$|V_{ud}|$ Determination and CKM Unitarity

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Ulifest – 03 Jul 2026

Based on: 2510.27648

Institute for Theoretical Particle Physics

Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

Outline

- Motivations
- Short-distance Contributions
  - Matching between SM and Weak Hamiltonian
  - Running of the Operators in EFT
  - Low-scale matching
- V_{ud} Determination
- Conclusions

The Standard Model

- $\mathcal{L}_{\text{Weak}} = \frac{g_2}{\sqrt{2}} \sum_i \bar{u}_L^i \gamma^\mu d_L^i W_\mu^+ + h.c. \rightarrow \mathcal{L}_{\text{Weak}} = \frac{g_2}{\sqrt{2}} \sum_{i,j} \boxed{V_{ij}} \bar{u}_L^{i\prime} \gamma^\mu d_L^{j\prime} W_\mu^+ + h.c.$

Cabibbo-Kobayashi-Maskawa (CKM) Matrix

- V_{ij} is unitary in SM $\longrightarrow \boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1}$

$$|V_{ub}| \ll |V_{ud}|, |V_{us}| \longrightarrow \boxed{|V_{ud}|^2} + \boxed{|V_{us}|^2} = 1 \longrightarrow \boxed{\cos^2\theta_C} + \boxed{\sin^2\theta_C} = 1$$

- (Semi-)Leptonic Decays of Hadrons ⇒ Electroweak Precision Test of SM

Weak Effective Hamiltonian $\quad \mathcal{H} = C_O(x)\, O(x), \quad O = (\bar{d}'\ u)_{V-A} \otimes (\bar{\nu}_\ell\ \ell)_{V-A}$

$d' = d, s$

Current Status of CKM First Row

- The ratio of leptonic $\Gamma(K_{\mu 2})/\Gamma(\pi_{\mu 2})$ ($\boldsymbol{K\ell 2}$) measures $|V_{us}/V_{ud}| = \tan\theta_C$

$$\frac{|V_{us}|}{|V_{ud}|}\boxed{\frac{f_K}{f_\pi}} = \left(\frac{\Gamma_{K\to\mu\nu(\gamma)} m_{\pi^\pm}}{\Gamma_{\pi\to\mu\nu(\gamma)} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\boxed{\frac{\delta R_{K\pi}}{2}}\right)$$

Decay constants from lattice QCD in Iso-symmetric limit

[FLAG '24] ($N_f = 2+1+1$)

QED and Iso-spin breaking corrections from lattice QCD

- $\delta R_{K\pi}$ = -1.26(14)% [Di Carlo et al, 2019]
- $\delta R_{K\pi}$ = -0.86(40)% [Boyle et al, 2023]
- Agreement with χ_{PT}
  $\delta R_{K\pi}$ = -1.12(21)% [Cirigliano, Neufeld, 2011]

- Experimental data dominated by KLOE and KLOE-2

$$\frac{f_{K^\pm}|V_{us}|}{f_{\pi^\pm}|V_{ud}|} = 0.27599(37)$$

[Moulson, 2017]

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23126(48)$$

Current Status of CKM First Row

- Semileptonic $K \to \pi \ell \nu_\ell$ ($\boldsymbol{K\ell 3}$) decays of Kaons measure $|V_{us}| = \sin\,\theta_C$ ($K_L e3, K_S e3, K^{\pm} e3,\ K_L \mu 3, K_S \mu 3, K^{\pm} \mu 3$)

$$\Gamma(K^0 \to \pi^- \ell^+ \nu_\ell(\gamma)) = \frac{G_F^2 m_K^5 C_K^2}{128\pi^3} |V_{us}|^2 S_{EW} |f_+^{K^0\pi^-}(0)|^2 I_{K^0\ell}^{(0)} \left(1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)$$

Short-distance universal EW corrections (Sirlin factor)

$f_+(0) = 0.9698(17)$ [FLAG '24] ($N_f = 2{+}1{+}1$)

- Unlike $K\ell 2$, no lattice complete lattice calculations of $\delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi}$ yet.
  - Newly proposed method: [Christ et al, '24] (QED_∞)

- Updated experimental and theoretical results yield [Seng et al, 2022]

$$|V_{us}| f_+(0) = 0.21634(38)(3) \longrightarrow \boxed{|V_{us}|_{K_{\ell 3}} = 0.22308(55)}$$

Current Status of CKM First Row

- $|V_{ud}|$ determination from superallowed nuclear β-decays and free neutron decays

$$|V_{ud}^{0^+\to 0^+}|^2 = \frac{K}{2G_F^2 \mathcal{F}t(1+\boxed{\Delta_R^V})}$$

$$|V_{ud}^n|^2 = \frac{K/\ln 2}{G_F^2 \mathcal{F}_n \tau_n (1+3\lambda^2)(1+\boxed{\Delta_R^V})}$$

Experimental determination of neutron lifetime τ_n and $\lambda = g_A/g_V$

- $K = 2\pi^3 \ln 2/m_e^5,\ G_F = G_\mu = 1.1663787(6) \times 10^-5 \mathrm{GeV}^2$

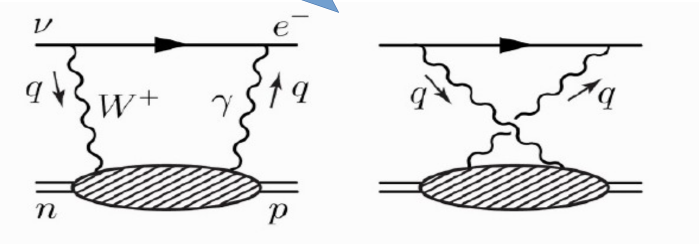
- $\boldsymbol{\Delta_R^V}$ are short-distance (transition independent, single-nucleon) universal radiative corrections (γW box diagrams)

- Marciano & Sirlin 2006: $\Delta_R^V = 0.02361(38)$
  - Dispersive evaluation of box diagrams

- Updated analysis with reduced uncertaintied quotes

$$\boldsymbol{\Delta_R^V = 0.02467(22)}$$ [Seng et al, '18]

  - Other recent studies find agreement [Cirigliano et al, '23]



Superallowed Nuclear β Decays

- The most precise determination of $|V_{ud}|$ comes from **superallowed nuclear beta-decays**

$0^+ \to 0^+$ Fermi transition: only vector coupling $g_V \propto G_F|V_{ud}|$

Outer corrections (depend on final Z and E_e)

$$\mathcal{F}t = ft(1+\delta'_R)(1+\underbrace{\delta_{\mathrm{NS}}-\delta_C}) = \frac{K}{2G_F^2|V_{ud}|^2(1+\Delta_R^V)}$$

Inner corrections: δ_{NS} nuclear effects in γW box diagrams
δ_C isospin-breaking corrections
[Seng et al., ‘18] [Gorchtein, ‘18]

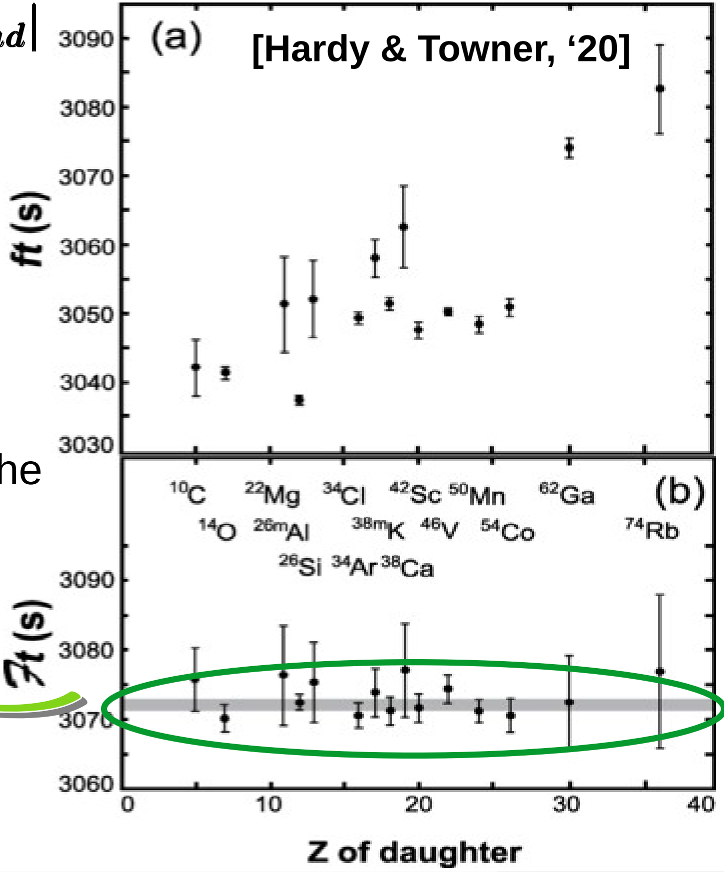
- $K = 2\pi^3 \ln 2/m_e^5,\ G_F = G_\mu = 1.1663787(6)\times 10^-5\mathrm{GeV}^2,\ f$ is the statistical rate function (depends on phase-space), *t* is the half-life

- $\mathcal{F}t$ is the “corrected” value (transition independent)

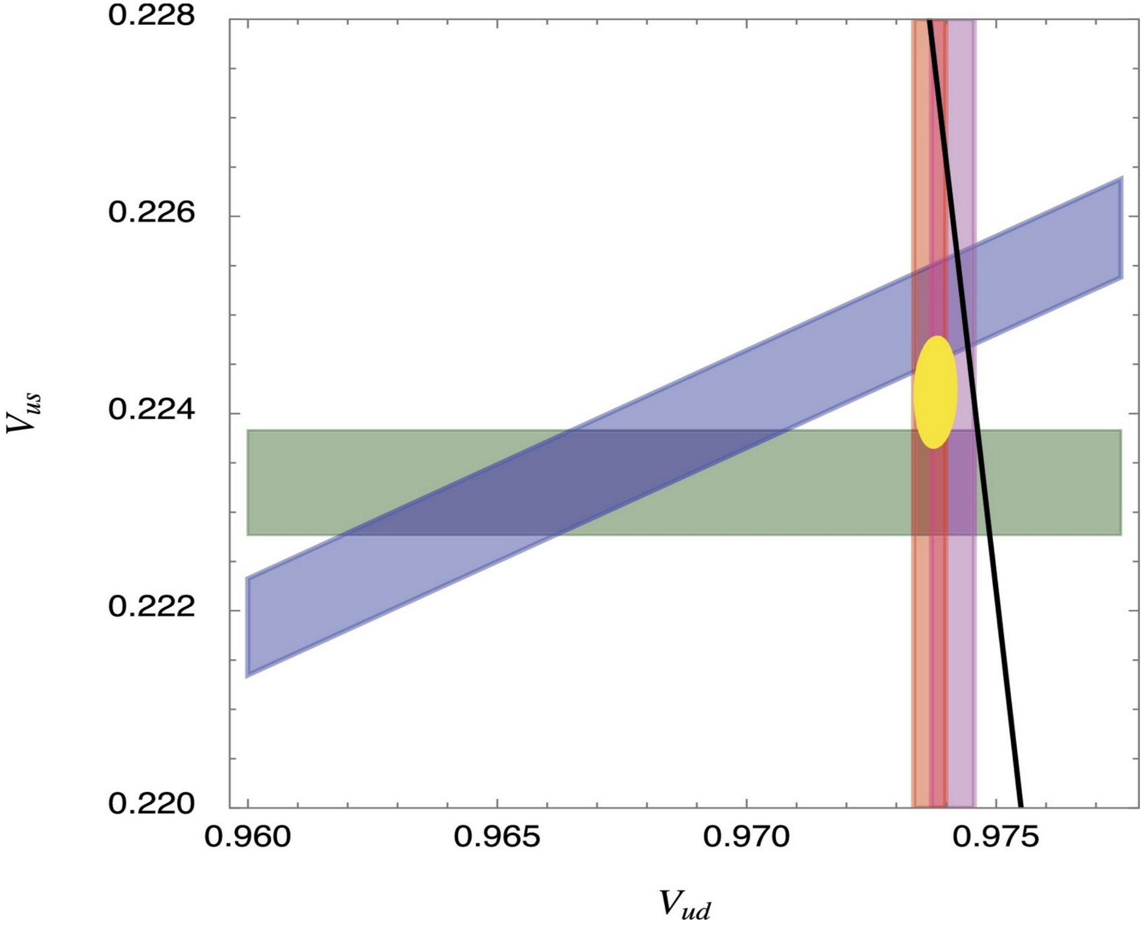
Averaged over 15 transitions

$$\overline{\mathcal{F}t} = 3072.24 \pm 1.85\ s$$

$$\boxed{|V_{ud}| = 0.97367(32)}$$



Current Status of CKM First Row



[Cirigliano et. al., '22]

- Three bands show
  - $K\ell 3$
  - $K\ell 2$
  - $\beta -$ decays
- Tensions in unitarity test
  $$\Delta_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}|^2 - 1$$
  - $\Delta_{\mathrm{CKM}}^{K\ell 3+K\ell 2} = -0.0164(63)\ [2.6\sigma]$
  - $\Delta_{\mathrm{CKM}}^{\beta+K\ell 3} = -0.00176(56)\ [3.1\sigma]$
  - $\Delta_{\mathrm{CKM}}^{\beta+K\ell 2} = -0.00098(58)\ [1.7\sigma]$

(B?)SM

- In recent years many analyses explored possible Beyond Standard Model scenarios to explain these anomalies

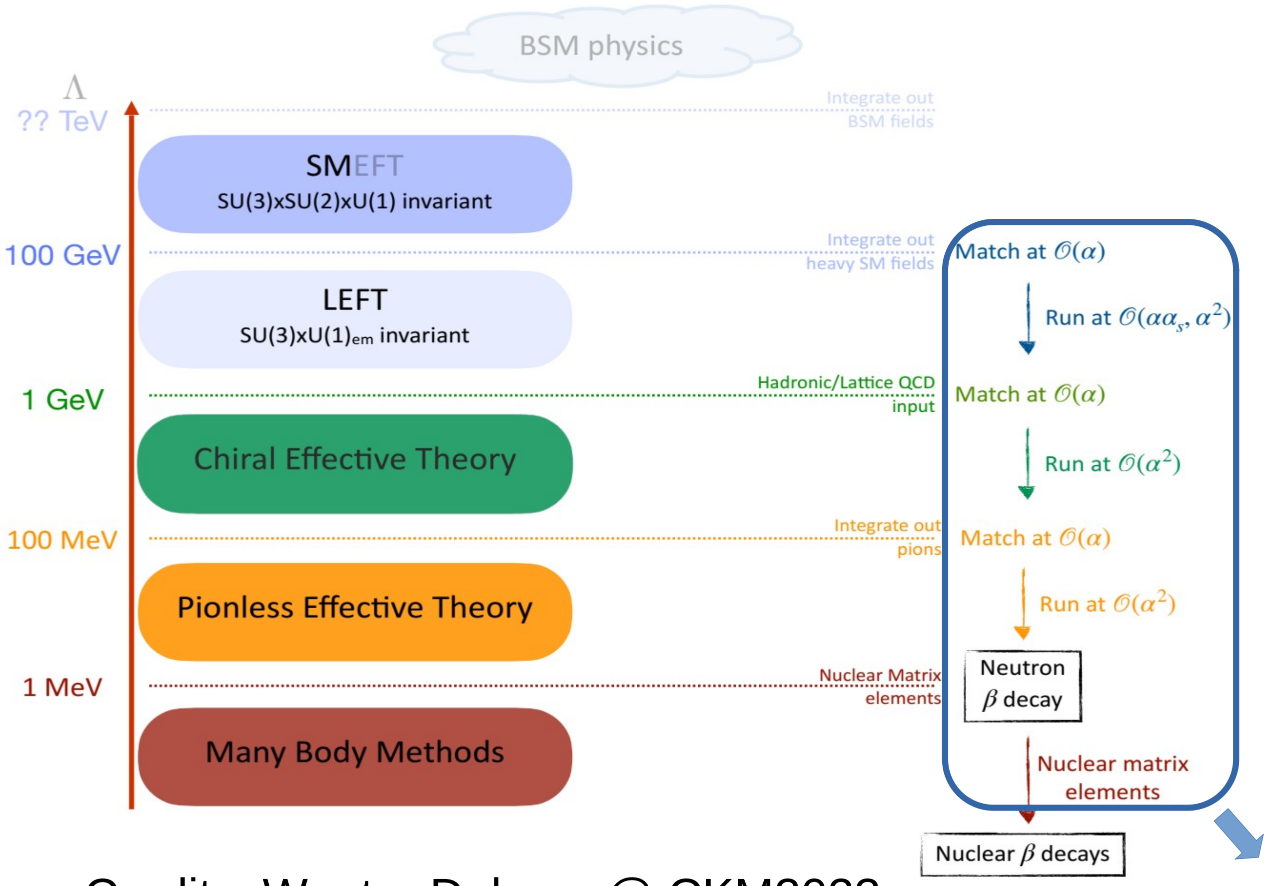
Vector-like fermions (e.g. Belfatto *et. al.*, **JHEP**10(2021)079), LFU Violation (e.g. Crivellin *et. al.*, **Phys. Rev. D** 103) or SMEFT (e.g. Alok *et. al.*, **Phys. Rev. D.** 108)

- Yet, in order to shed light on the nature of anomalies, more precise theoretical predictions are needed!

High-scale & low-scale higher-order effects

Higher-order corrections to β decays

EFT For Neutron Decays



Match @ $\mathcal{O}(\alpha, \alpha\,\alpha_s)$ $\Lambda \simeq M_W$

Run @ $\mathcal{O}(\alpha^2, \alpha\,\alpha_s^2)$

Match @ $\mathcal{O}(\alpha, \alpha\,\alpha_s)$ $\Lambda \simeq m_N$

[Cirigliano et. al., ‘23]

Credits: Wouter Dekens @ CKM2023

High-scale Matching

- At scales $\mu \ll M_W$, SM approximated by a Hamiltonian comprising a single charged current-current operator

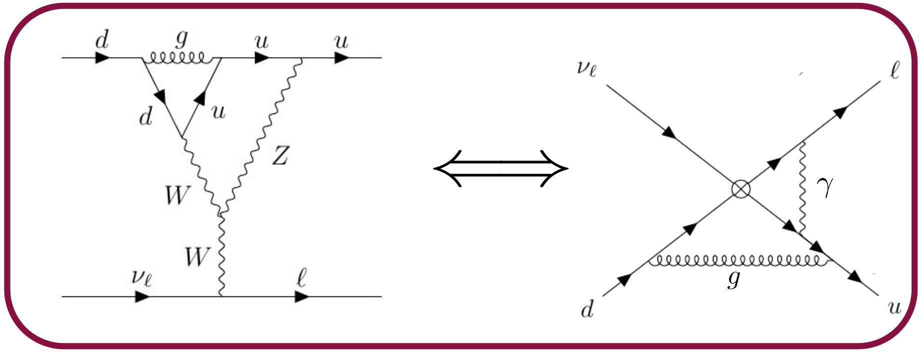
$$\mathcal{H} \propto C_O(x)\, O(x), \quad O = (\bar{d}\, u)_{V-A} \otimes (\bar{\nu}_\ell\, \ell)_{V-A}$$

Tree level Wilson Coefficient: $C_O = G_F |V_{ud}|$

- First corrections at $\mathcal{O}(\alpha)$ [Gambino & Haisch, 2001], [Brod & Gorbahn, 2008]

$$C_O = C_O^{(0)} + \frac{\alpha}{4\pi}\left(C^{(e)} + \frac{\alpha_s}{4\pi} C^{(es)}\right)$$

- We compute the two-loop $\mathcal{O}(\alpha\alpha_s)$ QEDxQCD EW corrections



Matching imposed at $\mu_W \simeq M_W$

Running of Wilson Coefficient C_O

Arbitrary renormalisation scale μ

Evolution Kernel

- $C_O^{\overline{\mathbf{MS}}}(\mu) = \mathcal{U}^{\overline{\mathbf{MS}}}(\mu, \mu_W)\, C_O^{\overline{\mathbf{MS}}}(\mu_W)$ $\quad\longleftarrow\quad \mathcal{U}^{\overline{\mathbf{MS}}}(\mu, \mu_W)$

Wilson coefficient at EW scale

- Renormalisation Group Equation (RGE)

$$\boxed{\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{U}^{\overline{\mathbf{MS}}} = \gamma_O^{\overline{\mathbf{MS}}} \mathcal{U}^{\overline{\mathbf{MS}}}}$$

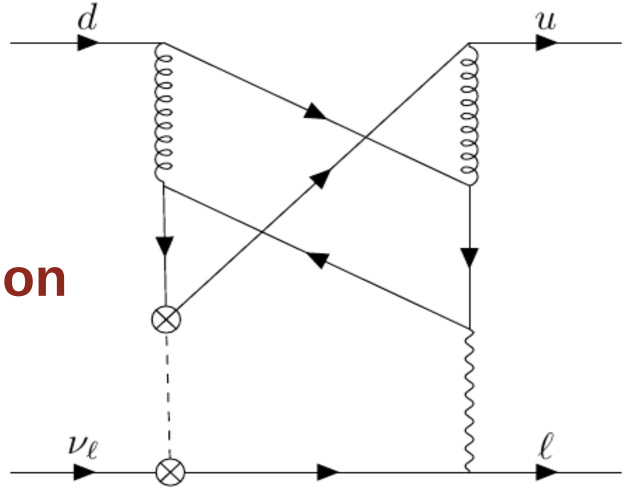
Anomalous dimension (ADM)

- $\gamma_O^{\overline{\mathbf{MS}}} \Longleftrightarrow \boxed{Z_O^{\overline{\mathbf{MS}}}}$

3-loop EFT renormalisation

The anomalous dimension is related to the single 1/ε poles in the renormalisation matrix

Known up to $\mathcal{O}(\alpha^2, \alpha\alpha_s)$ $\quad \gamma_O = \frac{\alpha}{4\pi}\left(\gamma^e + \frac{\alpha}{4\pi}\gamma^{ee} + \frac{\alpha_s}{4\pi}\gamma^{es} + \boldsymbol{\frac{\alpha_s^2}{16\pi^2}\gamma^{ess}}\right)$



χPT Description of Neutron Decay

Heavy nucleon fields

Nucleon velocity

Isospin matrix

- $$\mathcal{L} \supset \sqrt{2}\, G_F V_{ud} \bar{e} \gamma^\mu_L \nu_e \bar{N}(g_V v^\mu - 2 g_A S^\mu)\tau^+ N$$

g_V and g_A are vector and axial coupling

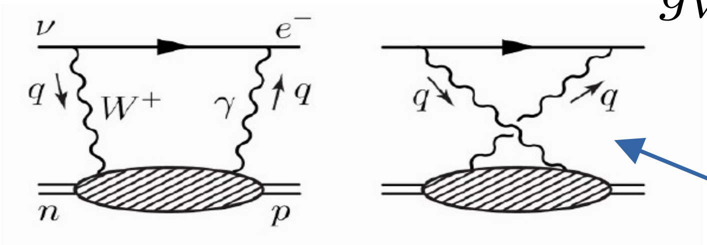
Nucleon Spin

Enters $0^+ \to 0^+$ transitions

- The vector coupling can be expressed as [Cirigliano et al, '23]

$$g_V = C_O \times \hat{m}$$

Matching coefficient between Weak EFT and χ_{PT}



$$\hat{m} = m(\mu, \mu_\chi)(1 + \overline{\Box}^V_{\text{Had}})$$

- $\hat{m}$ receives non-pertutbative contributions from γW box diagrams

Matching Beyond Leading-Log

- Isoscalar contributions to $\Box_{\gamma W}$: $T_0^{\mu\nu}(q^2) = \int \mathrm{d}^d x\, e^{iq\cdot x}\mathrm{T}\left[J_W^\mu(x) J_{V,0}^\nu(0)\right]$ $\quad Q_+ = Q_u + Q_d$

$$J_W^\mu = \bar{d}\gamma^\mu P_L u \qquad J_{V,0}^\nu = Q_+/2(\bar{u}\gamma^\nu u + \bar{d}\gamma^\nu d)$$

- We introduce intermediate scheme: subtraction of UV limit through OPE

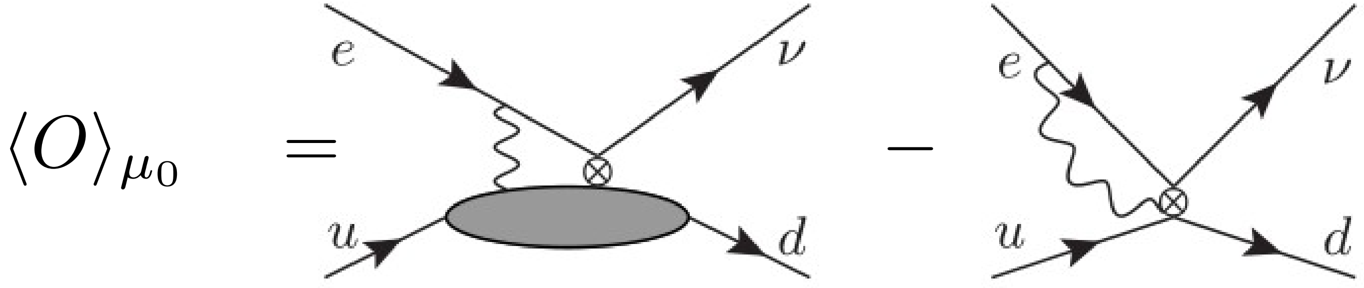
$$T_{S,0}^{\mu\nu}(q^2) = \frac{Q_+ q_\rho}{2(q^2-\mu_0^2)}\left\{\left[1 - \frac{\alpha_s}{\pi}\mathbf{1}_\epsilon\right]\boldsymbol{o}^{\mu\rho\nu} + \left[1 - \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{\mu_0^2-q^2}\right)^\epsilon\left(\frac{28}{3}+12\epsilon\right)\right]\boldsymbol{e}^{\mu\rho\nu}\right\}$$

Shift q^2 by a hard scale μ_0

$$\mathbf{1}_\epsilon = \frac{7+9\epsilon}{3}\left(\frac{\mu^2}{\mu_0^2-q^2}\right)^\epsilon - \frac{4}{3} \xrightarrow{d\to 4} 1 \qquad \boldsymbol{o}^{\mu\rho\nu} = 2i\varepsilon^{\mu\nu\rho\sigma}\bar{d}\gamma^\sigma P_L u \qquad \boldsymbol{e}^{\mu\rho\nu} = \bar{d}(\gamma^\mu\gamma^\rho\gamma^\nu - \gamma^\nu\gamma^\rho\gamma^\mu)P_L u - \left(1+\frac{4\alpha_s}{3\pi}\right)\boldsymbol{o}^{\mu\rho\nu}$$

- This form of OPE recorvers the correct 4-dimensional behaviour [Cirigliano et al, ‘23]

Matching Beyond Leading-Log



- Match to $\overline{\text{MS}}$ according to $Z^{s.c.}_{O,i} Z_{i,j} \langle Q_j \rangle = \langle O \rangle_{\mu_0}$ Inclusion of evanescent operator

Main advantage of intermediate scheme – not needed at LL-QCD accuracy

- $$m(\mu, \mu_\chi) = \left(Z^{s.c.}_{O,O}\right)^{-1} = 1 + \frac{\alpha}{4\pi} Q_e Q_+ \left[m^e + \frac{\alpha_s}{4\pi} m^{es}\right]$$

Agreement with [Cirigliano et al, ‘23]

New results – to be combined with 3L ADM γ^{ess}

Renormalisation Group Improvement

- $C^{(f)}(\mu) = \overbrace{J_f(\mu) u_f(\mu) u_f^{-1}(\mu_0) J_f^{-1}(\mu_0)}^{\mathcal{U}^{\overline{MS}}} C^{(f)}(\mu_0)$

$$J_f(\mu) = 1 + \frac{\alpha^{(f)}(\mu)}{4\pi}\delta J_e + \frac{\alpha}{4\pi}\frac{\alpha_s^{(f)}(\mu)}{4\pi}\delta J_{es}$$

NLL: $\sim \alpha(\alpha \ln)^n \quad \alpha\alpha_s(\alpha_s \ln)^n$

$$u_f(\mu) = \alpha^{(f)}(\mu)^{\frac{\gamma^e}{2\beta_0}} \alpha_s^{(f)}(\mu)^{-\frac{\alpha}{4\pi}\frac{\gamma^{es}}{2\beta_{0,s}}}$$

LL: $\sim (\alpha \ln)^n \quad \alpha(\alpha_s \ln)^n$

- We define scheme independent quantities
  - $\hat{C}_5 = u_5^{-1}(\mu) j_5^{-1}(\mu) C^{(5)}(\mu)$
  - $\hat{M}_{f\downarrow} = u_{f-1}^{-1}(\mu) j_{f-1}^{-1}(\mu) j_f(\mu) u_f(\mu)$
  - $\hat{M}_{\mu_\chi} = \left[1 + \overline{\Box}^V_{\mathrm{Had}}(\mu_0)\right] m_{\mu_0}(\mu, \mu_\chi) j_3(\mu) u_3(\mu)$

Cancellation of scheme and scale dependence can be checked at each order

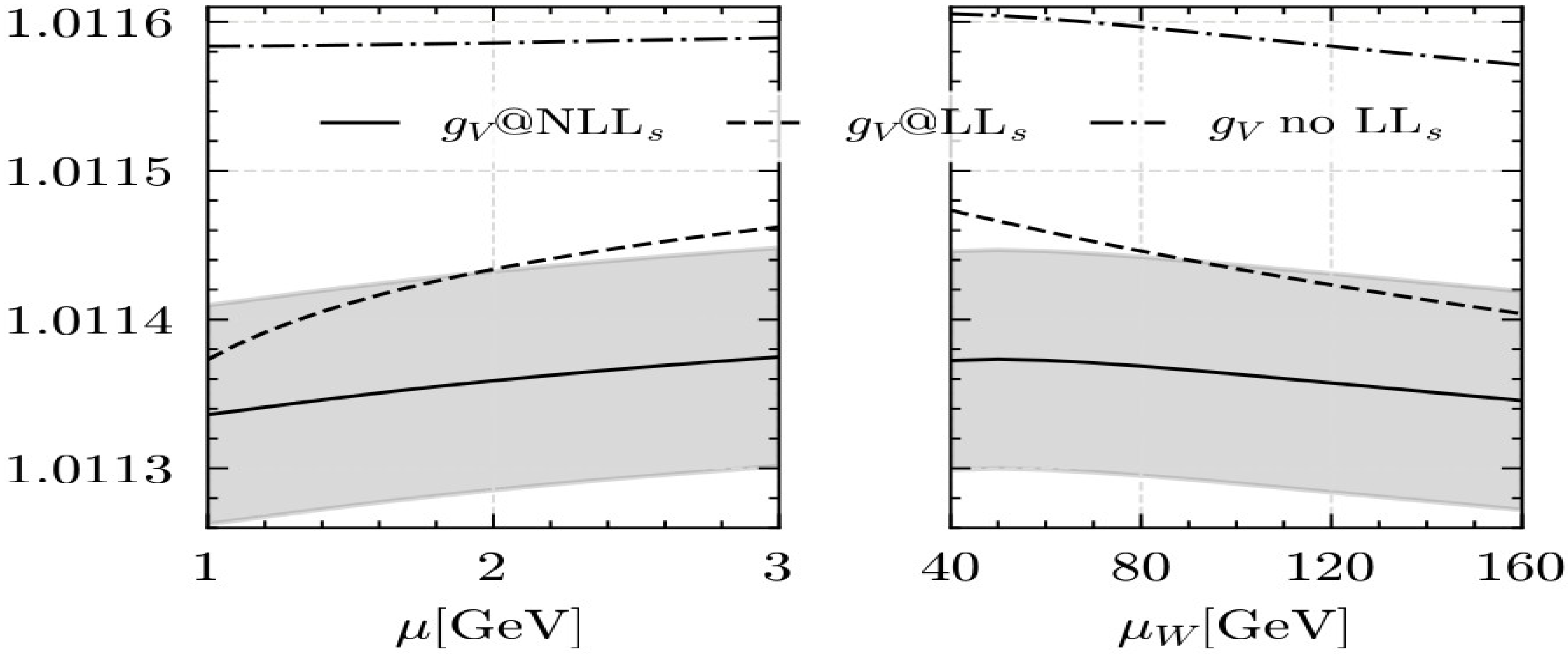
$$g_V(\mu_\chi) = \hat{M}_{\mu_\chi} \hat{M}_{4\downarrow} \hat{M}_{5\downarrow} \hat{C}_5$$

$$\Delta_R^V = g_V^2(m_n)\left(1 + \frac{5\alpha(m_n)}{8\pi}\right) - 1$$

[Cirigliano et al., '23]

Renormalisation Group Improvement

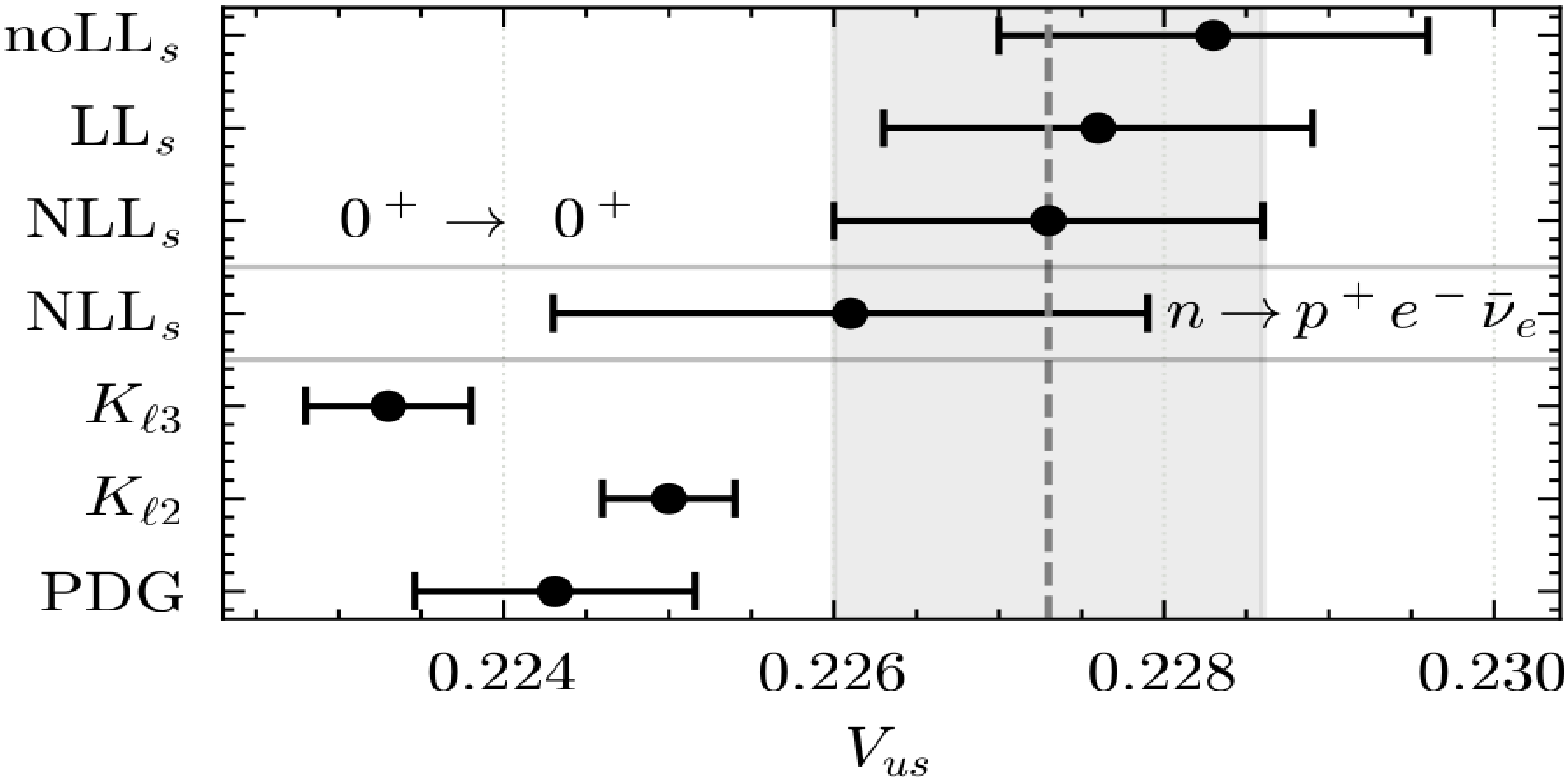
- We use lattice determination $\square_{\gamma W}^{V} = 1.490(73) \times 10^{-3}$ [Ma et al., '23]



$$(g_V - 1) \cdot 10^6 \overset{\mathbf{LLs}}{=} 11426^{+60}_{-57} \xrightarrow{\mathbf{NLLs}} 11355^{+24}_{-23}$$

1st Row CKM Unitarity

V_{us} from V_{ud} + unitarity



Conclusions

Conclusions

- Recent analyses have shown tensions up to 3σ in the unitarity test of the 1st row of CKM More precise SM predictions are needed to shed light on the nature of these tensions.

- Improved matching between the SM and the Weak EFT $\Longrightarrow$ Extraction of Wilson Coefficient at $\mathcal{O}(\alpha\ \alpha_s)$

- Improved running of the renormalised Wilson Coefficient in the $\overline{\text{MS}}$ – 3 loop ADM

- Higher-order QCD corrections to low-scale matchcing

- Updated value of universal radiative corrections Δ_R^V and analyses on the impact on the determination of V_{ud} from nuclear decays – test of CKM unitarity

Thank You! **Happy Birthday, Uli!**

Back Up

Non Perturbative Input

- Last ingredient to determine g_V is given by

$$\overline{\Box}^{V}_{\mathrm{Had}}(\mu_0) = -e^2 \int \frac{i d^4 q}{(2\pi)^4} \frac{\nu^2 + Q^2}{Q^4} \frac{T_3(\nu, Q^2)}{2 m_N \nu} + e^2 \int \frac{i d^4 q}{(2\pi)^4} \frac{2}{3} \frac{\nu^2 + Q^2}{Q^6 + \mu_0^2 Q^4} \left(1 - \frac{\alpha_s(\mu_0^2)}{\pi}\right)$$

One-dimensional integral over Nachtmann moment [Seng et al., '18]

Split into two regions

Low $Q^2=-q^2$ $(\lesssim Q_0^2 = 2\ \mathrm{GeV}^2)$

$$\Box^{V<}_{\gamma W}(Q_0^2)$$

High $Q^2=-q^2$ $(\gtrsim Q_0^2 = 2\ \mathrm{GeV}^2)$

DIS

$$\frac{\alpha}{8\pi}\left[1 - \frac{\alpha_s(\mu_0)}{\pi}\right] \log \frac{\mu_0^2}{Q_0^2}$$

Non Perturbative Input

Reduced Hadronic Uncertainty in the Determination of V_{ud}

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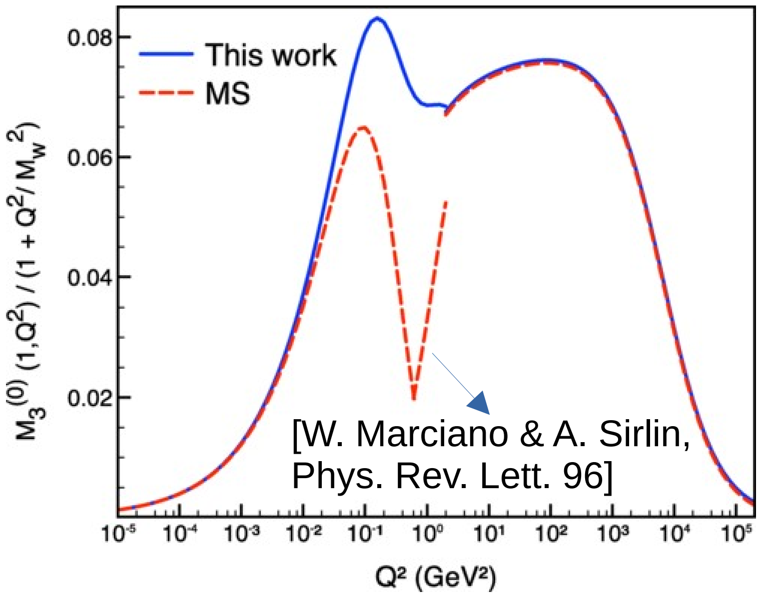
(Received 16 August 2018; published 14 December 2018)

We analyze the universal radiative correction Δ_R^V to neutron and superallowed nuclear β decay by expressing the hadronic γW-box contribution in terms of a dispersion relation, which we identify as an integral over the first Nachtmann moment of the γW interference structure function $F_3^{(0)}$. By connecting the needed input to existing data on neutrino and antineutrino scattering, we obtain an updated value of $\Delta_R^V = 0.02467(22)$, wherein the hadronic uncertainty is reduced. Assuming other standard model theoretical calculations and experimental measurements remain unchanged, we obtain an updated value of $|V_{ud}| = 0.97370(14)$, raising tension with the first row Cabibbo-Kobayashi-Maskawa unitarity constraint. We comment on ways current and future experiments can provide input to our dispersive analysis.

DOI: 10.1103/PhysRevLett.121.241804



$$\Box_{\gamma W}^{V<} = 1.62(10)\times 10^{-3}$$

Non Perturbative Input

Lattice QCD Calculation of Electroweak Box Contributions to Superallowed Nuclear and Neutron Beta Decays

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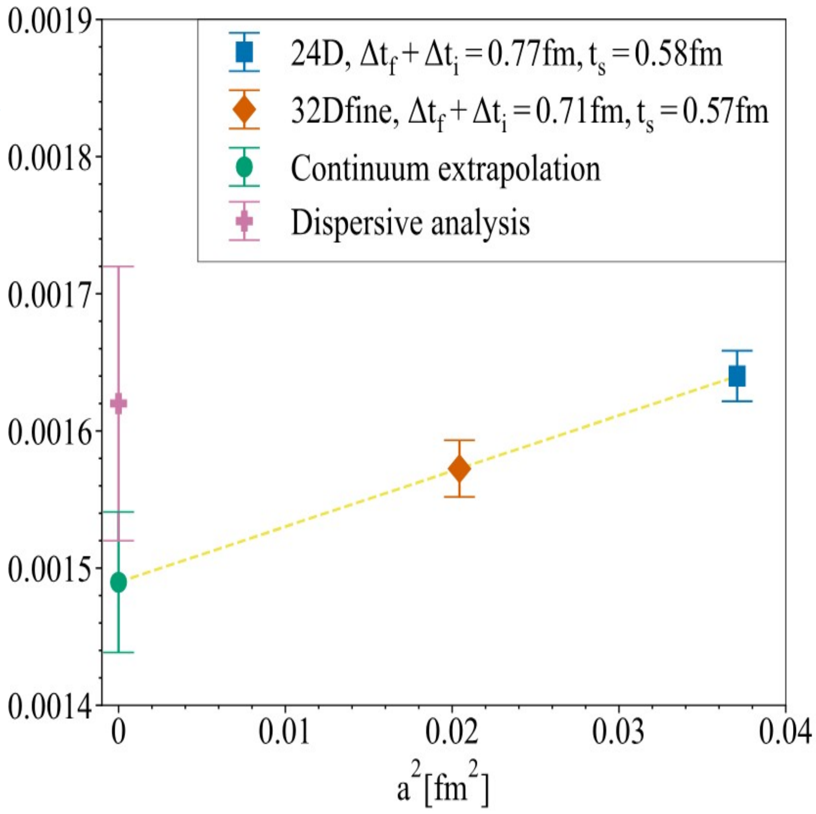
(Received 1 September 2023; revised 23 January 2024; accepted 16 April 2024; published 8 May 2024)

We present the first lattice QCD calculation of the universal axial γW-box contribution $\square_{\gamma W}^{VA}$ to both superallowed nuclear and neutron beta decays. This contribution emerges as a significant component within the theoretical uncertainties surrounding the extraction of $|V_{ud}|$ from superallowed decays. Our calculation is conducted using two domain wall fermion ensembles at the physical pion mass. To construct the nucleon four-point correlation functions, we employ the random sparsening field technique. Furthermore, we incorporate long-distance contributions to the hadronic function using the infinite-volume reconstruction method. Upon performing the continuum extrapolation, we arrive at $\boxed{\square_{\gamma W}^{VA} = 3.65(7)_{\mathrm{lat}}(1)_{\mathrm{PT}} \times 10^{-3}}$. Consequently, this yields a slightly higher value of $|V_{ud}| = 0.97386(11)_{\mathrm{exp}}(9)_{\mathrm{RC}}(27)_{\mathrm{NS}}$, reducing the previous 2.1σ tension with the CKM unitarity to 1.8σ. Additionally, we calculate the vector γW-box contribution to the axial charge g_A, denoted as $\square_{\gamma W}^{VV}$, and explore its potential implications.

DOI: 10.1103/PhysRevLett.132.191901



$$\square_{\gamma W}^{V<} = 1.490(73) \times 10^{-3}$$