Collaborative Research Center TRR 257

P H

Particle Physics Phenomenology after the Higgs Discovery

Research Training Group
Physics of the Heaviest
Particles at the LHC

RWTH AACHEN UNIVERSITY

# Complete NLO corrections to full off-shell $t\bar{t}W^+$ production

## The same-sign dilepton channel


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With funding from the:

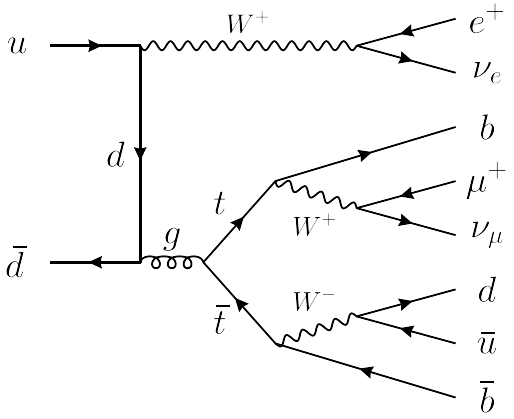
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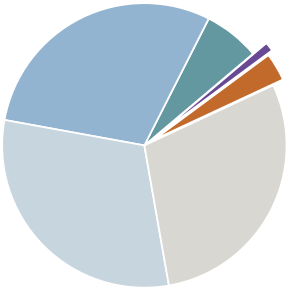
# Why study the same-sign dilepton channel?




- Two prompt leptons of the same sign suppress the large opposite-sign $t\bar{t}$ background.
- Majorana particles can produce same-sign leptons: examples include supersymmetric gluinos and heavy Majorana neutrinos.
- $t\bar{t}W$ is an important **SM background** in these searches and in multilepton measurements of $t\bar{t}H$ and $t\bar{t}t\bar{t}$.

Barnett–Gunion–Haber, PLB 315 (1993) 349; Almeida et al., PLB 400 (1997) 331.

# Why study the same-sign dilepton channel?



$t\bar{t}W^+ : \quad W^+W^+W^-$

| | |
|---|---|
| Same-sign dileptons | $3.18\%$ |
| Three leptons | $1.03\%$ |
| Opposite-sign dileptons | $6.36\%$ |
| One lepton | $29.62\%$ |
| Fully hadronic | $30.63\%$ |
| At least one $W \to \tau\nu$ | $29.18\%$ |

$$\frac{\mathcal{B}(2\ell_{\mathrm{SS}})}{\mathcal{B}(3\ell)} \simeq 3.1$$

More signal events than in $3\ell$, while retaining the distinctive same-sign signature.

**The hadronic $W$ decay adds two jets: jet reconstruction and QCD radiation become central.**

Direct $\ell = e, \mu$, before acceptance; $\tau$ decays are a separate category. Chart calculated from PDG 2025 $W$ branching fractions.

# Experimental viewpoint

Inclusive $pp \to t\bar{t}W^+ + t\bar{t}W^-$ at $13\,\mathrm{TeV}$

| | Cross section [fb] |
|---|---|
| NNLO QCD + EW reference | $745 \pm 50_{\mathrm{scale}} \pm 13_{\text{2-loop}} \pm 19_{\mathrm{PDF},\alpha_s}$ |
| ATLAS | $880 \pm 80$ |
| CMS | $938 \pm 45_{\mathrm{stat}} \pm 52_{\mathrm{syst}}$ |

ATLAS: $1.4\sigma$; CMS: $\simeq 2.2\sigma$ above the reference prediction.

CMS estimate: experimental and reference uncertainties combined in quadrature.

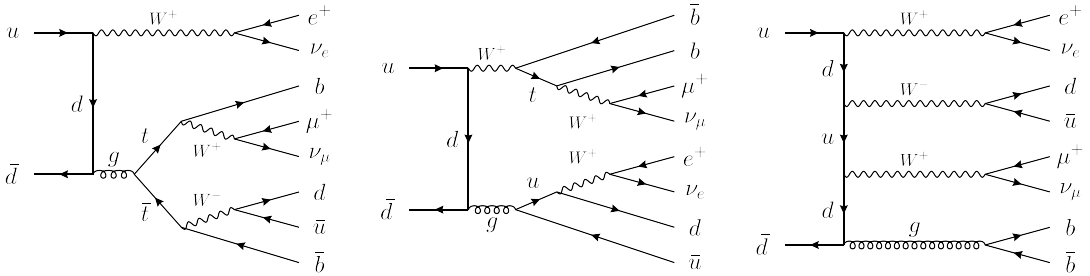
**We supply precise and accurate differential distributions.**

Complete NLO predictions for the full final state under fiducial cuts, resolving changes in shape and the tails of dimensionful distributions.

Measurements: ATLAS, JHEP 05 (2024) 131; CMS, JHEP 03 (2026) 083.
Reference used in the comparisons: Buonocore et al., PRL 131 (2023) 231901.

# Precision predictions for an experimentally important signature

$$pp \longrightarrow e^+\nu_e\,\mu^+\nu_\mu\,b\bar{b}\,jj + X$$




Two top resonances | One top resonance | No top resonance

Entering the precision era of physics demands an accurate description of the measured final state.

Interference, spin correlations and Breit–Wigner effects are included.

**Full off-shell:** especially important near kinematical edges and in BSM-sensitive high-$p_T$ tails, increasingly probed at the LHC and HL-LHC.

**Complete NLO:** all possible Born orders and all their NLO QCD and EW corrections.

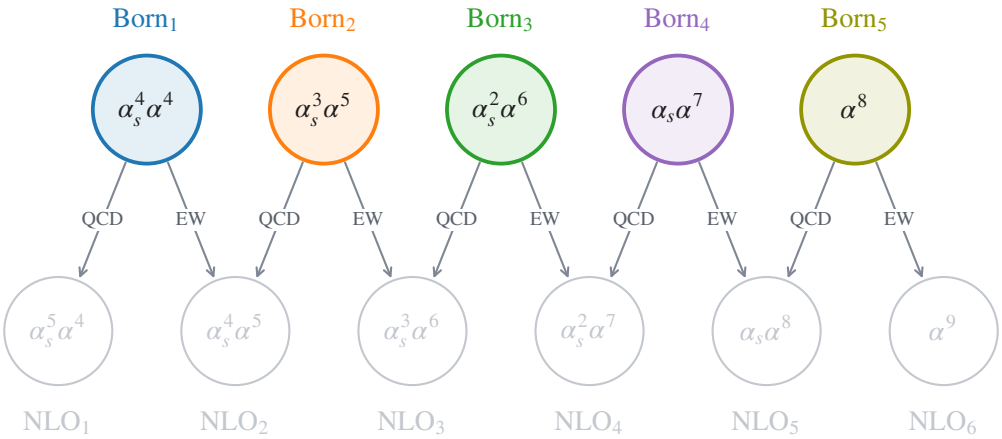
# Theory status

| Description | Accuracy | References |
|---|---|---|
| Production / NWA decays | NLO QCD | Campbell–Ellis, JHEP 07 (2012) 052 |
| Stable $t\bar{t}W$ | Complete NLO | Frederix–Pagani–Zaro, JHEP 02 (2018) 031 |
| $t\bar{t}W + 0, 1, 2$ jets | NLO QCD + parton shower | Frederix–Tsinikos, JHEP 11 (2021) 029 |
| Stable $t\bar{t}W$ | Approximate NNLO QCD + EW | Buonocore et al., PRL 131 (2023) 231901 |
| Stable $t\bar{t}W$ | NNLO QCD | Becchetti et al., arXiv:2606.09503 (2026) |
| Full off-shell $3\ell$ | NLO QCD | Bevilacqua et al., JHEP 08 (2020) 043<br>Denner–Pelliccioli, JHEP 11 (2020) 069 |
| Full off-shell $3\ell$ | Combined NLO QCD + EW | Denner–Pelliccioli, EPJC 81 (2021) 354 |
| **Full off-shell $2\ell_{\mathrm{SS}}$** | **Complete NLO** | **This calculation: all Born and NLO orders** |

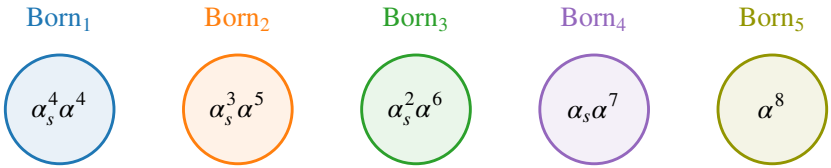
**New for $2\ell_{\mathrm{SS}}$: even the full off-shell NLO QCD calculation was previously unavailable.**

NNLO two-loop input: approximate finite term (2023); generalised leading-colour limit (2026).

# Complete NLO: the Born orders



# Born contributions: resonances and interference




**Born$_1$: QCD background**

No resonant top quarks; at most two resonant $W$ bosons.

**Born$_3$: dominant signal**

QCD production with two top and three $W$ resonances gives resonant enhancement.

**Born$_5$: purely EW**

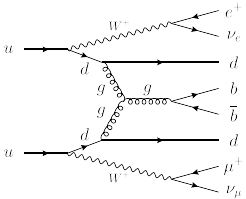
Two top resonances are possible, but the contribution is suppressed by $\alpha \ll \alpha_s$.

**Pure interferences:** Born$_2$ combines the amplitudes of Born$_1$ and Born$_3$;
Born$_4$ combines the amplitudes of Born$_3$ and Born$_5$.

Born$_3$ also receives interference between the highest-QCD and pure-EW amplitudes.

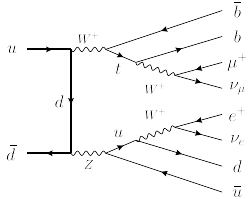
# Example diagrams at the Born level

$\text{Born}_1$ : $\mathcal{O}(\alpha_s^4\alpha^4)$



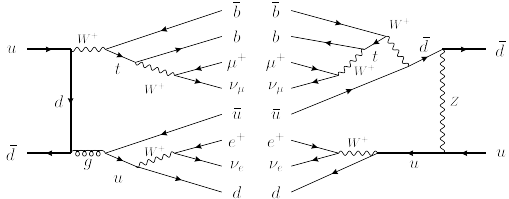
$\mathcal{M}^{(0)} \sim g_s^4 g^4$

$\text{Born}_5$ : $\mathcal{O}(\alpha^8)$



$\mathcal{M}^{(0)} \sim g^8$

$\text{Born}_4$ : $\mathcal{O}(\alpha_s\alpha^7)$



$\mathcal{M}_L^{(0)} \sim g_s^2 g^6$ $\quad$ $\mathcal{M}_R^{(0)} \sim g^8$

Crossed channel in $\text{Born}_1$:

$$uu \longrightarrow e^+\nu_e\,\mu^+\nu_\mu\,b\bar{b}\,dd$$

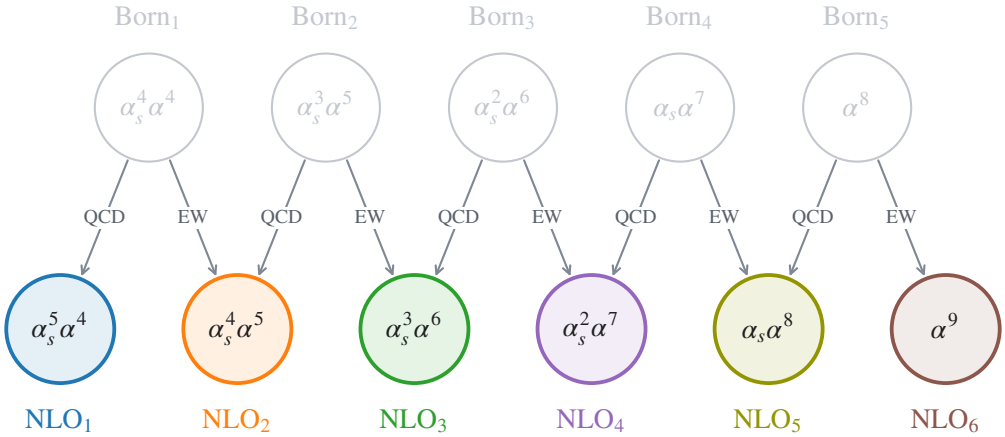
All possible light-quark crossings of

$$u\bar{d} \longrightarrow e^+\nu_e\,\mu^+\nu_\mu\,b\bar{b}\,\bar{u}d,$$
$$u\bar{d} \longrightarrow e^+\nu_e\,\mu^+\nu_\mu\,b\bar{b}\,\bar{c}s.$$

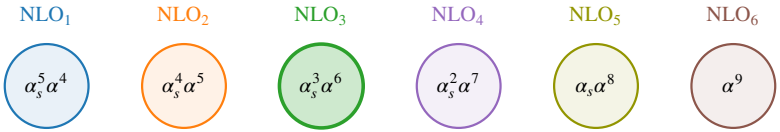
Interference contribution:

$$2\,\mathrm{Re}\Big[\mathcal{M}_L^{(0)}\mathcal{M}_R^{(0)*}\Big] \subset \text{Born}_4$$

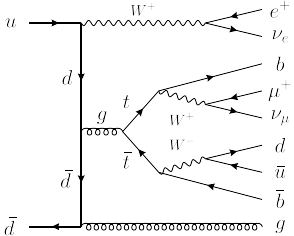
# Complete NLO: the NLO orders



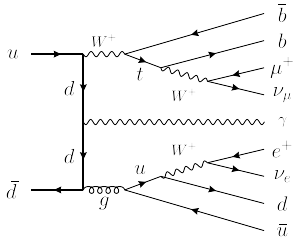
# NLO contributions: real radiation and interference




$\mathrm{NLO}_3$ includes the standard NLO QCD corrections to the dominant $\mathrm{Born}_3$ contribution.




$|\mathcal{M}_g|^2 \sim \alpha_s^3\alpha^6 \qquad (\mathrm{NLO}_3)$




$|\mathcal{M}_\gamma|^2 \sim \alpha_s^2\alpha^7 \qquad (\mathrm{NLO}_4)$

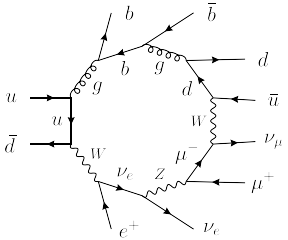
Real radiation includes gluons, light quarks, and photons.

Real-emission amplitudes with the same external particles can interfere across coupling orders.

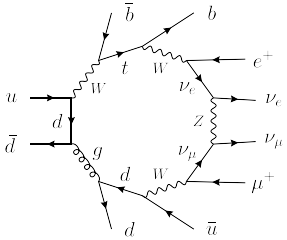
# NLO contribution of one-loop amplitudes

Loop–Born interference gives a diverse pattern of contributions across the NLO orders.

| Loop × Born | $g_s^4 g^4$ | $g_s^2 g^6$ | $g^8$ |
|---|---|---|---|
| $g_s^6 g^4$ | $\mathrm{NLO}_1$ | $\mathrm{NLO}_2$ | $\mathrm{NLO}_3$ |
| $g_s^4 g^6$ | $\mathrm{NLO}_2$ | $\mathrm{NLO}_3$ | $\mathrm{NLO}_4$ |
| $g_s^2 g^8$ | $\mathrm{NLO}_3$ | $\mathrm{NLO}_4$ | $\mathrm{NLO}_5$ |
| $g^{10}$ | $\mathrm{NLO}_4$ | $\mathrm{NLO}_5$ | $\mathrm{NLO}_6$ |



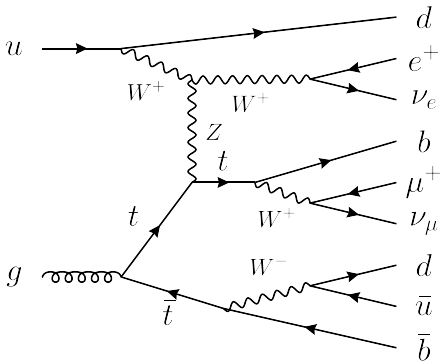

$\mathcal{M}^{(1)} \sim g_s^4 g^6$




$\mathcal{M}^{(1)} \sim g_s^2 g^8$

**Tensor integrals up to 10 points and rank 5 enter the calculation.**

# Two mechanisms behind large subleading corrections




$\mathrm{NLO}_5: \ \mathcal{O}(\alpha_s\alpha^8)$

$tW$ scattering: Frederix–Pagani–Zaro, JHEP 02 (2018) 031.
EW logarithms: Denner–Pozzorini, EPJC 18 (2001) 461.

## Real-emission enhancement

$$tW^+ \longrightarrow tW^+$$

Scattering topologies and large quark–gluon luminosities enhance $\mathrm{NLO}_5$.

## EW Sudakov logarithms

$$-\frac{\alpha}{4\pi s_W^2}\log^2\left(\frac{Q^2}{M_W^2}\right)$$

Massive $W$ and $Z$ bosons in loops generate negative logarithmic corrections to $\mathrm{NLO}_4$, enhanced at large $Q$.

# Computational framework

Full off-shell matrix elements in the complex-mass scheme

**RECOLA**

Born and real-emission matrix elements
One-loop amplitudes and tensor coefficients
COLLIER: tensor integrals
CutTools + OneLOop: OPP alternative

**HELAC-Dipoles**

QCD and QED infrared subtraction
Nagy–Soper subtraction for QCD radiation
QED dipoles for photon radiation
Local and integrated subtraction terms

**Phase-space integration:** KALEU + PARNI
Adaptive multi-channel integration and importance sampling

**Integrated cross sections and differential distributions**

RECOLA: Actis et al., CPC 214 (2017) 140; COLLIER: Denner et al., CPC 212 (2017) 220.
HELAC-Dipoles: Czakon et al., JHEP 08 (2009) 085; Nagy–Soper: Bevilacqua et al., JHEP 10 (2013) 204.
QED extension: Stremmer–Worek, JHEP 07 (2024) 091.

# Fiducial cross sections: every contribution

At least one light-jet pair with $|M_{jj} - M_W| < Q_{\text{cut}} = 20\,\text{GeV}$.

$\sigma_i/\sigma_{\text{Born}_3}$ [%], $\mu_0 = H_T/3$

| Contribution | With $Q_{\text{cut}}$ | Without |
|---|---|---|
| $\text{Born}_1$ | $+0.009$ | $+0.279$ |
| $\text{Born}_2$ | $+0.00010$ | $+0.045$ |
| $\text{Born}_3$ | 100 | 100 |
| $\text{Born}_4$ | $-0.00042$ | $+0.021$ |
| $\text{Born}_5$ | $+0.984$ | $+1.16$ |
| $\text{NLO}_1$ | $+0.088$ | $+0.371$ |
| $\text{NLO}_2$ | $+0.008$ | $+0.014$ |
| $\text{NLO}_3$ | $+4.33$ | $+60.33$ |
| $\text{NLO}_4$ | $-3.88$ | $-3.56$ |
| $\text{NLO}_5$ | $+13.96$ | $+18.67$ |
| $\text{NLO}_6$ | $-0.007$ | $-0.006$ |

| $\sigma$ [fb] | With $Q_{\text{cut}}$ | Without |
|---|---|---|
| LO | $1.2402(9)^{+24.1\%}_{-18.1\%}$ | $1.2746(9)^{+24.2\%}_{-18.1\%}$ |
| Born | $1.4431(8)^{+24.1\%}_{-18.1\%}$ | $1.4785(8)^{+24.1\%}_{-18.1\%}$ |
| NLO QCD | $1.5050(9)^{+2.7\%}_{-5.1\%}$ | $2.357(1)^{+15.4\%}_{-12.7\%}$ |
| Complete NLO | $1.650(1)^{+3.5\%}_{-6.0\%}$ | $2.583(1)^{+15.7\%}_{-13.0\%}$ |

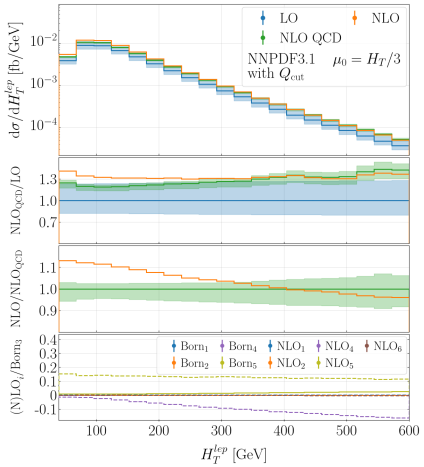
$$\sigma_{\text{NLO}_{\text{QCD}}} = \sigma_{\text{Born}} + \sigma_{\text{NLO}_3},$$

$$\sigma_{\text{NLO}} = \sigma_{\text{Born}} + \sum_{i=1}^{6} \sigma_{\text{NLO}_i}.$$

**Complete NLO relative to NLO QCD: $+9.7\%$ with the window, $+9.6\%$ without.**

MC errors in parentheses; seven-point scale bands. LO: $\Gamma_t^{\text{LO}}$; Born/NLO: $\Gamma_t^{\text{NLO}}$.

# Differential distributions




$$H_T^\ell = p_{T,e^+} + p_{T,\mu^+}$$

**Formally subleading NLO contributions**

Relative to $\mathrm{Born}_3$, from low $H_T^\ell$ to the tail:

$$\mathrm{NLO}_4: \quad -1\% \longrightarrow -16\%,$$

$$\mathrm{NLO}_5: \quad +15\% \longrightarrow +11\%.$$

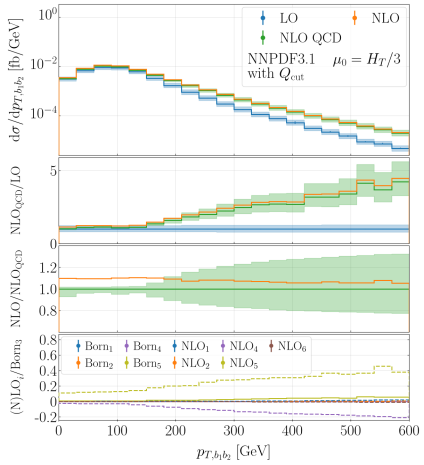
$\mathrm{NLO}_1$, $\mathrm{NLO}_2$ and $\mathrm{NLO}_6$ each remain below $0.5\%$ in magnitude.

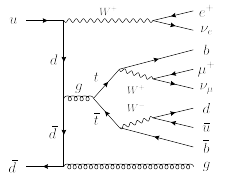
**Complete NLO relative to NLO QCD**

$$\underbrace{+13\%}_{\text{low } H_T^\ell} \longrightarrow \underbrace{-4\%}_{\text{high } H_T^\ell}$$

**The positive fiducial correction is not a constant rescaling of the spectrum.**

# Differential distributions




Initial-state radiation in $\mathrm{NLO}_3$.

$$p_{T,b_1b_2} = |\vec{p}_{T,b_1} + \vec{p}_{T,b_2}|$$

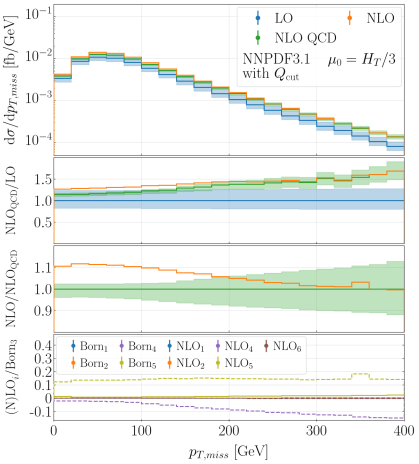
Hard radiation allows the heavy system to recoil; the $b$ jets inherit this boost.

NLO QCD / LO grows from about $1$ to about $4$ in the tail.

Hard-radiation recoil shapes the high-$p_T$ tail and is only LO accurate.

**Complete NLO adds a further $5$–$10\%$ across the spectrum.**

# Differential distributions




$$p_T^{\rm miss} = |\vec{p}_{T,\nu_e} + \vec{p}_{T,\nu_\mu}|$$

**In the high-$p_T^{\rm miss}$ region**

$$\frac{d\sigma_{\rm NLO_4}}{d\sigma_{\rm Born_3}} \simeq -15\%,$$

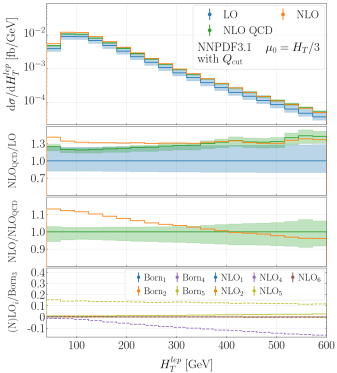
$$\frac{d\sigma_{\rm NLO_5}}{d\sigma_{\rm Born_3}} \simeq +14\%.$$

The complete NLO correction relative to NLO QCD falls from about $+12\%$ at low missing momentum to nearly zero in the tail.

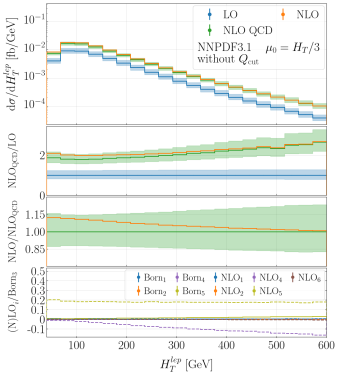
**Cancellations between various subleading contributions impact the shape of various observables.**

# Differential distributions without the $W$-mass window

With the $W$-mass window




Without the $W$-mass window




Without the window, $\text{NLO}_3/\text{Born}_3$ grows from about $60\%$ to $120\%$; $\text{NLO}_{\text{QCD}}/\text{LO} \simeq 1.8$–$2.6$.

**Including NLO QCD is essential to ensure perturbative stability.**

Complete NLO relative to NLO QCD in the high-$H_T^{\ell}$ region: about $-4\%$ with the window, but $+1\%$ without.

# Conclusions

$$pp \longrightarrow e^+\nu_e\,\mu^+\nu_\mu\,b\bar{b}\,jj + X$$

- First full off-shell NLO QCD and complete NLO prediction for this same-sign dilepton final state: **five Born orders and six NLO orders**.
- Formally subleading corrections increase the integrated fiducial cross section by about **10% relative to NLO QCD**, for both selections.
- The differential effects are not a uniform rescaling: enhanced $tW$-scattering mechanisms compete with negative EW Sudakov corrections, producing **shape changes and cancellations**.
- These effects matter in the high-$p_T$ regions used in searches for heavy new particles. The calculation retains the complete resonance structure, interference and finite-width effects in these regions.

# Fiducial setup and the hadronic-$W$ selection

$$pp \to e^+\nu_e\,\mu^+\nu_\mu\,b\bar{b}\,jj + X, \qquad \sqrt{s} = 13.6\,\text{TeV}$$

**Resolved objects**

$$p_{T,\ell} > 20\,\text{GeV}, \qquad |y_\ell| < 2.5,$$
$$p_{T,j},\ p_{T,b} > 25\,\text{GeV}, \quad |y_j|, |y_b| < 2.5.$$

At least two light jets and two $b$ jets.
Anti-$k_T$ jets with $R = 0.4$.

NNPDF3.1 NLO with LUXqed
$\alpha_s(M_Z) = 0.118$.

**Hadronic-$W$ mass window**
At least one light-jet pair that fulfills

$$|M_{jj} - M_W| < Q_{\text{cut}} = 20\,\text{GeV}.$$

Compare with the same selection **without** this requirement.

**Central scale:** $\mu_R = \mu_F = H_T/3$
Seven-point scale variation.

$$H_T = p_{T,e^+} + p_{T,\mu^+} + p_T^{\text{miss}} + p_{T,b_1} + p_{T,b_2} + p_{T,j_1} + p_{T,j_2}.$$

# Input parameters and prediction definitions

$$m_t = 172.5\,\text{GeV},$$
$$\Gamma_t^{\text{LO}} = 1.4580658\,\text{GeV},$$
$$\Gamma_t^{\text{NLO}} = 1.3518591\,\text{GeV}.$$

| | Mass [GeV] | Width [GeV] |
|---|---|---|
| $W$ (OS) | 80.377 | 2.085 |
| $Z$ (OS) | 91.1876 | 2.4955 |
| $H$ | 125 | $4.07 \times 10^{-3}$ |

$$G_\mu = 1.1663787 \times 10^{-5}\,\text{GeV}^{-2}.$$

Complex masses; $W, Z$ inputs converted to pole parameters.

$$M_V = \frac{M_V^{\text{OS}}}{\sqrt{1 + (\Gamma_V^{\text{OS}}/M_V^{\text{OS}})^2}}.$$

$$\text{Born} = \sum_{i=1}^{5} \text{Born}_i,$$
$$\text{NLO}_{\text{QCD}} = \text{Born} + \text{NLO}_3,$$
$$\text{NLO} = \text{Born} + \sum_{i=1}^{6} \text{NLO}_i.$$

LO: tree prediction with $\Gamma_t^{\text{LO}}$;
Born: tree reference with $\Gamma_t^{\text{NLO}}$.

# Integrated cross sections: with $Q_{\text{cut}}$, $\mu_0 = H_T/3$

| | Order | $\sigma_i$ [fb] | Ratio to $\text{Born}_3$ |
|---|---|---|---|
| $\text{Born}_1$ | $\mathcal{O}(\alpha_s^4\alpha^4)$ | $+0.000125(3)$ | $+0.00874\%$ |
| $\text{Born}_2$ | $\mathcal{O}(\alpha_s^3\alpha^5)$ | $+0.000001(3)$ | $+0.000104\%$ |
| $\text{Born}_3$ | $\mathcal{O}(\alpha_s^2\alpha^6)$ | $+1.4289(3)$ | $+100.0000\%$ |
| $\text{Born}_4$ | $\mathcal{O}(\alpha_s\alpha^7)$ | $-0.000006(1)$ | $-0.000423\%$ |
| $\text{Born}_5$ | $\mathcal{O}(\alpha^8)$ | $+0.0141(6)$ | $+0.9844\%$ |
| $\text{NLO}_1$ | $\mathcal{O}(\alpha_s^5\alpha^4)$ | $+0.001265(6)$ | $+0.0885\%$ |
| $\text{NLO}_2$ | $\mathcal{O}(\alpha_s^4\alpha^5)$ | $+0.000113(3)$ | $+0.00790\%$ |
| $\text{NLO}_3$ | $\mathcal{O}(\alpha_s^3\alpha^6)$ | $+0.0619(5)$ | $+4.3292\%$ |
| $\text{NLO}_4$ | $\mathcal{O}(\alpha_s^2\alpha^7)$ | $-0.0554(1)$ | $-3.8761\%$ |
| $\text{NLO}_5$ | $\mathcal{O}(\alpha_s\alpha^8)$ | $+0.1994(9)$ | $+13.9567\%$ |
| $\text{NLO}_6$ | $\mathcal{O}(\alpha^9)$ | $-0.00010(9)$ | $-0.00688\%$ |
| LO | | $1.2402(9)^{+24.07\%}_{-18.11\%}$ | 0.8679 |
| Born | | $1.4431(8)^{+24.06\%}_{-18.10\%}$ | 1.0099 |
| $\text{NLO}_{\text{QCD}}$ | | $1.5050(9)^{+2.67\%}_{-5.07\%}$ | 1.0532 |
| Complete NLO | | $1.650(1)^{+3.46\%}_{-5.95\%}$ | 1.1549 |

MC errors in parentheses; seven-point scale envelope on aggregate rows.

# Integrated cross sections: with $Q_{\text{cut}}$, $\mu_0 = E_T/3$

| | Order | $\sigma_i$ [fb] | Ratio to $\text{Born}_3$ |
|---|---|---|---|
| $\text{Born}_1$ | $\mathcal{O}(\alpha_s^4\alpha^4)$ | $+0.000114(3)$ | $+0.00836\%$ |
| $\text{Born}_2$ | $\mathcal{O}(\alpha_s^3\alpha^5)$ | $+0.000001(2)$ | $+0.000054\%$ |
| $\text{Born}_3$ | $\mathcal{O}(\alpha_s^2\alpha^6)$ | $+1.3606(3)$ | $+100.0000\%$ |
| $\text{Born}_4$ | $\mathcal{O}(\alpha_s\alpha^7)$ | $-0.000006(1)$ | $-0.000439\%$ |
| $\text{Born}_5$ | $\mathcal{O}(\alpha^8)$ | $+0.0140(6)$ | $+1.0272\%$ |
| $\text{NLO}_1$ | $\mathcal{O}(\alpha_s^5\alpha^4)$ | $+0.001328(6)$ | $+0.0976\%$ |
| $\text{NLO}_2$ | $\mathcal{O}(\alpha_s^4\alpha^5)$ | $+0.000127(3)$ | $+0.00936\%$ |
| $\text{NLO}_3$ | $\mathcal{O}(\alpha_s^3\alpha^6)$ | $+0.1636(5)$ | $+12.0214\%$ |
| $\text{NLO}_4$ | $\mathcal{O}(\alpha_s^2\alpha^7)$ | $-0.0528(1)$ | $-3.8830\%$ |
| $\text{NLO}_5$ | $\mathcal{O}(\alpha_s\alpha^8)$ | $+0.194(1)$ | $+14.2581\%$ |
| $\text{NLO}_6$ | $\mathcal{O}(\alpha^9)$ | $-0.00008(9)$ | $-0.00592\%$ |
| LO | | $1.1813(9)^{+23.59\%}_{-17.84\%}$ | 0.8683 |
| Born | | $1.3746(8)^{+23.58\%}_{-17.83\%}$ | 1.0104 |
| $\text{NLO}_{\text{QCD}}$ | | $1.5382(8)^{+4.38\%}_{-6.50\%}$ | 1.1306 |
| Complete NLO | | $1.681(1)^{+5.64\%}_{-7.22\%}$ | 1.2353 |

MC errors in parentheses; seven-point scale envelope on aggregate rows.

# Integrated cross sections: without $Q_{\text{cut}}$, $\mu_0 = H_T/3$

| | Order | $\sigma_i$ [fb] | Ratio to $\text{Born}_3$ |
|---|---|---|---|
| $\text{Born}_1$ | $\mathcal{O}(\alpha_s^4\alpha^4)$ | $+0.00407(3)$ | $+0.2791\%$ |
| $\text{Born}_2$ | $\mathcal{O}(\alpha_s^3\alpha^5)$ | $+0.00066(1)$ | $+0.0451\%$ |
| $\text{Born}_3$ | $\mathcal{O}(\alpha_s^2\alpha^6)$ | $+1.4565(3)$ | $+100.0000\%$ |
| $\text{Born}_4$ | $\mathcal{O}(\alpha_s\alpha^7)$ | $+0.00030(2)$ | $+0.0206\%$ |
| $\text{Born}_5$ | $\mathcal{O}(\alpha^8)$ | $+0.0169(6)$ | $+1.1628\%$ |
| $\text{NLO}_1$ | $\mathcal{O}(\alpha_s^5\alpha^4)$ | $+0.0054(2)$ | $+0.3709\%$ |
| $\text{NLO}_2$ | $\mathcal{O}(\alpha_s^4\alpha^5)$ | $+0.00021(8)$ | $+0.0142\%$ |
| $\text{NLO}_3$ | $\mathcal{O}(\alpha_s^3\alpha^6)$ | $+0.8788(6)$ | $+60.3320\%$ |
| $\text{NLO}_4$ | $\mathcal{O}(\alpha_s^2\alpha^7)$ | $-0.0519(2)$ | $-3.5604\%$ |
| $\text{NLO}_5$ | $\mathcal{O}(\alpha_s\alpha^8)$ | $+0.2720(5)$ | $+18.6726\%$ |
| $\text{NLO}_6$ | $\mathcal{O}(\alpha^9)$ | $-0.0001(1)$ | $-0.00561\%$ |
| LO | | $1.2746(9)^{+24.15\%}_{-18.14\%}$ | 0.8751 |
| Born | | $1.4785(8)^{+24.12\%}_{-18.13\%}$ | 1.0151 |
| $\text{NLO}_{\text{QCD}}$ | | $2.357(1)^{+15.36\%}_{-12.74\%}$ | 1.6184 |
| Complete NLO | | $2.583(1)^{+15.70\%}_{-12.95\%}$ | 1.7733 |

MC errors in parentheses; seven-point scale envelope on aggregate rows.

# Integrated cross sections: without $Q_{\text{cut}}$, $\mu_0 = E_T/3$

| | Order | $\sigma_i$ [fb] | Ratio to $\text{Born}_3$ |
|---|---|---|---|
| $\text{Born}_1$ | $\mathcal{O}(\alpha_s^4\alpha^4)$ | $+0.00473(3)$ | $+0.3406\%$ |
| $\text{Born}_2$ | $\mathcal{O}(\alpha_s^3\alpha^5)$ | $+0.00082(1)$ | $+0.0588\%$ |
| $\text{Born}_3$ | $\mathcal{O}(\alpha_s^2\alpha^6)$ | $+1.3875(3)$ | $+100.0000\%$ |
| $\text{Born}_4$ | $\mathcal{O}(\alpha_s\alpha^7)$ | $+0.00034(2)$ | $+0.0247\%$ |
| $\text{Born}_5$ | $\mathcal{O}(\alpha^8)$ | $+0.0169(6)$ | $+1.2191\%$ |
| $\text{NLO}_1$ | $\mathcal{O}(\alpha_s^5\alpha^4)$ | $+0.0049(3)$ | $+0.3509\%$ |
| $\text{NLO}_2$ | $\mathcal{O}(\alpha_s^4\alpha^5)$ | $-0.0001(1)$ | $-0.00394\%$ |
| $\text{NLO}_3$ | $\mathcal{O}(\alpha_s^3\alpha^6)$ | $+0.9745(6)$ | $+70.2330\%$ |
| $\text{NLO}_4$ | $\mathcal{O}(\alpha_s^2\alpha^7)$ | $-0.0495(2)$ | $-3.5700\%$ |
| $\text{NLO}_5$ | $\mathcal{O}(\alpha_s\alpha^8)$ | $+0.2669(5)$ | $+19.2397\%$ |
| $\text{NLO}_6$ | $\mathcal{O}(\alpha^9)$ | $-0.0001(1)$ | $-0.00584\%$ |
| LO | | $1.2160(9)^{+23.72\%}_{-17.89\%}$ | 0.8764 |
| Born | | $1.4103(8)^{+23.68\%}_{-17.87\%}$ | 1.0164 |
| $\text{NLO}_{\text{QCD}}$ | | $2.3847(9)^{+16.99\%}_{-13.55\%}$ | 1.7188 |
| Complete NLO | | $2.607(1)^{+17.14\%}_{-13.67\%}$ | 1.8789 |

MC errors in parentheses; seven-point scale envelope on aggregate rows.