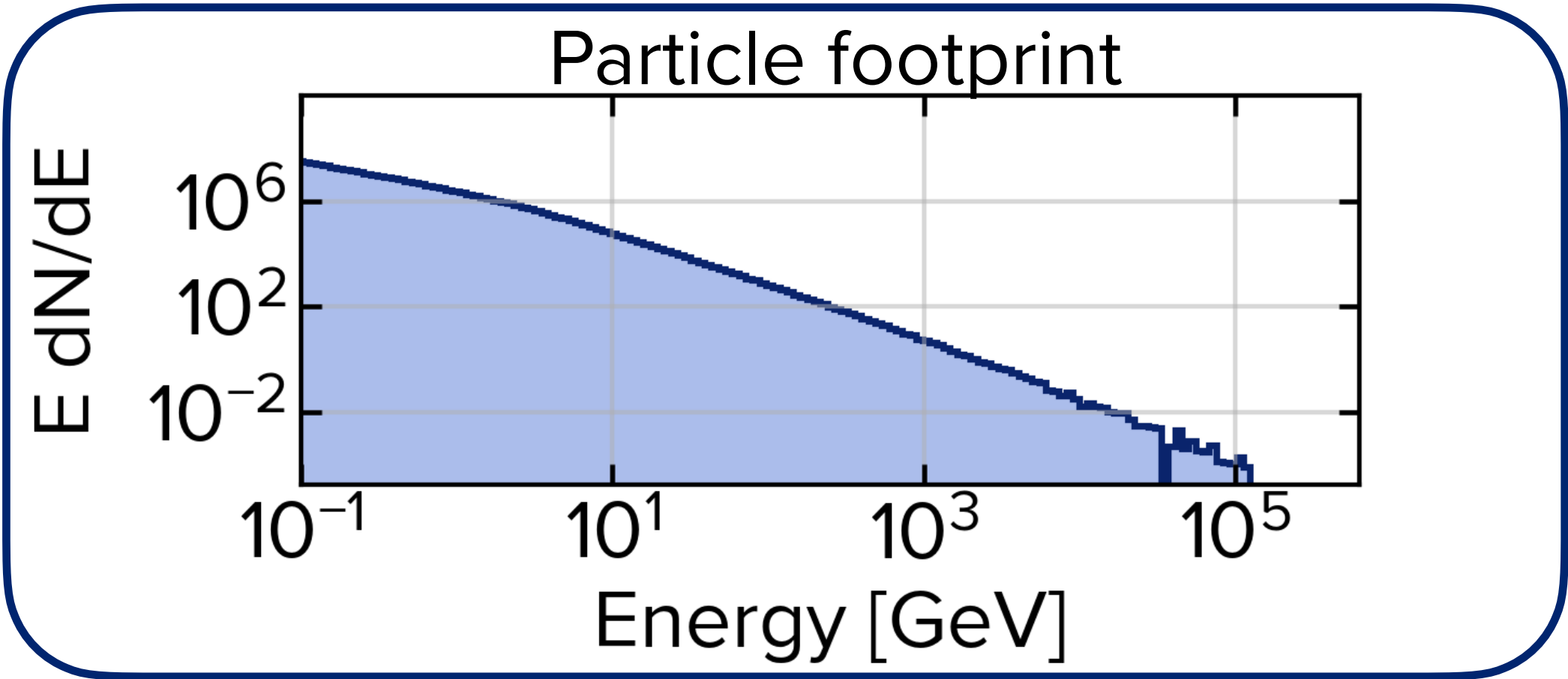
# Longitudinal Profile

▶ NKG longitudinal parameterisation:

$$N(X, E_i) = \frac{0.31}{\sqrt{\ln(E_i/E_c)}} e^{\frac{X}{X_0}(1 - 1.5 \ln(s))}$$

*K. Greisen (1956)*

- ▶  $E_c = 78.6 \text{ MeV}$
  - ▶  $X_0 = 36.08 \text{ g/cm}^2$
- $$s(X, E_i) = \frac{3X/X_0}{X/X_0 + 2 \ln(E_i/E_c)}$$



# Longitudinal Profile

► NKG longitudinal parameterisation:

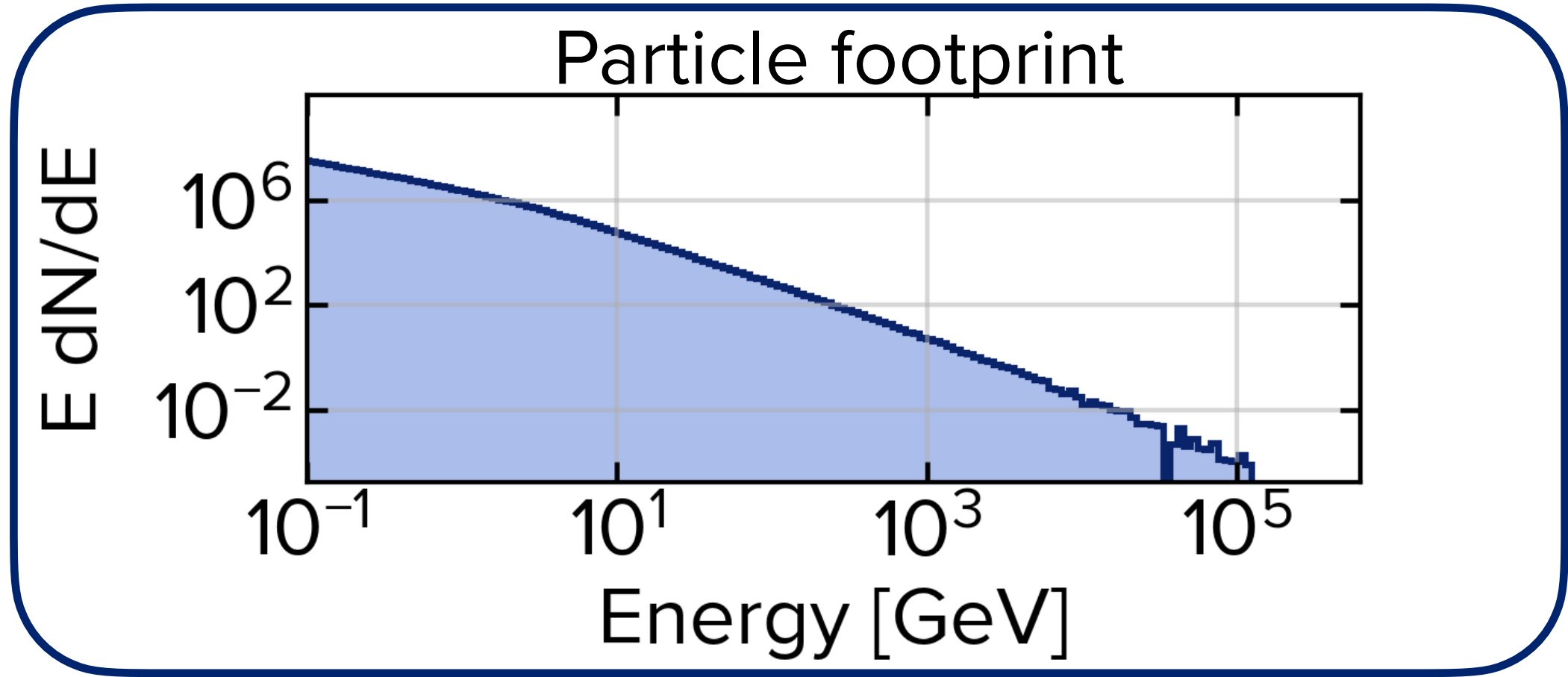
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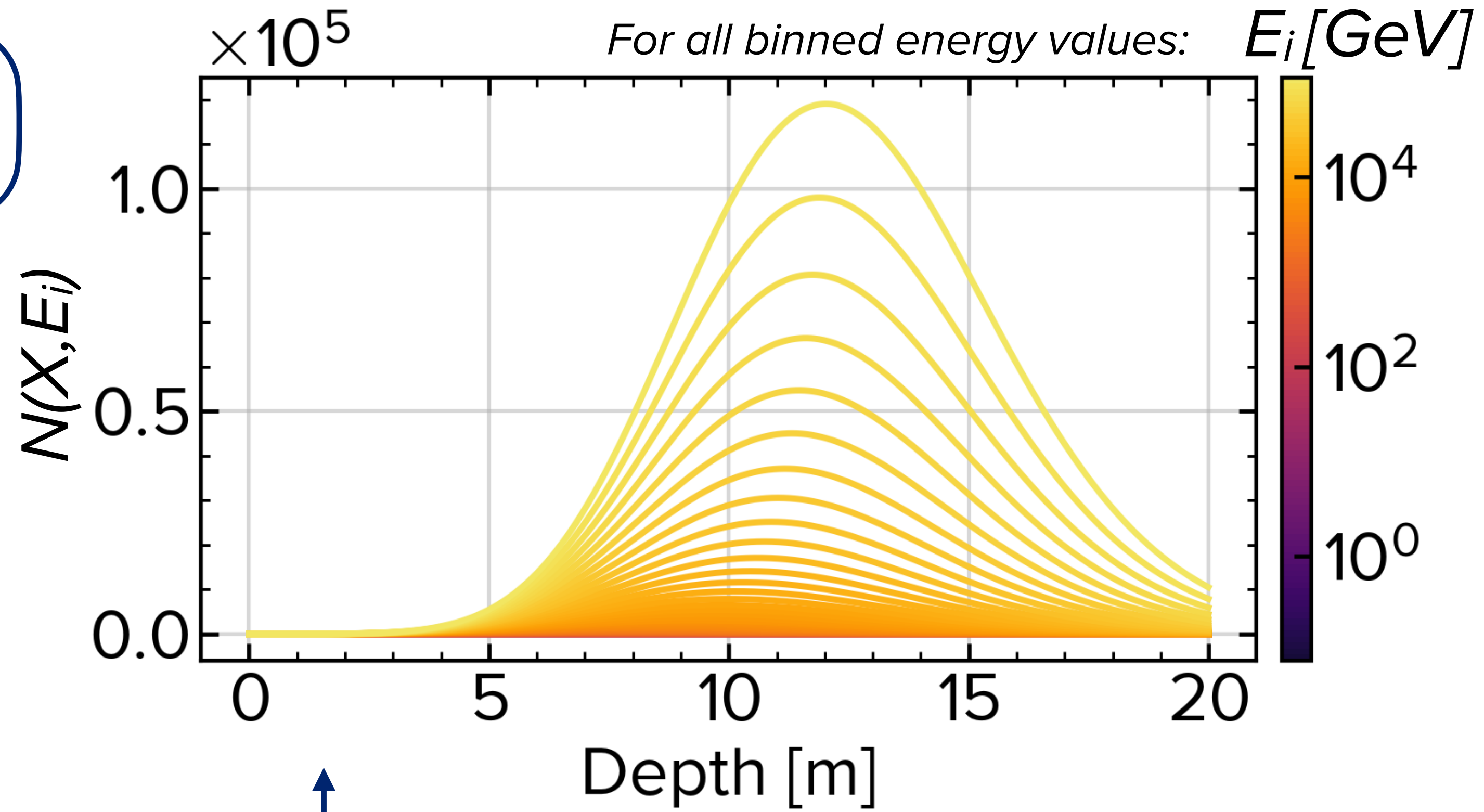
►  $X_0 = 36.08$  g/cm<sup>2</sup>

$$s(X, E_i) = \frac{3X/X_0}{X/X_0 + 2 \ln(E_i/E_c)}$$



Calculate for each bin, using the bin energy and number of counts:

$$N_i = N_{bin,i} \cdot N(X, E_i)$$



# Longitudinal Profile

► NKG longitudinal parameterisation:

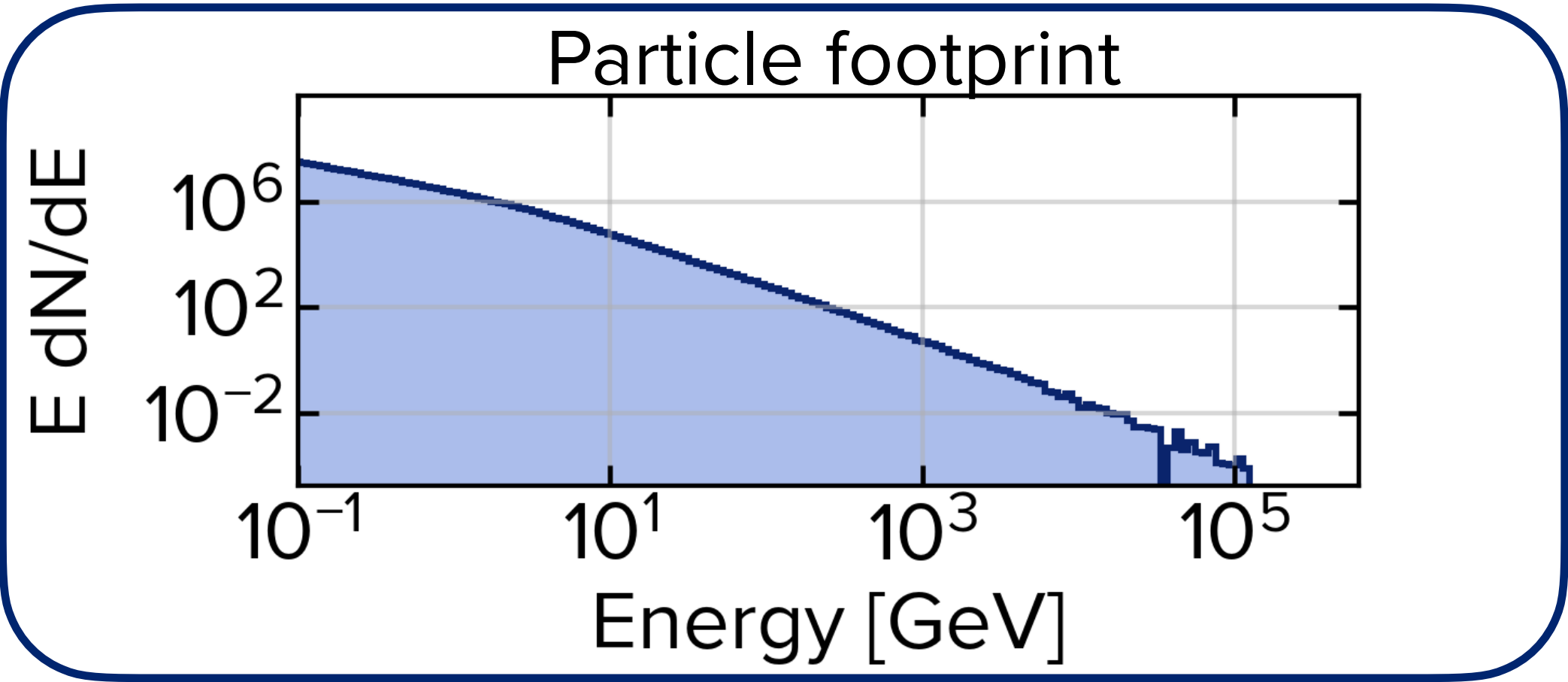
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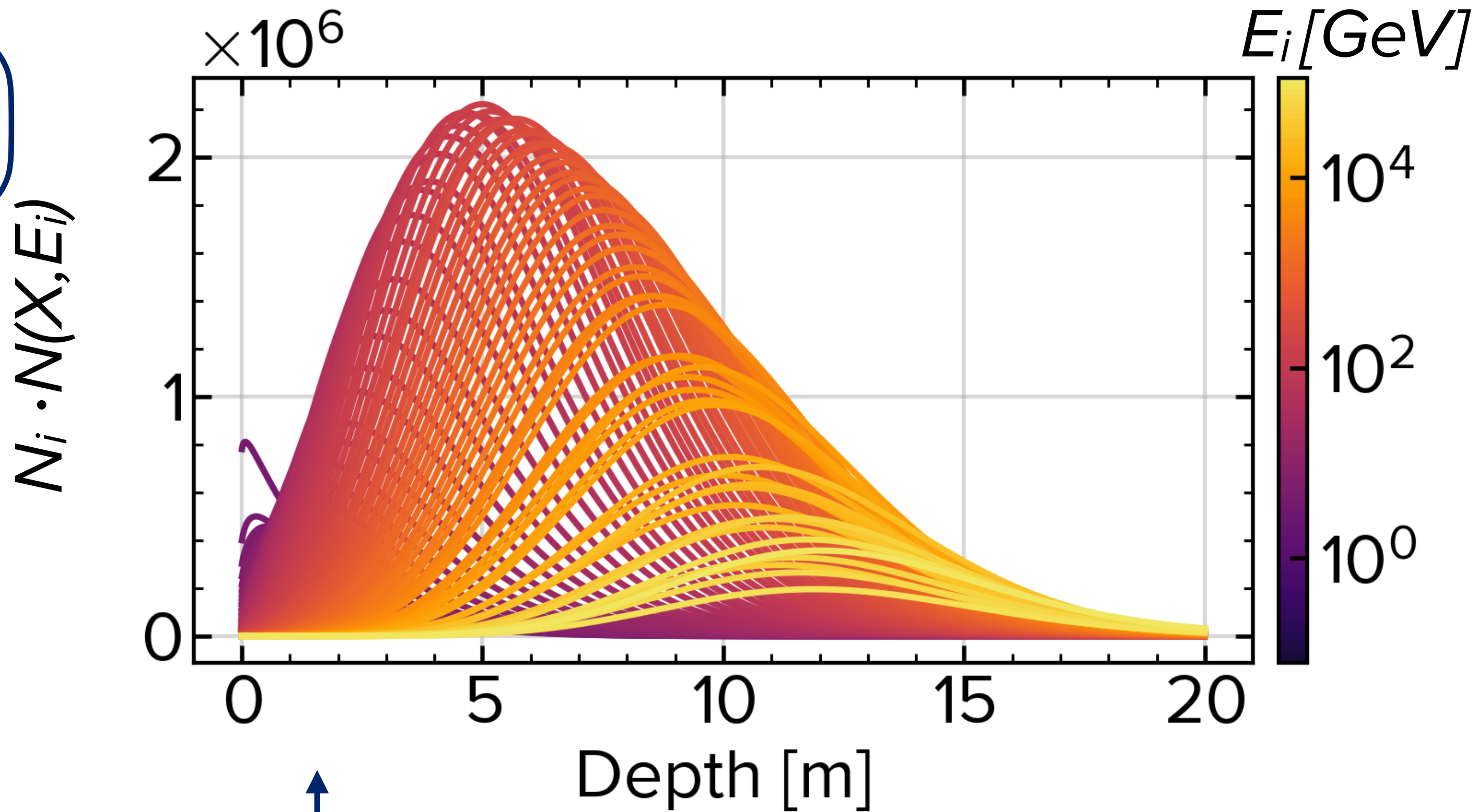
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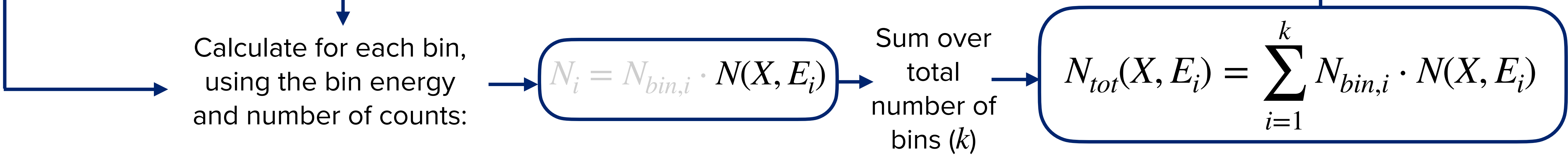
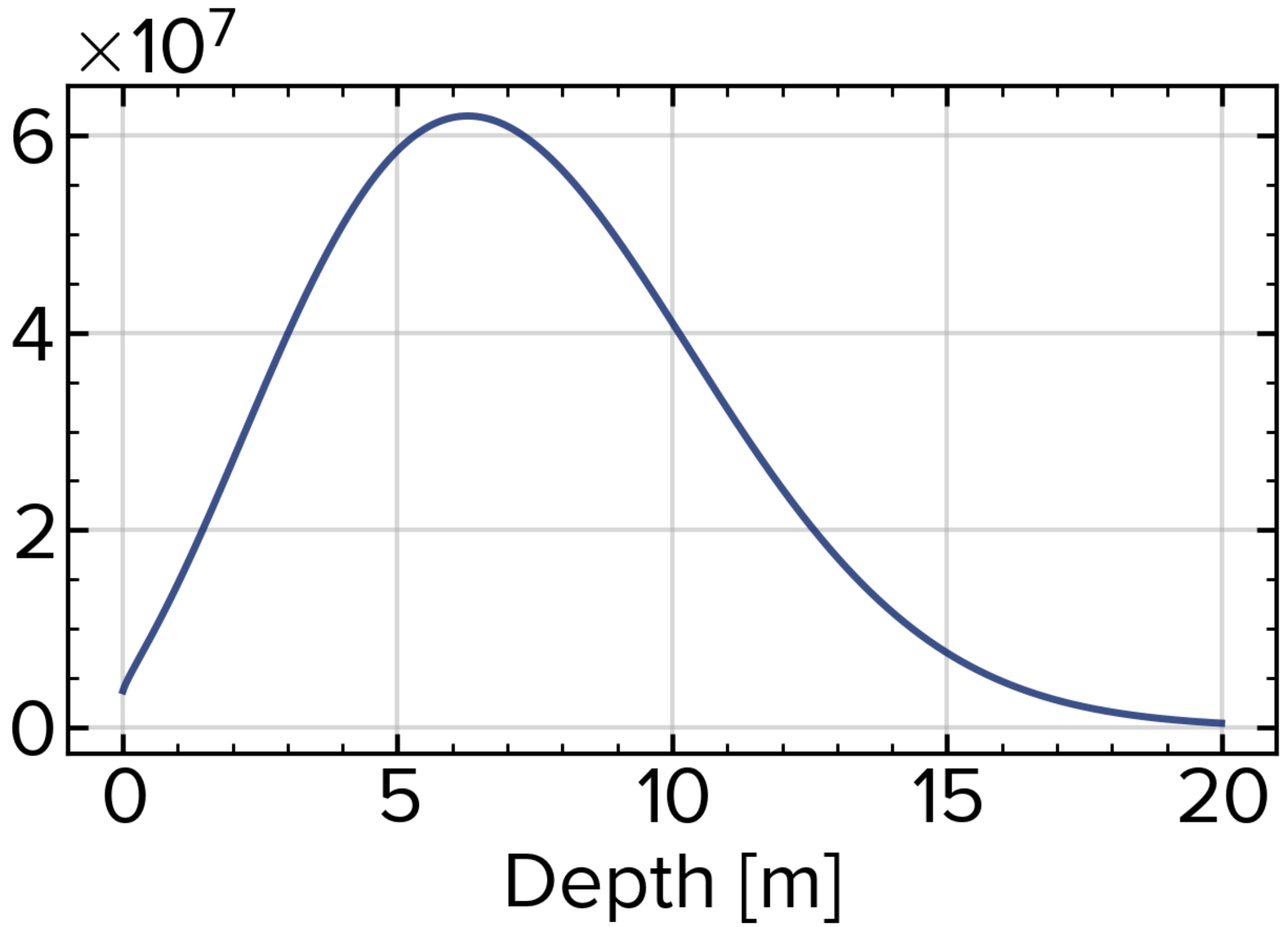
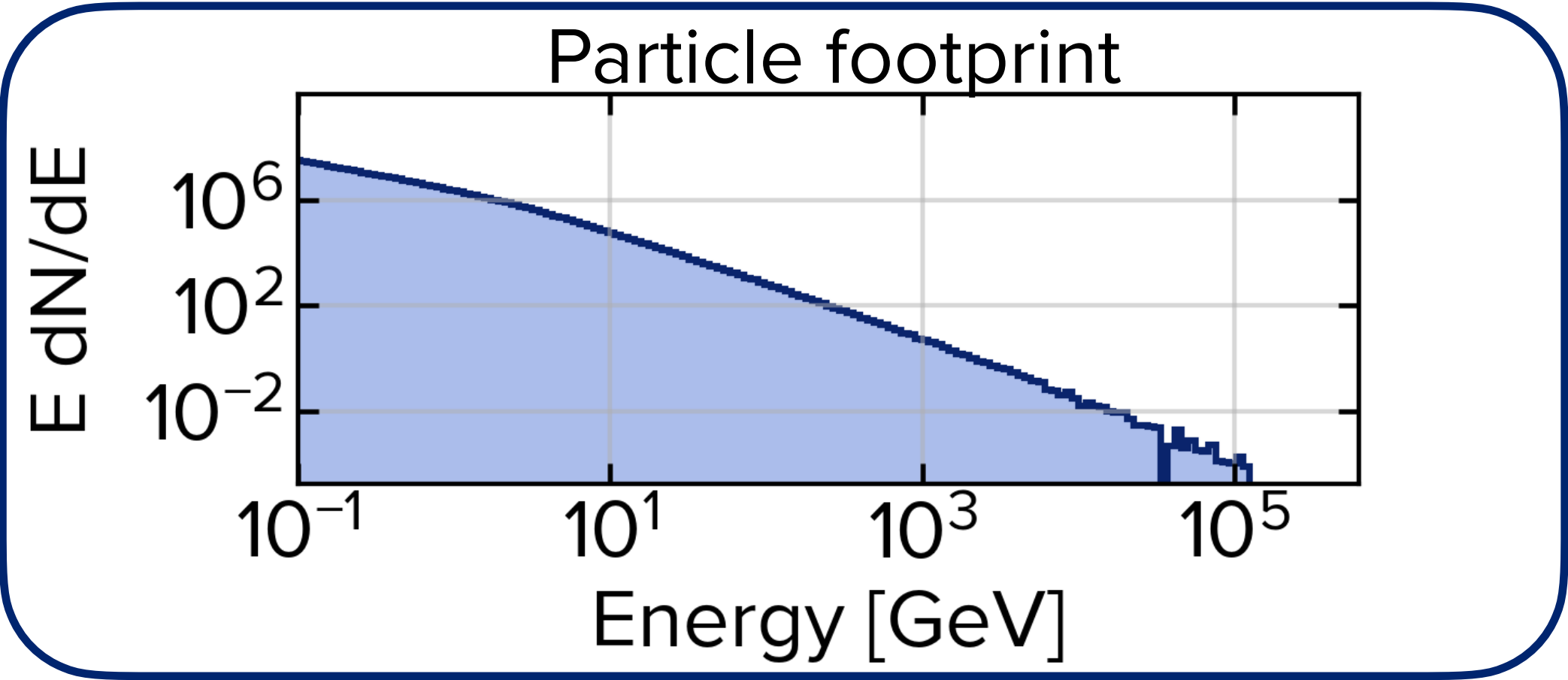
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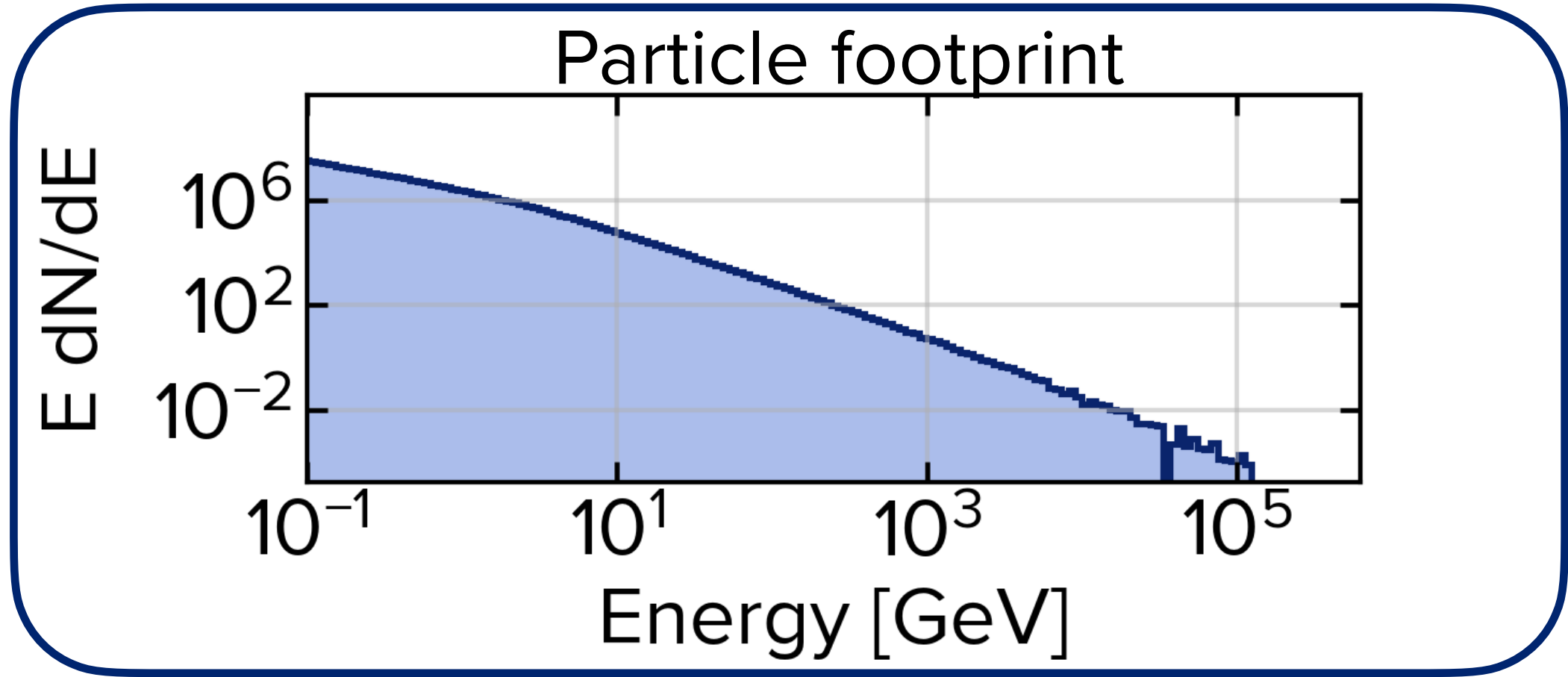
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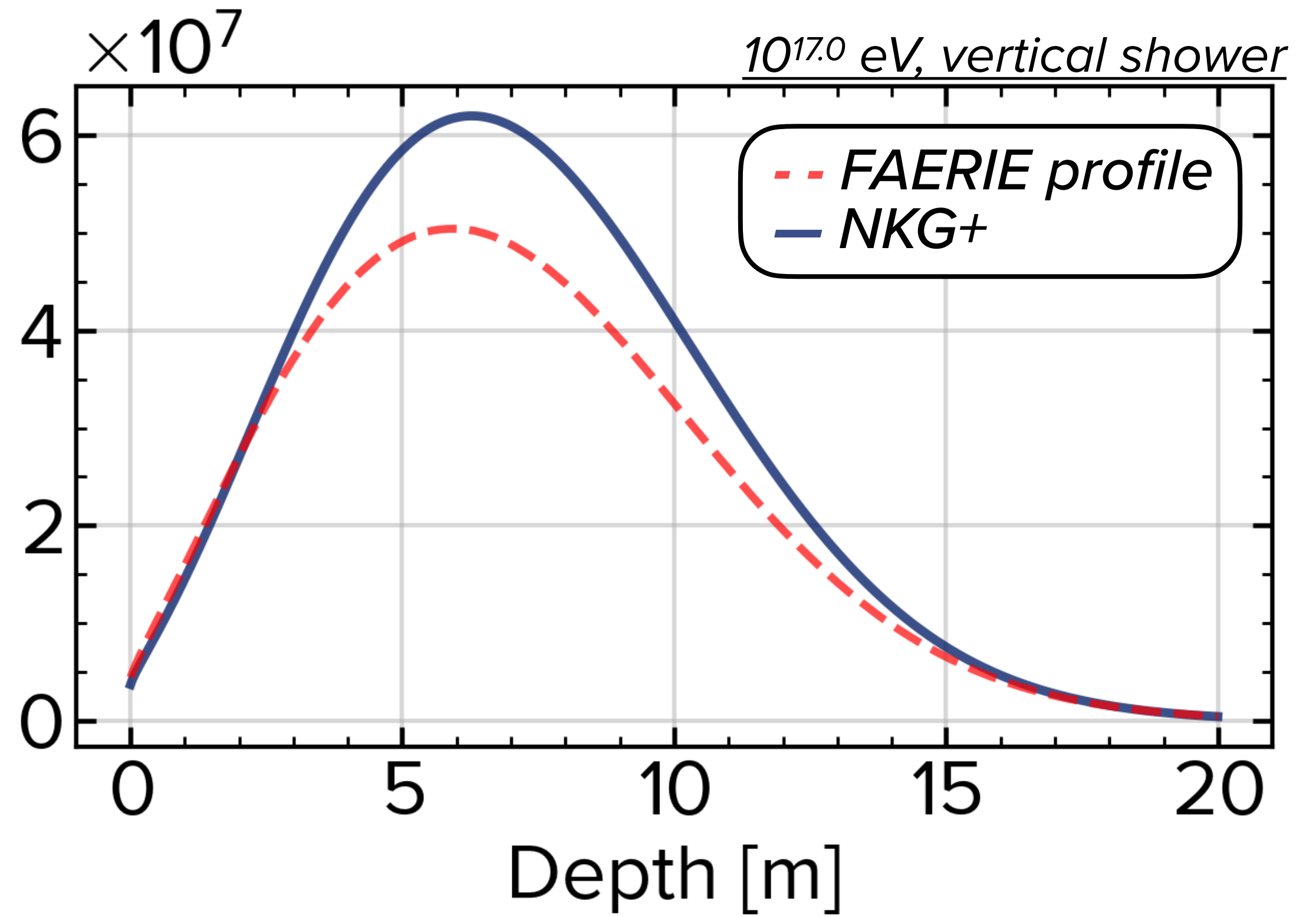


Calculate for each bin, using the bin energy and number of counts:

$$N_i = N_{bin,i} \cdot N(X, E_i)$$

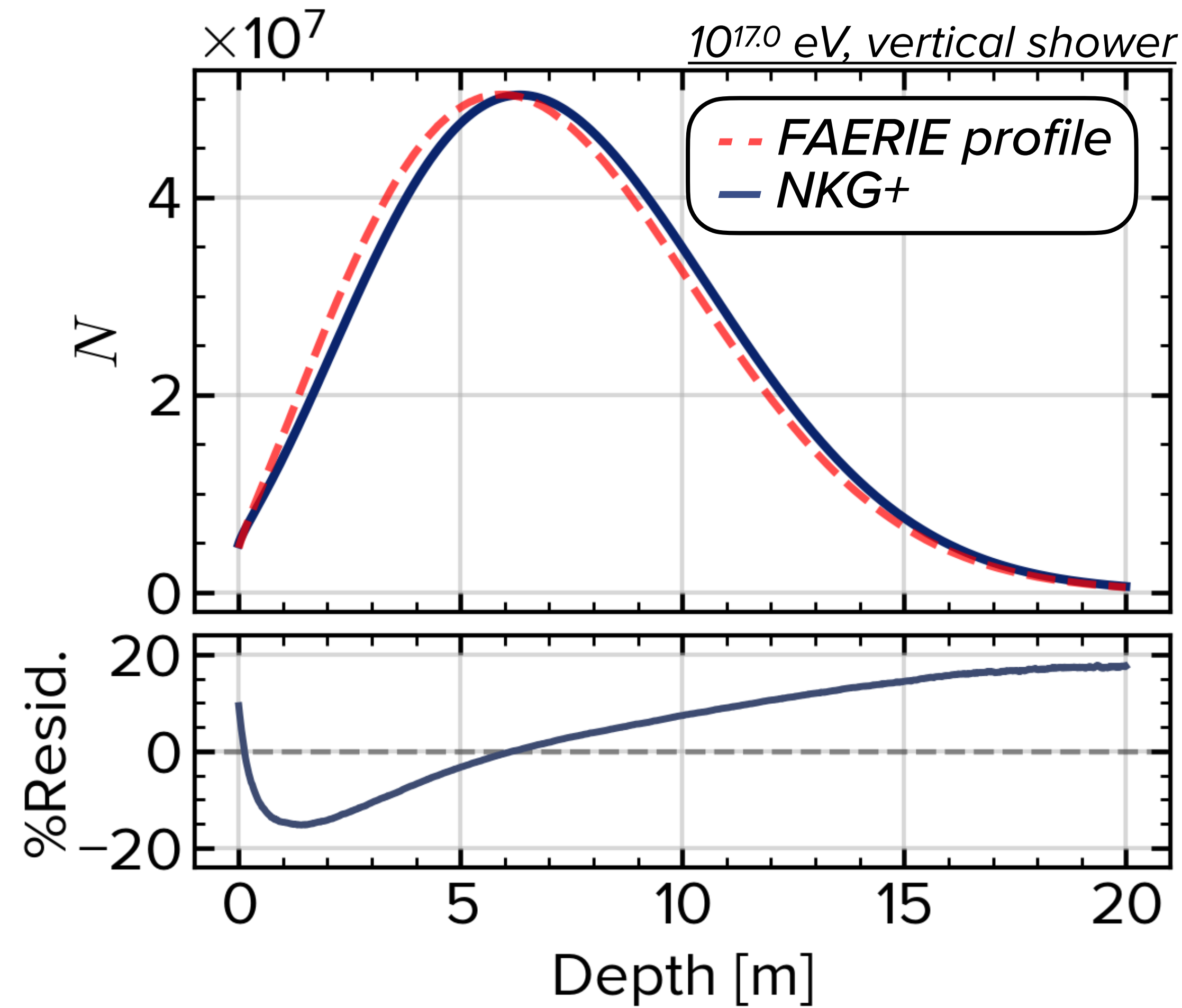
Sum over total number of bins ( $k$ )

$$N_{tot}(X, E_i) = \sum_{i=1}^k N_{bin,i} \cdot N(X, E_i)$$



# Longitudinal Profile

► Exploring fit modifications:



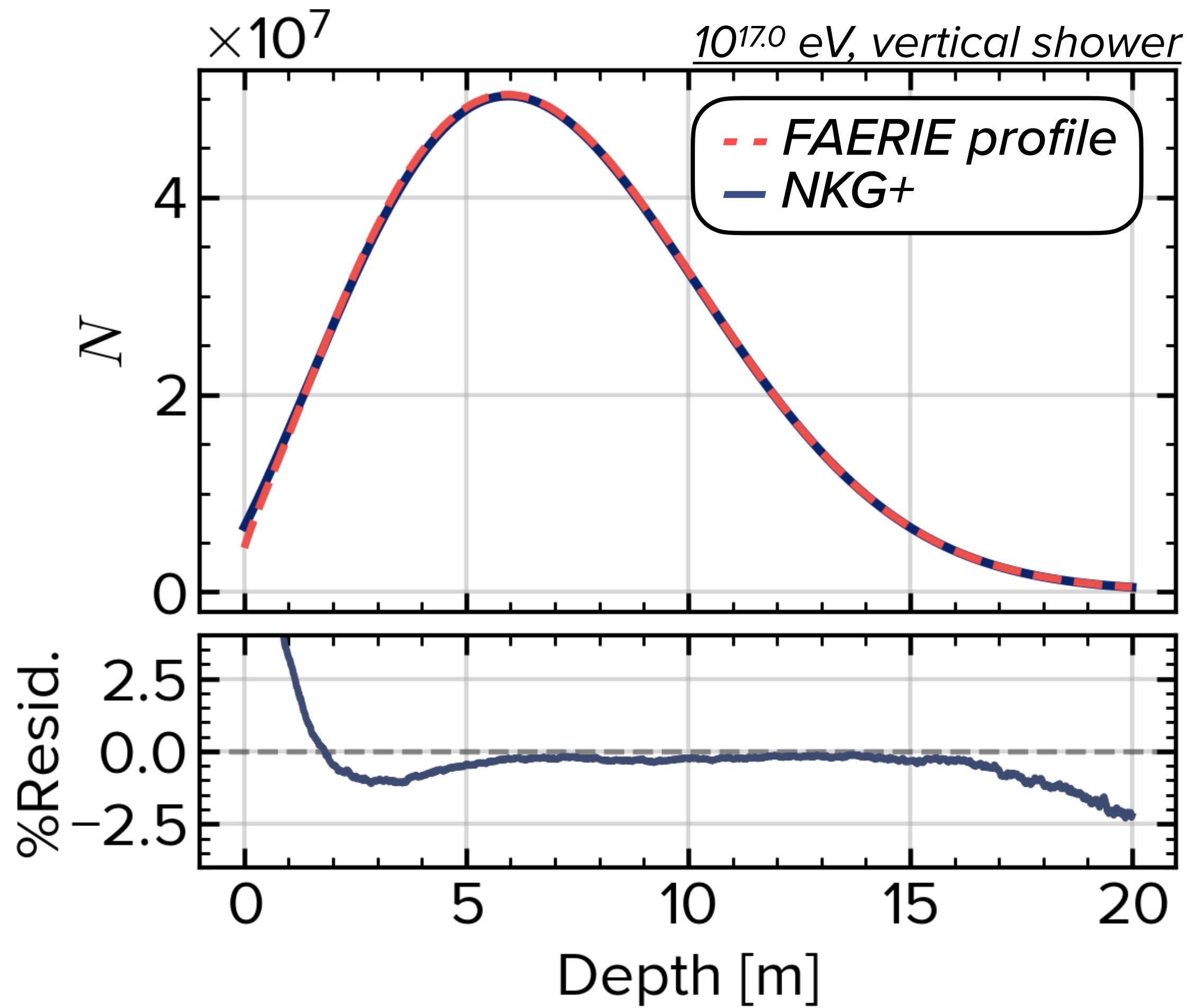
► Modified critical energy ( $E_c$ ) and radiation length ( $X_0$ ) values

Fit constants:

$$X_0 = 37.9 \text{ g/cm}^2$$
$$E_c = 97 \text{ MeV}$$

# Longitudinal Profile

► Exploring fit modifications:



- Modified critical energy ( $E_c$ ) and radiation length ( $X_0$ ) values
- Introduce *shift* parameter,  $X_1$

$$N_{tot} = \sum_{i=1}^k N_{i,bin} \cdot (N(X', E_i))$$

$$X' = X - X_1$$

Fit constants:

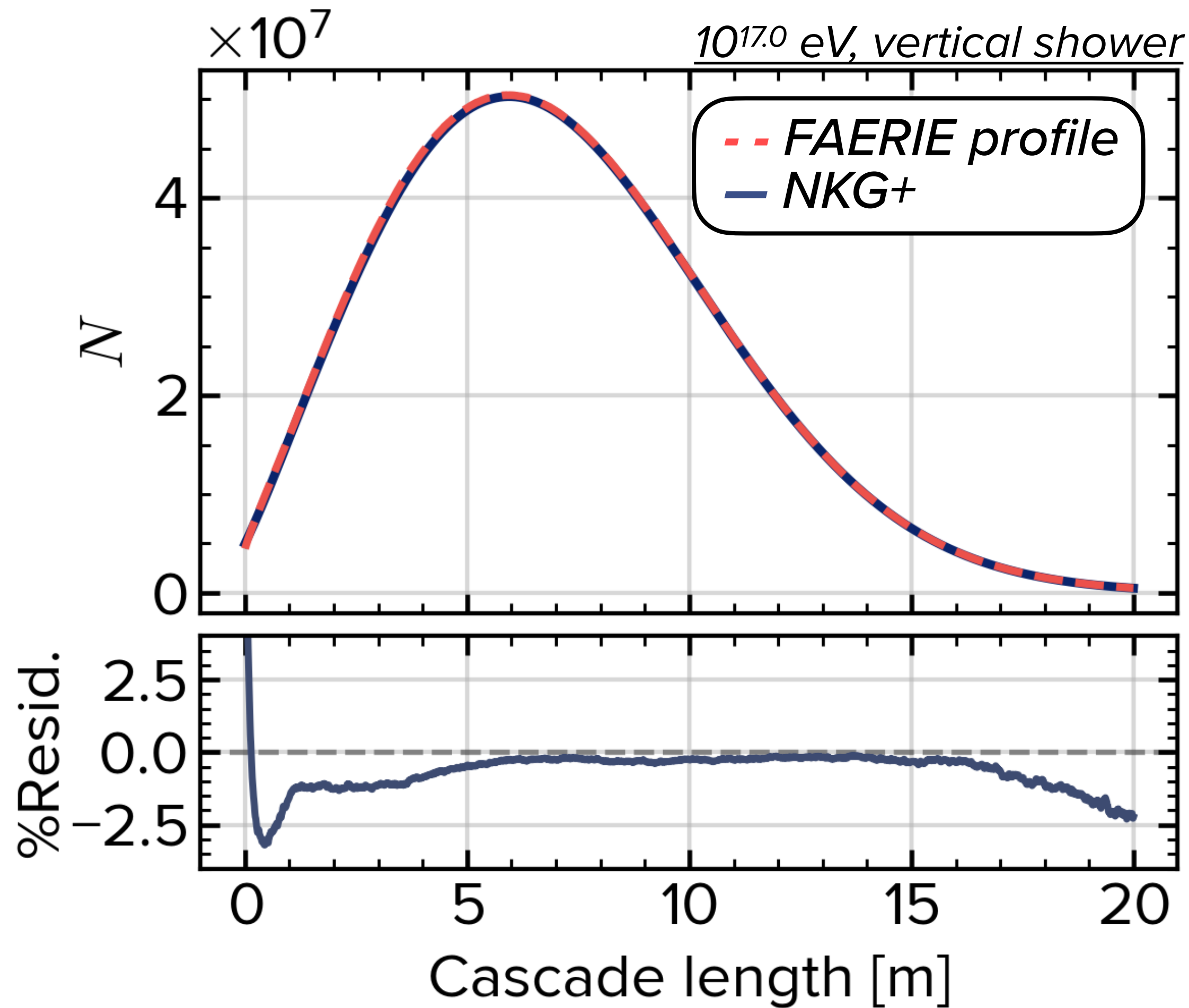
$$X_0 = 37.9 \text{ g/cm}^2$$

$$E_c = 97 \text{ MeV}$$

$$X_1 = 18 \text{ g/cm}^2$$

# Longitudinal Profile

► Exploring fit modifications:



► Modified critical energy ( $E_c$ ) and radiation length ( $X_0$ ) values

► Introduce *shift* parameter,  $X_1$

$$N_{tot} = \sum_{i=1}^k N_{i,bin} \cdot (N(X', E_i))$$

$$X' = X - X_1$$

► Add early cut off, of form

$$w(X) = 1 - be^{-aX}$$

$$N_{tot} = w(X') \cdot \sum_{i=1}^k N_{i,bin} \cdot (N(X', E_i))$$

Fit constants:

$$X_0 = 37.9 \text{ g/cm}^2$$

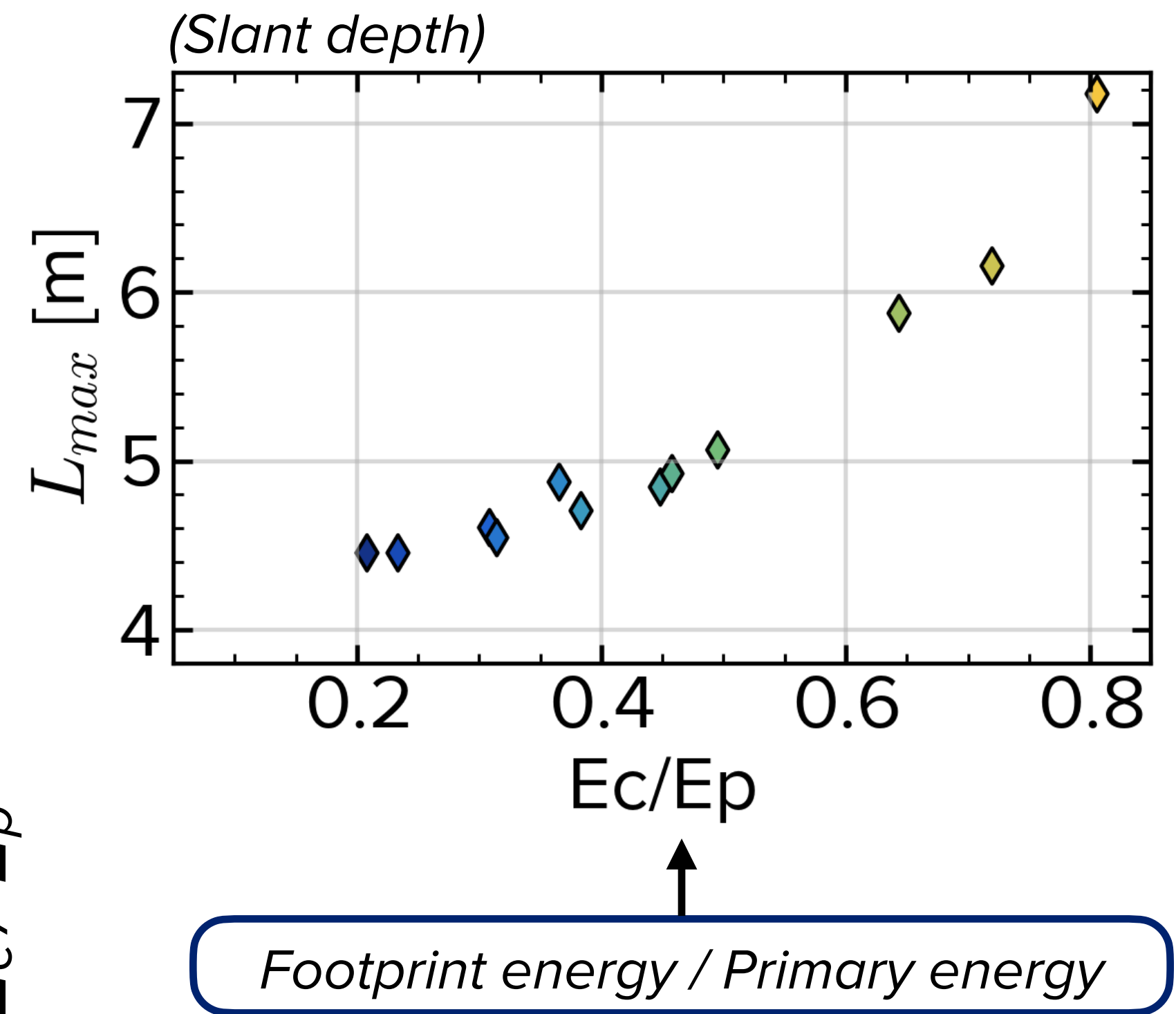
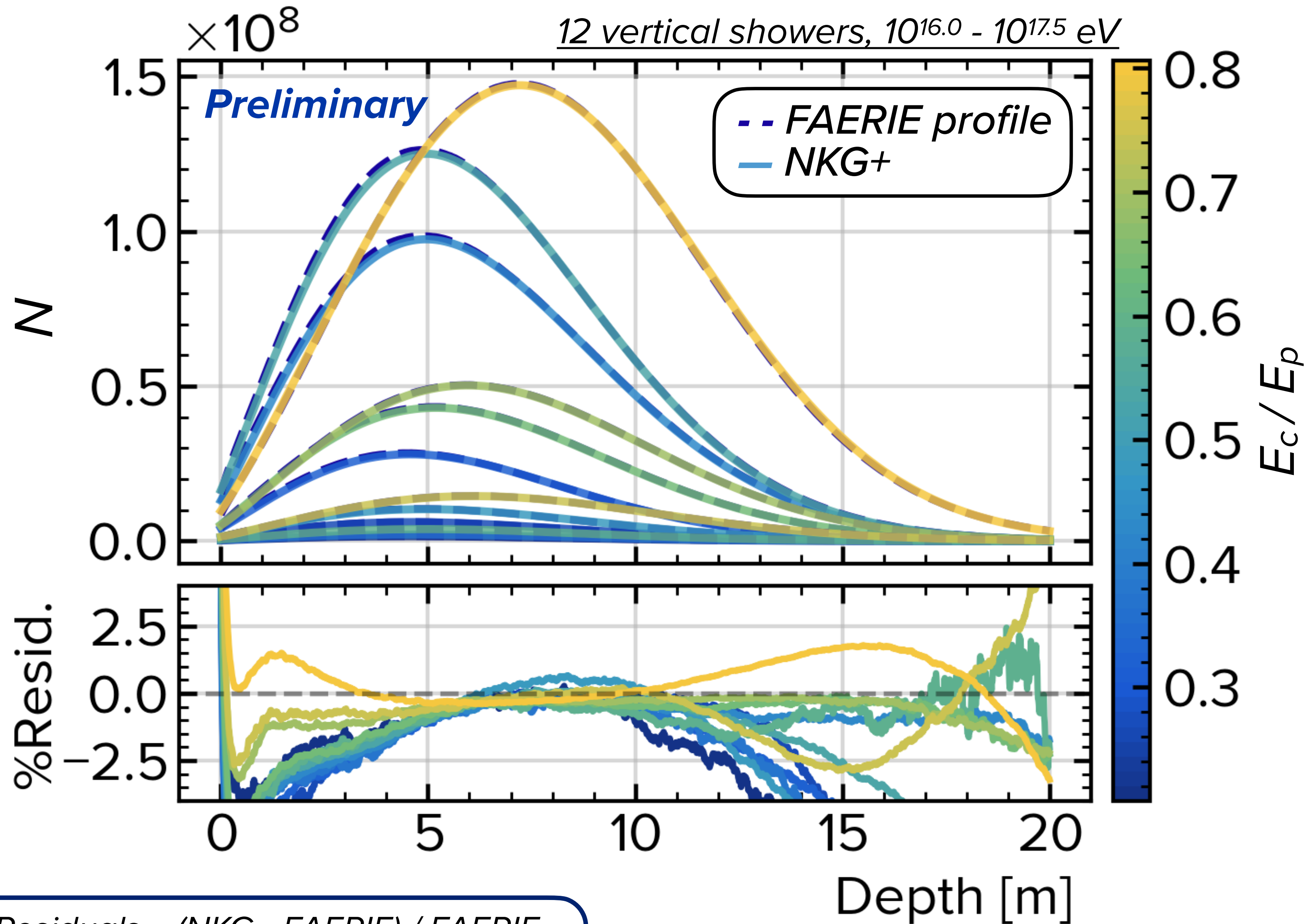
$$E_c = 97 \text{ MeV}$$

$$X_1 = 18 \text{ g/cm}^2$$

$$a = 0.05$$

$$b = 0.67$$

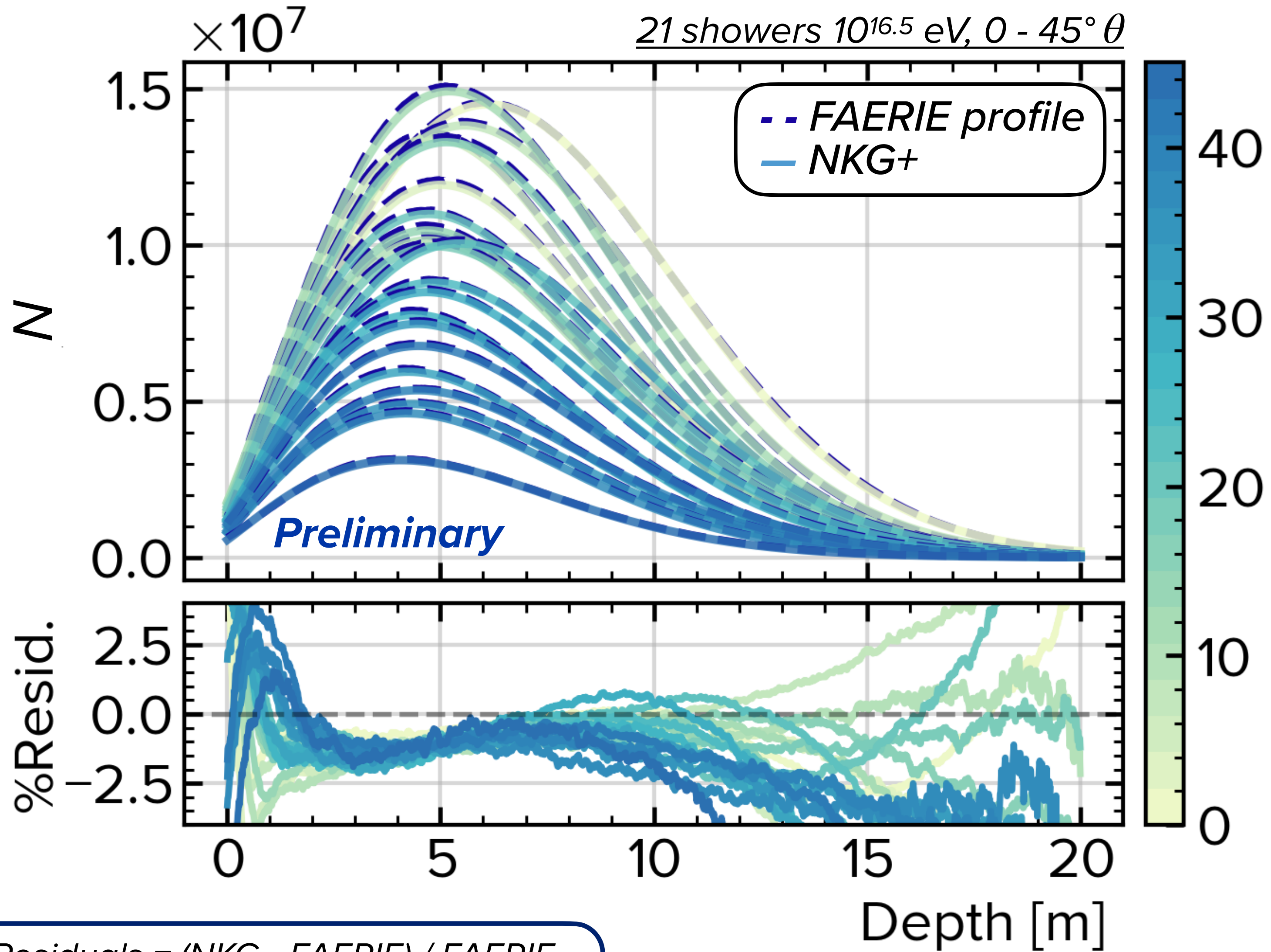
# Vertical showers



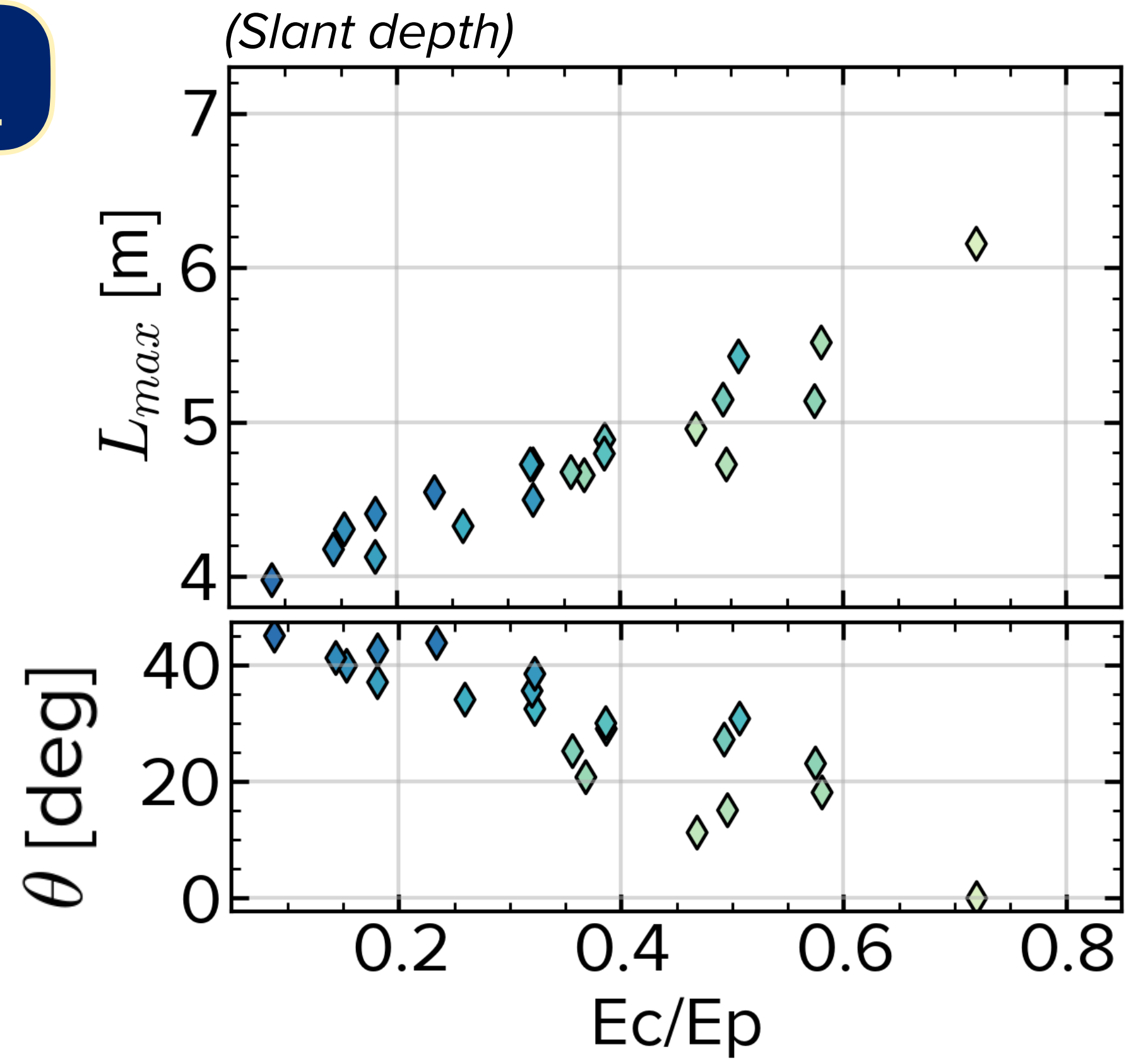
Footprint energy / Primary energy

$\% \text{ Residuals} = (NKG - FAERIE) / FAERIE$

# Inclined showers

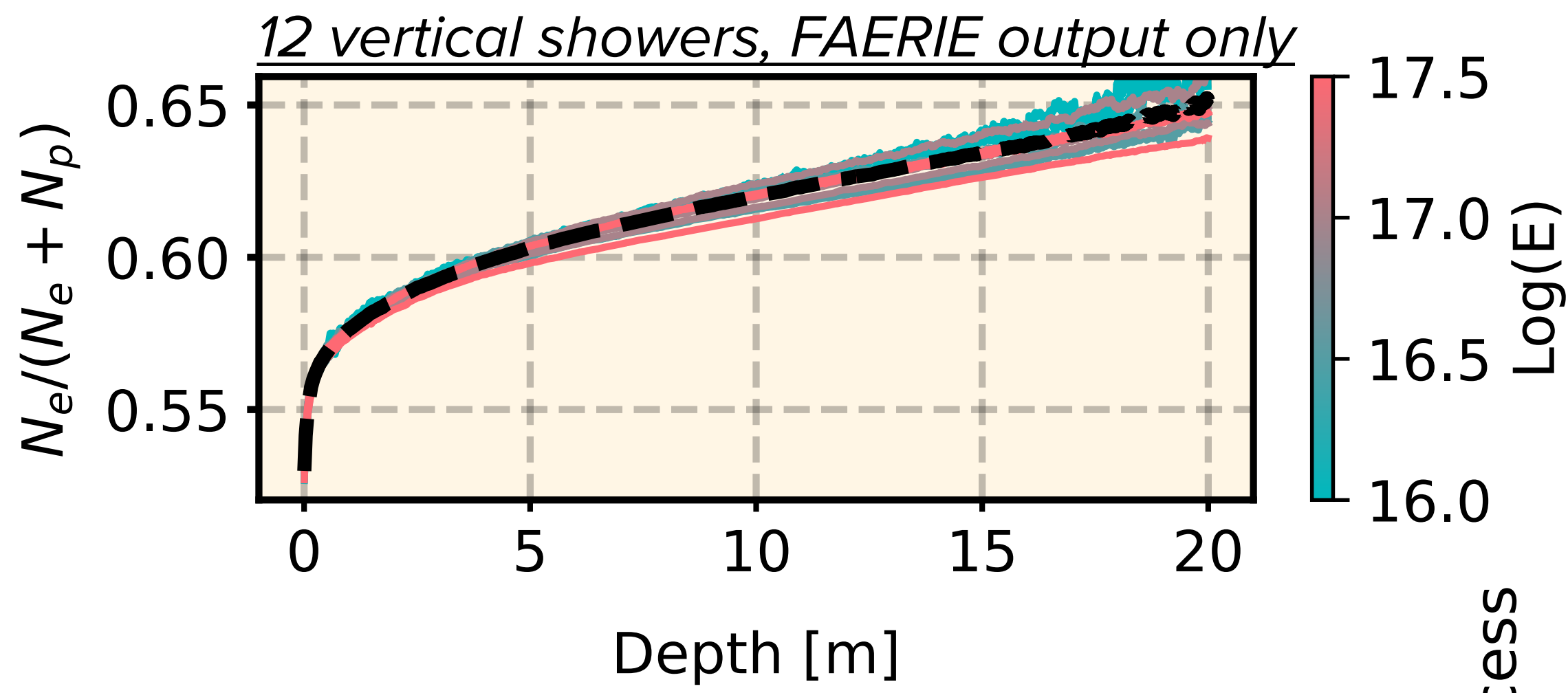
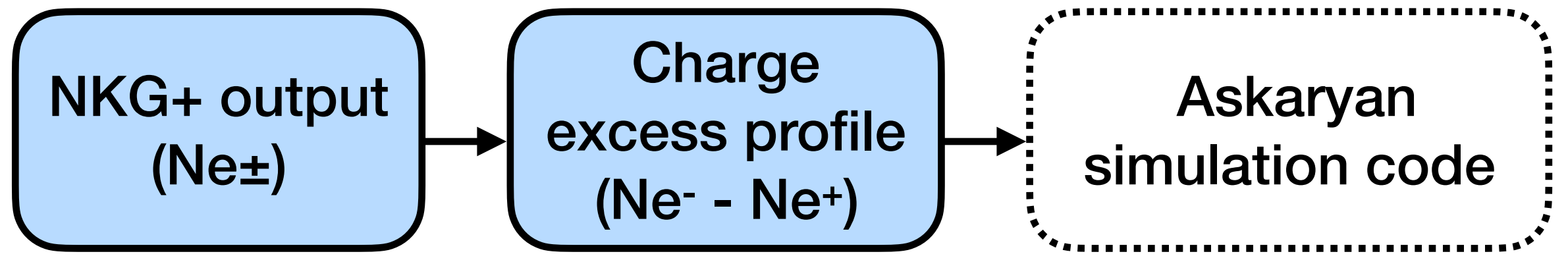


$\% \text{ Residuals} = (NKG - FAERIE) / FAERIE$

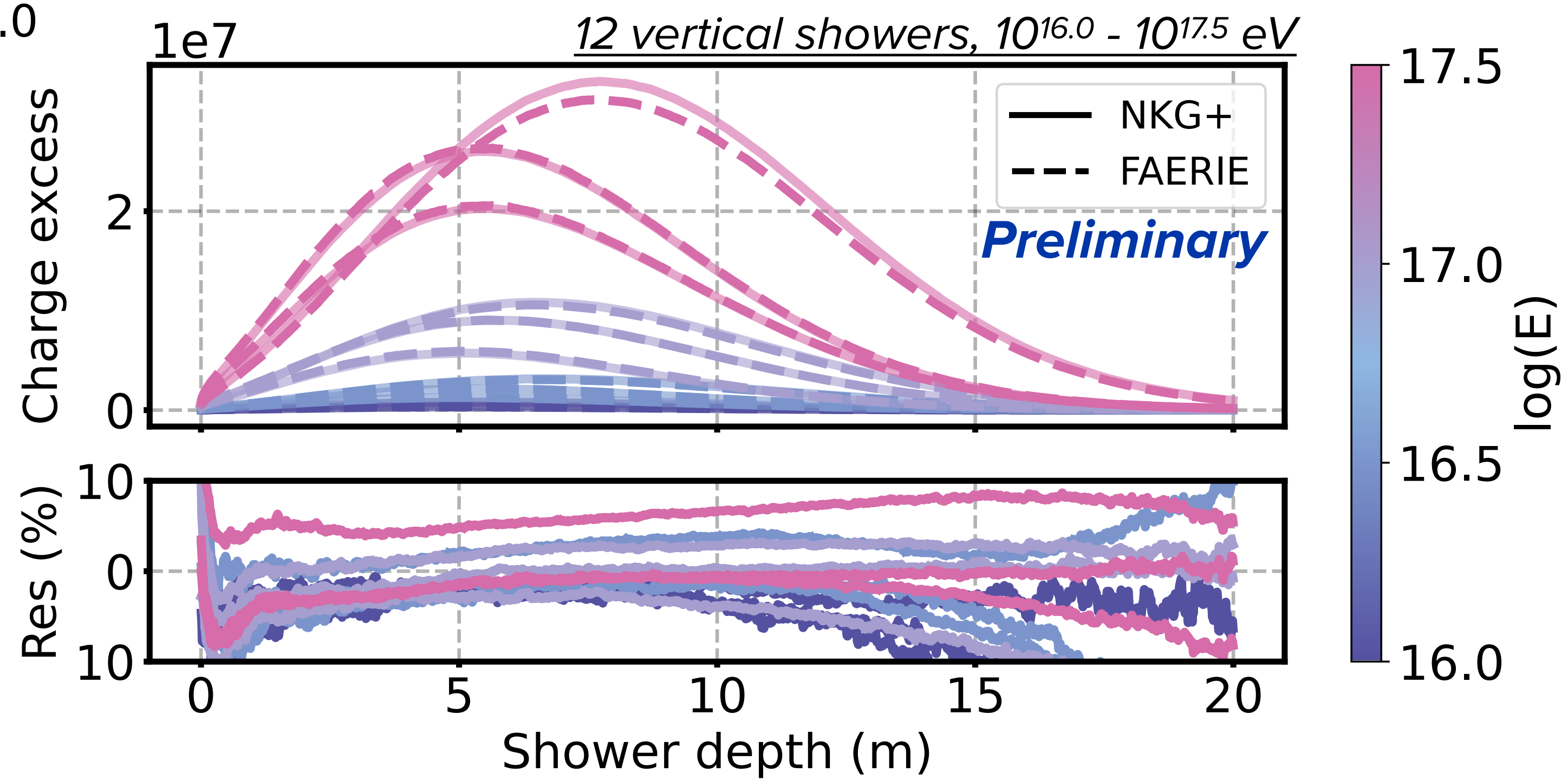


- Askaryan application: utilise the NKG+ longitudinal profile as input to external code

→ *Askaryan*



Credit: J. Stoffels



→ Simulation of the Askaryan signal, with this method, currently being investigated by Jethro Stoffels

- ▶ Radar application: utilise the NKG+ 2D profile as input to *MARES* code, generate resulting radar signal

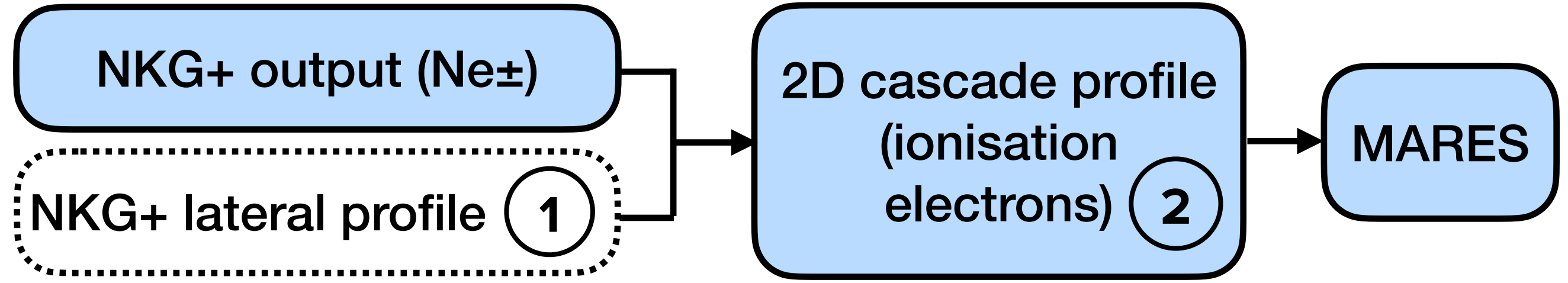
→ *Radar*

*E. Huesca  
Santiago et al.,  
2024*

- 1 Lateral profile of the in-ice CR (*WIP*)

$$w(r, s) = \frac{1}{r_0} \frac{\Gamma(4.5 - s)}{\Gamma(s)\Gamma(4.5 - 2s)} \left(\frac{r}{r_0}\right)^{s-1} \left(\frac{r}{r_0} + 1\right)^{s-4.5}$$

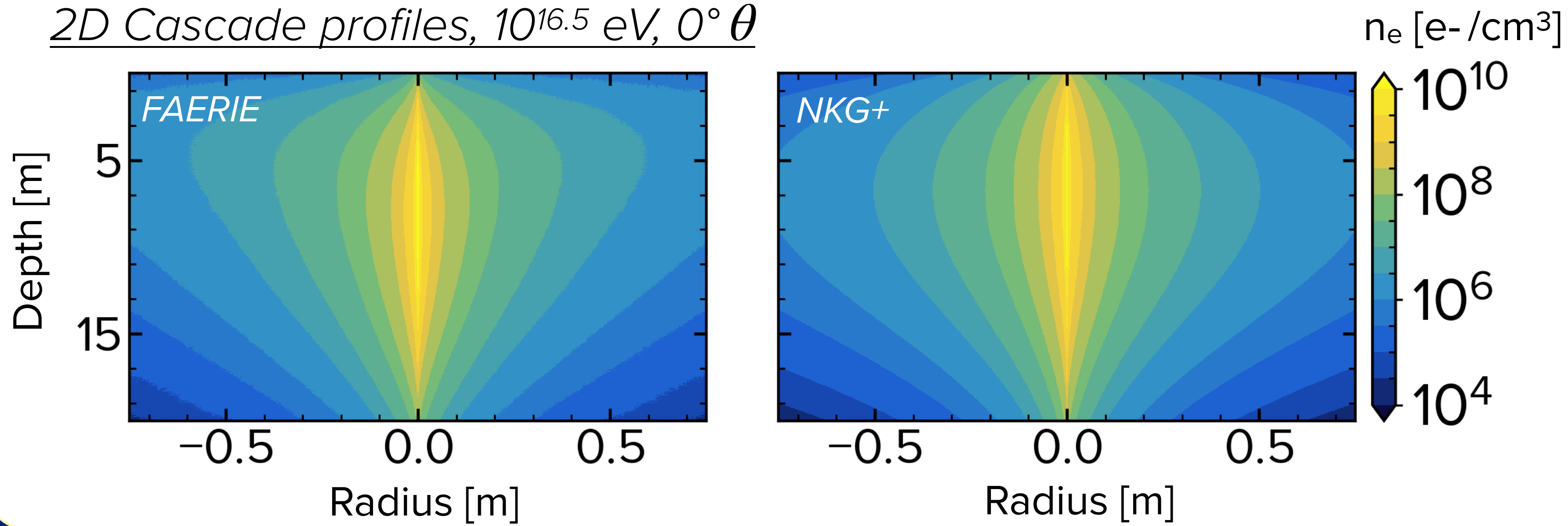
*K. Werner, K. De Vries, O. Scholten (2012)*



- 2 Convert from  $N_{e\pm}$  to the ionisation trail electrons

$$N_{e,ion} = \frac{dE/dX}{\langle E_{ion} \rangle} \rho_{ice} \quad [e^-/cm]$$

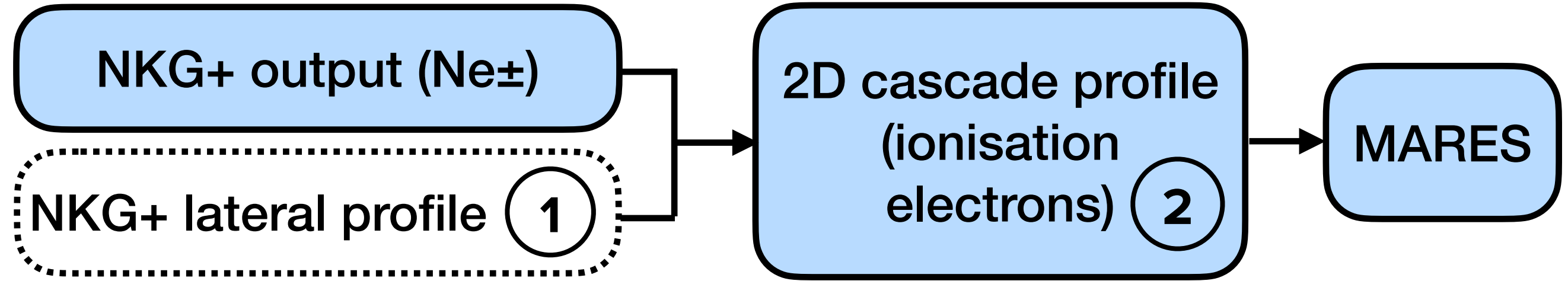
*2D Cascade profiles,  $10^{16.5}$  eV,  $0^\circ \theta$*



- ▶ Radar application: utilise the NKG+ 2D profile as input to *MARES* code, generate resulting radar signal

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*E. Huesca  
Santiago et al.,  
2024*



- 1 Lateral profile of the in-ice CR (*WIP*)

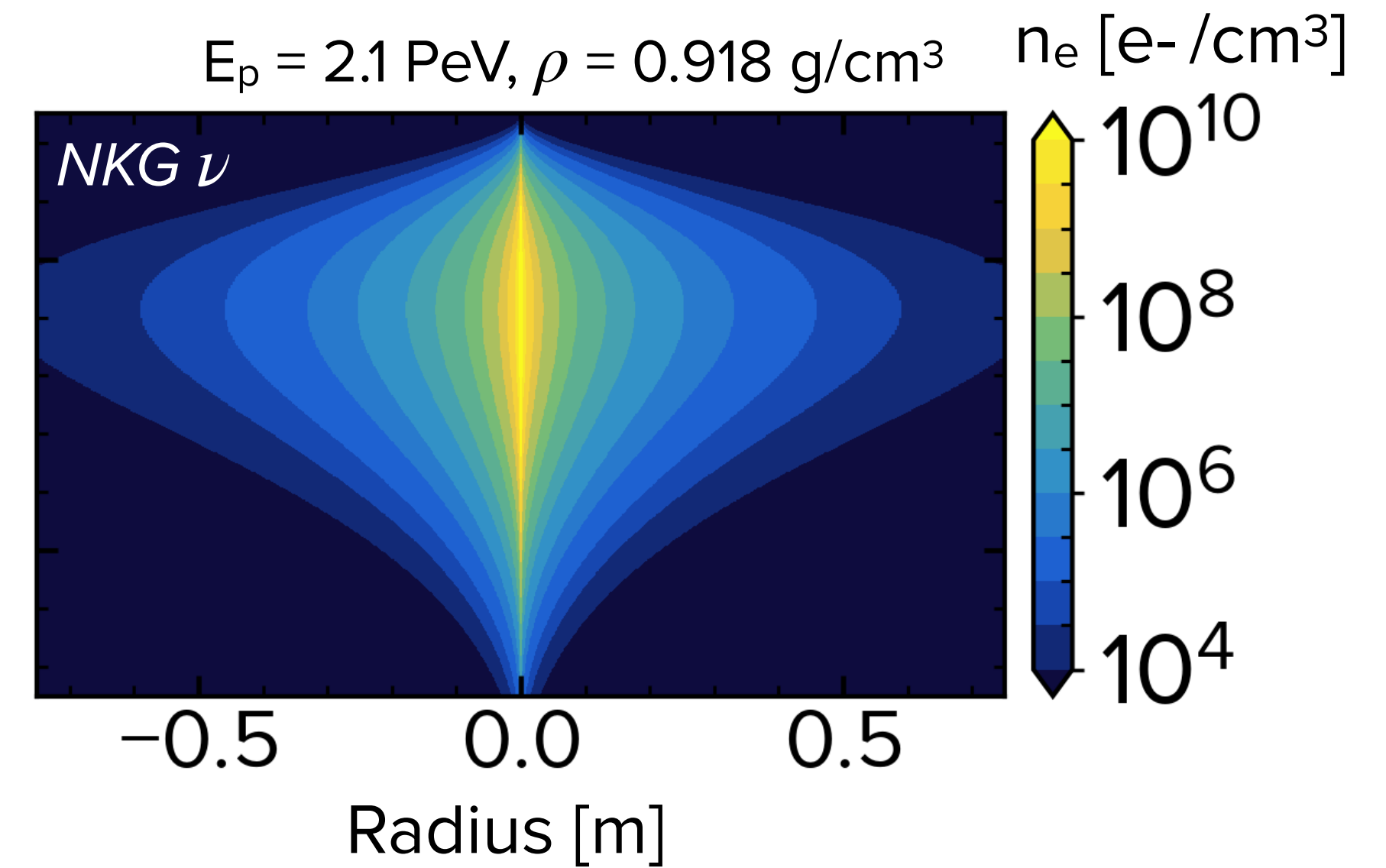
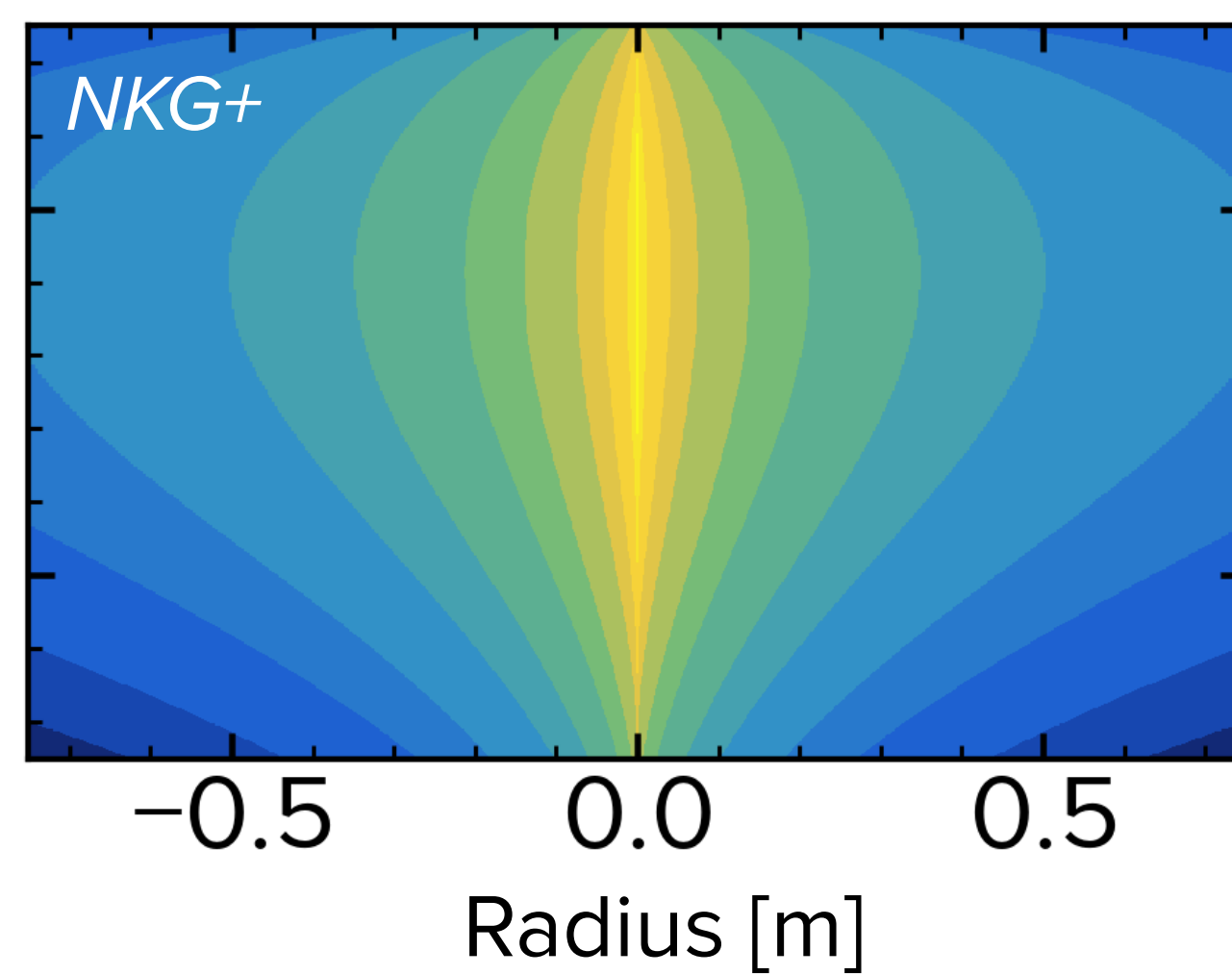
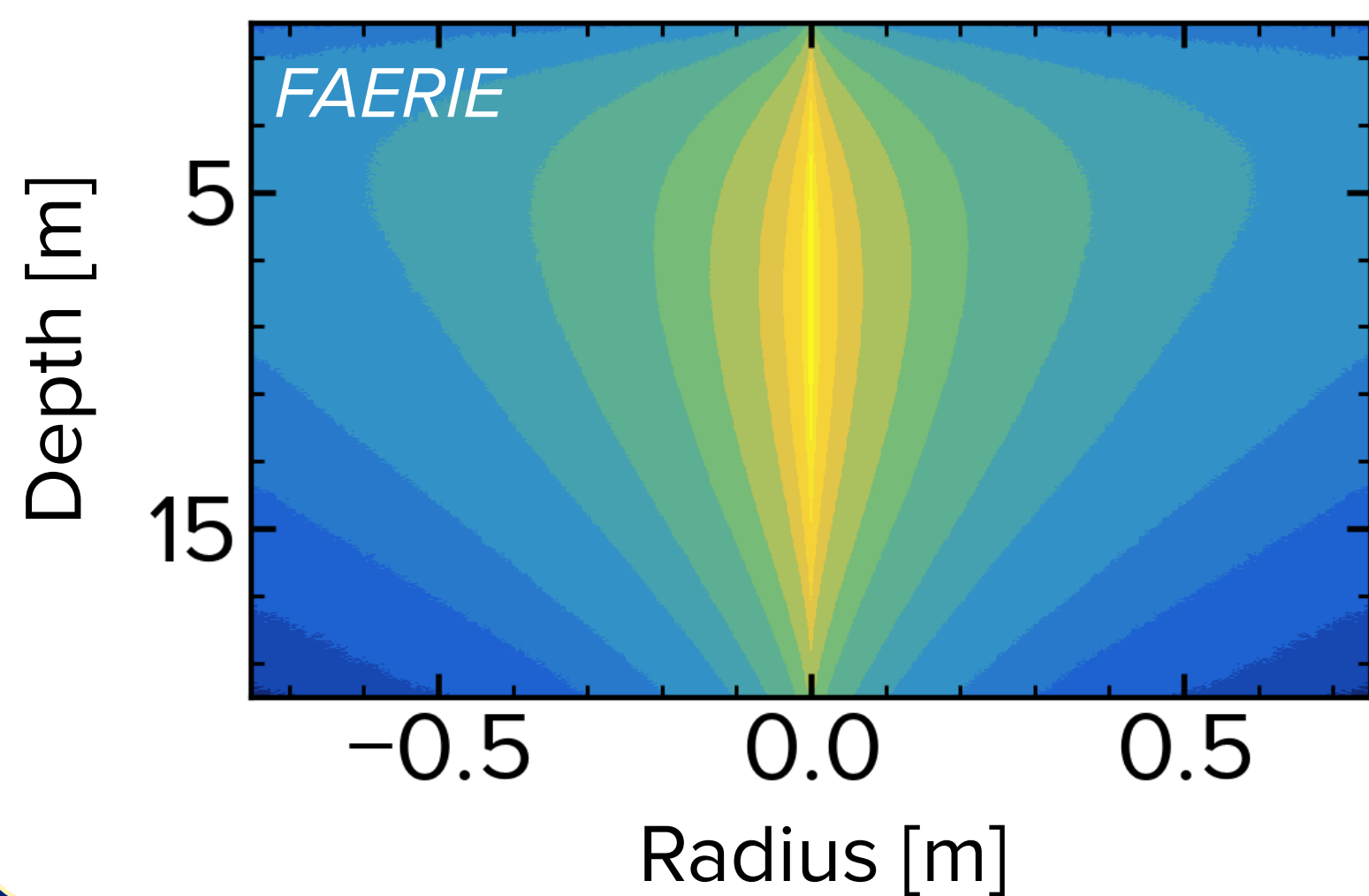
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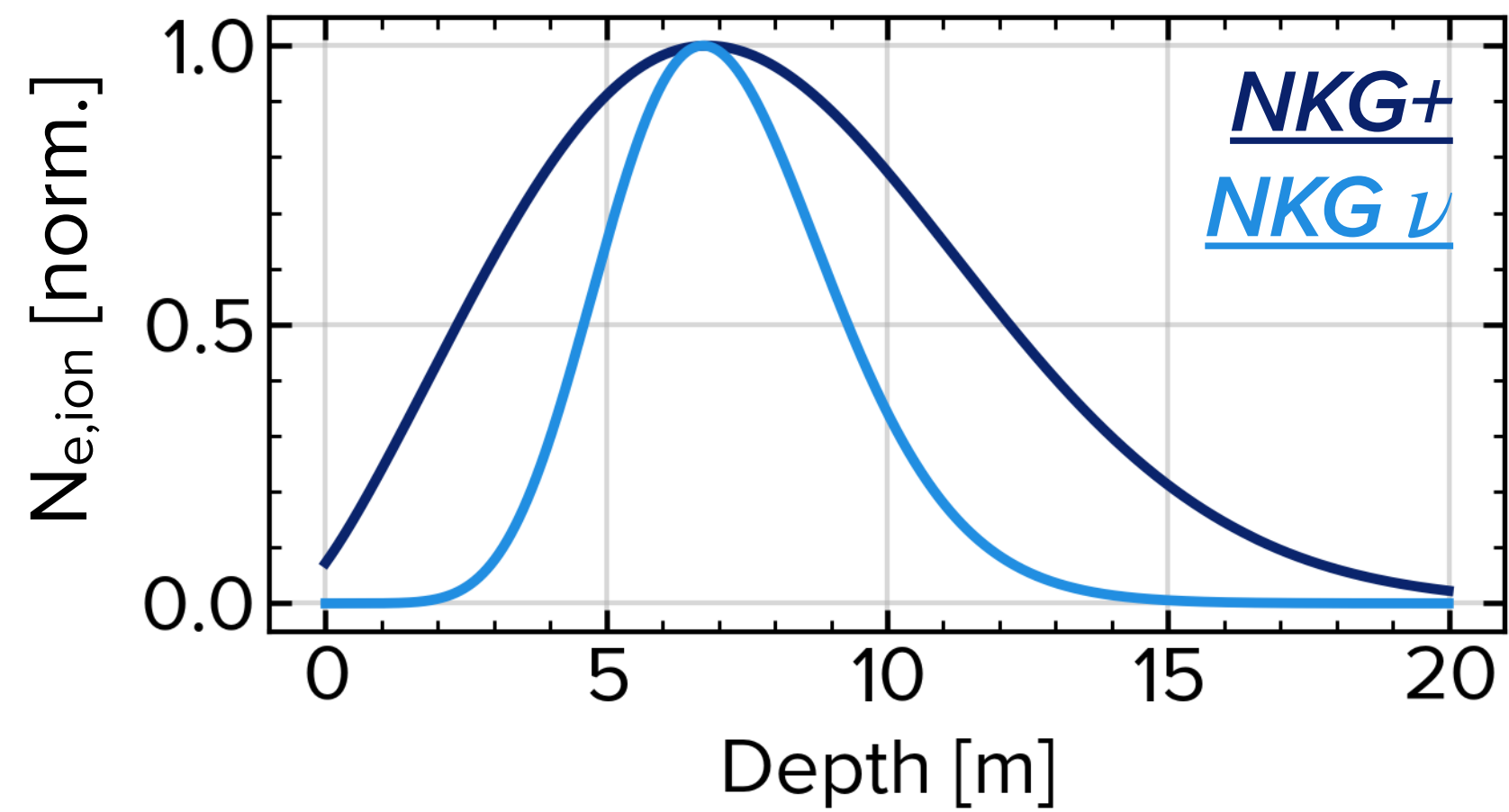
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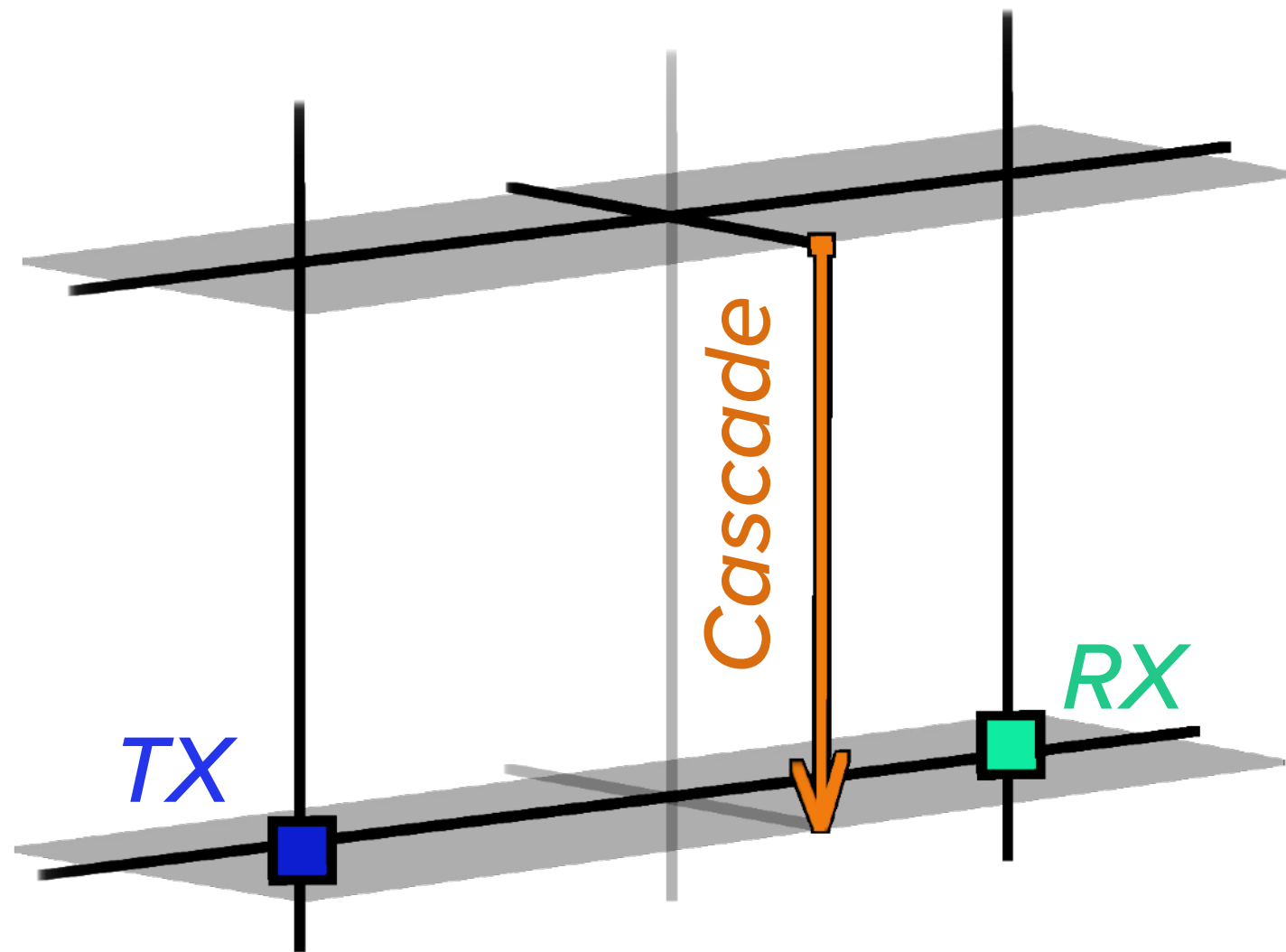
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*2D Cascade profiles,  $10^{16.5}$  eV,  $0^\circ \theta$*

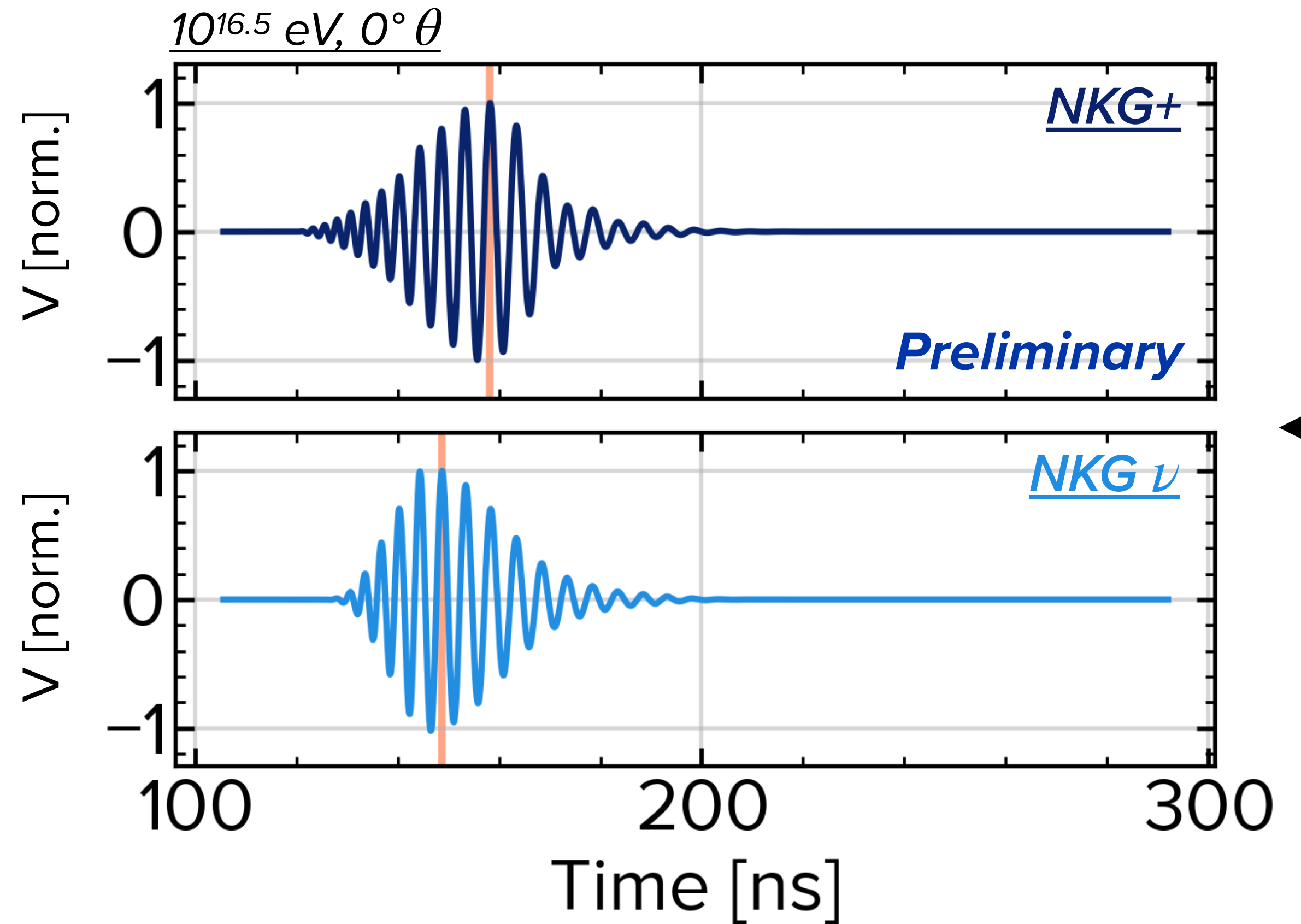
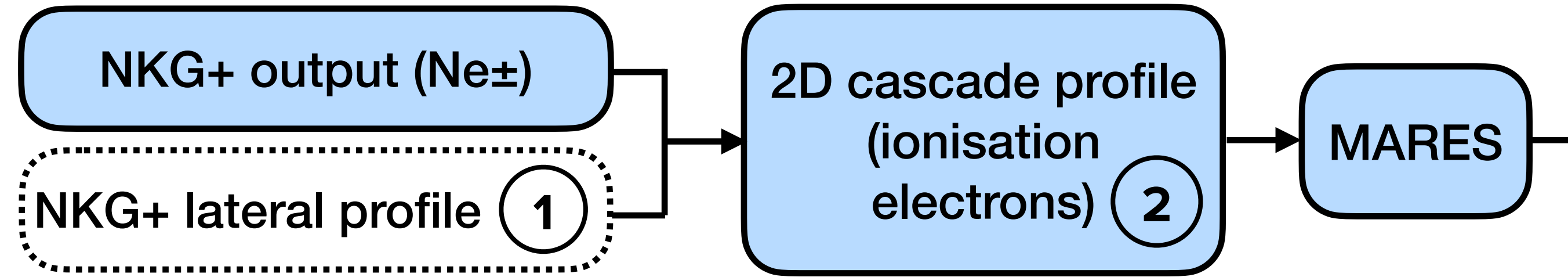




- ▶ CR cascades exhibit wider profiles, less variation with primary energy



## → Radar



# Summary

- ▶ Exploring method to quickly generate in-ice CRs resulting from a *CORSIKA* air-shower footprint

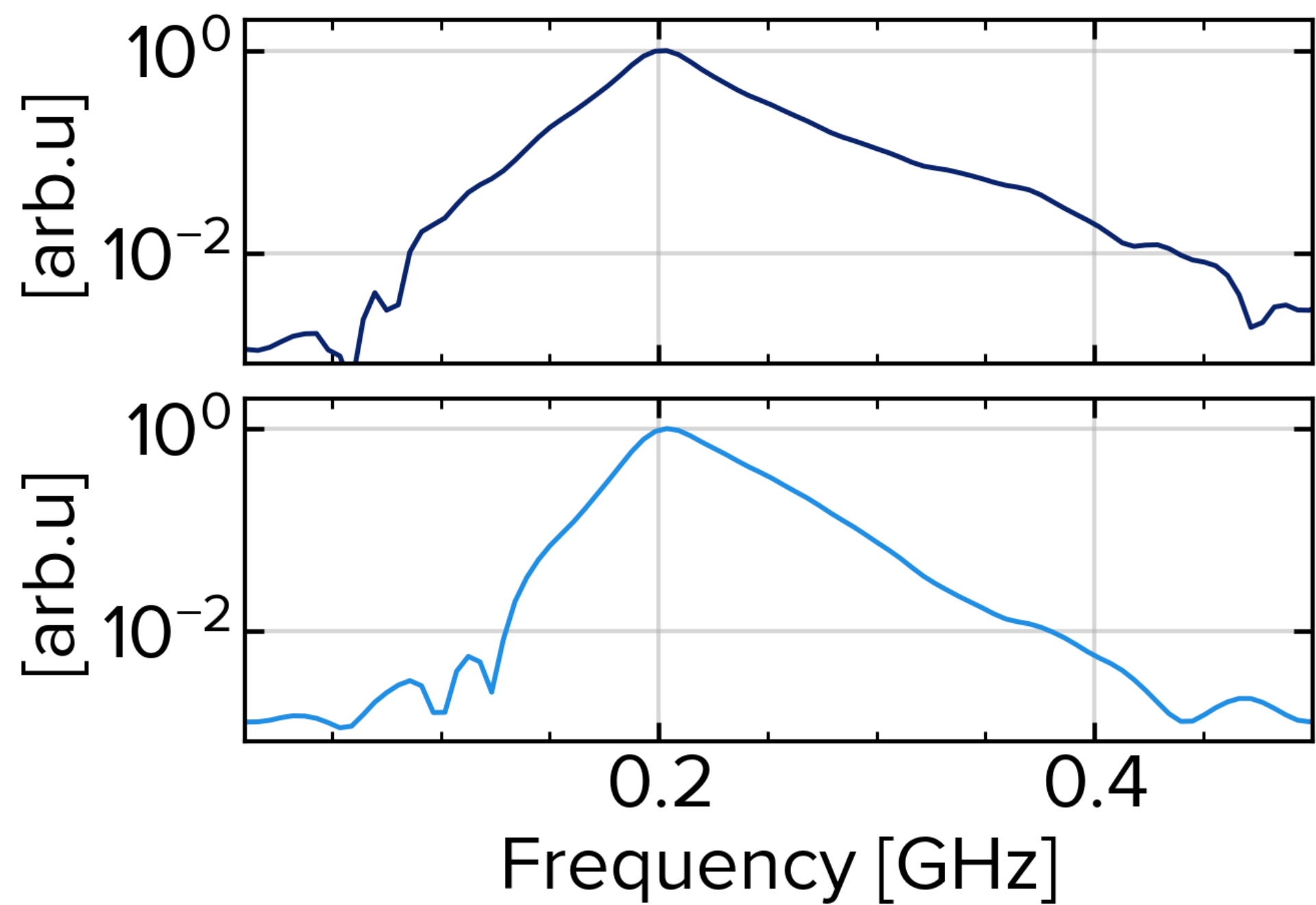
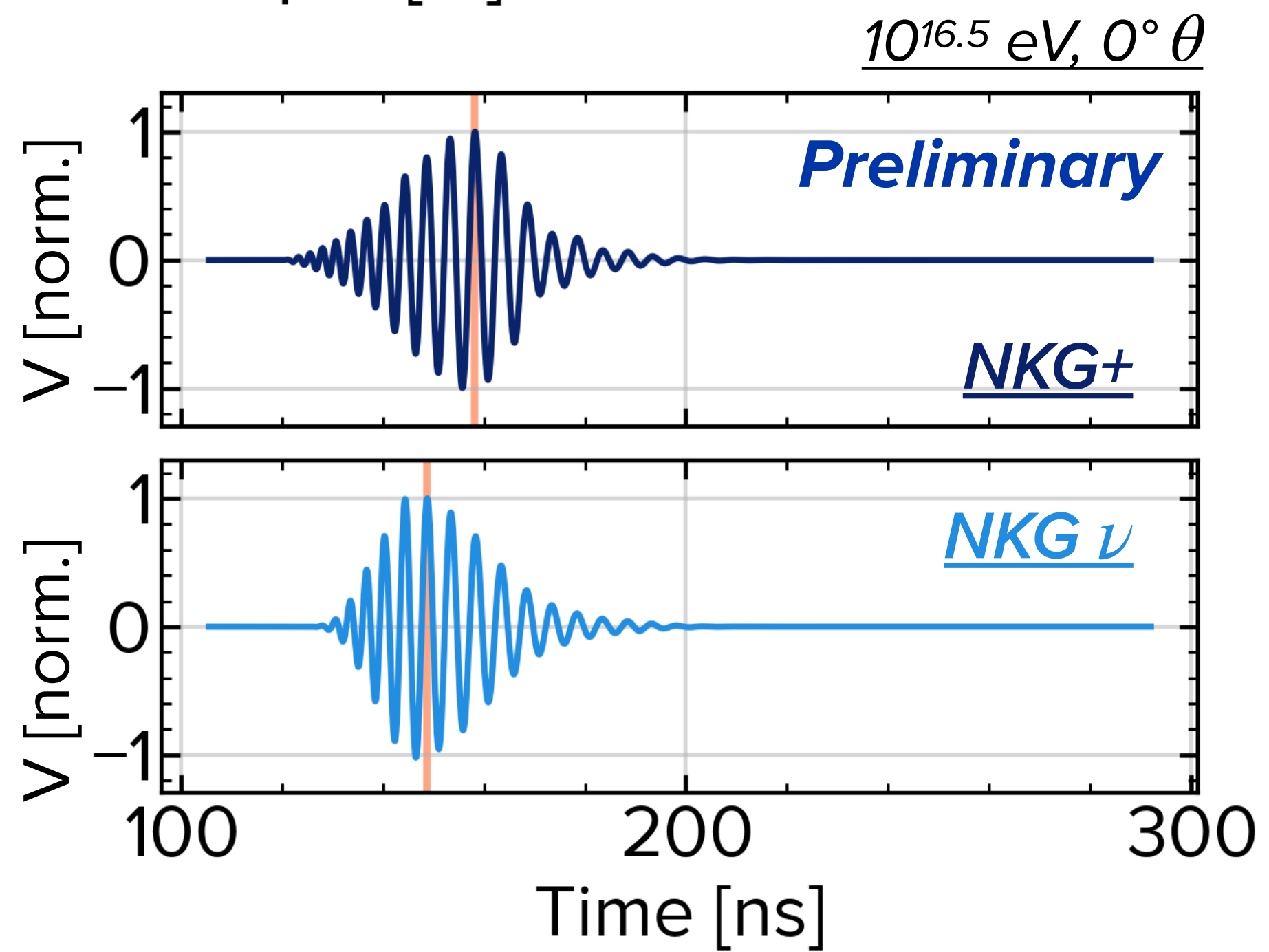
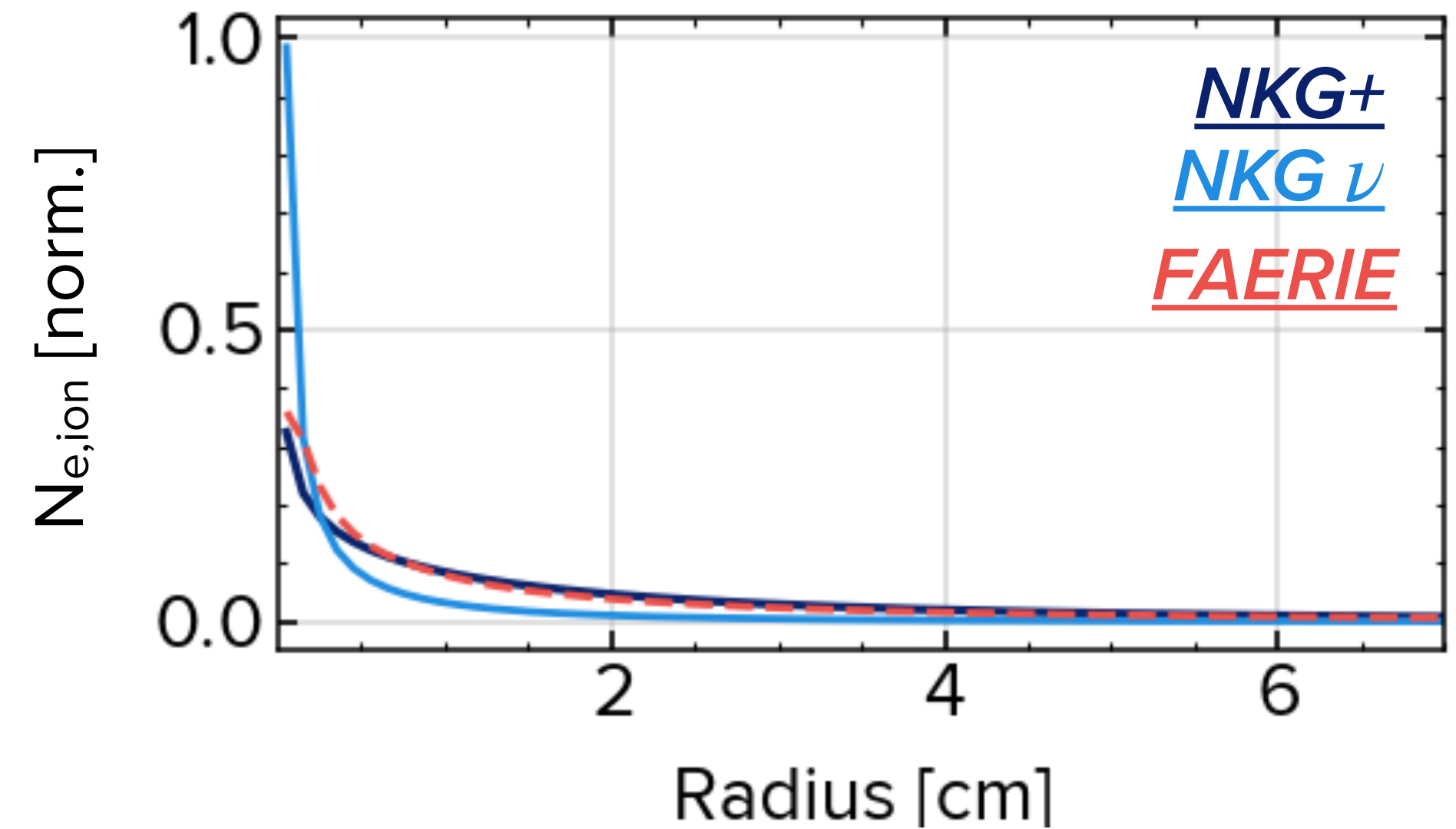
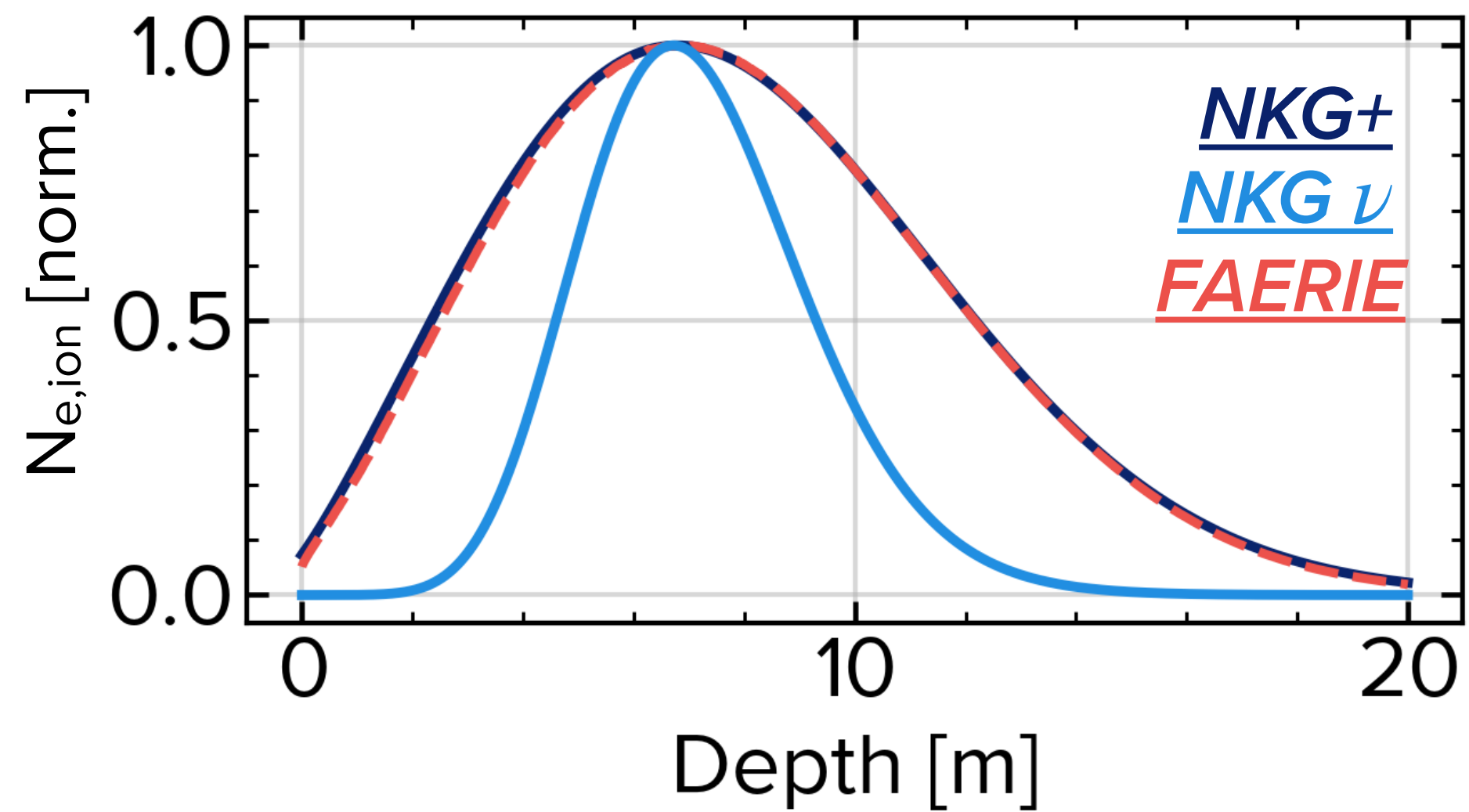
## Next steps:

- ▶ Finalise MARES implementation for use in RET-CR analyses
- ▶ Continue investigating use in simulating resulting Askaryan signals

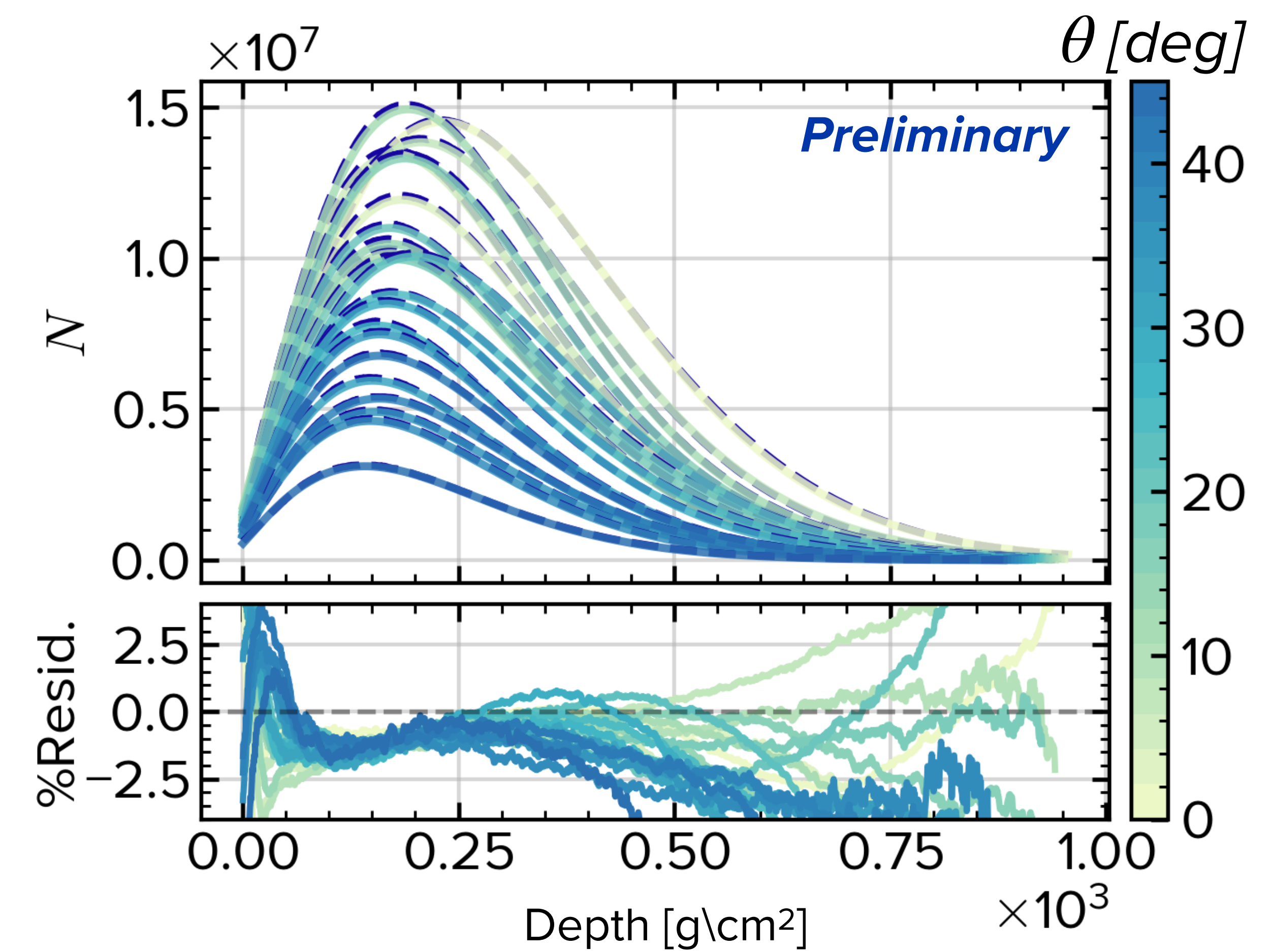
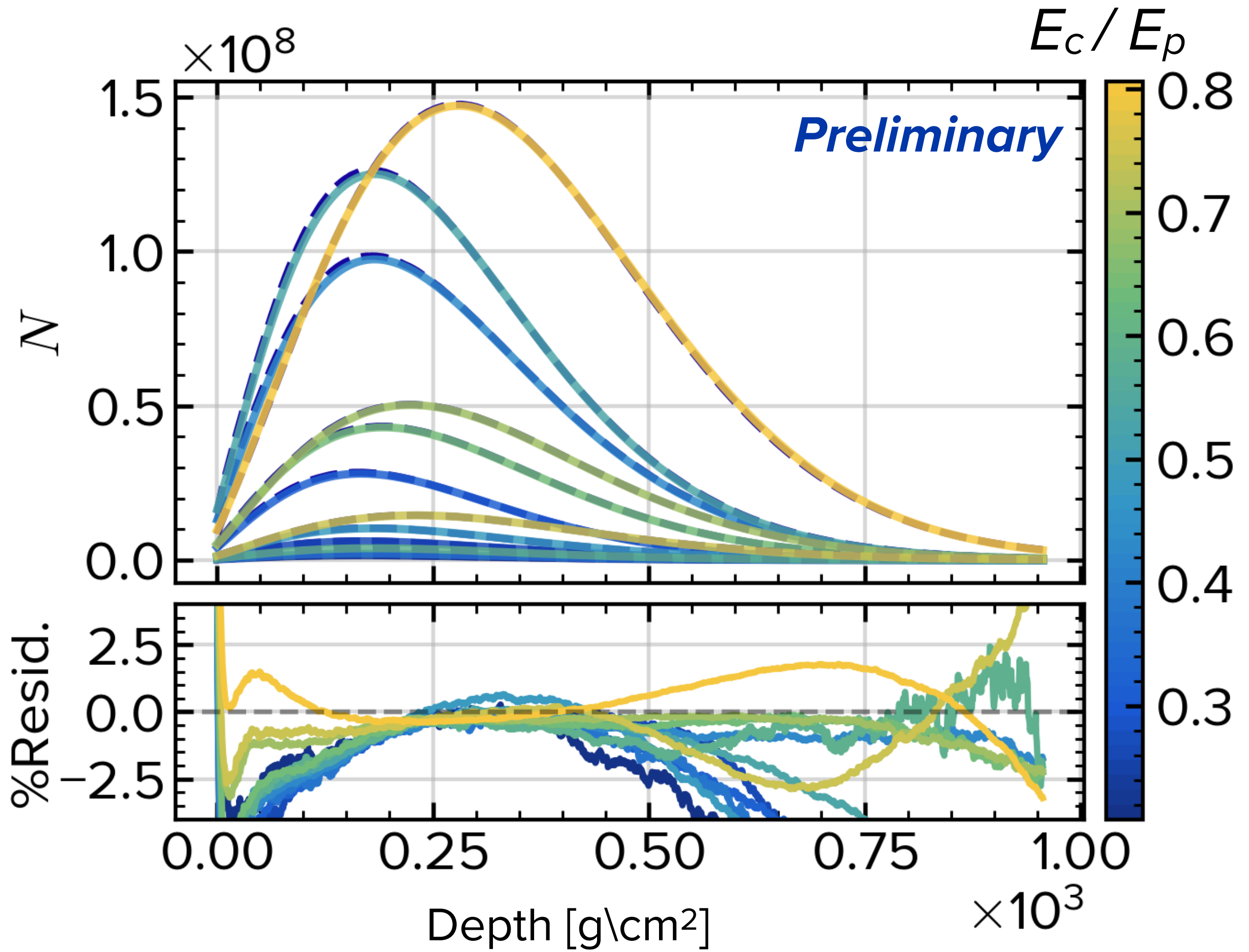




# 2D Profiles



# Longitudinal Profiles



→ Askaryan

