

Short-flow-time expansion of the CMDM operator

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Group: R. Harlander
22.09.2026
Aachen

Motivation

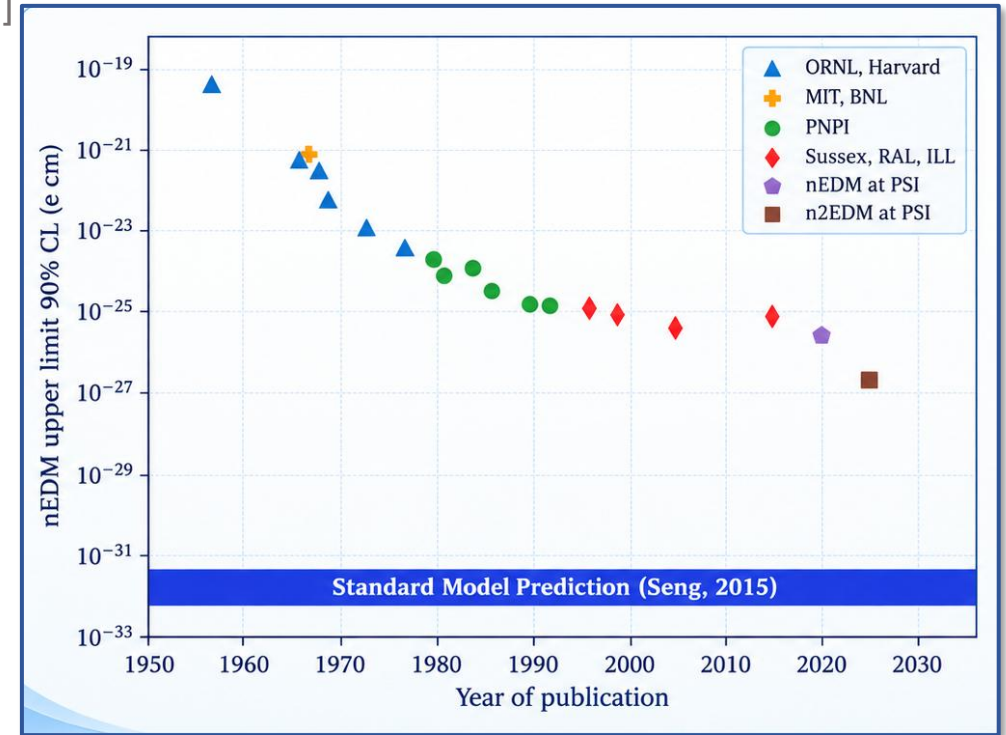
- Baryon asymmetry of the universe $\longrightarrow$ BSM CP-violation
[Sakharov '67]

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- Electric Dipole Moments (EDMs) are interesting probes

$$\mathcal{O}_{\text{EDM}} \sim (\bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi) F^{\mu\nu}$$



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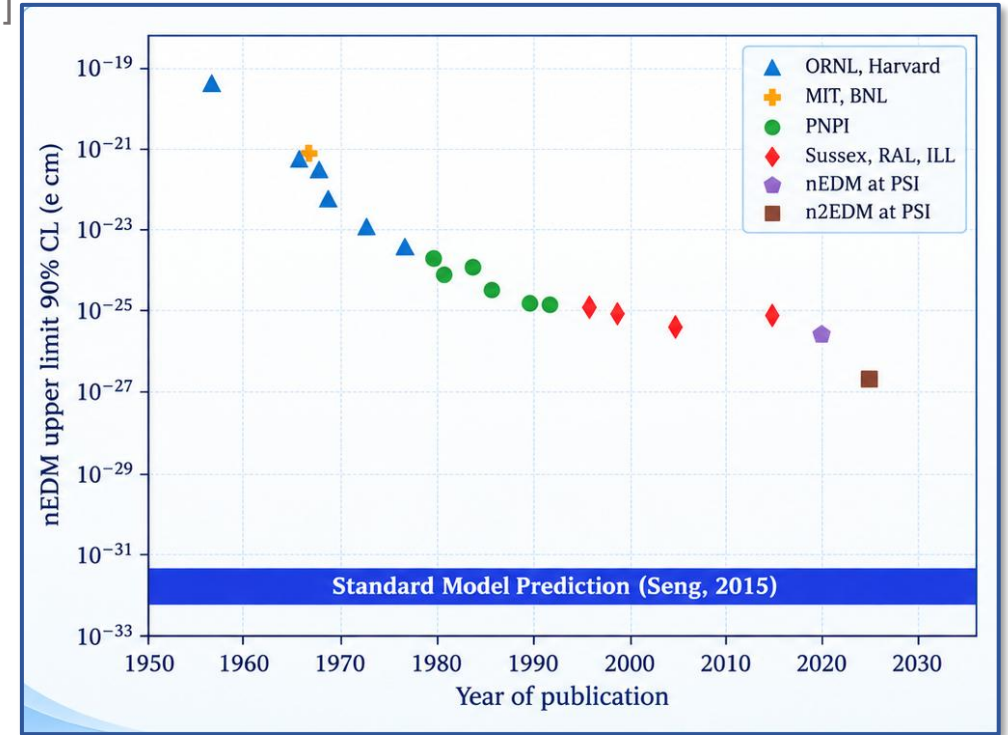
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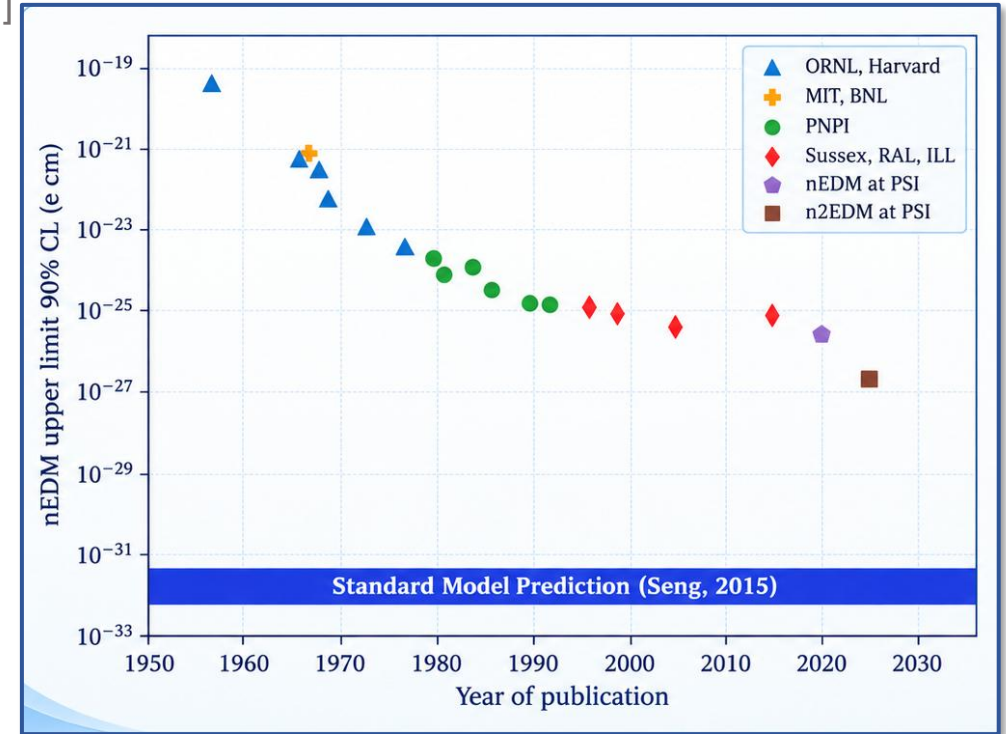
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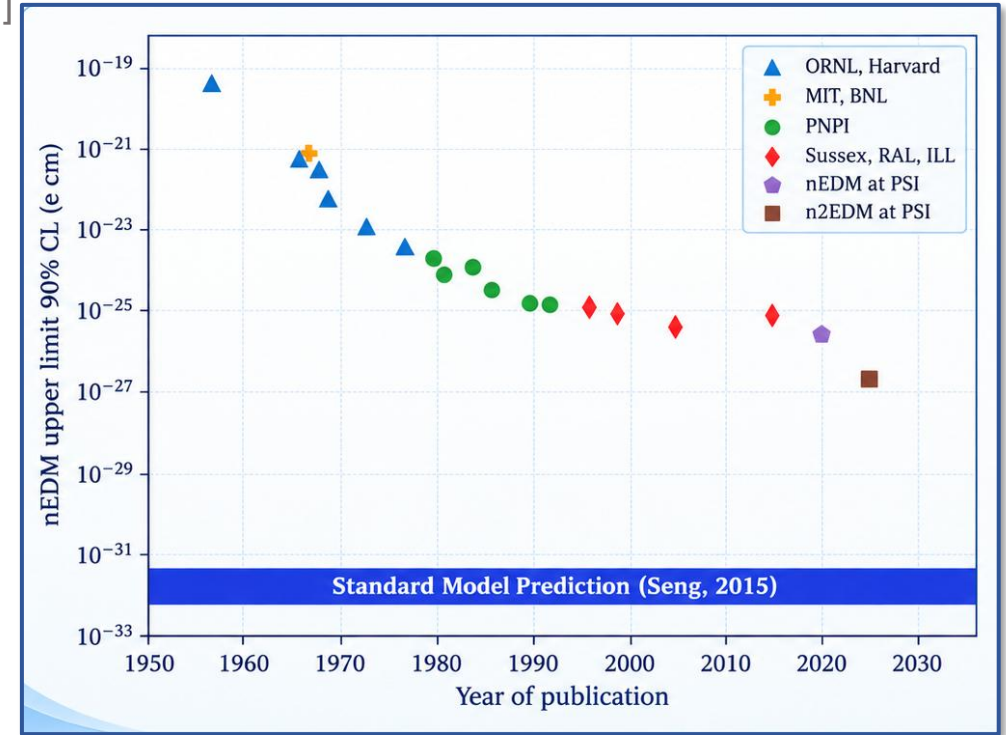
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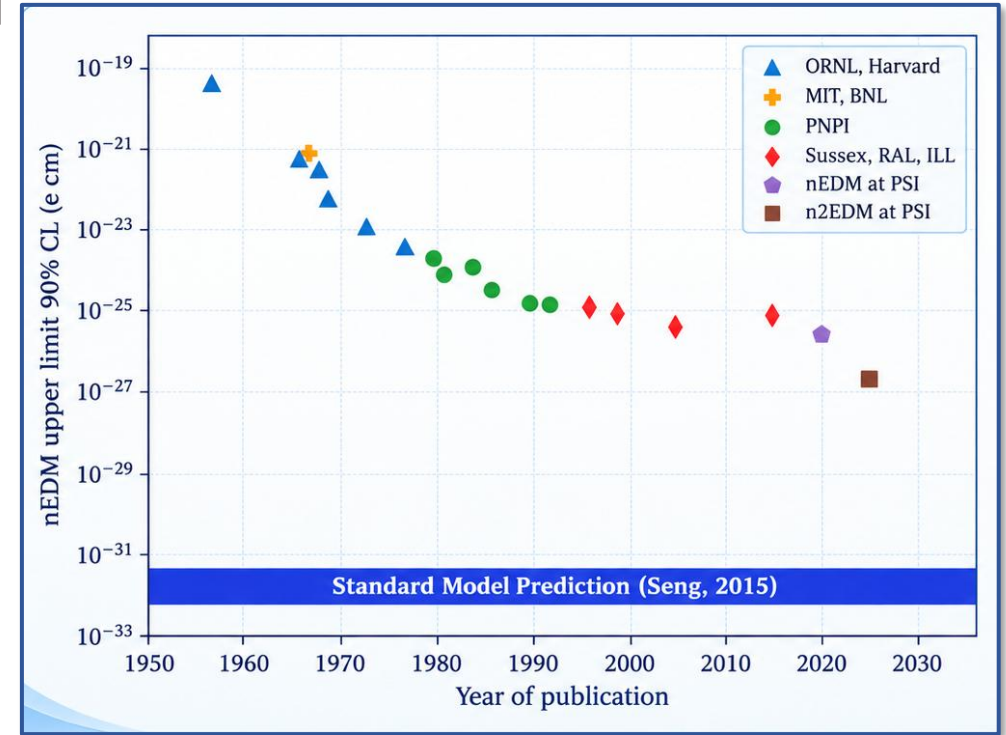
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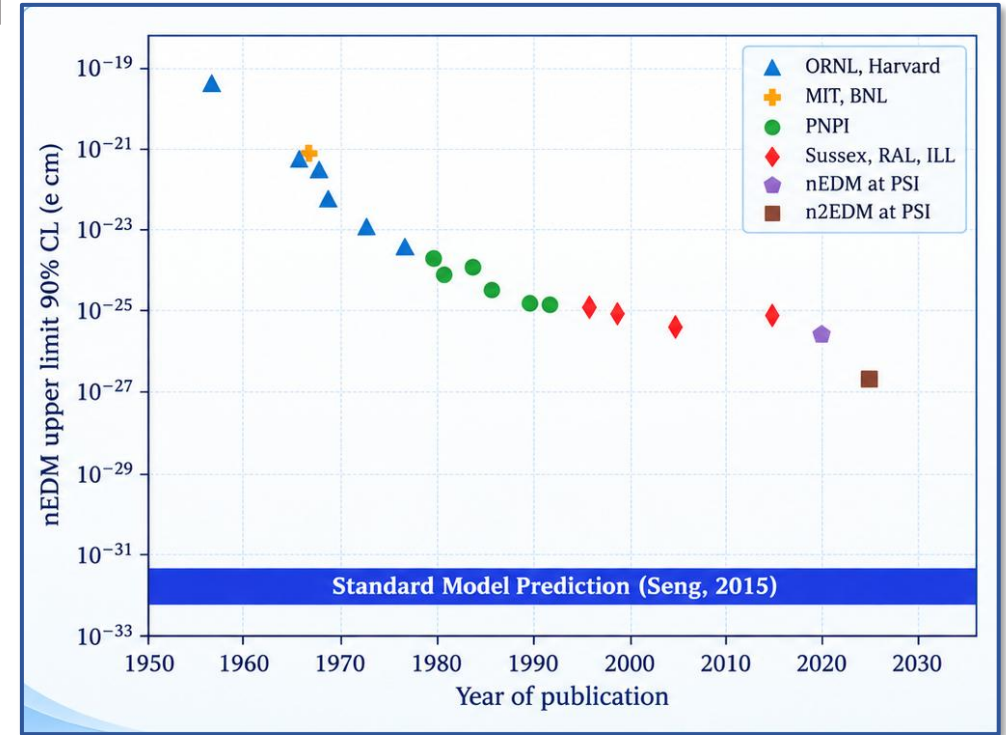
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chromo MDM



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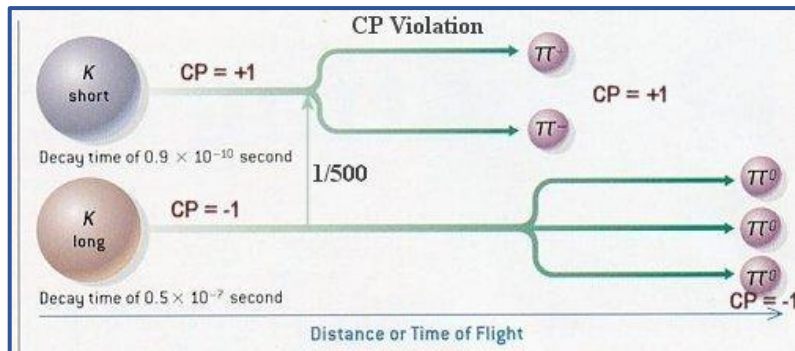
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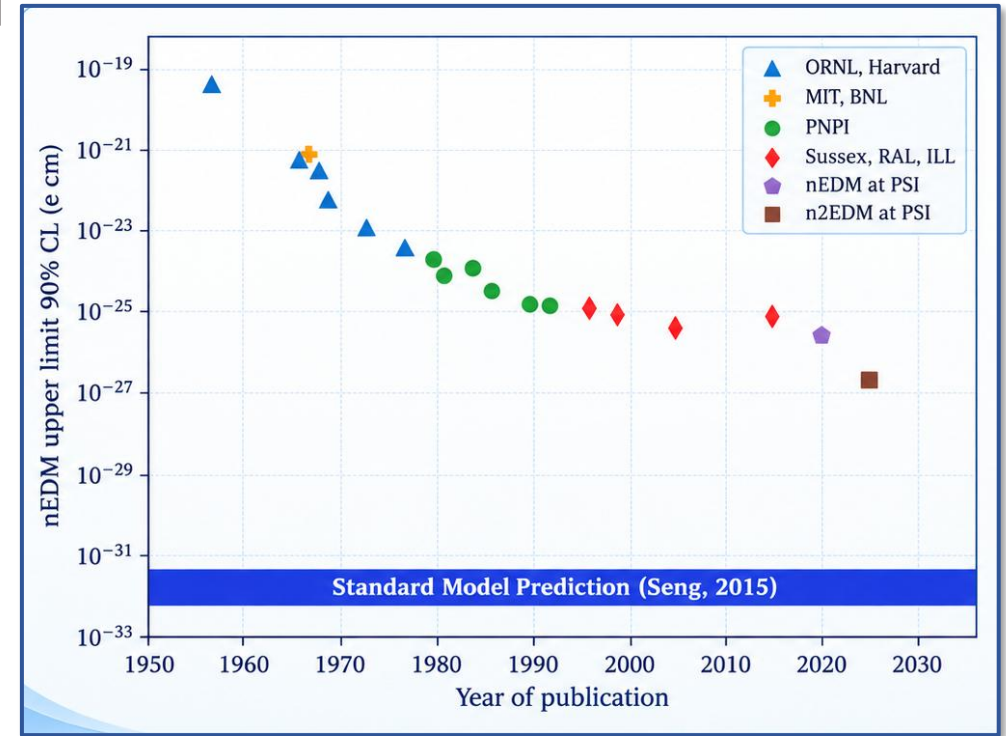
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[Paul '16]



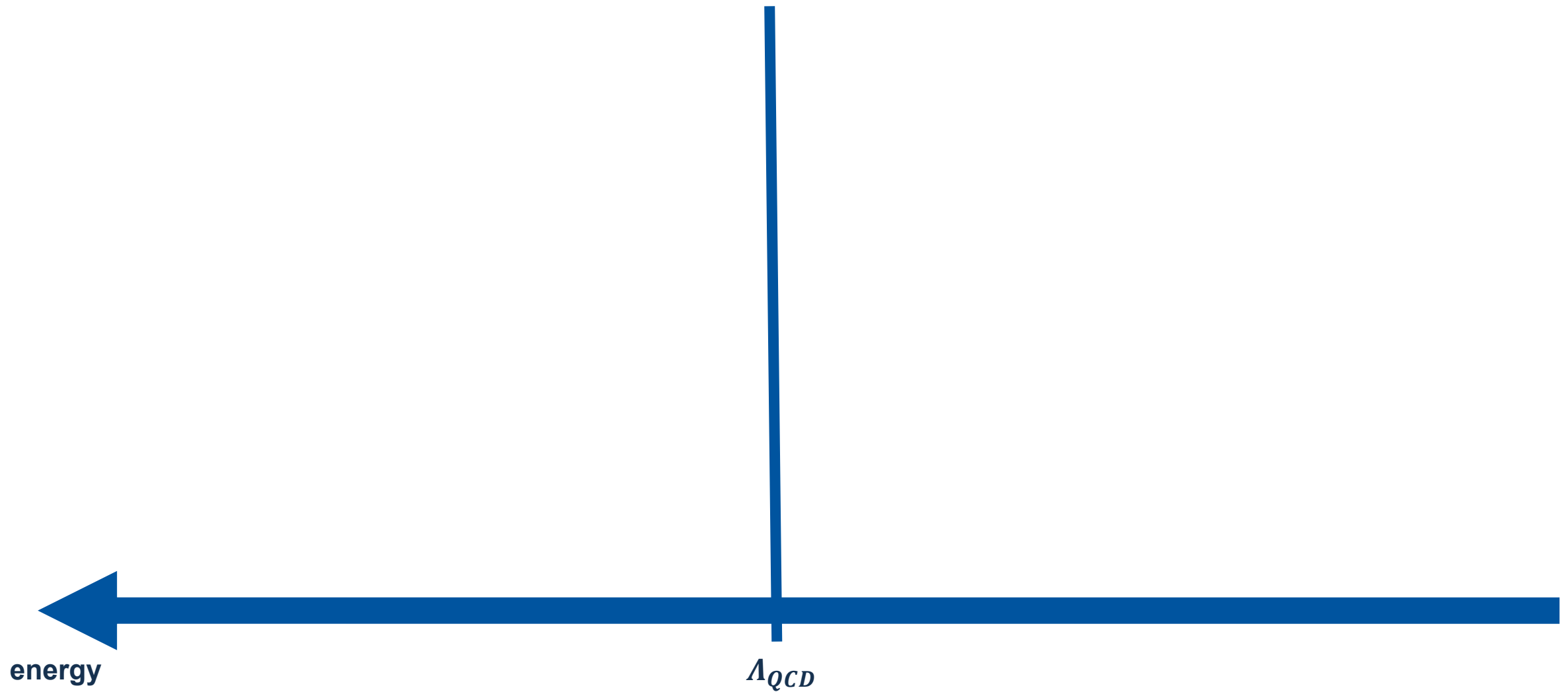
CMDM relevant on its own!!

- $\longrightarrow$ (In-)direct CP-violation through neutral Kaons
- $\longrightarrow$ SMEFT: top physics
- $\longrightarrow$ Effective electroweak hamiltonian
- $\dots$

chromo MDM

Motivation

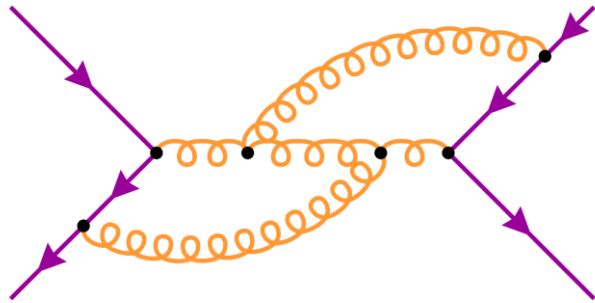
$$R = \sum_i C_i \langle \mathcal{O}_i \rangle$$



Motivation

$$R = \sum_i C_i \langle \mathcal{O}_i \rangle$$

perturbative



regulator:

deviation to 4D: $D = 4 - 2\epsilon$

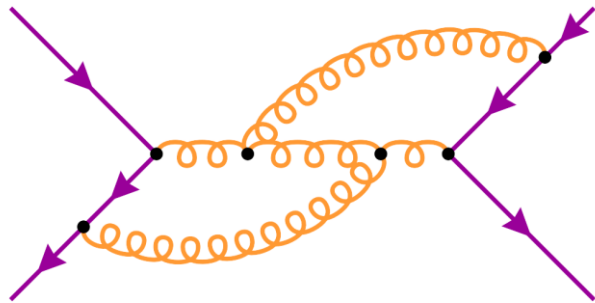
energy

Λ_{QCD}

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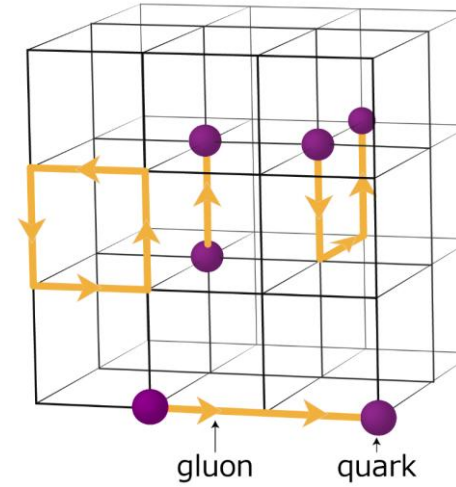
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lattice spacing: a

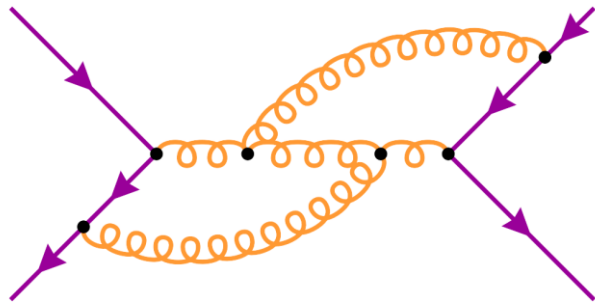
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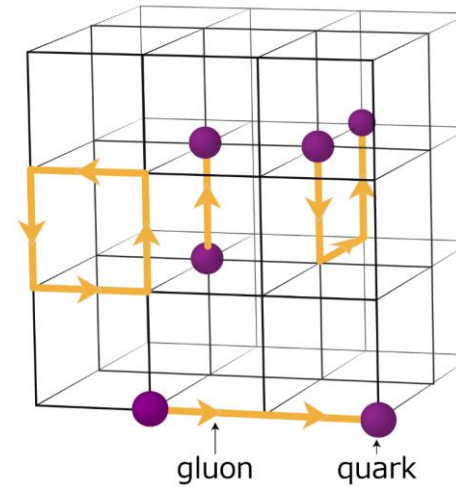
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$$\frac{1}{\epsilon} \longleftrightarrow \frac{1}{a}$$

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Gradient flow

[Lüscher '10; Lüscher, Weisz '11]

$$R = \sum_i C_i \langle \mathcal{O}_i \rangle = \sum_i \tilde{C}_i(t) \langle \tilde{\mathcal{O}}_i(t) \rangle$$

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non-perturbative

$$\begin{aligned} \frac{\partial}{\partial t} B_\mu(x, t) &= \mathcal{D}_\nu G_{\nu\mu}(x, t) \\ B_\mu(x, 0) &= A_\mu(x) \end{aligned}$$

$$\tilde{B}_\mu(p, t) \sim e^{-tp^2} \tilde{A}_\mu(p)$$

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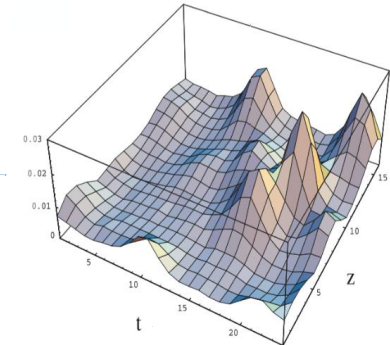
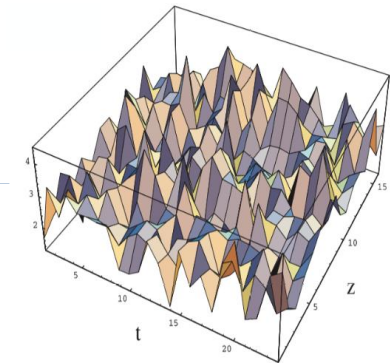
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$$\int \frac{d^D p}{(p^2)^n} = UV\text{-divergent}$$

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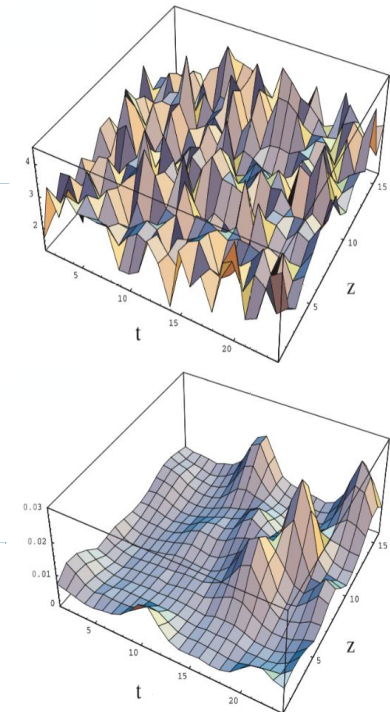
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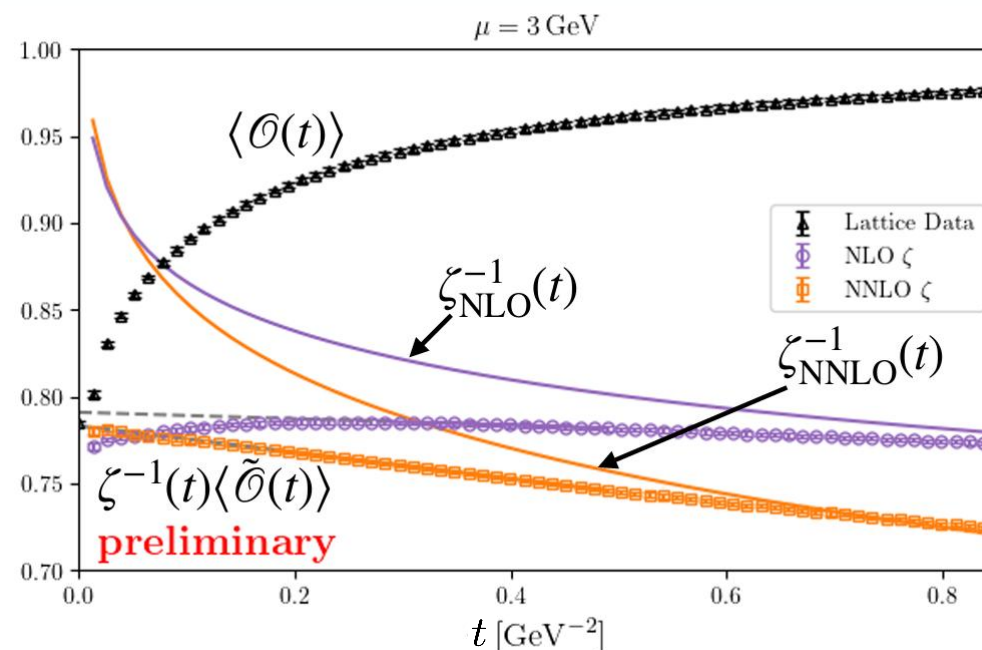
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Example: Heavy-Meson-Mixing



[Black et al. '26]

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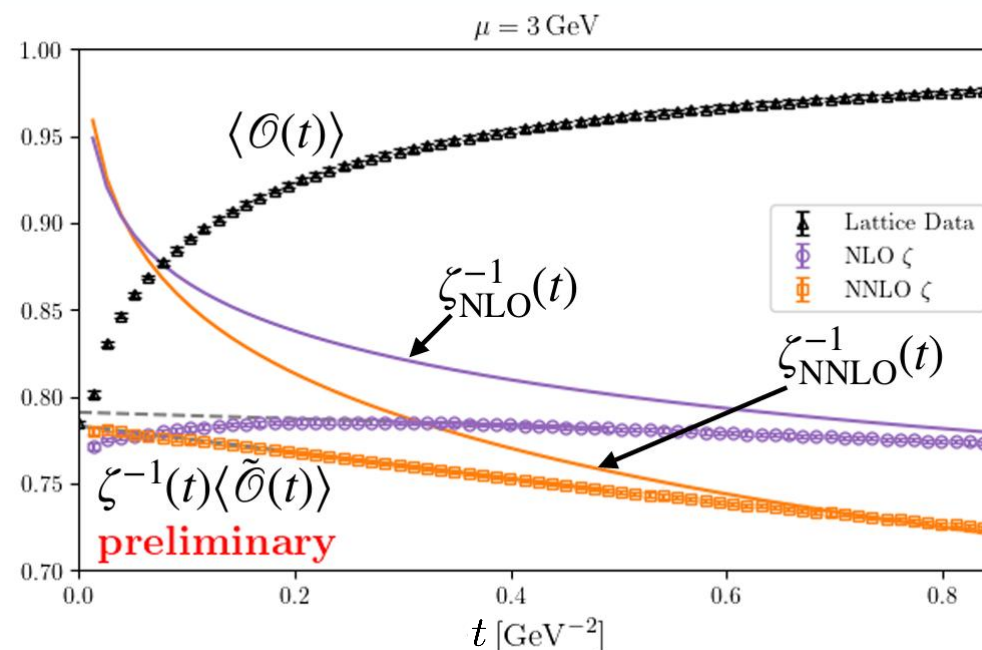
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How do we calculate the matching matrix?

Example: Heavy-Meson-Mixing



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Perturbative gradient flow

[Lüscher '10; Lüscher, Weisz '11]

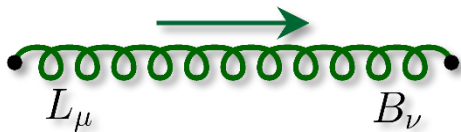
$$\mathcal{L}_{flow} = \mathcal{L}_{QCD} + \int dt \left[L_\mu \frac{\partial}{\partial t} B_\mu - L_\mu \mathcal{D}_\nu G_{\nu\mu} \right]$$

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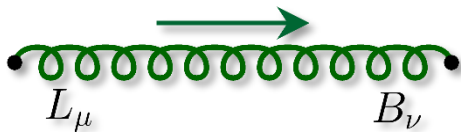
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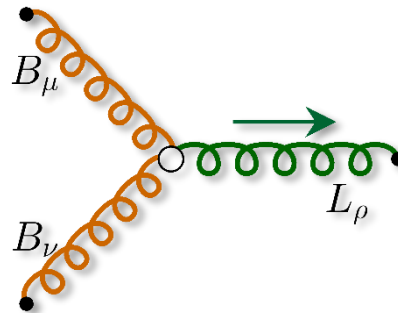
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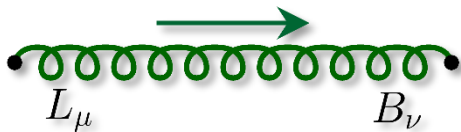
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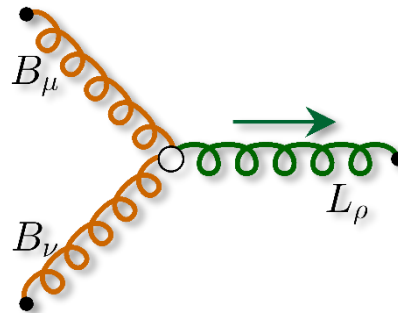
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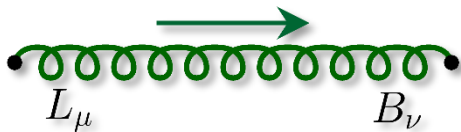
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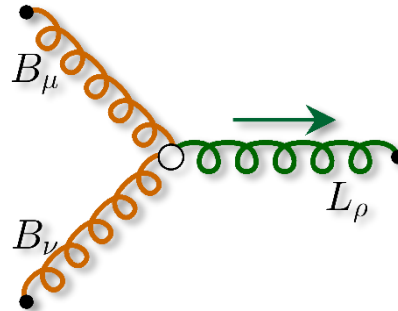
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+ QCD vertices + quarks + ...

Method of projectors

$$\tilde{\mathcal{O}}_i(t) = \sum_j \zeta_{ij}(t) \mathcal{O}_j$$

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- 4) **Normalize to LO**

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- 5) **Set all external scales to zero**

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Nuisance operators: $\tilde{\mathcal{O}}_i(t) = \sum_j \zeta_{ij}(t) \mathcal{O}_j + \sum_k \zeta_{ik}^{\mathcal{N}}(t) \mathcal{N}_k$

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$$P_{\text{CM}}[\dots] = \frac{1}{N} \frac{\partial}{\partial p_1^\mu} \left\langle \bar{\psi}_i \psi_j A_\nu^a(p_1) \left| \sigma_{\mu\nu} T_{ji}^a \dots \right| 0 \right\rangle_{p_i=0, m=0}^{\text{1PI, amp}}$$

Nuisance operators: $\tilde{\mathcal{O}}_i(t) = \sum_j \zeta_{ij}(t) \mathcal{O}_j + \sum_k \zeta_{ik}^{\mathcal{N}}(t) \mathcal{N}_k$

**Operators that vanish on-shell
(EOM, BRST - exact)**

Method of projectors

$$\tilde{\mathcal{O}}_i(t) = \sum_j \zeta_{ij}(t) \mathcal{O}_j$$

$$P_j[\mathcal{O}_i] = \delta_{ij} \quad \longrightarrow \quad P_j[\tilde{\mathcal{O}}_i(t)] = \zeta_{ij}(t)$$

Determine the projectors:

- 1) Choose one rule from the operator $\longrightarrow$ final states
- 2) Use Dirac/flavor structure from operator
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- 4) Normalize to LO
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QCD at Dimension 5

- QCD with n_f flavours: $\psi = (\psi_1, \dots, \psi_{n_f})$

[Bhattacharya et al. '16]

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$$\mathcal{E}_+[\Lambda] = \bar{\psi} \left[\{M, \Lambda\} \not{D} - \overleftarrow{\not{D}} \{M, \Lambda\} + \{M, \{M, \Lambda\}\} \right] \psi,$$

$$\mathcal{E}_-[\Lambda] = \bar{\psi} \left[[M, \Lambda] \not{D} + \overleftarrow{\not{D}} [M, \Lambda] - [M, [M, \Lambda]] \right] \psi,$$

$$\mathcal{E}_2[\Lambda] = \bar{\psi} \left[\Lambda \not{D}^2 + \overleftarrow{\not{D}}^2 \Lambda - \{M^2, \Lambda\} \right] \psi,$$

$$\mathcal{E}_3[\Lambda] = \bar{\psi} \left[\overleftarrow{\not{D}} \Lambda \not{D} + M \Lambda M \right] \psi,$$

$$\mathcal{N}_1[\Lambda] = \partial^2 (\bar{\psi} \Lambda \psi),$$

$$\mathcal{N}_2[\Lambda] = g \bar{\psi} \left[\Lambda \not{A} \not{D} - \overleftarrow{\not{D}} \not{A} \Lambda + \{M, \Lambda\} \not{A} \right] \psi.$$

Renormalization

$$\int \frac{d^D p}{(p)^n} \sim \frac{1}{\epsilon_{\text{UV}}} + \frac{1}{\epsilon_{\text{IR}}} = 0$$

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$$\left. \begin{aligned} \int \frac{d^D p}{(p)^n} &\sim \frac{1}{\epsilon_{\text{UV}}} + \frac{1}{\epsilon_{\text{IR}}} = 0 \\ \zeta_{ij}(t) &\sim \int \frac{d^D p}{(p)^n} e^{-tp^2} \sim \frac{1}{\epsilon_{\text{IR}}} = -\frac{1}{\epsilon_{\text{UV}}} \end{aligned} \right\} \begin{aligned} P_j[\tilde{\mathcal{O}}_i(t)] &= \zeta_{ij}^{\text{B}}(t, \epsilon) \\ \longrightarrow \zeta_{ij}^{\text{R}}(t) &= \sum_k \zeta_{ik}^{\text{B}}(t, \epsilon) [Z^{-1}(\epsilon)]_{kj} \end{aligned}$$

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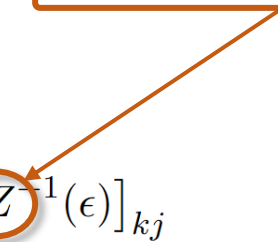


$$P_j[\tilde{\mathcal{O}}_i(t)] = \zeta_{ij}^B(t, \epsilon)$$

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Renormalization matrix of unflowed theory:

$$\mathcal{O}^R = Z \mathcal{O}^B$$



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$$\tilde{\mathcal{O}}_1[\Lambda] = \sum_{j=1}^{13} \zeta_{1j}^{\text{R}}(t) \mathcal{O}_j[\Lambda]$$

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Renormalization

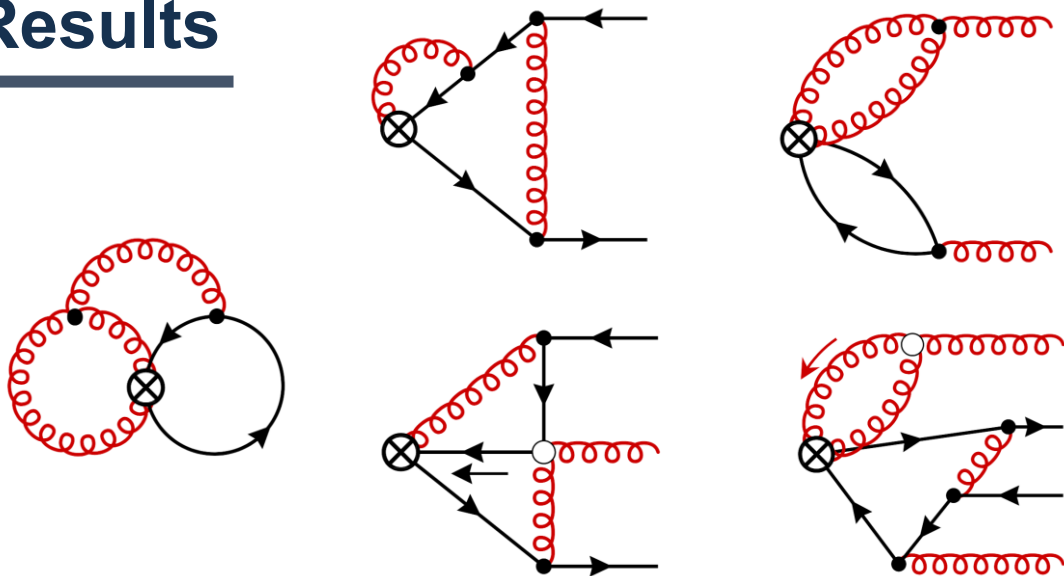
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Computational Setup:
 Qgraf $\longrightarrow$ tapir/exp $\longrightarrow$ FORM $\longrightarrow$ Integral Reduction

See Robert's talk

Results



$$\tilde{\mathcal{O}}_1[\Lambda] = \sum_{j=1}^{13} \zeta_{1j}^{\text{R}}(\textcolor{brown}{t}) \mathcal{O}_j[\Lambda]$$

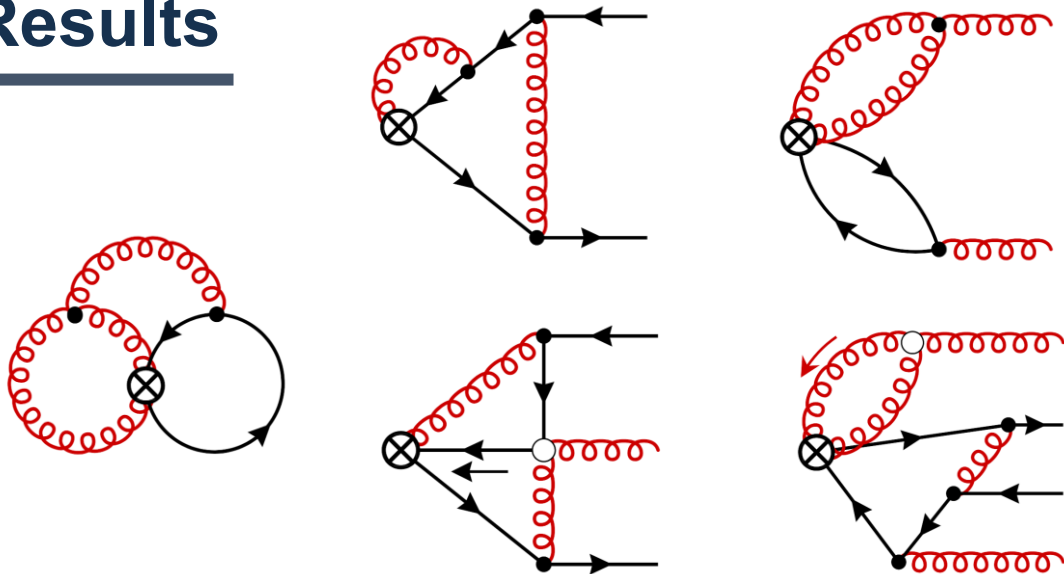
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External legs	# Diagrams	
	$\tilde{\mathcal{O}}_1(t)$	$\tilde{\mathcal{O}}_5(t)$
$\emptyset$	157 (24)	258 (48)
$\bar{\psi}, \psi$	222 (30)	375 (60)
A, A	497 (0)	772 (0)
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$$\begin{aligned} \zeta_{12}^{\text{R}} = & -a_s n_f T_{\text{R}} \left\{ 1 + L_{\mu t} + a_s \left[C_A \left(\frac{35}{48} + \frac{1}{8} \zeta_2 + 6 \ln 2 - \frac{13}{4} \ln 3 + \text{Li}_2 \left(\frac{1}{4} \right) \right) \right. \right. \\ & + C_F \left(\frac{101}{24} + \frac{9}{4} \ln 2 - \frac{39}{8} \ln 3 - \frac{9}{4} \text{Li}_2 \left(\frac{1}{4} \right) \right) \\ & \left. \left. + L_{\mu t} \left[\frac{3}{4} C_A + C_F \left(\frac{15}{8} - \ln 2 - \frac{3}{4} \ln 3 \right) \right] + L_{\mu t}^2 \left(-\frac{1}{4} C_A + C_F \right) \right] \right\} \end{aligned}$$

$$a_s \equiv \frac{\alpha_s}{\pi} = \frac{g^2}{4\pi^2}$$

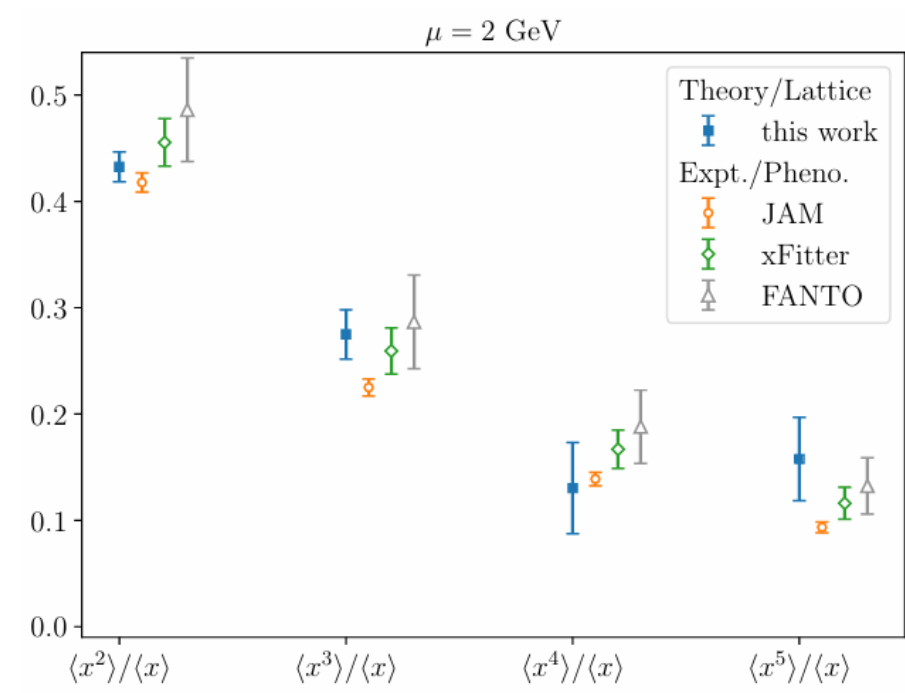
$$L_{\mu t} \equiv \log \left(\frac{\mu^2}{\mu_t^2} \right), \quad \text{with} \quad \mu_t^2 = \frac{e^{-\gamma_E}}{2t}$$

Conclusion and outlook

- Gradient flow $\longrightarrow$ powerful tool to combine lattice and continuum physics

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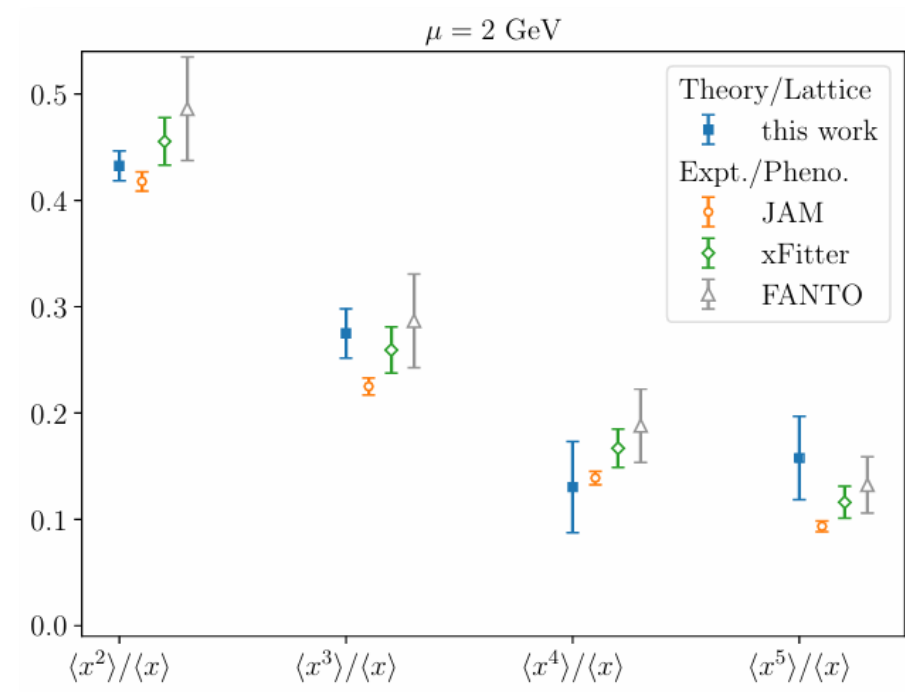
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[Francis et al. '25]

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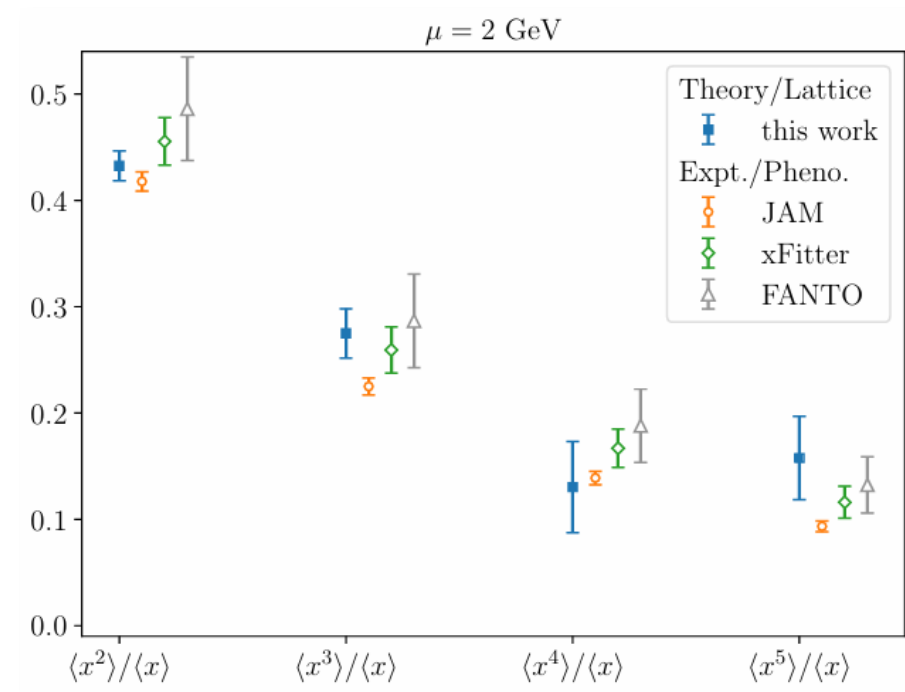
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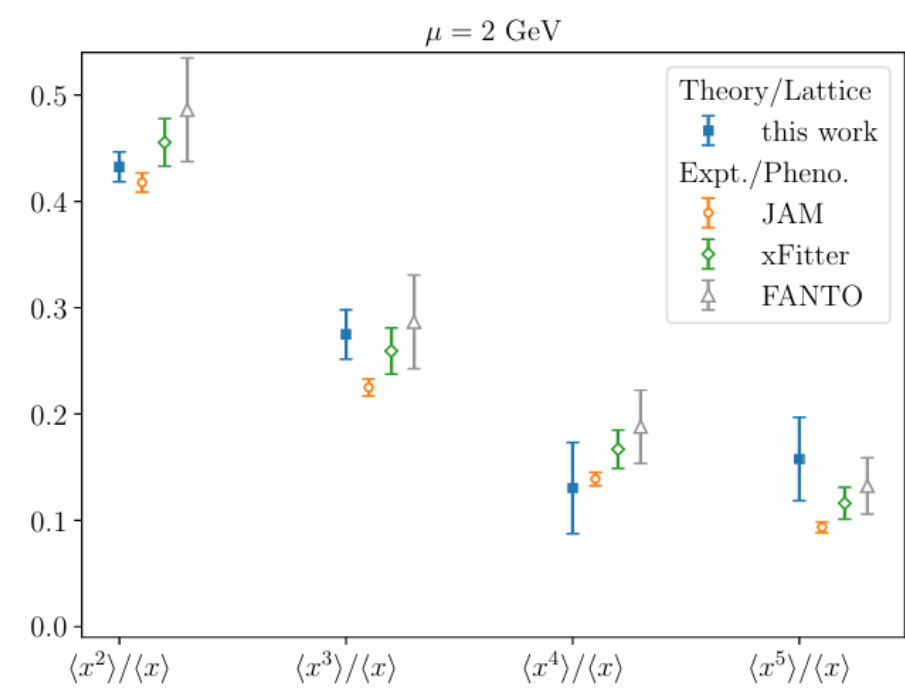
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 - ...
- CP-even dimension-5 QCD at NNLO in the gradient flow done!
 - $\longrightarrow$ flowed anomalous dimension



[Francis et al. '25]

Conclusion and outlook

- Gradient flow $\longrightarrow$ powerful tool to combine lattice and continuum physics
 - $\longrightarrow$ previously unreachable first principle calculations possible (PDFs)
 - $\longrightarrow$ unmatched precision (Heavy-Meson-Mixing)
 - ...
- CP-even dimension-5 QCD at NNLO in the gradient flow done!
 - $\longrightarrow$ flowed anomalous dimension
- CP-odd sector is coming $\longrightarrow$ stay tuned!



[Francis et al. '25]