

The $B^+ - B_0^s$ Lifetime Differences @ NNLO

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Based on: [2604.24841](#)

Outline

- Introduction
- Heavy Quark Expansion
- Next-to-Next-to-Leading Order Calculation
- Phenomenology
- Summary & Conclusions

- Lifetimes are fundamental properties of elementary particles. We consider the lifetimes of hadrons containing one heavy quark, which decays through weak interactions
- Experimentally, we observe

B-mesons

	$B_d = (bd)$	$B^+ = (bu)$	$B_s = (bs)$
Lifetime (ps)	1.517(4)	1.638(4)	1.516(6)
$\tau(X)/\tau(B_d)$	1	1.076(4)	0.999(6)

D-mesons

	$D^0 = (\bar{u}c)$	$D^+ = (dc)$	$D_s^+ = (\bar{s}c)$
Lifetime (ps)	0.4103(10)	1.033(5)	0.5012(22)
$\tau(X)/\tau(D^0)$	1	2.510(15)	1.222(6)

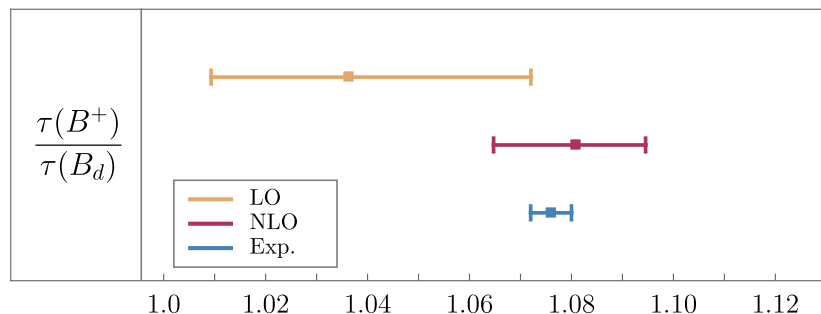
b-baryons

	$\Lambda_b = (udb)$	$\Xi_b^0 = (usb)$	$\Xi_b^- = (dsb)$	$\Omega_b^- = (ssb)$
Lifetime (ps)	1.467(9)	1.477(32)	1.570(23)	1.64(16)
$\tau(X)/\tau(B_d)$	0.964(7)	0.974(32)	1.035(23)	1.08(16)

[Lenz, 2014]

- All lifetimes of the same order (ps)
- Lifetimes of *b*-hadrons within $\sim 10\%$
- Can we explain this? $\implies$ **Yes! With Heavy Quark Expansion**

- **Heavy Quark Expansion (HQE)** provides a theoretical framework to systematically study decays of hadrons containing one heavy quark
- HQE is at the basis of many calculations in Flavour Physics, e.g. $\Delta\Gamma$ determination in the neutral B_s and B_d system [\[Gerlach, Nierste, Reeck, Shtabovenko, Steinhauser 2025\]](#)
[\[Nierste, Reeck, Shtabovenko, Steinhauser 2025\]](#)
- Lifetimes ratios are theoretically clean, with more complicated contributions cancelling in the decay widths difference  BSM effects expected to be below 1‰ level
- We can use precise theoretical predictions of lifetimes to test the HQE at higher orders in perturbation theory



$$\tau(B^+)/\tau(B_d)|_{\text{LO}} = 1.036^{+0.036}_{-0.027}$$

$$\tau(B^+)/\tau(B_d)|_{\text{NLO}} = 1.081^{+0.014}_{-0.016}$$

$$\tau(B^+)/\tau(B_d)|_{\text{exp.}} = 1.076 \pm 0.004$$

[\[Black, Lang, Lenz, Wüthrich, 2024\]](#)

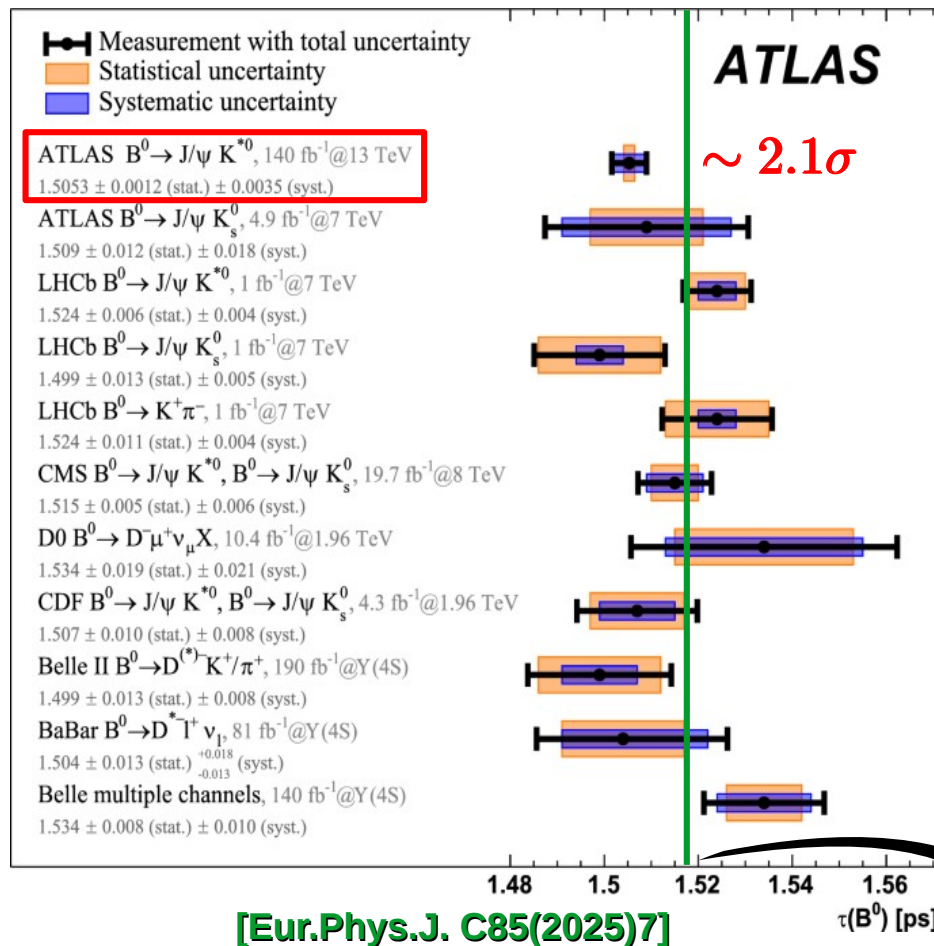
[\[Egner, Fael, Lenz, Piscopo, Rusov, Schönwald, Steinhauser, 2024\]](#)

 Theory predictions are behind

<i>b</i> -hadron species	average lifetime	lifetime ratio
B^0	1.517 ± 0.004 ps	
B^+	1.638 ± 0.004 ps	$B^+/B^0 = 1.076 \pm 0.004$
B_s^0	1.520 ± 0.005 ps	$B_s^0/B^0 = 1.0032 \pm 0.0032$
B_{sL}	1.429 ± 0.006 ps	
B_{sH}	1.622 ± 0.008 ps	

- Experimental averages from Heavy Flavour Averaging Group (**HFLAV 2024**)
- B^0 lifetime :
 - $B \rightarrow J/\psi K_S$, $B \rightarrow J/\psi K^{*0}$
ATLAS/LHCb/CMS Run 1 data
 - $B \rightarrow D^{(*)-} \pi^+$ at Belle II
- B^+ lifetime :
 - Mostly LHCb Run 1 data from the channel $B \rightarrow J/\psi K^+$
- Lifetime ratio consistent with HQE predictions

Recent ATLAS Results



- $B \rightarrow J/\psi K^{*0}$ 2.45 million events – Run 2

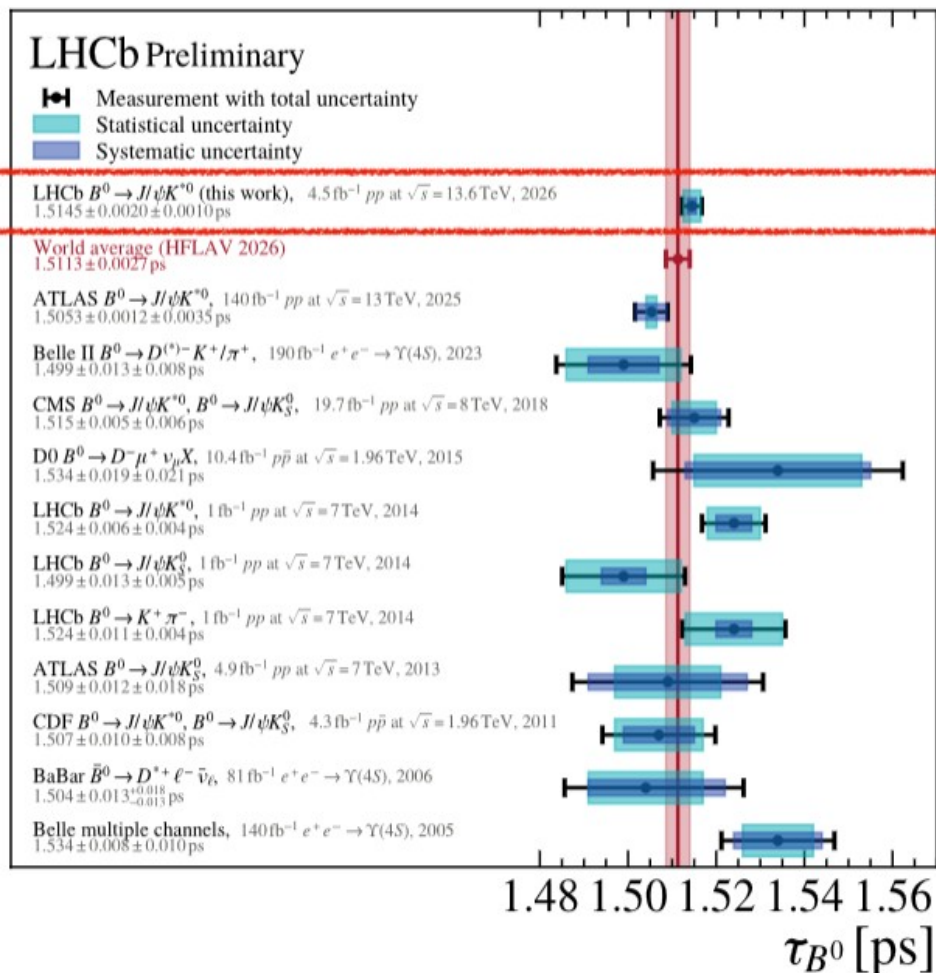
- Simultaneous fit to mass and decay time

$$\tau_{B^0} = 1.5053 \pm 0.0012(\text{stat.}) \pm 0.0035(\text{syst.}) \text{ ps.}$$

- Limited by systematic uncertainty
- New results expected soon (see Franz Muheim's talk at **FPCP26**)
- Experimental uncertainty of $\tau(B^+)/\tau(B^0)$ expected to be reduced by ~ 2

HFLAV 2024 average

Recent LCHb Results



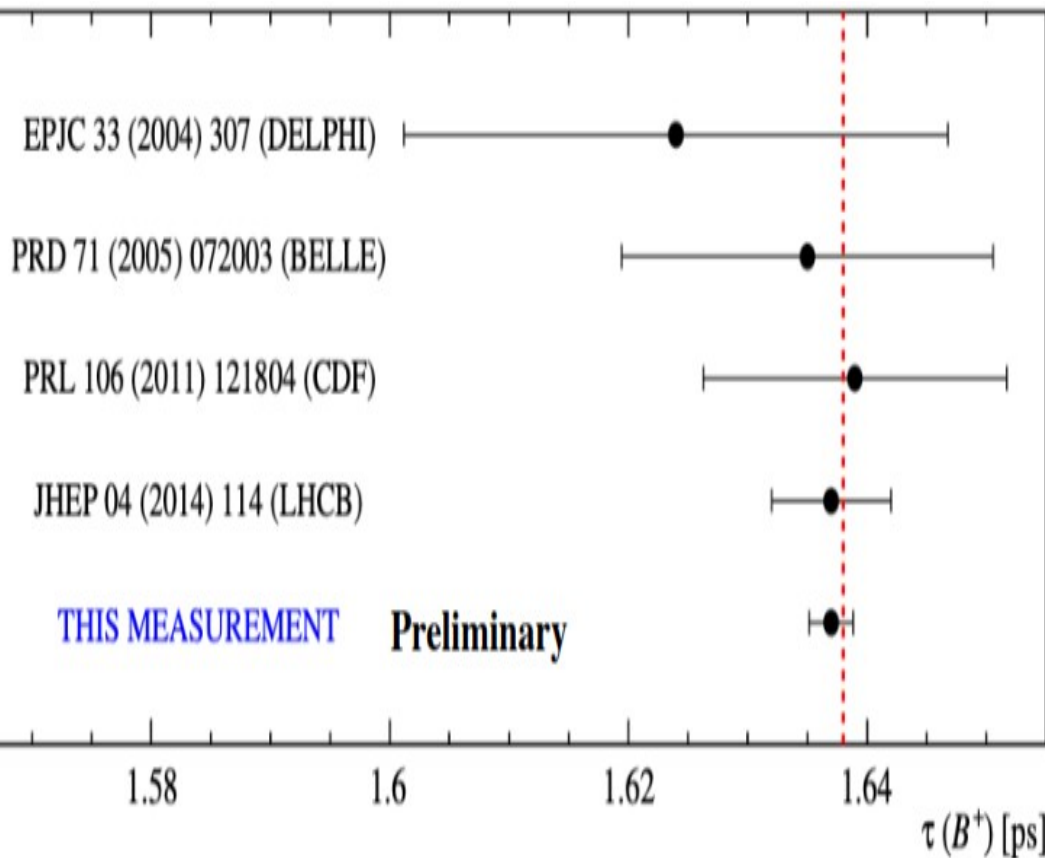
- $B \rightarrow J/\psi K^{*0}$ Run 3 data
- Simultaneous fit to mass and decay time
- Most precise determination of the B^0 lifetime in any B^0 modes

$$\tau_{B^0} = 1.5145 \pm 0.0020(\text{stat.}) \pm 0.0010(\text{syst.}) \text{ ps.}$$

- Dominated by statistical uncertainty

Credits: Ozlem Ozelik's talk @ ICHEP26

Recent LCHb Results



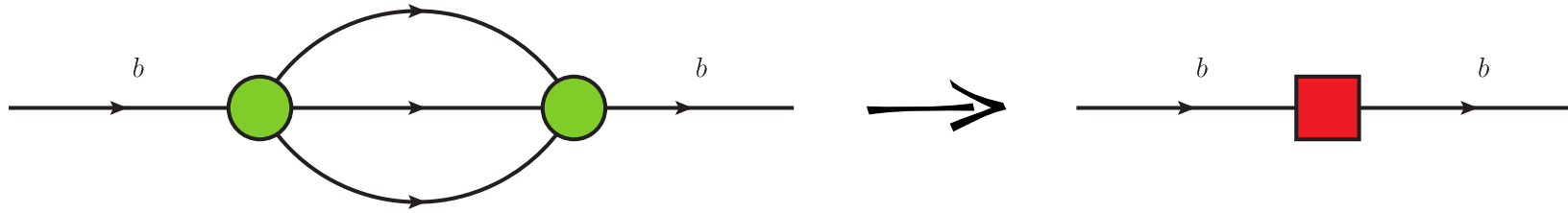
- $B^+ \rightarrow J/\psi K^+$ Run 3 data
- Simultaneous fit to mass and decay time
- Most precise determination of the B^+ lifetime in this channel

$$\tau_{B^+} = 1.6370 \pm 0.0014(\text{stat.}) \pm 0.0014(\text{syst.}) \text{ ps.}$$

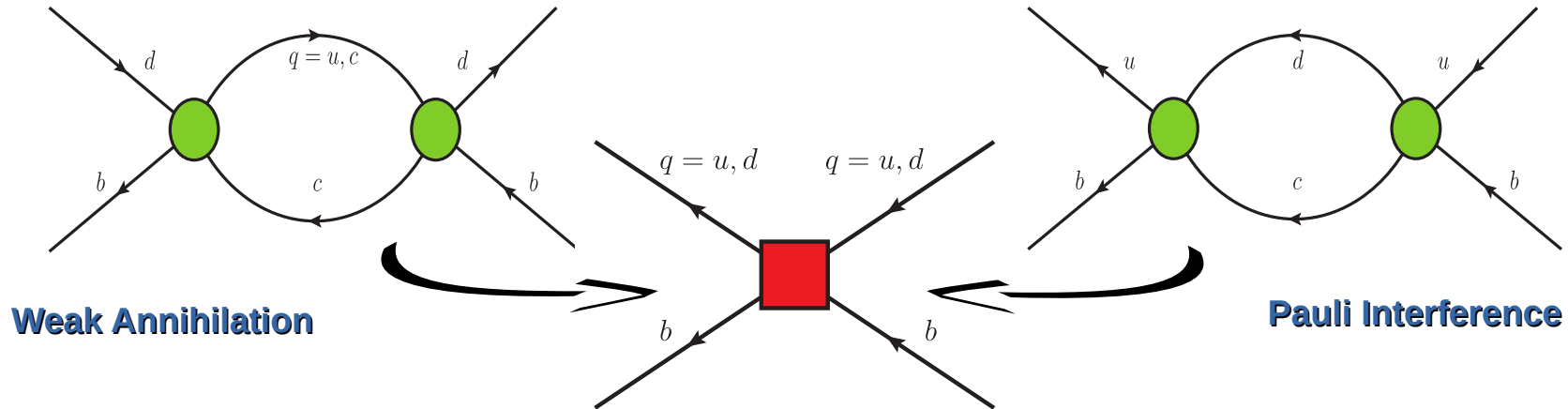
- Combining the two (preliminary) results

$$\frac{\tau_{B^+}}{\tau_{B^0}} = 1.0809 \pm 0.0017(\text{stat.}) \pm 0.0007(\text{syst.}) \text{ ps.}$$

Credits: Ozlem Ozelik's talk @ ICHEP26



HQE



- The first step to compute $\Gamma(H_b)$ is to perform an OPE integrating out the heavy W boson that mediates the weak decays

↪ Effective $|\Delta B| = 1$ local Hamiltonian

$$H = \frac{G_F}{\sqrt{2}} V_{cb}^* \sum_{\substack{u'=u,c \\ d'=d,s}} V_{u'd'} \left[C_1(\mu_1) Q_1^{u'd'}(\mu_1) + C_2(\mu_1) Q_2^{u'd'}(\mu_1) \right]$$

↪ μ_1 renormalisation scale

Contributions from penguin operators are not shown

- The Wilson coefficients $C_{1,2}$ encode the physics above the scale μ_1
- We work in the “historical” basis

↪ **Mixing under Fierz transformation**

$$Q_1^{u'd'} = (\bar{b}_i c_j)_{V-A} (\bar{u}'_j d'_i)_{V-A}$$

$$Q_2^{u'd'} = (\bar{b}_i c_i)_{V-A} (\bar{u}'_j d'_j)_{V-A}$$

- We relate the total decay rate $\Gamma(H_b)$ to the Hamiltonian H using the optical theorem

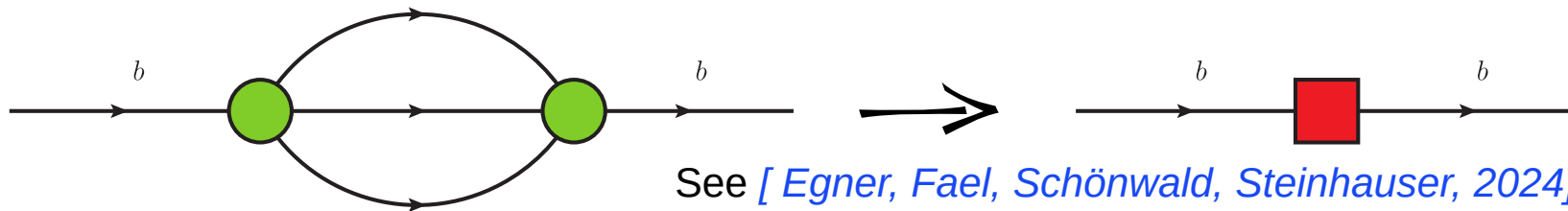
$$\Gamma(H_b) = \frac{1}{2M_{H_b}} \langle H_b | \mathcal{T} | H_b \rangle \quad \mathcal{T} = \text{Im } i \int d^4x \, T \{ H(x) H(0) \}$$

Non-local correlator !

- HQE exploits the hierarchy of scales $m_b \gg \Lambda_{\text{QCD}}$

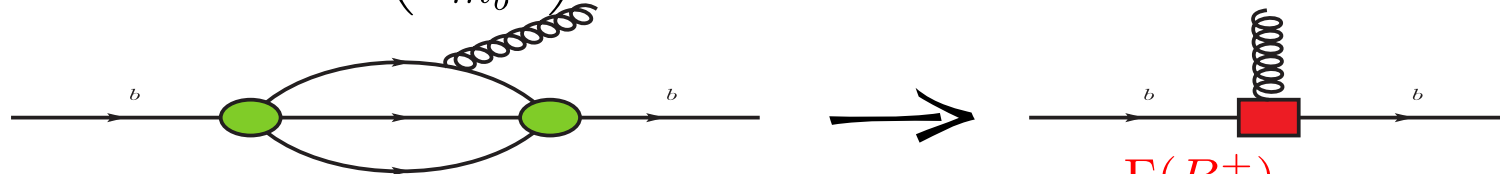
$$\mathcal{T} = \mathcal{T}^0 + \mathcal{T}^2 + \mathcal{T}^3 + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^4 \quad \text{where} \quad \mathcal{T}^i \sim 1/m_b^i$$

- Leading order $\Rightarrow$ Free b-quark decay $\rightarrow$ All lifetimes are the same



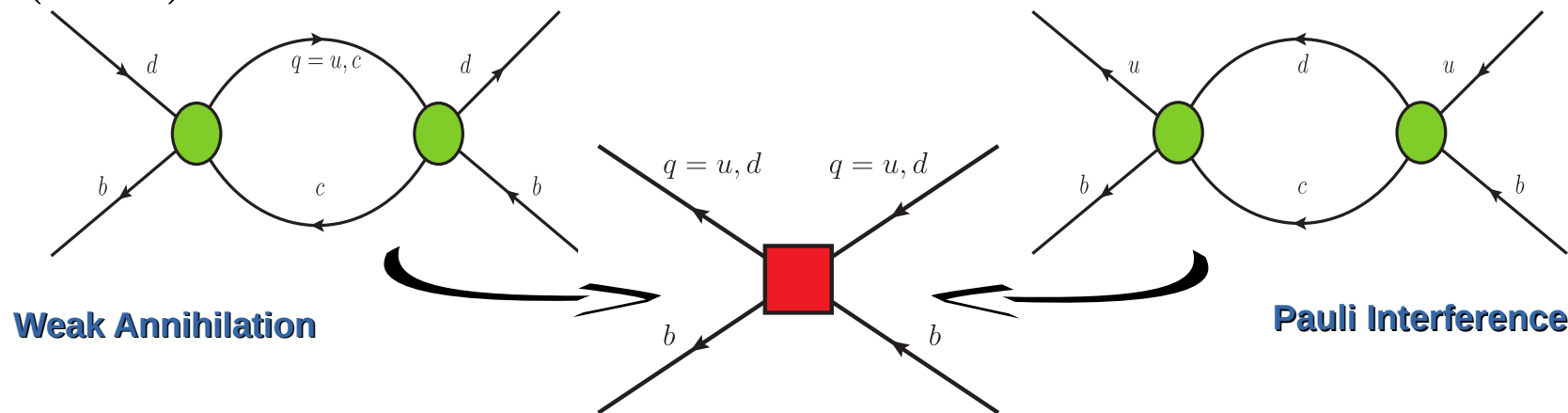
See [Egner, Fael, Schönwald, Steinhauser, 2024]

- First corrections arise at $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2$ from chromomagnetic interactions

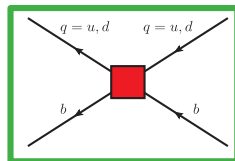
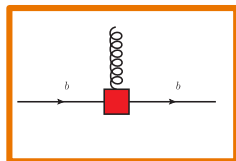
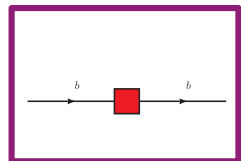


- Strong isospin $\Rightarrow$ Negligible contributions to $\tau(B^+)/\tau(B_d)$ $\frac{\Gamma(B^+)}{\Gamma(B_d^0)} \approx 1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^3$

- At $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^3 \Rightarrow$ weak interaction of b-quarks with (light) valence quarks



- Optical theorem:** $\Gamma(H_b) = \frac{1}{2M_{H_b}} \langle H_b | \mathcal{T} | H_b \rangle$ $\mathcal{T} = \text{Im } i \int d^4x e^{iqx} \text{T} \{ H(x) H(0) \}$



Non-local Correlator !

$$m_b \gg \Lambda_{\text{QCD}}$$

Effective $|\Delta B| = 1$ local Hamiltonian

$$\mathcal{T} = \boxed{\mathcal{T}^0} + \boxed{\mathcal{T}^2} + \boxed{\mathcal{T}^3} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^4$$

Same for all hadrons
Isospin singlets

Negligible

$$H = \frac{G_F}{\sqrt{2}} V_{cb}^* \sum_{\substack{u'=u,c \\ d'=d,s}} V_{u'd'} \left[C_1(\mu_1) Q_1^{u'd'}(\mu_1) + C_2(\mu_1) Q_2^{u'd'}(\mu_1) \right]$$

μ_1 renormalisation scale

- We focus on contributions from dimension-6 operators $\Rightarrow \mathcal{T}^3 = \boxed{\mathcal{T}^u} + \boxed{\mathcal{T}^d}$

Weak Annihilation
Pauli Interference

$$\bullet \mathcal{T}^u = \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi} \left[|V_{ud}|^2 (F^u Q^d + F_S^u Q_S^d + G^u T^d + G_S^u T_S^d) + |V_{cd}|^2 (F^c Q^d + F_S^c Q_S^d + G^c T^d + G_S^c T_S^d) \right]$$

$$\bullet \mathcal{T}^d = \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi} \left[(F^d Q^u + F_S^d Q_S^u + G^d T^u + G_S^d T_S^u) \right]$$

[Buras, Weisz 1990] $Q_1^{u'd'} = (\bar{b}_i c_j)_{V-A} (\bar{u}'_j d'_i)_{V-A}$
 "Historical" basis:
 $Q_2^{u'd'} = (\bar{b}_i c_i)_{V-A} (\bar{u}'_j d'_j)_{V-A}$

Other CKM suppressed contributions are not shown

- We match the amplitudes onto a set of effective local $|\Delta B|=0$ operators

$$Q^q = (\bar{b} q)_{V-A} (\bar{q} b)_{V-A}$$

$$Q_S^q = (\bar{b} q)_{S-P} (\bar{q} b)_{S+P}$$

$$T^q = (\bar{b} T^a q)_{V-A} (\bar{q} T^a b)_{V-A}$$

$$T_S^q = (\bar{b} T^a q)_{S-P} (\bar{q} T^a b)_{S+P}$$

- Using the isospin relation $\langle B_d | Q^{u,d} | B_d \rangle = \langle B^+ | Q^{d,u} | B^+ \rangle$

$$\frac{\tau(B^+)}{\tau(B_d)} - 1 = \tau(B^+) [\Gamma(B_d) - \Gamma(B^+)] = \frac{G_F^2 m_b^2 |V_{cb}|^2}{12\pi} f_B^2 M_B \left(|V_{ud}|^2 \vec{F}^u + |V_{cd}|^2 \vec{F}^c - \vec{F}^d \right) \cdot \vec{B}$$

Wilson coefficients $\vec{F}^q(z, \mu_0) = \begin{pmatrix} F^q(z, \mu_0) \\ F_S^q(z, \mu_0) \\ G^q(z, \mu_0) \\ G_S^q(z, \mu_0) \end{pmatrix}$

$z = \frac{m_c^2}{m_b^2}$

[Black, Lang, Lenz, Wüthrich, 2024]

Non-perturbative

Bag parameters

$$\vec{B}(\mu_0) = \begin{pmatrix} B_1(\mu_0) \\ B_2(\mu_0) \\ \epsilon_1(\mu_0) \\ \epsilon_2(\mu_0) \end{pmatrix}$$

- Hadronic matrix elements

$$\langle B^+ | (Q^u - Q^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 B_1(\mu_0)$$

$$\langle B^+ | (Q_S^u - Q_S^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 B_2(\mu_0)$$

$$\langle B^+ | (T^u - T^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 \epsilon_1(\mu_0)$$

$$\langle B^+ | (T_S^u - T_S^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 \epsilon_2(\mu_0)$$

$|\Delta B| = 0$ renormalisation scale

NNLO QCD Calculations

[Nogueira, 1991]



Diagram generation based on model files

[Gerlach, Herren, Lang, 2022]
[Seidensticker, 1999]



Implementation of Feynman rules & mapping over topologies

[Vermaseren, 2000]

[Reeck, Shtabovenko, Steinhauser, 2024]

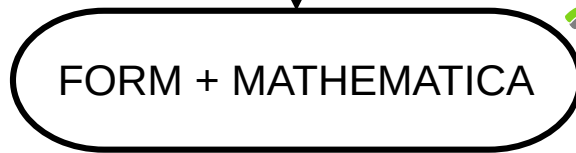


Reduction of Feynman amplitudes in terms of scalar integrals

[Lange, Usovitsch, Wu, 2023]



Integration-by-parts reduction to master integrals



Implementation of master integrals & renormalisation

- Physics mediated by heavy particle can be written at low scales as

$$\mathcal{H} = \sum_i C_i \hat{Q}_i$$


Wilson coefficients encode the heavy physics

- Wilson coefficients C_i mix upon renormalisation according to

$$C_j^b = Z_{ij} C_i \quad Z_{ij} = \delta_{ij} + \sum_{k=1}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^k Z_{ij}^{(k)} \quad Z_{ij}^{(k)} = \sum_{l=0}^k \frac{1}{\epsilon^l} Z_{ij}^{(k,l)}$$

- Scale dependence is governed by the anomalous dimension matrix (**ADM**)



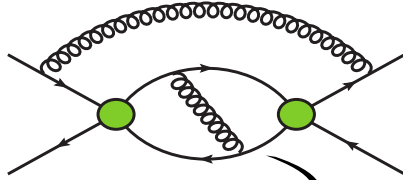
$$\mu \frac{dC_i}{d\mu} = \gamma_{ji} C_j$$

Renormalisation Group Equations (RGEs)

$$\Longleftarrow \gamma_{ij} = Z_{ik} \mu \frac{d}{d\mu} Z_{kj}^{-1}$$


The anomalous dimension is related to the single $1/\epsilon$ poles in the renormalisation matrix

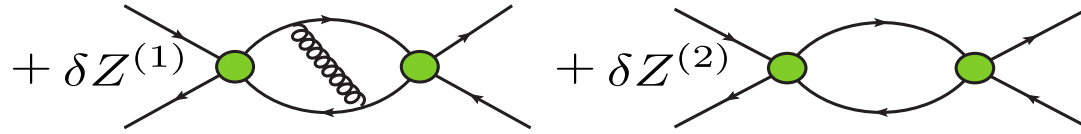
$|\Delta B| = 1$



• $\mathcal{O}(2k)$ diagrams

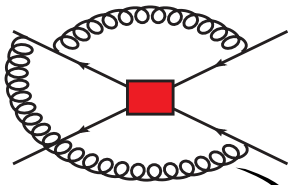
- MIs from recent projects:
[Reeck, Shtabovenko, Steinhauser, 2024]

↘ Log-linear series in $x = m_c/m_b$



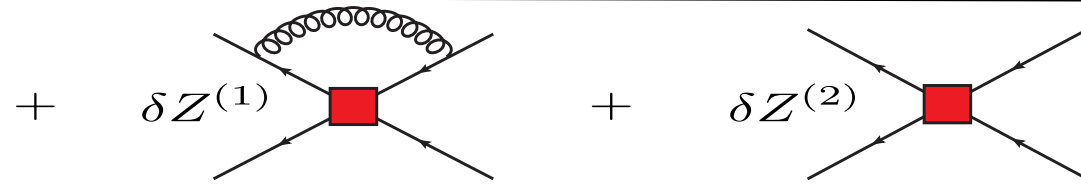
- Lower order counter-terms
- Mixing matrix up to $\mathcal{O}(\alpha_s^2)$. Appropriate choice of Evanescent operators: [Egner, Fael, Schönwald, Steinhauser, 2024]
- $\overline{\text{MS}}$ renormalisation of charm quark mass
- on-shell renormalisation of bottom quark mass

$\Delta B = 0$



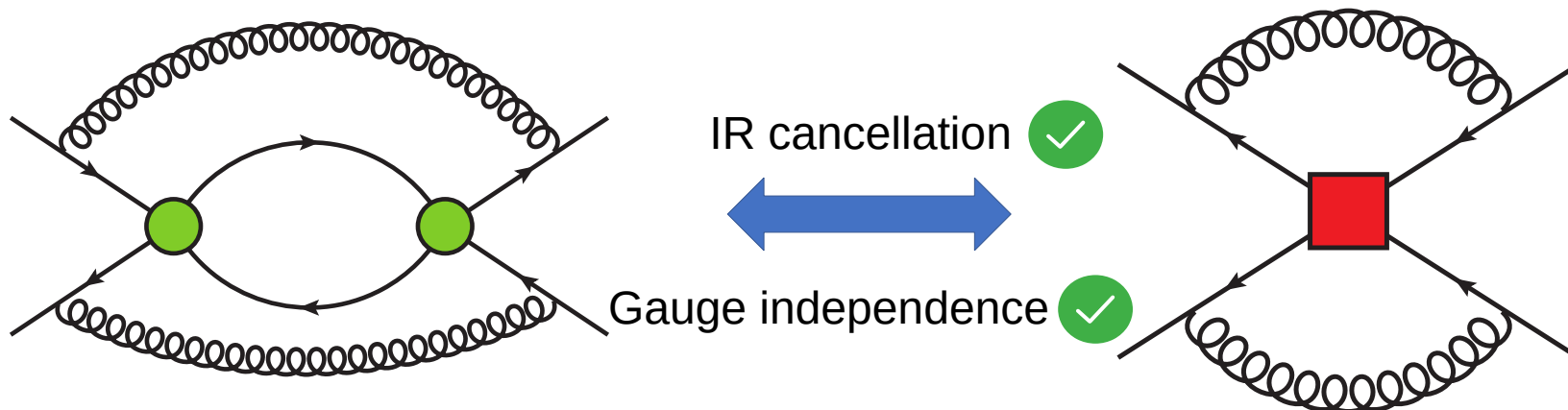
• $\mathcal{O}(1k)$ diagrams

- MIs from recent projects:
[Reeck, Shtabovenko, Steinhauser, 2024]



- Mixing matrix up to $\mathcal{O}(\alpha_s)$. NNLO calculation in:
[FM, Nierste, Reeck, Steinhauser, 2026],
[Aebischer, Morell, Pesut, Virto, 2025],
[Black, Harlander, Kohnen, Lange, Rago, Shindler, Witzel, 2026]
- on-shell renormalisation of bottom quark mass

- All results have been checked within our collaboration
- Additional strong checks such as gauge independence and cancellation of IR divergences
- We also checked the independence of the Wilson coefficients $F, \dots, G_S$ on μ_1 :
 - We perform the matching at $\mu_1 \rightarrow F_{ij}(x, \mu_1)$
 - We use Renormalisation Group Equation $F_{ij}(x, \mu_1) \longrightarrow F_{ij}(x, \mu_1, \mu_0)$ $\curvearrowright \mu_0 \sim \bar{m}_b(\bar{m}_b)$



Phenomenology

Lifetimes Ratio

- The lifetime ratio is then given by

$$\frac{\tau(B^+)}{\tau(B_d)} - 1 = \tau(B^+) [\Gamma(B_d) - \Gamma(B^+)]$$

We take the experimental value from the **PDG**

- We recall the formula for the width difference

$$\Gamma(B_d) - \Gamma(B^+) = \frac{G_F^2 m_b^2 |V_{cb}|^2}{12\pi} f_B^2 M_B \left(|V_{ud}|^2 \vec{F}^u + |V_{cd}|^2 \vec{F}^c - \vec{F}^d + \dots \right) \cdot \vec{B}$$

Additional CKM suppressed contributions

CKMfitter

FLAG

Function of $\bar{x} = \frac{\bar{m}_c(\mu_1)}{\bar{m}_b(\mu_1)}$

Bag parameters from HQET sum rules

$\mu_0 = \bar{m}_b(\bar{m}_b) = 4.163(16) \text{ GeV}$

[Black, Lang, Lenz, Wüthrich, 2024]

Different choices of scheme:

- Pole
- $\overline{\text{MS}}$
- PS

Perturbative conversion from pole scheme to $\mathcal{O}(\alpha_s^2)$

We use these for our predictions

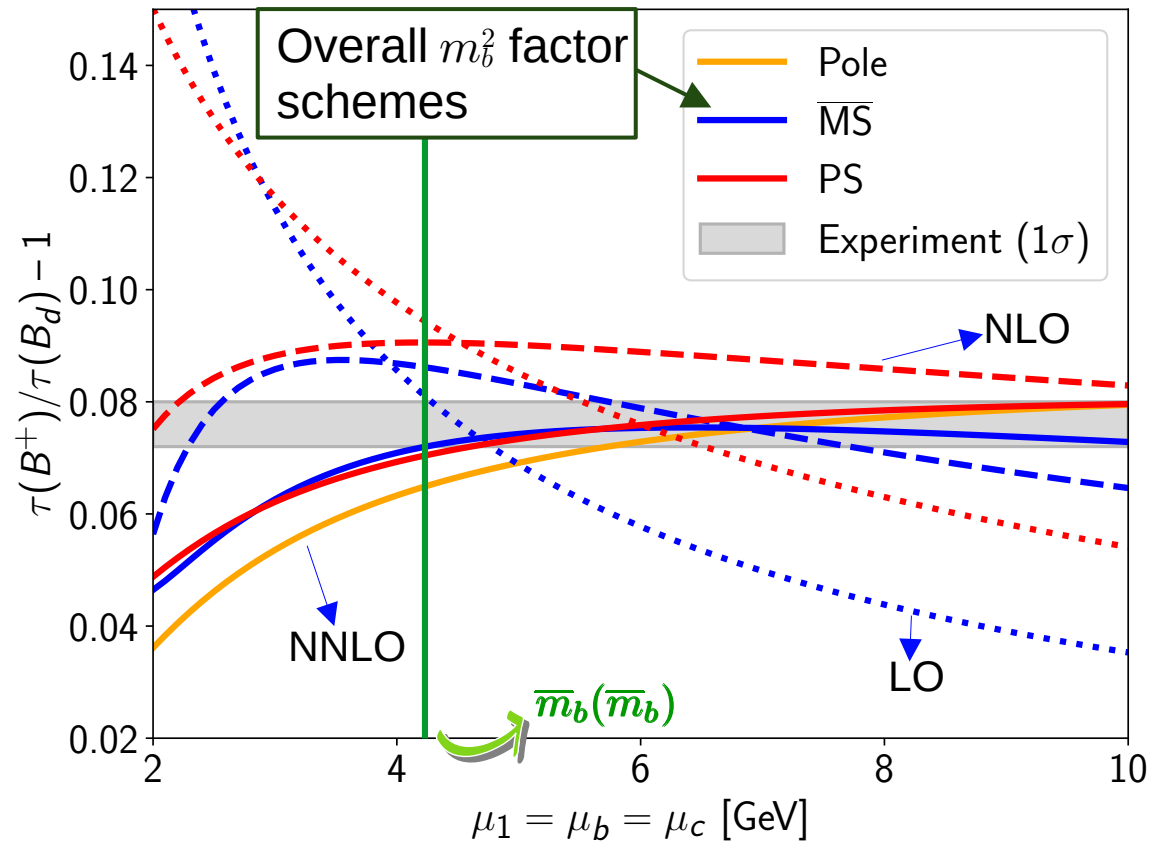
- We show the ratio $\tau(B^+)/\tau(B_d) - 1$ as a function of μ_1 .
- We estimate the scale uncertainty by varying μ_1 in [2.1 GeV, 8.4 GeV]
- We observe an increment in the scale dependence for both $\overline{\text{MS}}$ and PS:

	NLO	NNLO
$\overline{\text{MS}}$	14% ↗	18%
PS	7% ↗	20%

- Predictions at NNLO between $\overline{\text{MS}}$ and PS are much closer together w.r.t. NLO case



Reduction of scheme dependence @ NNLO



[Credits: Pascal Reek]

- For our final predictions, we combine the results in $\overline{\text{MS}}$ and PS and obtain

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.072 \pm 0.024 \quad (\text{NNLO})$$

- To be compared to the experimental value

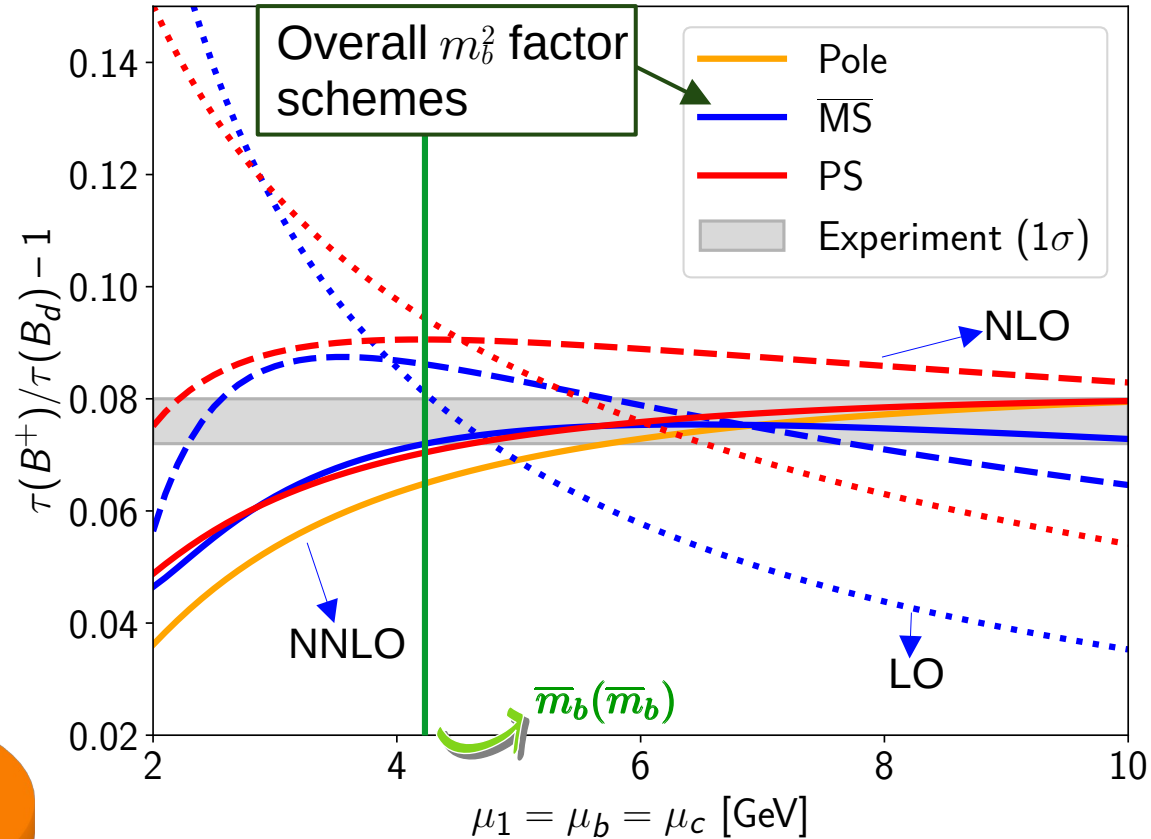
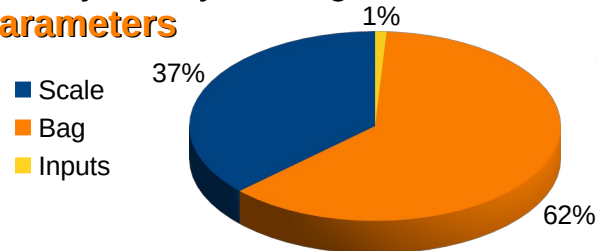
$$\frac{\tau(B^+)}{\tau(B_d)} = 1.076 \pm 0.004 \quad (\text{exp.})$$

HFLAV

- Good agreement with experimental value, but larger ($\sim 6\times$) uncertainties in theory predictions

Our results show the validity of the HQE !

- Theoretical uncertainty mainly coming from **scale** and **bag parameters**



[Credits: Pascal Reeck]

D Mesons

New Lattice Results



Heavy-Meson Bag Parameters using Gradient Flow

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(Dated: March 31, 2026)

We demonstrate the use of the gradient flow combined with the short flow-time expansion (GF+SFTX) as a renormalization procedure for four-quark operator matrix elements and associated bag parameters relevant to neutral heavy-meson mixing ($\Delta Q = 2$) and heavy-meson lifetimes ($\Delta Q = 0$). Using six RBC/UKQCD 2+1-flavor domain-wall fermion ensembles, we calculate for a charm-strange system with physical quark masses flowed bag parameters and match them to the $\overline{\text{MS}}$ scheme using perturbative SFTX coefficients up to next-to-next-to-leading order in QCD. We employ a multi-scale matching strategy and a renormalization-group improved flow-time evolution which allows for a reliable estimate of systematic uncertainties. For a fictitious neutral D_s meson, we obtain the $\Delta Q = 2$ $\overline{\text{MS}}$ bag parameter $B_1^{\overline{\text{MS}}}(3 \text{ GeV}) = 0.7673(123)$, consistent with existing short-distance D^0 mixing determinations. For the $\Delta Q = 0$ lifetime-ratio operator basis, we find the $\overline{\text{MS}}$ results $B_1^{\overline{\text{MS}}}(3 \text{ GeV}) = 1.0524(97)$, $B_2^{\overline{\text{MS}}}(3 \text{ GeV}) = 0.9621(71)$, $\epsilon_1^{\overline{\text{MS}}}(3 \text{ GeV}) = -0.2275(76)$, and $\epsilon_2^{\overline{\text{MS}}}(3 \text{ GeV}) = -0.0005(8)$. We provide conversion formulae to re-express these results for an arbitrary choice of evanescent operators. These results demonstrate that GF+SFTX can deliver precise determinations of dimension-six four-quark operators and establish a framework for future lattice computations including more complex operator bases, where the challenge of power-divergent mixing is shifted to the continuum and handled in the SFTX.

Siegen

Aachen

Karlsruhe

Next-to-next-to-leading QCD corrections to the $B^+-B_d^0$, D^+-D^0 , and $D_s^+-D^0$ lifetime ratios

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ABSTRACT: The total decay widths of heavy mesons can be systematically calculated in terms of an expansion in the two parameters $1/m_Q$ and $\alpha_s(m_Q)$, where $Q = c, b$ denotes a heavy quark. The dominant contributions to meson lifetime splittings stem from terms which are suppressed by $1/m_Q^3$ with respect to the leading universal contribution to the total decay width. We calculate three-loop contributions of order α_s^2/m_q^3 to the lifetime ratios $\tau(B^+)/\tau(B_d^0)$, $\tau(D^+)/\tau(D^0)$, and $\tau(D_s^+)/\tau(D^0)$ in the limit of exact isospin and V-spin symmetry, respectively. Furthermore, we present new α_s/m_q^3 corrections to the Cabibbo-suppressed terms in $\tau(B^+)/\tau(B_d^0)$. Combining our perturbative coefficients with hadronic matrix elements calculated from Heavy Quark Effective Theory sum rules, we find $\tau(B^+)/\tau(B_d^0) = 1.072 \pm 0.024$. Using hadronic matrix elements from a recent lattice QCD calculation we find $\tau(D^+)/\tau(D^0) = 2.344 \pm 0.170$ and $\tau(D_s^+)/\tau(D^0) = 1.289 \pm 0.042$. We find good agreement of our predictions with experimental data, which constitutes a successful probe of the calculations of hadronic matrix elements and permits estimates of the unknown $1/m_Q^4$ contributions as well as the V-spin breaking terms in $\tau(D_s^+)/\tau(D^0)$.

Synergies Within The CRC!

- We can adapt our matching coefficients to study lifetime ratios in the D mesons system

$$\tau(D^+)/\tau(D^0) - 1$$

- Isospin doublet, like $(B^+, B_d) \Rightarrow$ Cancellation of Isospin singlet contributions
- $\Lambda_{\text{QCD}}/m_c \sim 0.3$, so that higher orders in the HQE are expected to give sizeable contributions

Dimension-7 operators



- We estimate their size comparing our predictions with data

$$\tau(D^+)/\tau(D^0) = 1.737 \pm 0.228 \quad (\text{NNLO})$$

$$\tau(D^+)/\tau(D^0) = 2.510 \pm 0.015 \quad (\text{exp.})$$

$$\tau(D_s^+)/\tau(D^0) - 1$$

- In contrast, (D_s^+, D^0) is a V-spin doublet $\Rightarrow$ Singlet contributions do not cancel exactly
- SU(3)-breaking corrections as $m_s - m_u \sim 0.3 \Lambda_{\text{QCD}}$ in addition to the dimension-7 contributions
- We estimate their combined size comparing our predictions with data

$$\tau(D_s^+)/\tau(D^0) = 1.135 \pm 0.131 \quad (\text{NNLO})$$

$$\tau(D_s^+)/\tau(D^0) = 1.222 \pm 0.006 \quad (\text{exp.})$$

Bag parameters from HQET sum rules results

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Dimension-7 operators



- We estimate their size comparing our predictions with data

$$\tau(D^+)/\tau(D^0) = 2.344 \pm 0.170 \quad (\text{NNLO})$$

$$\tau(D^+)/\tau(D^0) = 2.510 \pm 0.015 \quad (\text{exp.})$$

$$\tau(D_s^+)/\tau(D^0) - 1$$

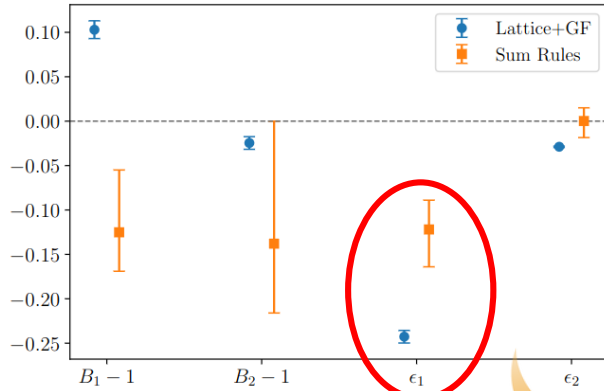
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$$\tau(D_s^+)/\tau(D^0) = 1.289 \pm 0.042 \quad (\text{NNLO})$$

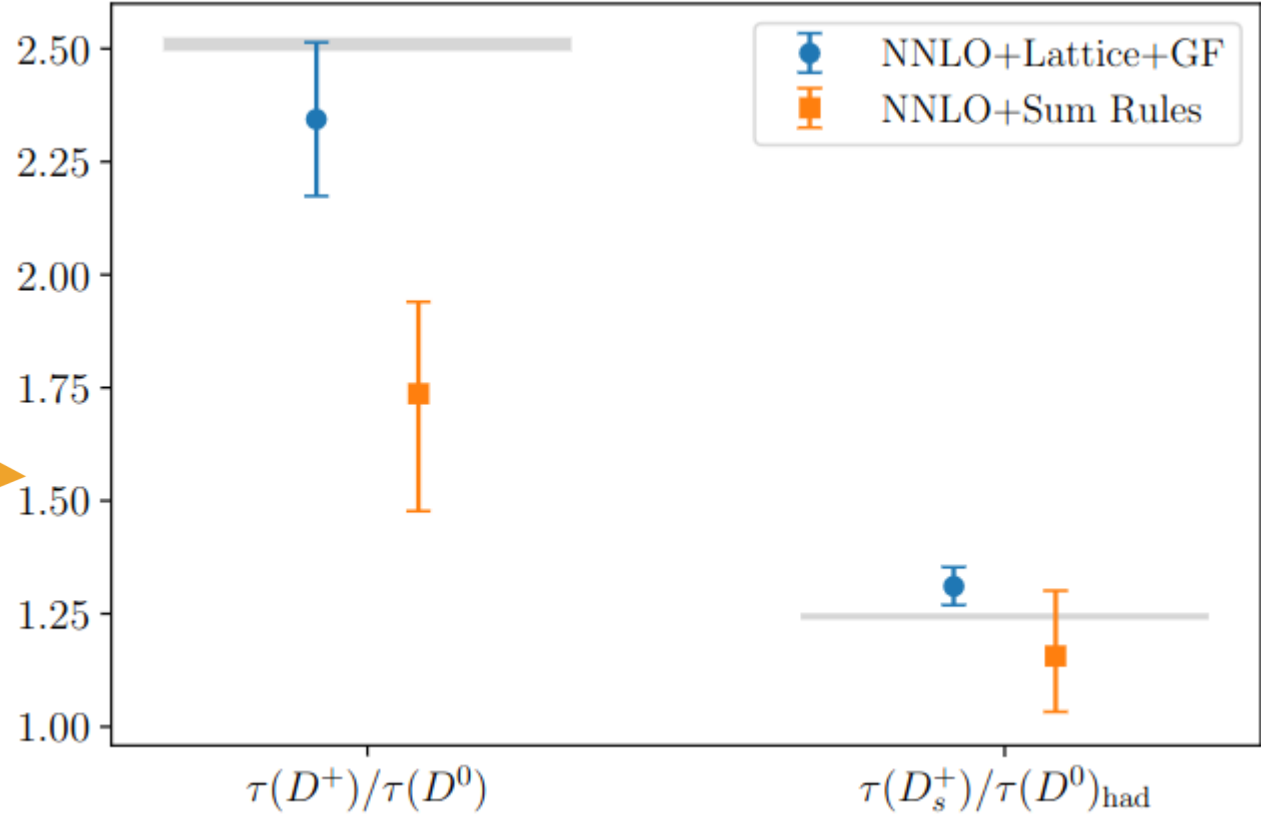
$$\tau(D_s^+)/\tau(D^0) = 1.222 \pm 0.006 \quad (\text{exp.})$$

Bag parameters from recent lattice results

Lifetimes Ratio



- ϵ_1 and ϵ_2 Bag parameters are smallest in size, but have largest Wilson coefficients



Credits: Matthew Black's theory talk @ FPCP26

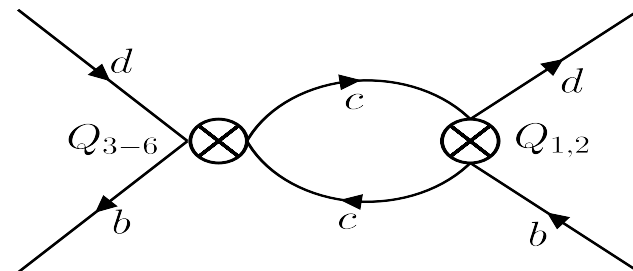
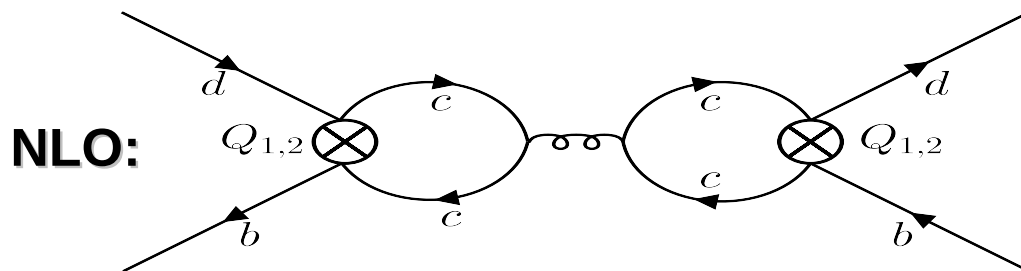
Summary & Conclusions

- Lifetimes ratios provide a clean theoretical ground to test HQE and compare with precise experimental results
- We focus on the dimension-6 operators contribution coming from spectator effects and perform matching to $|\Delta B|=0$ operators to NNLO accuracy in QCD
- Our results show the validity of the framework and support the HQE as the correct method to calculate lifetime differences between heavy hadrons



Application of the method to observable with high sensitivity to BSM: $B_{d,s} - \bar{B}_{d,s}$

- Future outlooks: Include penguin-like contributions @ NNLO and study $\tau(B_s)/\tau(B_d)$



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Thank You!

Backup

- The first step to compute $\Gamma(H_b)$ is to perform an OPE integrating out the heavy W boson that mediates the weak decays

↪ Effective $|\Delta B| = 1$ local Hamiltonian

$$H = \frac{G_F}{\sqrt{2}} V_{cb}^* \sum_{\substack{u'=u,c \\ d'=d,s}} V_{u'd'} \left[C_1(\mu_1) Q_1^{u'd'}(\mu_1) + C_2(\mu_1) Q_2^{u'd'}(\mu_1) \right]$$

↪ μ_1 renormalisation scale

We focus on current-current operators only

- The Wilson coefficients $C_{1,2}$ encode the physics above the scale μ_1
- We work in the “historical” basis

↪ **Mixing under Fierz transformation**

$$Q_1^{u'd'} = (\bar{b}_i c_j)_{V-A} (\bar{u}'_j d'_i)_{V-A}$$

$$Q_2^{u'd'} = (\bar{b}_i c_i)_{V-A} (\bar{u}'_j d'_j)_{V-A}$$

- We relate the total decay rate $\Gamma(H_b)$ to the Hamiltonian H using the optical theorem

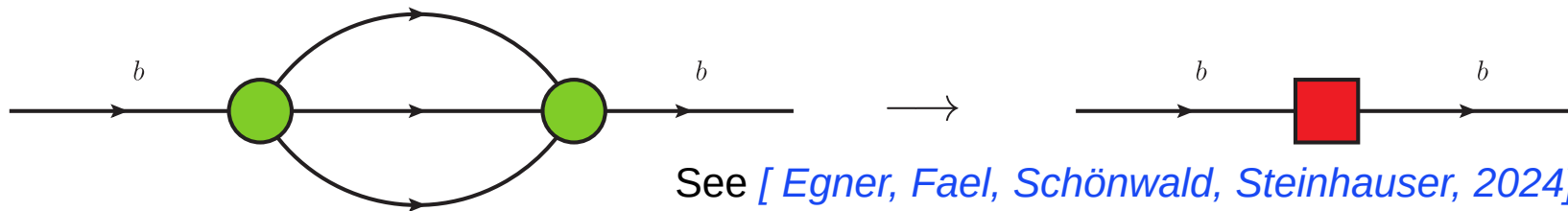
$$\Gamma(H_b) = \frac{1}{2M_{H_b}} \langle H_b | \mathcal{T} | H_b \rangle \quad \mathcal{T} = \text{Im } i \int d^4x \, T \{ H(x) H(0) \}$$

Non-local correlator !

- HQE exploits the hierarchy of scales $m_b \gg \Lambda_{\text{QCD}}$

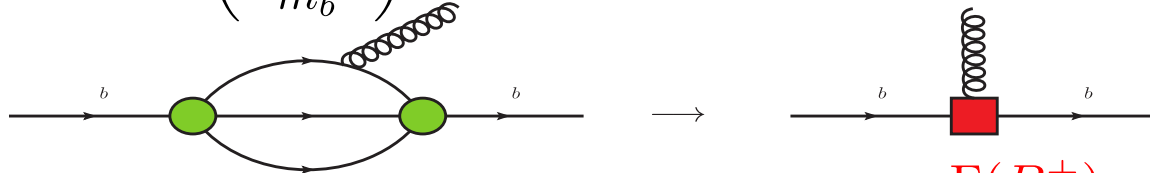
$$\mathcal{T} = \mathcal{T}^0 + \mathcal{T}^2 + \mathcal{T}^3 + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^4 \quad \text{where} \quad \mathcal{T}^i \sim 1/m_b^i$$

- Leading order $\Rightarrow$ Free b-quark decay $\rightarrow$ All lifetimes are the same



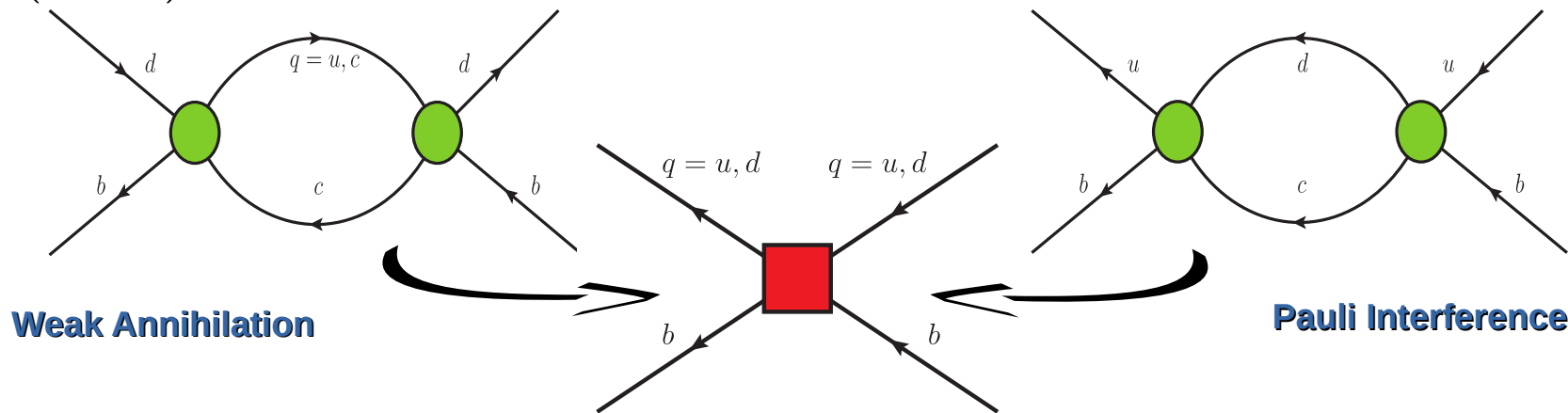
See [\[Egner, Fael, Schönwald, Steinhauser, 2024 \]](#)

- First corrections arise at $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2$ from chromomagnetic interactions



- Strong isospin $\Rightarrow$ Negligible contributions to $\tau(B^+)/\tau(B_d)$ $\frac{\Gamma(B^+)}{\Gamma(B_d^0)} \approx 1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^3$

- At $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^3 \Rightarrow$ weak interaction of b-quarks with (light) valence quarks



- We match the amplitudes onto a set of effective local $|\Delta B|=0$ operators

$$Q^q = (\bar{b} q)_{V-A} (\bar{q} b)_{V-A}$$

$$Q_S^q = (\bar{b} q)_{S-P} (\bar{q} b)_{S+P}$$

$$T^q = (\bar{b} T^a q)_{V-A} (\bar{q} T^a b)_{V-A}$$

$$T_S^q = (\bar{b} T^a q)_{S-P} (\bar{q} T^a b)_{S+P}$$

[Beneke, Buchalla, Greub, Lenz, Nierste, 2002]

- The decay rate is then decomposed as $\mathcal{T}^3 = \boxed{\mathcal{T}^u} + \boxed{\mathcal{T}^d}$

Weak Annihilation

Pauli Interference

$$\bullet \mathcal{T}^u = \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi} [|V_{ud}|^2 (F^u Q^d + F_S^u Q_S^d + G^u T^d + G_S^u T_S^d) \\ + |V_{cd}|^2 (F^c Q^d + F_S^c Q_S^d + G^c T^d + G_S^c T_S^d)]$$

$$\bullet \mathcal{T}^d = \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi} [(F^d Q^u + F_S^d Q_S^u + G^d T^u + G_S^d T_S^u)]$$

 Matching to $|\Delta B|=1$

- Hadronic matrix elements

$$\langle B_d^0 | Q^{u,d} | B_d^0 \rangle = \langle B^+ | Q^{d,u} | B^+ \rangle$$

$$\langle B^+ | (Q^u - Q^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 B_1(\mu_0)$$

$$\langle B^+ | (Q_S^u - Q_S^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 B_2(\mu_0)$$

$$\langle B^+ | (T^u - T^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 \epsilon_1(\mu_0)$$

$$\langle B^+ | (T_S^u - T_S^d)(\mu_0) | B^+ \rangle = f_B^2 M_B^2 \epsilon_2(\mu_0)$$

$|\Delta B| = 0$ renormalisation scale

- Non-perturbative evaluation of the bag parameters

Isospin-breaking combinations

Preliminary studies

Lattice QCD

HQET Sum Rules

[Di Pierro, Sachrajda, 1998]

[Di Pierro, Sachrajda, Michael, 1999]

[Becirevic, 2001]

[Lin, Detmold, Meinel, 2022]

[Black, Harlander, Lange, Rago, Shindler, Witzel, 2023]

[Black, Harlander, Lange, Rago, Shindler, Witzel, 2024]

[Black, Harlander, Kohnen, Lange, Rago, Shindler, Witzel, 2026]

[Kirk, Lenz and Rauh, 2017]

[Black, Lang, Lenz, Wüthrich, 2024] **We use this**

[King, Lenz, Rauh, 2021]

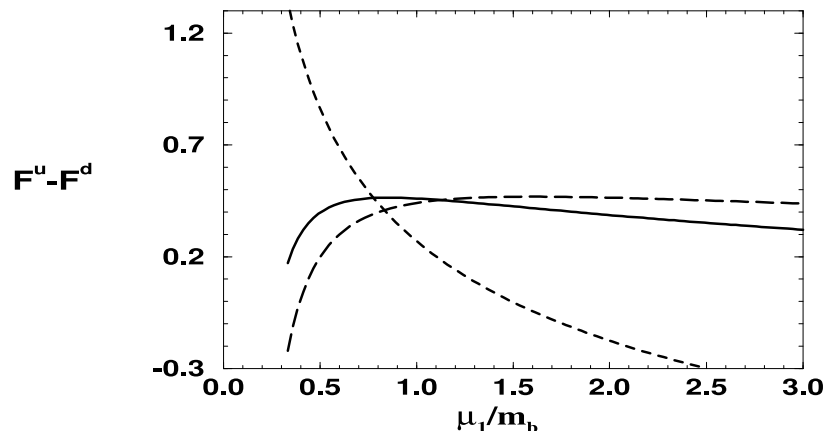
We use this for D mesons

- Next-to-Leading order calculation of the Wilson coefficients

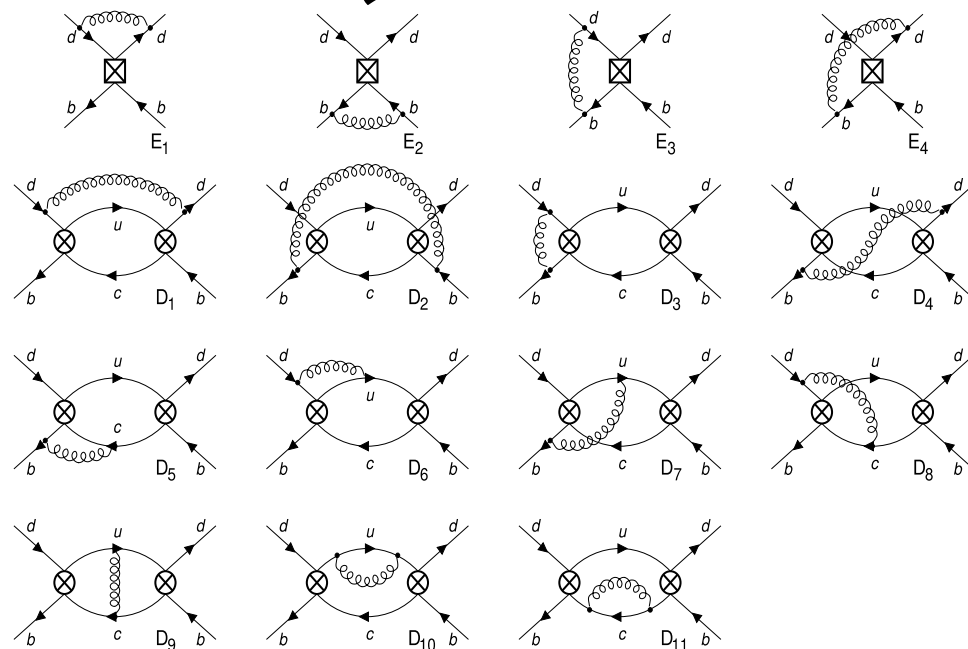
$$m_c = 0$$

- [Ciuchini, Franco, Lubicz, Mescia, 2001]
- [Franco, Lubicz, Mescia, Tarantino, 2002]
- [Beneke, Buchalla, Greub, Lenz, Nierste, 2002]

No contribution from $c\bar{c}$ intermediate states at NLO



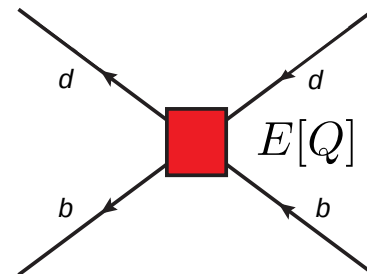
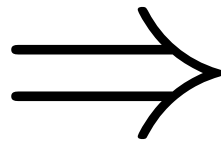
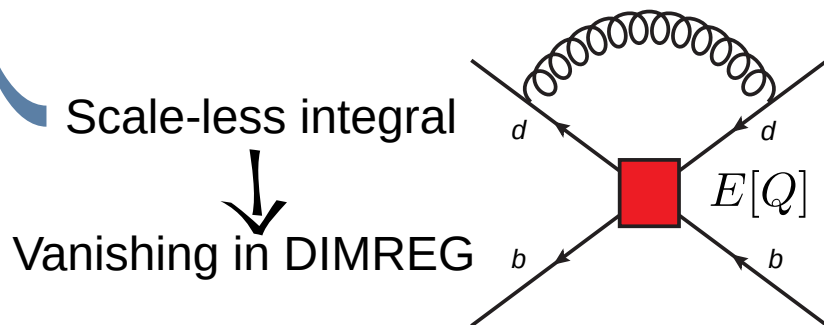
Feynman diagrams



Matching & Wilson Coefficients



- We perform the matching between the “full” $|\Delta B|=1$ theory and the effective $|\Delta B|=0$ theory in $d=4-2\epsilon$
- We set $p_{d,u}=0$ and $p_b^2=m_b^2 \Rightarrow$ We generate **IR spurious divergences**
- The IR behaviour of the two theories is the same $\Rightarrow$ cancellation of $1/\epsilon_{\text{IR}}$ poles
- Yet, spurious poles give finite contributions when they multiply $\mathcal{O}(\epsilon)$ parts of evanescent structures $\Rightarrow$ **Evanescent operators must be included in the matching**



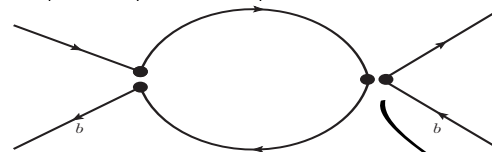
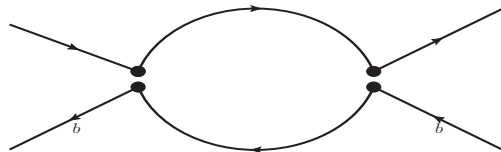
Uncanceled
counter-terms

$|\Delta B|=1$ Preliminaries



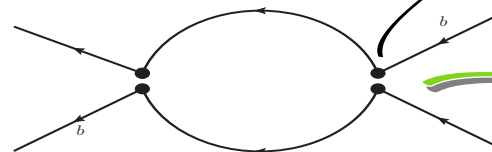
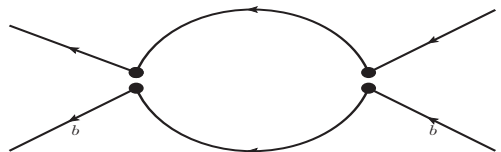
- Leading CKM contribution @ NNLO $\rightarrow |V_{cd}|=0$
- “Full theory” given by $|\Delta B|=1$ contributions $\Rightarrow$ “Historical” basis operators $Q_{1,2}$
- We can define $Q_1^{q_1 q_2 q_3} = (\bar{b}^i q_1^j)_{V-A} (\bar{q}_2^j q_3^i)_{V-A}$ $Q_2^{q_1 q_2 q_3} = (\bar{b}^i q_1^i)_{V-A} (\bar{q}_2^j q_3^j)_{V-A}$
- $\Gamma^{q_1 q_2 q_3} = \frac{1}{2M_{H_b}} \sum_{i,j=1,2} \left(\frac{G_F |\lambda_{q_1 q_2 q_3}|}{\sqrt{2}} \right)^2 C_i^\dagger C_j \text{Im } i \int d^4 x e^{iqx} \langle H_b | T \{ Q_i^{\dagger q_1 q_2 q_3}(x) Q_j^{q_1 q_2 q_3}(0) \} | H_b \rangle$
 $\lambda_{q_1 q_2 q_3} = V_{q_1 b}^* V_{q_2 q_3}$
- We want to match onto an operator basis with $(\bar{b} \cdots q) \otimes (\bar{q} \cdots b)$

WA



Fierzed operators

PI



More complicated to evaluate

- We actually compute the following amplitude

$$\tilde{\Gamma}^{q_1 q_2 q_3} = \frac{1}{2M_{H_b}} \sum_{i,j=1,2} \left(\frac{G_F |\lambda_{q_1 q_2 q_3}|}{\sqrt{2}} \right)^2 \tilde{C}_i^\dagger C_j \text{Im } i \int d^4 x e^{iqx} \langle H_b | T \{ Q_i^{\dagger q_3 q_2 q_1}(x) Q_j^{q_1 q_2 q_3}(0) \} | H_b \rangle$$

$\tilde{C}_i$ Wilson coefficient of $Q_i^{q_3 q_2 q_1}$

How do we recover the expression for $\Gamma^{q_1 q_2 q_3}$?

- In $d=4$ $Q_1^{q_1 q_2 q_3} = (\bar{b}^i q_1^j)_{V-A} (\bar{q}_2^j q_3^i)_{V-A} = (\bar{b}^i q_3^i)_{V-A} (\bar{q}_2^j q_1^j)_{V-A} = Q_2^{q_3 q_2 q_1}$
- $$\Gamma^{q_1 q_2 q_3} = \tilde{\Gamma}^{q_1 q_2 q_3} \Big|_{\tilde{C}_1 \rightarrow C_2, \tilde{C}_2 \rightarrow C_1}$$

- What about $d \neq 4$?

- Wilson coefficients $C_{1,2}$ mix upon renormalisation according to

$$\bullet C_j^b = Z_{ij} C_i \quad \bullet Z_{ij} = \delta_{ij} + \sum_{k=1}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^k Z_{ij}^{(k)} \quad \bullet Z_{ij}^{(k)} = \sum_{l=0}^k \frac{1}{\epsilon^l} Z_{ij}^{(k,l)}$$

- We define the anomalous dimension matrix (ADM)

$$\mu \frac{dC_i}{d\mu} = \gamma_{ji} C_j \quad \text{for } i,j=1,2 \quad \Longleftarrow \quad \gamma_{ij} = Z_{ik} \mu \frac{d}{d\mu} Z_{kj}^{-1}$$

- As shown in [\[Buras, Weisz, 1990\]](#), Fierz symmetry can be restored order by order in α_s by imposing

$$\gamma_{11} = \gamma_{22} \quad \& \quad \gamma_{12} = \gamma_{21}$$

- How do we fix these conditions?

- In DIMREG we have the presence of the so-called Evanescent operators
- Their $d \rightarrow 4$ limit is fixed, but they can be defined up to arbitrary choices of $\mathcal{O}(\epsilon)$ terms
- At NNLO, we encounter the following first and second generation evanescents

$$E^{(1)}[Q_1^{q_1 q_2 q_3}] = (\bar{b}^i \gamma^{\mu_1 \mu_2 \mu_3} P_L q_1^j) (\bar{q}_2^j \gamma_{\mu_1 \mu_2 \mu_3} P_L q_3^i) - (16 - 4\epsilon + A_2) Q_1^{q_1 q_2 q_3}$$

$$E^{(1)}[Q_2^{q_1 q_2 q_3}] = (\bar{b}^i \gamma^{\mu_1 \mu_2 \mu_3} P_L q_1^i) (\bar{q}_2^j \gamma_{\mu_1 \mu_2 \mu_3} P_L q_3^j) - (16 - 4\epsilon + A_2) Q_2^{q_1 q_2 q_3}$$

$$E^{(2)}[Q_1^{q_1 q_2 q_3}] = (\bar{b}^i \gamma^{\mu_1 \mu_2 \mu_3 \mu_4 \mu_5} P_L q_1^j) (\bar{q}_2^j \gamma_{\mu_1 \mu_2 \mu_3 \mu_4 \mu_5} P_L q_3^i) - (256 - 224\epsilon + B_1) Q_1^{q_1 q_2 q_3}$$

$$E^{(2)}[Q_2^{q_1 q_2 q_3}] = (\bar{b}^i \gamma^{\mu_1 \mu_2 \mu_3 \mu_4 \mu_5} P_L q_1^i) (\bar{q}_2^j \gamma_{\mu_1 \mu_2 \mu_3 \mu_4 \mu_5} P_L q_3^j) - (256 - 224\epsilon + B_2) Q_2^{q_1 q_2 q_3}$$

$$A_2 = -4 \quad B_1 = -\frac{45936}{115} \quad B_2 = -\frac{115056}{115} \quad [\text{Egner, Fael, Schönwald, Steinhauser, 2024}]$$