Scale Invariance and the Hierarchy Problem

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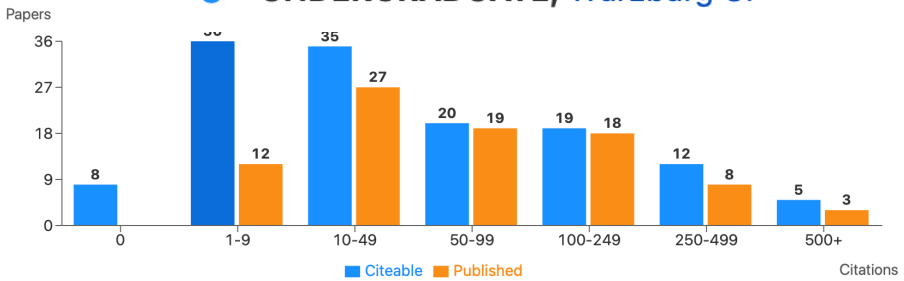
Exploring new physics with flavour, KIT, July 2-3, 2026

Email: Many of you already know: Uli turns 60!

Ulrich Nierste

- 2005-present
  SENIOR, KIT, Karlsruhe, TTP
- 2002-2005
  JUNIOR, Fermilab
- 2000-2001
  POSTDOC, CERN
- 1998-2000
  POSTDOC, Fermilab
- 1996-1998
  POSTDOC, DESY
- 1992-1995
  PHD, Munich, Tech. U.
- **UNDERGRADUATE,** Wurzburg U.



- 2006-present
  SENIOR, Heidelberg, Max Planck Inst.
- 1993-2006
  SENIOR, Munich, Tech. U.
- 1991-1993
  JUNIOR, Heidelberg U.
- 1989-1991
  POSTDOC, CERN
- 1987-1989
  POSTDOC, Fermilab
- 1984-1987
  PHD, Munich, Max Planck Inst.
- 1984
  UNDERGRADUATE, Munich U.

➔ 1 joint paper 2005

Reasons for Scale Invariance

Why scale invariance?

- Theory with mass terms from the beginning

  ➔ **no chance to explain their origin**

- 4d QFT allows to calculate only differences

  ➔ $\mathcal{L}$ should not contain dimension-full parameters

  ➔ Scale invariance (or beyond 4d QFT)

Scale invariance is hardly broken by scale anomaly

Callan ’70, Symanzik ’70

Massless scalar theories exist in perturbation theory

Lowenstein+Zimmermann ’75, ’76

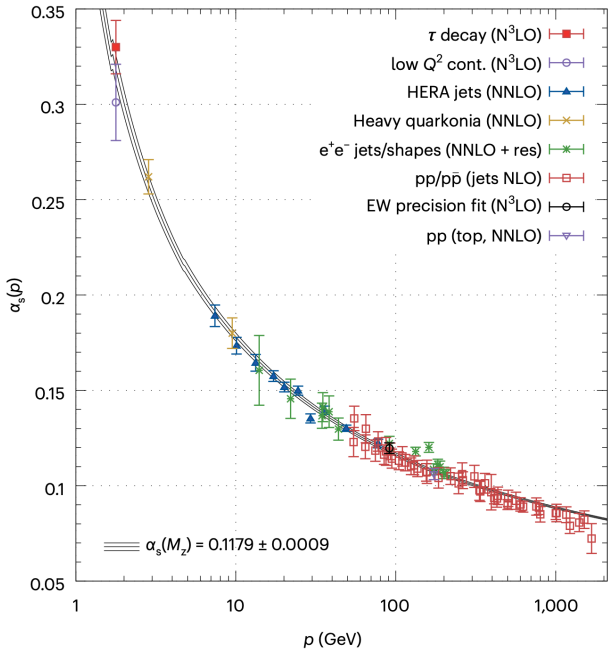
Role Models of Scale Invariance

Chiral QCD

Dimensional transmutation:

➔ scale generation → Λ_{QCD}

Λ_{QCD} (one scale) fixed by measurement
Scale ratios (spectrum) calculable



➔ Scale invariance violated by quark masses
⇔ understand masses in a scale invariant setting?

Scale invariant scalar QCD (fermions → mass-less scalar)

➔ dimensional transmutation – generation of a scale

More known Cases of Scale Genesis

Perturbative

$\langle \phi \rangle \neq 0$ Coleman+Weinberg,'73, Gildener+Weinberg,'76

Nonperturbative

$\langle \bar{\psi}\psi \rangle \neq 0$ Nambu+Jona-Lasinio,'60,'61

$\langle S^\dagger S \rangle \neq 0$ Osterwalder+Seiler, 78, Fradkin+Shenker,'79

$\langle F^2_{\alpha\beta} \rangle \neq 0$ Shifman, Vainshtein+Zakharov, '79

By boundary condition

Ferreira,Hill+Ross,'16;'18

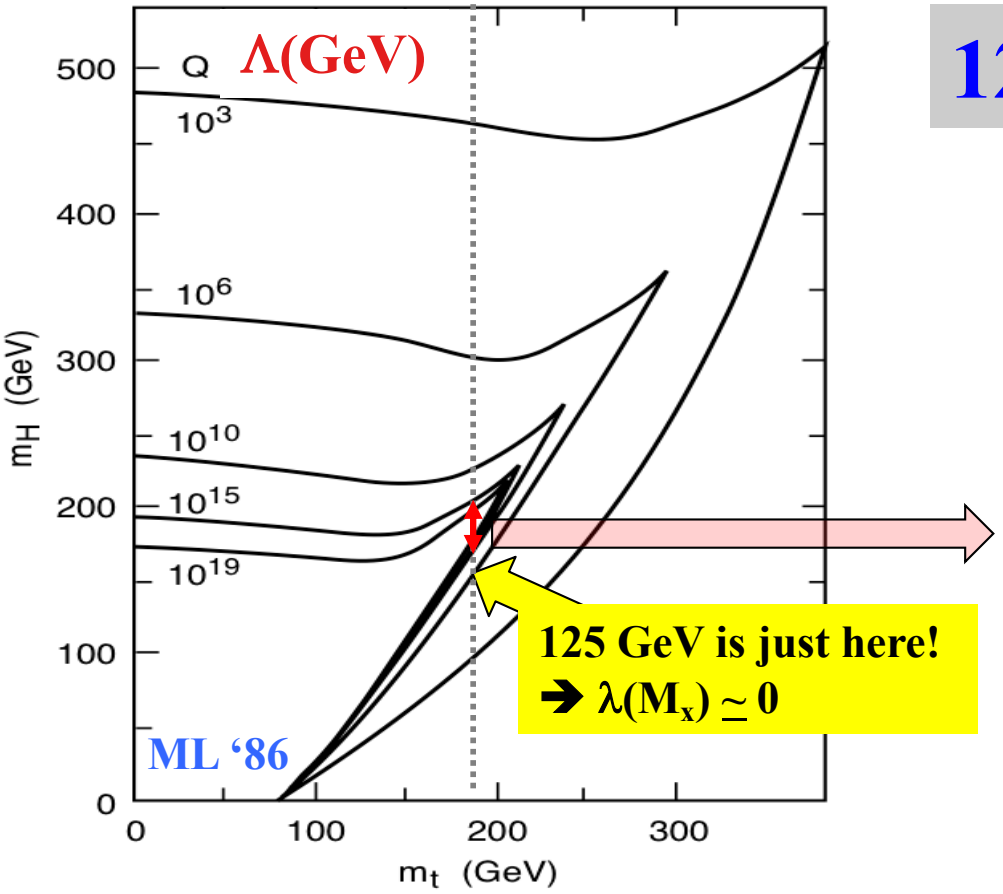
By gauge fixing in Weyl gauge symmetry

Ghilencea+Lee,'18

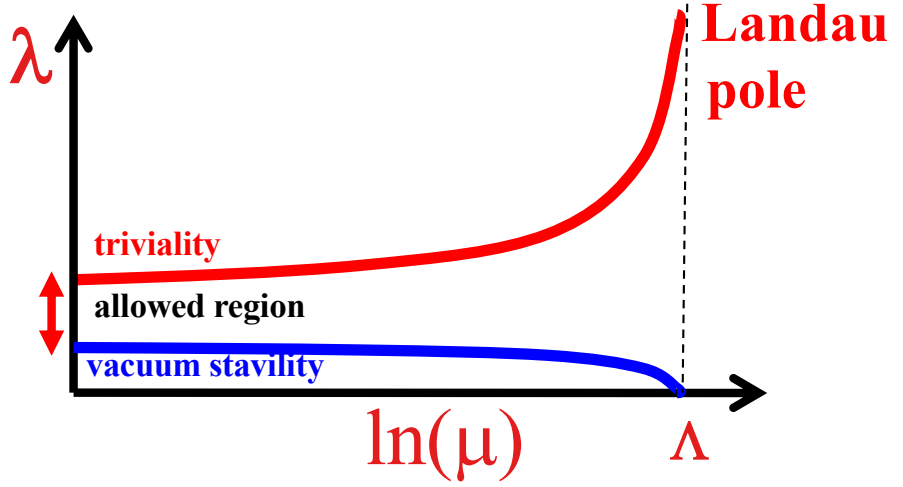
A remarkable Coincidence in the SM

➔ SM is a renormalizable QFT like QED w/o hierarchy problem

➔ **Cutoff "Λ" has no meaning ➔ triviality, vacuum stability**



126 GeV < m_H < 174 GeV



➔ **SM quantum corrections OK over large scals distances**

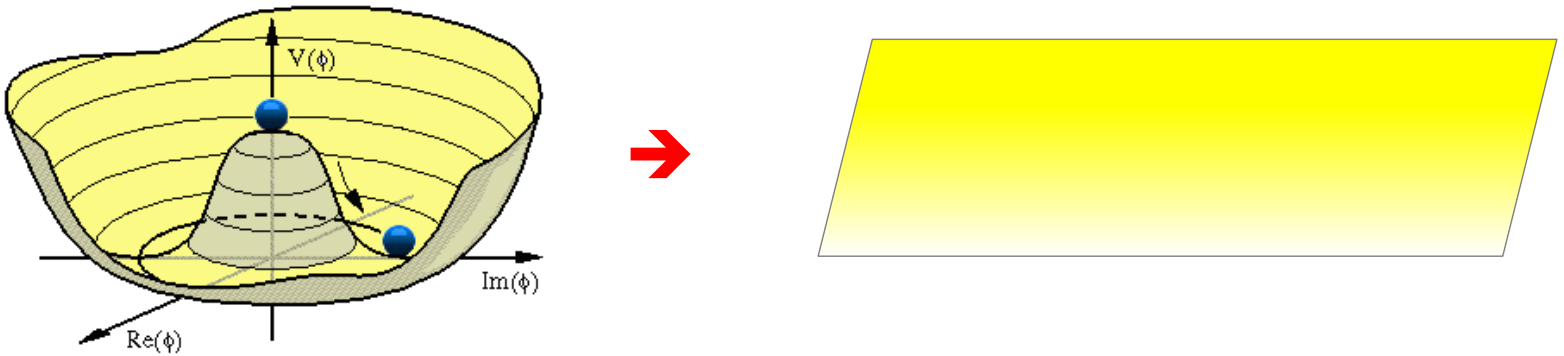
Important observation:

- **a remarkable relation between the weak scale, m_t, m_H, gauge couplings and Λ**
- **not quadratic divergences! log divergences ←→ β-functions ←→ scale anomaly!**

Is there a Message?

- $\lambda(\Lambda) \simeq 0$? ➔remarkable cancellation of logs ←➔ CA~ β-functions
  Λ, M_{weak}, gauge, Higgs & Yukawa couplings are unrelated

- remember: μ is the only single scale of the SM ➔ special role
  ➔ if in addition $\mu^2 = 0$ ➔ $V(\Lambda) \simeq 0$
  ➔ flat Mexican hat (<1%) at very high scale Λ (←➔ Planck scale)



➔ scale/conformal (or shift) symmetry + log running as solution to the HP?

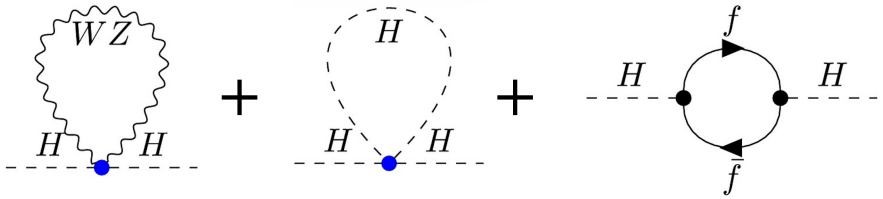
➔ combined conformal & EW symmetry breaking

- conceptual issues
- minimal realizations ←➔ SM seems to know about high scales ➔ bottom-up

Hierarchy Problems (the usual Story)

Stability of vastly different scales under quantum corrections?

SM:

$$\delta M^2_H =$$ 

$$\delta M_H^2 = \frac{\Lambda^2}{32\pi^2 V^2}\left(6M_W^2 + 3M_Z^2 + 3M_H^2 - 12M_t^2\right) \quad \sim \Lambda^2 \gg M^2_H$$

- SM is renormalizable – no cutoff – no problem
  ➔ Veltman condition is meaningless
- it becomes a problem if Λ stands for new physics = embedding
- embeddings require to talk about individual thresholds
  ➔ quadratic sensitivity to some mean embedding scale Λ
  ➔ not RG invariant, no protective symmetry – at best partial

SM+: massive neutrinos

- **Dirac neutrino masses** – no problem (Yukawa couplings)

- **Majorana neutrino masses** → more scales M_i

  generates another HP problem for large M even if y_ν is tiny

from see-saw: $y_\nu^2 = M m_\nu / v^2$ → $\delta m_H^2 \simeq \frac{y_\nu^2}{16\pi^2} M^2$

avoiding the problem: $M \lesssim 10^7 - 10^8$ GeV

←→ see-saw, leptogenesis, ...

And not to forget the 0th order hierarchy problem:

- why are (tree level) scales vastly different?
- why at all scales in the Lagrangian?

The Problem: More than one EXPLICIT Scale

- Renormalizable QFT with two scalars φ , Φ with masses m, M and a hierarchy $m \ll M$
- $\varphi^+\varphi$ and $\Phi^+\Phi$ are singlets ➔ must interact via portal term ➔ $\lambda_{mix}(\varphi^+\varphi)(\Phi^+\Phi)$ in addition to φ^4 and Φ^4
- Quantum corrections $\sim M^2$ drives m to the (heavy) scale M ➔ vastly different explicit scalar scales are generically unstable

- **Since SM Higgs exists ➔ problem: embedding with a 2nd scalar**
  - **gauge extensions: LR, PS, GUTs → must be broken…**
  - **even for SUSY GUTS → doublet-triplet splitting...**
  - **also for fashionable Higgs-portal scenarios...**
- **Ways out:**
  - **No Higgs (classically composite theories @EW scale), ...**
  - **Symmetry: SUSY, ...**

  ➔ here: scale symmetry = no explicit scale at all!

The conformal Anomaly & Scale Invariance

Trace of the energy-momentum tensor Θ^{μ}_{μ}
Callan, Coleman+Jackiw'70, Chanowitz+Ellis,'73, ...

$\Theta^{\mu}_{\mu} = 0$ ➔ conformal symmetry
$\Theta^{\mu}_{\mu} \neq 0$ ➔ conformal anomaly if zero at the classical level

Scalar QED (no scalar mass term)**:**

$$= \frac{\beta_e}{2e} F^2_{\alpha\beta} + \beta_{\lambda_\varphi} (\varphi^\dagger \varphi)^2$$

IMPORTANT:
anomaly does not induce a mass term!

➔ **Question:** But how can scale symmetry be broken
without creating a scalar mass term in the Lagrangian?

The massless Coleman Weinberg Potential

Massless scalar QED:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} |D_\mu \phi|^2 - \frac{\lambda}{4!} |\phi|^4$$

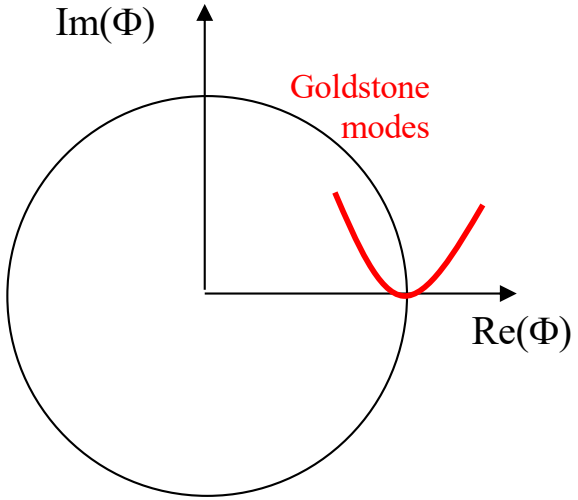
$$V_{\text{eff}} = \frac{3e^4}{64\pi^2} \phi_c^4 \left(\ln \frac{\phi_c^2}{\langle \phi \rangle^2} - \frac{1}{2} \right)$$

↳ $\beta_\lambda \Leftrightarrow$ scale anomaly

➔ **scale generation: V_{eff} has a non-vanishing minimum even though $m^2=0$**

looks like spontaneous symmetry breaking $<\Phi> \neq 0$:
but scale symmetry is explicitely broken

- ➔ without mass term in the Lagrangian
- ➔ anomaly does not require $m^2\Phi^2$ in Lagrangian
- ➔ no renormalization of m^2 ➔ no hierachy problem
- ➔ Goldstone directions like in SSB



The Hierarchy Problem and Scale Invariance

Massless scalar QED
Massless scalar QCD
...

➔ **no hierarchy problem** – just one scale generated by dimensional transmutation

➔ Hierarchy problem emerges in scale invariant theories if more scalar scales are generated

For illustration: add two real scalar fields σ and s ➔ Gildener-Weinberg mechanism

$$V_{\rm GW}(s,\sigma) = \frac{1}{4}\left(\lambda_\sigma \sigma^4 + \lambda_s s^4 + \lambda_{s\sigma}\sigma^2 s^2\right) + \frac{1}{4}\left(\lambda_{s\varphi}\, s^2 + \lambda_{\sigma\omega}\,\sigma^2\right)\varphi^\dagger\varphi$$

➔ $$\Delta\Theta^\alpha_\alpha = -\frac{1}{4}\left(\beta_{\lambda_\sigma}\sigma^4 + \beta_{\lambda_s} s^4 + \beta_{\lambda_{s\sigma}}\sigma^2 s^2\right) - \frac{1}{4}\left(\beta_{\lambda_{s\varphi}}\, s^2 + \beta_{\lambda_{\sigma\varphi}}\,\sigma^2\right)\varphi^\dagger\varphi$$

chose parameters such with a mass hierarch for the involved scalars

➔ **unless portal couplings are tiny (tuned) one obtains all masses of similar magnitude**

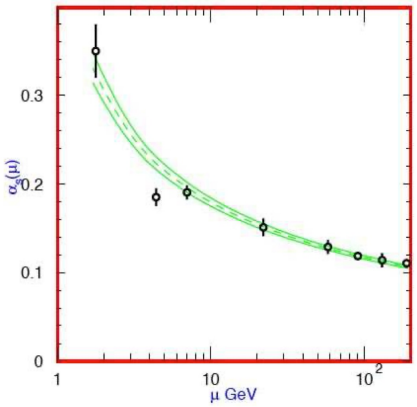
➔ multiple scale generation implies that all scales are similar – no big hierarchies

➔ **how to solve the hierarchy problem?**

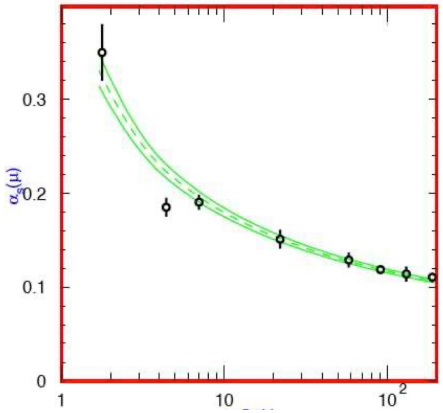
Avoiding the Cross-Talk of Scalar Scales

- chiral QCD and a second (orthogonal) copy of QCD: $SU(3)_1 \otimes SU(3)_2$
- two decoupled gluons
- chiral fermions in both sectors $\psi_1 = (3,1)$ and $\psi_2 = (1,3)$

QCD



hidden QCD



scale?

➔ dimensional transmutation: gauge coupling g_i ➔ scales

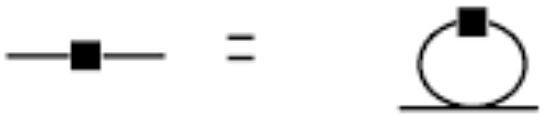
$\Lambda_{QCD} \approx$ 200-300 MeV Λ_{hidden} = arbitrary

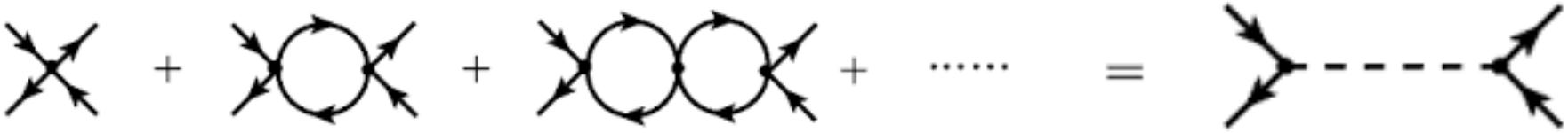
- no fundamental scale
- no hierarchy problem: two renormalizable sectors with no scale
- scale ratio determined by initial (dimension-less) gauge couplings

➔ log running ←→ expect exponential scale ratios (linear in couplings)

Is there induced Cross-Talk of Scales?

- describe chiral symmetry breaking in QCD in NJL approximation

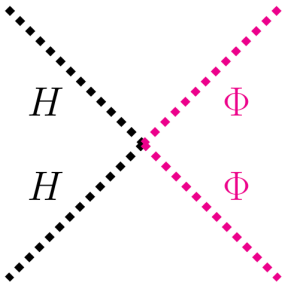
gap eqn. 



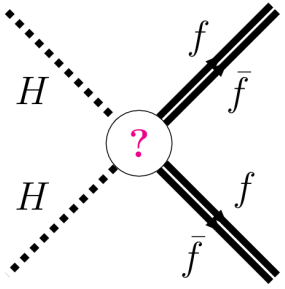
➔ chiral symmetry breaking ➔ "effective" scalar:
3 massless Goldstone Bosons (←→ pions) + 1 massive scalar σ

question: does this introduce a portal in our $SU(3)_1 \otimes SU(3)_2$ example?
- naive: yes – there is no symmetry that forbids a portal ~ $\lambda\ (\sigma_1^+\sigma_1)\ (\sigma_2^+\sigma_2)$
- correct: no!

fundamental scalars
→ portal must exist



composite/effective
→ depends on quantum numbers of constituents



➔ **unavoidable cross talk if there exists more than one fundamental scalar field**

Adding Gravity

Einstein gravity

- has a scale: M_{Pl} ➔ isn't that against scale invariance?
- is not renormalizable... ➔ does not allow meaningful statements about loops
- seems to suggest that all masses are $O(M_{Pl})$ once quantum corrections are calculable

- There exist various generalizations of gravity with Einstein limit
- **scale invariant / R^2 / conformal gravity** are among them ➔ calculable quantum effects
- how to generate the Planck mass?
  ➔ scale invariant (quadratic) gravity + Jordan, Brans-Dicke mechanism

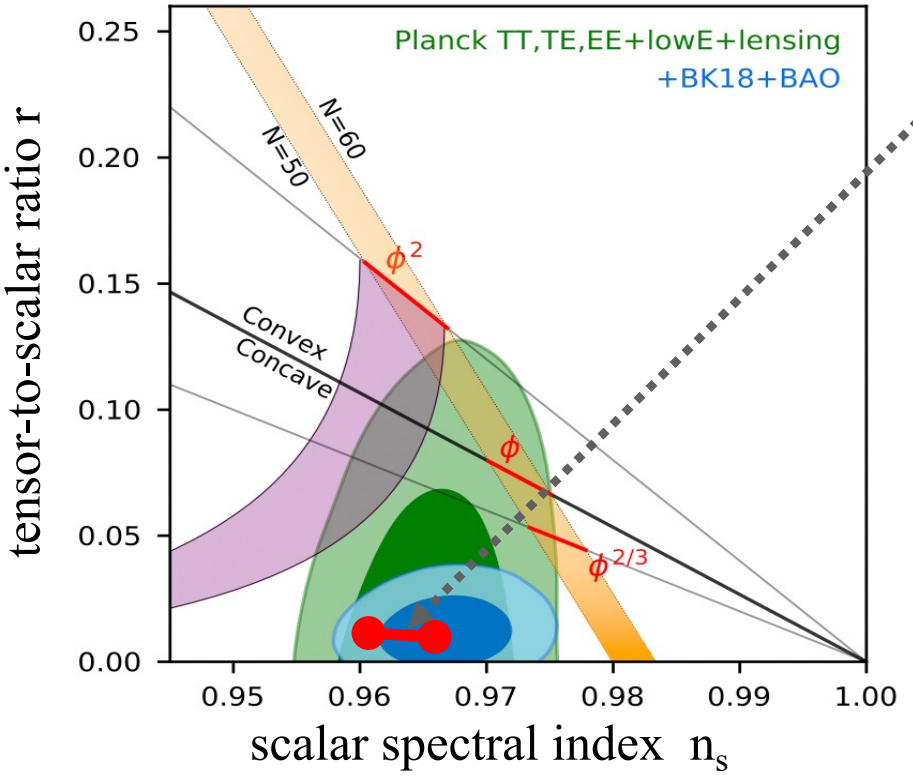
$$\frac{\mathcal{L}_{QG}}{\sqrt{-g}} = \gamma R^2 - \kappa C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \qquad (C_{\mu\nu\alpha\beta} \text{ is the Weyl tensor})$$

adding a massless a scalar field S allows an extra term ~ S^+SR

if S develops a VEV ➔ $\langle S^+S \rangle R = \frac{M_{Pl}^2}{2} R$

Idea: create VEV by dimensional transmutation of a scale invariant theory!

Indications for Scale Invariance in Cosmology



Starobinsky (R^2) inflation

Starobinsky '80, Mukhanov+Chibisov '81

$$\frac{\mathcal{L}_{\mathbf{SB}}}{\sqrt{-g}} = -\frac{M_{\mathrm{Pl}}^2}{2} R + \gamma R^2$$

Einstein gravity
$G = 1/M^2_{Pl}$

$\gamma \approx 10^9$

scale breaking ⇔ scale invariant

- Inflation in early Universe dominated by γR^2 term ⇔ scale invariance!
- Breaking ←→ today's Universe

➔ scale invariant versions of gravity (power counting renormalizable)
➔ generate M_{Pl} from dimensional transmutation

Putting it all together

Ingredients:

- SM in scale invariant limit ($\mu^2 = 0$)
- scale invariant gravity
- a new (scale invariant) chiral gauge sector: G ≈ QCD, but orthogonal representations

de Boer, Kubo, ML, Reinig

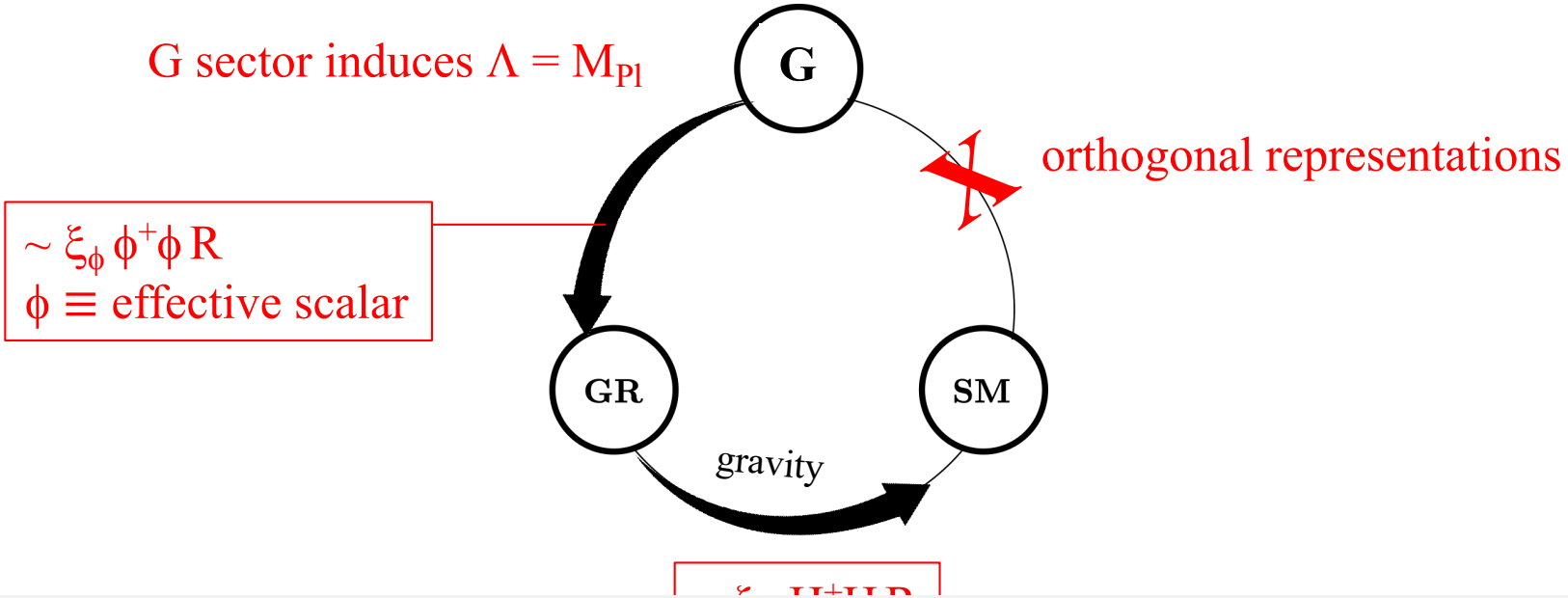
Choose G-sector such that it condenses like QCD, but at the Planck scale

➔ effective scalar degree of freedom acting acting Brans-Dicke like ➔ M_{Pl}

➔ orthogonality of the G-sector does not lead to direct interaction with SM sector

➔ gravity, once massive, does talk to the SM sector

➔ no portal for composite scalars!



The Model

Gauge group: SM $\otimes$ G G ≡ non abelian gauge group

$$\mathcal{L}_{\text{matter}} = \mathcal{L}_{\text{SMGF}} + D_\mu H^\dagger D^\mu H - \lambda_H (H^\dagger H)^2 - \frac{1}{2}\,\text{Tr}\, F^2 + \bar{\psi} i \not{D} \psi$$

LSMGF: gauge and fermionic parts of the SM
H: fundamental Higgs field; no mass term; $\mu^2=0$
F: field-strength tensor of G= SU(n_c)
ψ: chiral fermions which couple only th G

} scale invariant

chiral condensate $\langle \bar{\psi}\psi \rangle \neq 0$ leads to a scale = M_{Pl} ➔ effective scalar field ϕ

add scale invariant gravity $\dfrac{\mathcal{L}_{QG}}{\sqrt{-g}} = \gamma R^2 - \kappa C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} + \xi_\phi \phi^+\phi R + \xi_H H^+ H R$

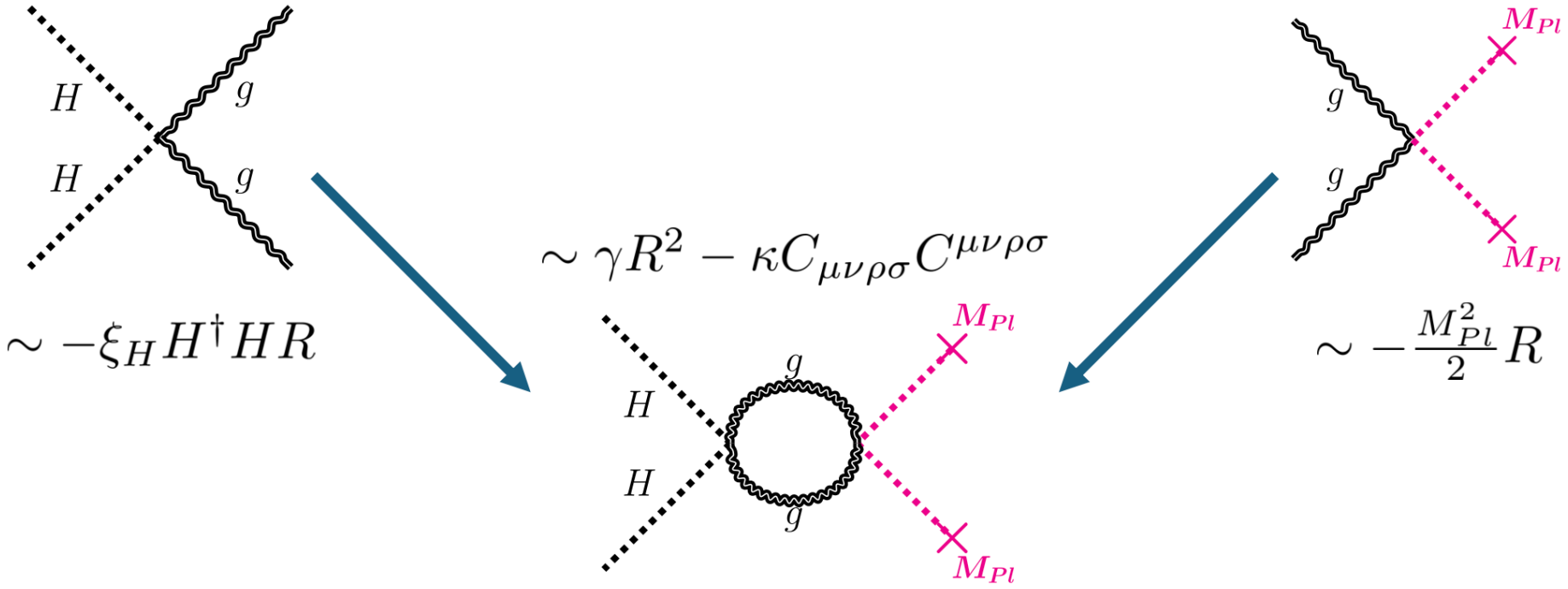
$\langle \phi^+\phi \rangle \neq 0$ generates Einstein gravity
$\xi_H H^+ H R$ generates EW scale

Induced Higg Mass

M_{Planck} is generated as an implicit (not as a hard) scale

➔ no portal between G-sector and SM

➔ scale communicated to the electroweak sector by a gravity induced portal term:



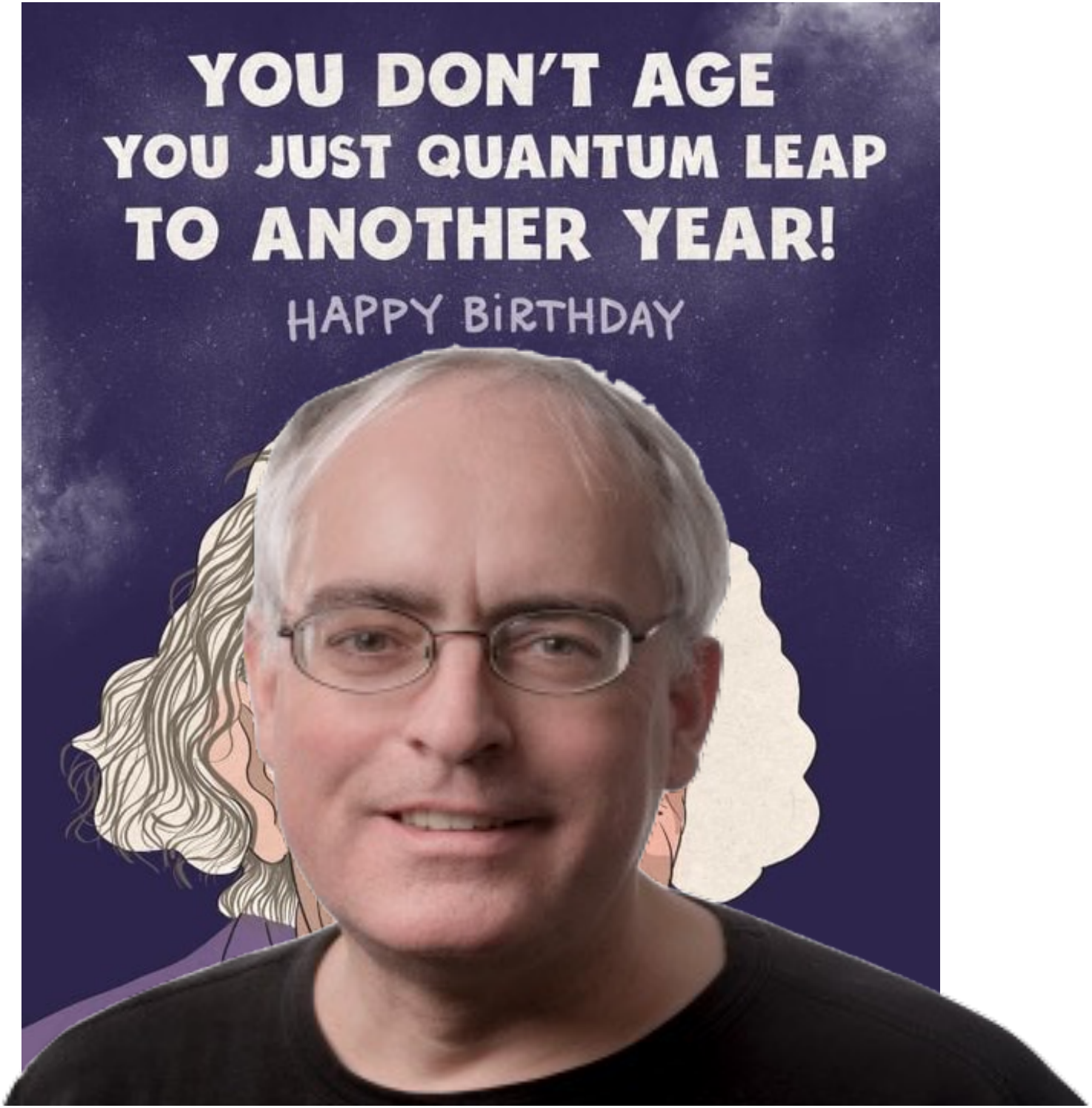
The big value of γ (and κ) acts like a tiny portal term ➔ hierachical scale ratio

Conclusions

- **The Standard Model**
  - works perfectly – no problems besides triviality, metastability
  - list of unanswered questions / problems ←→ BSM
  - lots of progress: DM, n's, GR waves, ... + many new ideas
  - hierarchy problem worsened due to the little hierarchy problem
  - remarkable coincidence of parameters: flat Higgs potential @HE
    → hints for scale invariance in SM and cosmology

- **Scale invariant sectors: anomaly and avoiding cross-talk (portal)**
  - scale anomaly does not induce a mass term in scale invariant theories (CW example)
  - hierarchy problem in scale invariant theories with multiple scales
  - avoid cross-talk (portals) by "orthogonal" sectors; here SM⊗G without overlaping reps

- **Adding scale invariant gravity**
  - renormalizable (ghost issue?)
  - unavoidable portal to fundamental and composite scalars
  - induced Planck scale by condensation in the G-sector
  - induced EW scale ≪ Planck scale ←→ scale mediation by gravity
  - interesting phenomenological consequences
  - it's a mechanism – many options for other realizations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YOU JUST QUANTUM LEAP
TO ANOTHER YEAR!
HAPPY BIRTHDAY