Update of the Standard-Model prediction for $\bar{B} \to X_s\gamma$

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Based on:

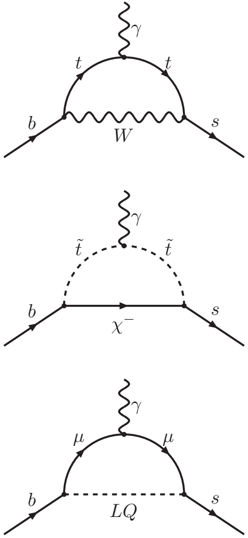
	K. Brune, L.-T. Moos, TH	2509.22564, JHEP01(2026)142
	M. Czakon, M. Czaja, M. Misiak, M. Niggetiedt, A. Rehman, K. Schönwald, M. Steinhauser, TH	to appear

UliFest, Karlsruhe, July 3rd, 2026

Motivation to study $\bar{B} \to X_s\gamma$

- X_s: Hadronic system with net-strangeness $S = -1$
  and excluding charm
- Dominant contribution: quark-level $b \to s\gamma$ transition
  - Flavour-changing neutral current (FCNC) process
  - Loop-induced in SM
  - GIM suppression lifted by large top-quark mass
  - Sensitive to new degrees of freedom via virtual effects
    - Standard candle for many new-physics analyses
    - Precision required on theoretical and experimental side
- Envisaged precision at Belle II is $\pm 2.6\%$ [Belle II Physics Book, 1808.10567]

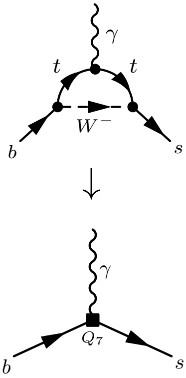
  $\Longrightarrow$ Goal: shrink theory uncertainty as much as possible



Weak effective Hamiltonian

- Integrate out heavy particles $W^\pm$, Z, H, t

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QED+QCD} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[-\sum_{i=1}^{2} C_i \sum_{p=u,c} \frac{V_{ps}^* V_{pb}}{V_{ts}^* V_{tb}} Q_i^p + \sum_{i=3}^{8} C_i Q_i \right] + \text{h.c.}$$



- Effective operators

$$\begin{aligned}
Q_1^p &= (\bar{s}_L \gamma_\mu T^a p_L)(\bar{p}_L \gamma^\mu T^a b_L), & Q_5 &= (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\sigma b_L) \textstyle\sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\sigma q), \\
Q_2^p &= (\bar{s}_L \gamma_\mu p_L)(\bar{p}_L \gamma^\mu b_L), & Q_6 &= (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\sigma T^a b_L) \textstyle\sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\sigma T^a q), \\
Q_3 &= (\bar{s}_L \gamma_\mu b_L) \textstyle\sum_q (\bar{q} \gamma^\mu q), & Q_7 &= \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}, \\
Q_4 &= (\bar{s}_L \gamma_\mu T^a b_L) \textstyle\sum_q (\bar{q} \gamma^\mu T^a q), & Q_8 &= \frac{g}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}.
\end{aligned}$$

- Supplemented by evanescent operators

- Wilson Coefficients C_i (effective couplings) carry effects of $W^\pm$, Z, H, t

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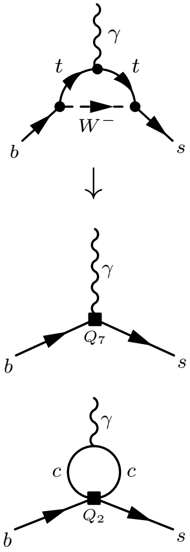

History

- Major updates
  - 2001: NLO $\mathcal{B}(\bar{B} \to X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}} = (3.60 \pm \underset{8.3\%}{\underset{\uparrow}{0.30}}) \times 10^{-4}$ [Gambino,Misiak hep-ph/0104034]

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    $\mathcal{B}_{s\gamma}$: CP- and isospin averaged branching ratio

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  - 2026: NNLO $\mathcal{B}_{s\gamma} =$?? [few more mins. :-)] [Misiak et al. 260n.nnnnn]

    exact m_c-dep. of $Q_{1,2} - Q_7$ interference, multi-body contributions to NLO, new normalization

Recent developments

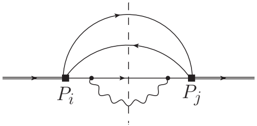
- Heavy-quark expansion

$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^{\rm parton}\gamma)_{E_\gamma > E_0} + \mathcal{O}(\Lambda_{\rm QCD}/m_b) \qquad \delta = 1 - \frac{2E_0}{m_b}$$

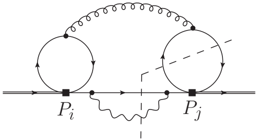
$$\begin{aligned}\Gamma(b \to X_s^{\rm parton}\gamma)_{E_\gamma > E_0} &= \Gamma(b \to s\gamma) + \Gamma(b \to sg\gamma) + \Gamma(b \to sq\bar{q}\gamma) + \Gamma(b \to sq\bar{q}g\gamma) + \ldots \\ &= \frac{G_F^2 m_b^5 \alpha_e |V_{ts}^* V_{tb}|^2}{32\pi^4} \sum_{i,j} C_i^{\rm eff\,*}(\mu)\, C_j^{\rm eff}(\mu)\, \hat{G}_{ij}(\mu, z_c, E_0)\end{aligned}$$

- Recent developments on perturbative side [non-pert. see later]
  - Multi-parton contributions $b \to s\, q\bar{q}\,(g)\,\gamma$ at NLO [Brune,Moos,TH'25]
  - Exact m_c-dependence of bare two-body $Q_{1,2} - Q_7$ interference at NNLO
    [Greub,Asatrian,Saturnino,Wiegand'23; Czakon,Czaja,Misiak,Niggetiedt,Rehman,Schönwald,Steinhauser,TH'23]
    [Fael,Lange,Schönwald,Steinhauser'23; Greub,Asatrian,Asatryan,Born,Eicher'24]
  - Exact m_c-dependence of complete $Q_{1,2} - Q_7$ interference at NNLO
    [Czakon,Czaja,Misiak,Niggetiedt,Rehman,Schönwald,Steinhauser,TH'26]
  - Partial $\mathcal{O}(\alpha_s^3)$-corrections to the $Q_7 - Q_7$ contribution [Fael's talk]

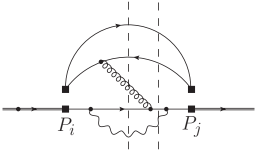
Multi-body contributions to NLO



[Kaminski,Misiak,Poradzinski'12]

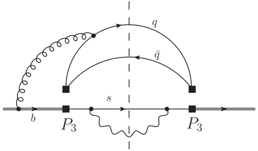


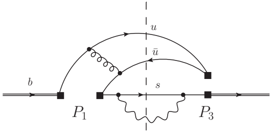
[Poradzinski,Virto,TH'14]



[Brune,Moos,TH'25]

- Partonic contributions of the type $b \to s\, q\bar{q}\,(g)\,\gamma$ with cut $E_\gamma \geq E_0$ on photon energy
  - $\sim$ 180 one-loop 4-body $b \to s\, q\bar{q}\,\gamma$ diagrams per interference
  - $\sim$ 400 tree-level 5-boby $b \to s\, q\bar{q}\, g\,\gamma$ diagrams per interference





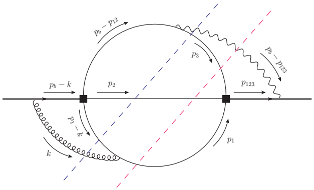
- Master formula: $$G_{ij}^{(1)}(\mu, E_0) = V_{ji} + V_{ij}^* + R_{ji} + S_{ji} + Z_\psi T_{ji} + Z_\psi T_{ij}^* + M_{ji} + M_{ij}^* + \sum_k \left\{ Z_{jk} T_{ki} + Z_{ik} T_{kj}^* \right\}$$

Multi-body contributions to NLO

- Stay differential in photon energy $\dfrac{2\,E_\gamma}{m_b} = \dfrac{2\,p_b \cdot p_\gamma}{m_b^2} \equiv 1 - z \equiv \bar{z}$, implement via factor $\delta(\bar{z} - s_{ij})$
- Reverse unitarity [Anastasiou,Melnikov'02]

$$-2\pi i\,\delta(A) = \frac{1}{A + i\eta} - \frac{1}{A - i\eta}$$

  - turns all integrals into (partially massive) four-loop propagators
- Seven topologies in total

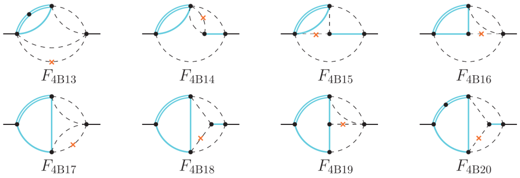


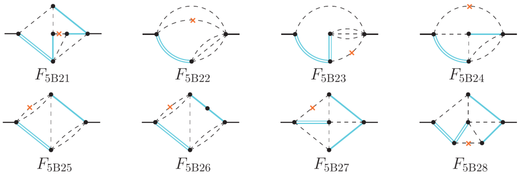
$$\mathcal{T}_2 = \Big\{ (p_b - p_{123})^2, 2p_b(p_b - p_{123}) - m_b^2\,\bar{z}, p_1^2, p_2^2, p_3^2, k^2, (p_3 - k)^2, (p_{123} - k)^2 - m_b^2, \\ (p_b - k)^2 - m_b^2, (p_b - p_{12})^2, (p_1 - k)^2, (p_b - p_{13})^2, (p_b - p_1)^2, (p_b - p_3)^2 \Big\}$$

- Integral reduction to $\sim 30 + 30$ four- and five-body master integrals

Multi-body contributions to NLO

- Techniques for computing masters:
  - Direct integration over D-dim. phase space
  - Differential equations, boudaries from
    - Asymptotic expansions (${}_pF_q$, Mellin-Barnes)
    - Integration over $\bar{z}$
- Everything analytic in terms of HPLs and GPLs
  - weights $\{0, \pm 1, \pm \frac{i}{\sqrt{3}}\}$
  - arguments $\{\bar{z}, i\sqrt{\frac{\bar{z}}{4-\bar{z}}}\}$





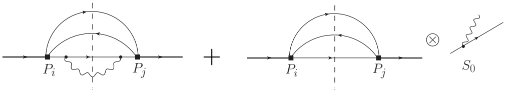
Multi-body contributions to NLO

- UV renormalization via standard procedure
- IR subtraction

$$\frac{\mathrm{d}\Gamma_m}{\mathrm{d}z} = \frac{\mathrm{d}\Gamma_\epsilon}{\mathrm{d}z} + \frac{\mathrm{d}\Gamma_{\text{shift}}}{\mathrm{d}z}$$

$$\begin{aligned} T_{ij} &= T_{ij}^{\text{bare}} + T_{ij}^{\text{no }\gamma} \otimes S_0\,, \qquad & R_{ij} &= R_{ij}^{\text{bare}} + R_{ij}^{\text{no }\gamma} \otimes S_0\,, \\ V_{ij} &= V_{ij}^{\text{bare}} + V_{ij}^{\text{no }\gamma} \otimes S_0\,, \qquad & S_{ij} &= T_{ij}^{\text{no }\gamma} \otimes S_1 \end{aligned}$$

$$S_0 = \frac{\alpha_e}{4\pi}\,\frac{2\pi e^{\gamma_E \epsilon}(1-\epsilon)\csc(\pi\epsilon)}{\Gamma(1-\epsilon)}\,\frac{1+\bar{z}_\gamma^2}{z_\gamma}\left(\frac{m_q z_\gamma}{\mu}\right)^{-2\epsilon}$$



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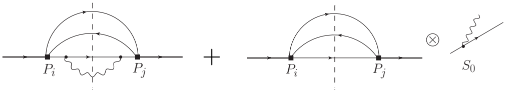


$$S_1 = \frac{\alpha_e\alpha_s}{(4\pi)^2}\,C_F\left(\frac{m_q}{\mu}\right)^{-4\epsilon}\left[\frac{1}{2\epsilon^2}\left(P^{(0)}_{q\to q}\otimes P^{(0)}_{q\to\gamma}\right)(z_\gamma) - \frac{1}{2\epsilon}R^{(1)}_{q\to\gamma}(z_\gamma)\right]$$

- Leading pole follows DGLAP splitting function

$$P^{(0)}_{q\to q}(z_\gamma) = \frac{1+z_\gamma^2}{(1-z_\gamma)_+} + \frac{3}{2}\,\delta(1-z_\gamma)\,, \qquad P^{(0)}_{q\to\gamma}(z_\gamma) = \frac{1+\bar{z}_\gamma^2}{z_\gamma}$$

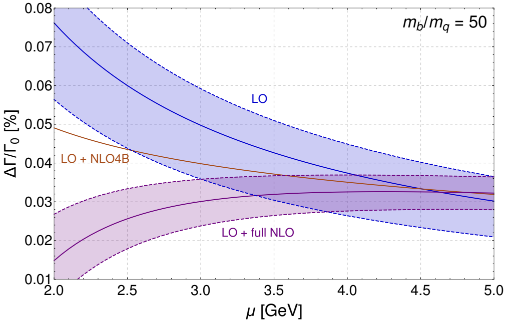
- Subleading contribution is given by

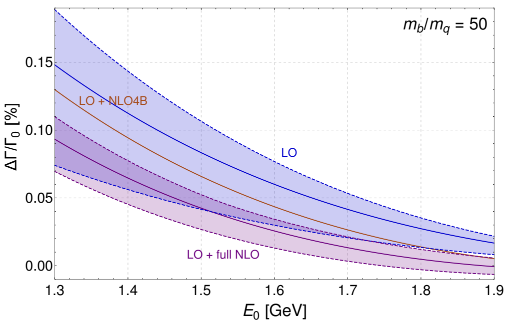
$$\begin{aligned} R^{(1)}_{q\to\gamma}(z_\gamma) = \frac{1+\bar{z}_\gamma^2}{z_\gamma}\Big(&16\ln(1-z_\gamma)\ln(z_\gamma) - 4\ln^2(1-z_\gamma) \\ &+\frac{16}{3}\pi^2 - 30\Big) \\ &- 6(z_\gamma-2)\ln^2(z_\gamma) + \frac{8(z_\gamma-2)^2}{z_\gamma}\ln(1-z_\gamma) \\ &- 2(z_\gamma-16)\ln(z_\gamma) + \frac{2\,(3z_\gamma+10)}{z_\gamma} \end{aligned}$$

Multi-body contributions to NLO

[Brune,Moos,TH'25]

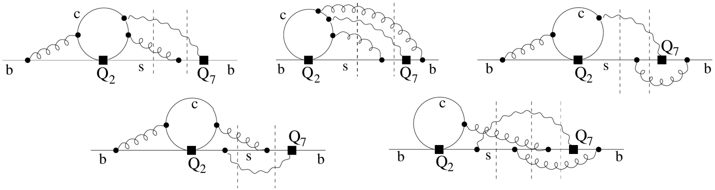
- Final result has expected features
  - Renormalization-group invariance through to NLO
  - Reduced uncertainty and scale dependence at NLO
- Final result is small
  - CKM-, Wilson coefficient-, and phase-space suppression
  - Partial cancellation between LO and NLO contributions





Exact m_c-dependence of $Q_{1,2} - Q_7$ interference at NNLO

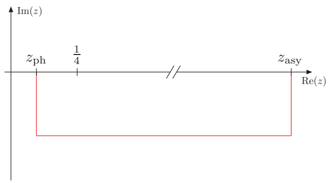
[Czakon,Czaja,Misiak,Niggetiedt,Rehman,Schönwald,Steinhauser,TH'26]



- So far: large m_c limit (extrapolation) [Misiak et al.'06] and $m_c = 0$ (interpolation) [Misiak et al.'15]
  and exact m_c-dep. of fermionic contribution [Misiak,Rehman,Steinhauser'20]
- To remove interpolation uncertainty $(\pm 3\%)$ need to perform calculation for physical value of m_c
- Several hundred 4-loop propagator diagrams with 2-, 3- and 4-particle cuts
- Result in few $\times 10^5$ four-loop, two-scale scalar integrals with unitarity cuts in $\mathcal{O}(500)$ families.
- Integral reduction (Kira 2.0, FIRE 6). Up to $\mathcal{O}(\text{weeks})$ of CPU and $\mathcal{O}(1\text{TB})$ of RAM per topology
  [Maierhöfer,Usovitsch,Uwer'17; Klappert,Lange,Maierhöfer,Usovitsch'20; Smirnov,Chukharev'19]
- $\mathcal{O}(500)$ master integrals

Exact m_c-dependence of $Q_{1,2} - Q_7$ interference at NNLO

- Establish DE in $z = \frac{m_c^2}{m_b^2}$: $\quad \frac{d}{dz} J_i = M_{ij} J_j$
- Solve for 2-, 3-, 4-particle cuts separately
- Boundary conditions as $z \to \infty$: single-scale integrals
  - ${}_pF_q$, Mellin-Barnes, PSLQ



- Use deep asymptotic expansion about $z = \infty$ as starting point for numerical evolution along contour in complex plane
  - use ODEint library, odepack
- Second method: BC at $m_c/m_b = 0.2$ with AMFlow [Liu,Ma'22], evolve DE with method "expand and match" [Fael,Lange,Schönwald,Steinhauser'21]
- For 2-particle cuts, also use direct calculation with AMFlow at physical point of m_c/m_b

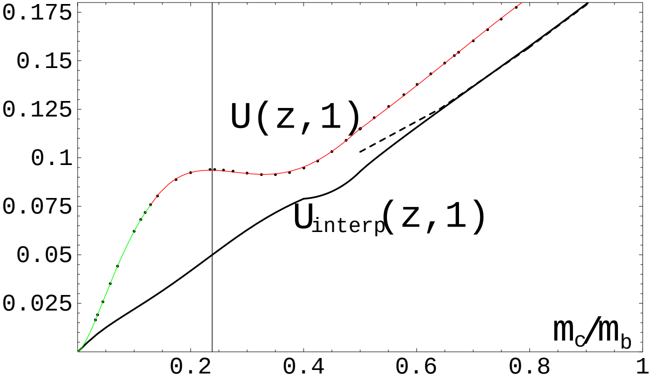
[Czakon,Czaja,Misiak,Niggetiedt,Rehman,Schönwald,Steinhauser,TH'23]

[see also Greub,Asatrian,Saturnino,Wiegand'23; Fael,Lange,Schönwald,Steinhauser'23; Greub,Asatrian,Asatryan,Born,Eicher'24]

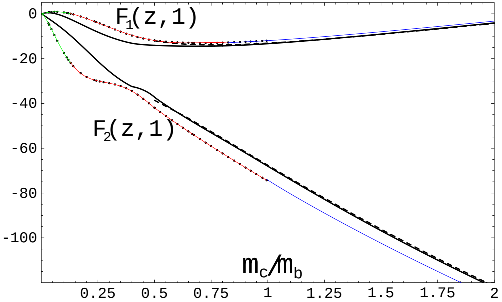
- Final numerical accuracy in physical region of z is ~ 10 digits

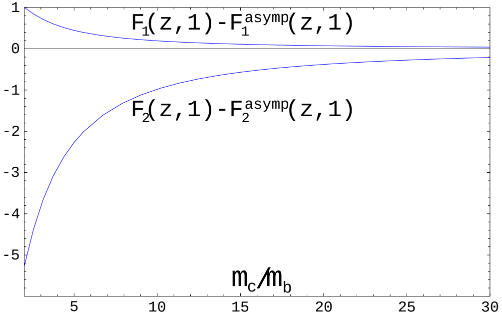
Exact m_c-dependence of $Q_{1,2} - Q_7$ interference at NNLO

$$\frac{\Delta\mathcal{B}_{s\gamma}}{\mathcal{B}_{s\gamma}^{\rm LO}} \simeq U(z,\delta) \equiv \frac{\alpha_s^2(\mu_b)}{8\pi^2} \frac{C_1^{(0)} F_1(z,\delta) + \left(C_2^{(0)} - \frac{1}{6} C_1^{(0)}\right) F_2(z,\delta)}{C_7^{(0)} - \frac{1}{3}C_3^{(0)} - \frac{4}{9}C_4^{(0)} - \frac{20}{3}C_5^{(0)} - \frac{80}{9}C_6^{(0)}}$$



$$\frac{\Delta\mathcal{B}_{s\gamma}}{\mathcal{B}^{\rm LO}_{s\gamma}} \simeq U(z,\delta) \equiv \frac{\alpha_s^2(\mu_b)}{8\pi^2}\, \frac{C_1^{(0)}F_1(z,\delta) + \left(C_2^{(0)} - \frac{1}{6}C_1^{(0)}\right)F_2(z,\delta)}{C_7^{(0)} - \frac{1}{3}C_3^{(0)} - \frac{4}{9}C_4^{(0)} - \frac{20}{3}C_5^{(0)} - \frac{80}{9}C_6^{(0)}}$$





$$F_1(z,1) - F_1^{\rm asymp}(z,1) \simeq (0.598\log^2(z) + 1.261\log(z) + 0.735)/z + \mathcal{O}(\log^2(z)/z^2))$$

$z = \frac{m_c^2}{m_b^2}$

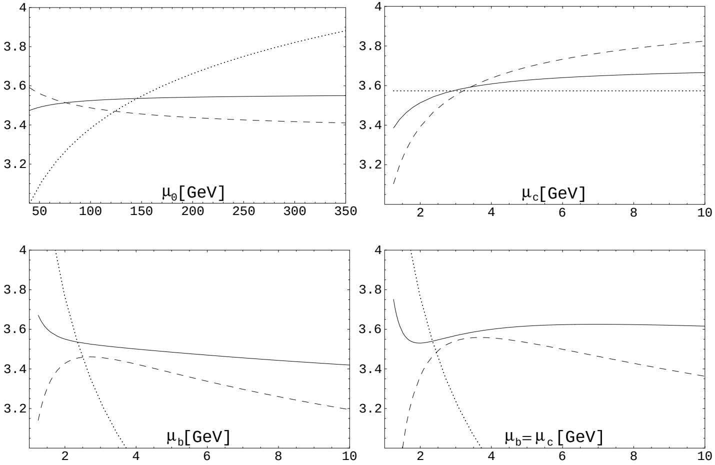
$$F_2(z,1) - F_2^{\rm asymp}(z,1) \simeq (-3.140\log^2(z) - 5.537\log(z) - 5.330)/z + \mathcal{O}(\log^2(z)/z^2))$$

Branching ratio: scale dependence

- μ_0: Matching scale, $\mu_0 \sim \mathcal{O}(M_W, m_t)$
- μ_c: Scale at which m_c is renormalized
- μ_b: Scale at which α_s and Wilson coefficients are evaluated and decay rate is computed.

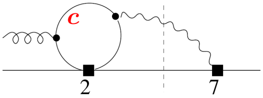
- dotted: LO
- dashed: NLO
- solid: NNLO

Vertical axes: BR in units of 10^{-4}



Nonpert. corrections: resolved photons in $Q_{1,2} - Q_7$

[see e.g. Voloshin'96; Buchalla,Isidori,Rey'97; Benzke,Lee,Neubert,Paz'10; Gunawardana,Paz'19; Benzke,Hurth'20; Bartocci,Böer,Hurth'24; Benzke,Garzelli,Hurth'25]



$$\Delta\Gamma \propto \left(C_2 - \tfrac{1}{6}C_1\right)C_7 \underbrace{\left[-\frac{\mu_G^2}{27m_c^2} + \frac{\Lambda_{17}}{m_b}\right]}_{\equiv -\frac{\kappa_V\,\mu_G^2}{27m_c^2}}$$

$$\Lambda_{17} = \tfrac{2}{3}\mathrm{Re}\int_{-\infty}^{\infty}\frac{d\omega_1}{\omega_1}\left[1 - F\left(\frac{m_c^2 - i\eta}{m_b\omega_1}\right) + \frac{m_b\omega_1}{12m_c^2}\right] h_{17}(\omega_1,\mu)$$

$F(x) = 4x\arctan^2(1/\sqrt{4x^2-1})$

- Soft function h_{17}

$$h_{17} = \int\frac{dr}{4\pi M_B}e^{-i\omega_1 r}\langle\bar{B}|(\bar{h}S_{\bar{n}})(0)\not{\bar{n}}i\gamma_\alpha^\perp\bar{n}_\beta(S_{\bar{n}}^\dagger gG_s^{\alpha\beta}S_{\bar{n}})(r\bar{n})$$
$$\times(S_{\bar{n}}^\dagger h)(0)|\bar{B}\rangle$$

- Model and constrain h_{17}

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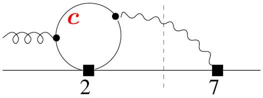
[see e.g. Voloshin'96; Buchalla,Isidori,Rey'97; Benzke,Lee,Neubert,Paz'10; Gunawardana,Paz'19; Benzke,Hurth'20; Bartocci,Böer,Hurth'24; Benzke,Garzelli,Hurth'25]



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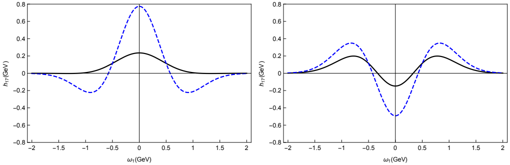
- Model and constrain h_{17}

- Sample class of model functions for h_{17}

$$h_{17} = e^{-\frac{\omega_1^2}{2\sigma^2}}\sum_n a_{2n}\,H_{2n}\left(\frac{\omega_1}{\sigma\sqrt{2}}\right)$$

- Constraints from moments, e.g. $\int d\omega_1 h_{17} = \frac{2}{3}\mu_G^2$



- Numerically [Gunawardana,Paz'19]

$$\Lambda_{17} \in [-24, 5]\ \text{MeV} \Rightarrow \kappa_V = 1.2 \pm 0.3$$

- Other recent studies find larger values and larger uncertainties for κ_V [Hurth et al.'20+]
  - But includes *partial* $\mathcal{O}(\Lambda_{\rm QCD}^2/m_b^2)$ effects

Anatomy of modifications

1	2	3	4	5	total
$+1.5\%$	-1.1%	$+1.1\%$	-0.2%	$+4.2\%$	$+5.5\%$

1. Change of normalization

from $$\mathcal{B}(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \mathcal{B}(\bar{B} \to X_c \ell \bar{\nu})_{\text{exp}} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\text{em}}}{\pi\, C} \left[P(E_0) + N(E_0) \right]$$

to $$\mathcal{B}_{s\gamma} = \frac{G_F^2\, \alpha_{\text{em}}\, m_{b,\text{kin}}^5\, \tau_{\text{av}}}{32\pi^4} \left| V_{ts}^* V_{tb} \right|^2 \left[\widetilde{P}(E_0) + \widetilde{N}(E_0) \right]$$

due to poor behaviour of perturbative series for $C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(\bar{B} \to X_c \ell \bar{\nu})}{\Gamma(\bar{B} \to X_u \ell \bar{\nu})}$ at $\mathcal{O}(\alpha_s^3)$

[Fael,Schönwald,Steinhauser'20; Chen,Li,Li,Wang,WangWu'23; Fael,Usovitsch'23; Chen,Chen,Guan,Ma'26]

and precise knowledge of $m_{b,\text{kin}}$ [Fael,Schönwald,Steinhauser'20]

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1. Change of normalization
2. Update all input parameters
3. Update treatment of non-perturbative resolved photon contributions [Gunawardana,Paz'19; see also Hurth et al.'20+]
4. Include all multi-body contributions to NLO [Kaminski,Misiak,Poradzinski'12; Poradzinski,Virto,TH'14; Brune,Moos,TH'25]
5. Replace interpolated NNLO function $U_{\text{interp.}}(z,1)$ by exact function $U(z,1)$

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- New SM theory prediction for CP- and isospin averaged BR for $E_\gamma > 1.6$ GeV

[Czakon,Czaja,Misiak,Niggetiedt,Rehman,Schönwald,Steinhauser,TH'26]

$$\mathcal{B}_{s\gamma} = (3.54 \pm 0.14) \times 10^{-4}$$

- Total uncertainty is $\pm 4\%$, comprised of
  - $\pm 3\%$ (higher orders at leading and subleading power)

    Retain $\pm 3\%$ despite flat μ-dependences, taking into account potentially sizeable but yet unknown higher-order effects in the power-suppressed, non-perturbative corrections
  - $\pm 2.7\%$ (parametric), added in quadrature

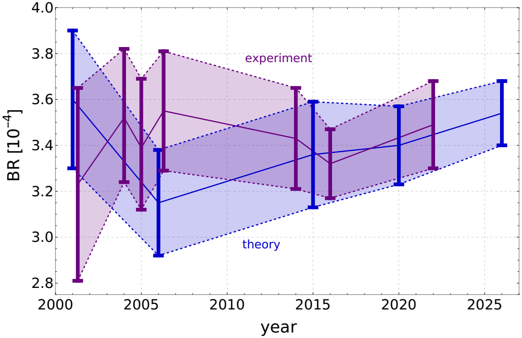
$\bar{B} \to X_s\gamma$: SM theory vs. experiment

- Good agreement between SM theory prediction and latest experimental average

$$\mathcal{B}_{s\gamma}^{\text{th., SM}} = (3.54 \pm 0.14) \times 10^{-4}$$

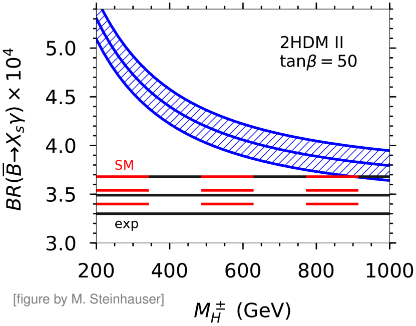
$$\mathcal{B}_{s\gamma}^{\text{exp.}} = (3.49 \pm 0.19) \times 10^{-4}$$

[PDG,HFLAV'22'24]



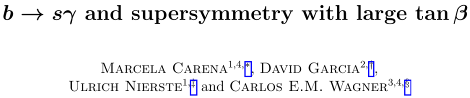
- Constraint on mass of charged Higgs boson in type-II 2HDM

[Ciuchini,Degrassi,Gambino,Giudice'97]



[figure by M. Steinhauser]

- Largely independent of $\tan\beta$ unless $\tan\beta \lesssim 2$
- For $\tan\beta \lesssim 2$ even stronger bounds



$$M_{H^\pm} > 675 \text{ GeV} \quad \text{at } 95\% \text{ C.L.}$$

[Czakon,Czaja,Misiak,Niggetiedt,Rehman,Schönwald,Steinhauser,TH'26]

Conclusion and outlook

- Conclusion
  - We computed
    - $Q_{1,2} - Q_7$ interferences at NNLO for physical value of m_c
    - Multi-parton corrections at NLO (formally completes NLO at leading power)
  - Result
    - has uncertainty of $4\% = \sqrt{(3\%)^2 + (2.7\%)^2}$
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- Outlook
  - Exact m_c-dependence at NNLO for $Q_{3-6} - Q_7$ interferences
  - Take cut on photon-energy in NNLO $Q_{1-6} - Q_7$ into account $\Rightarrow$ additional scale
  - Continue $\mathcal{O}(\alpha_s^3)$ [cf. Fael's talk]
  - Complete $\Lambda_{\text{QCD}}^2/m_b^2$ resolved-photon corrections in $Q_{1,2} - Q_7$ interference
  - Update resolved photons in $Q_8 - Q_8$ interference [Hurth,Szafron'23]

Backup slides