Models and Anomalies from Fermilab to Karlsruhe

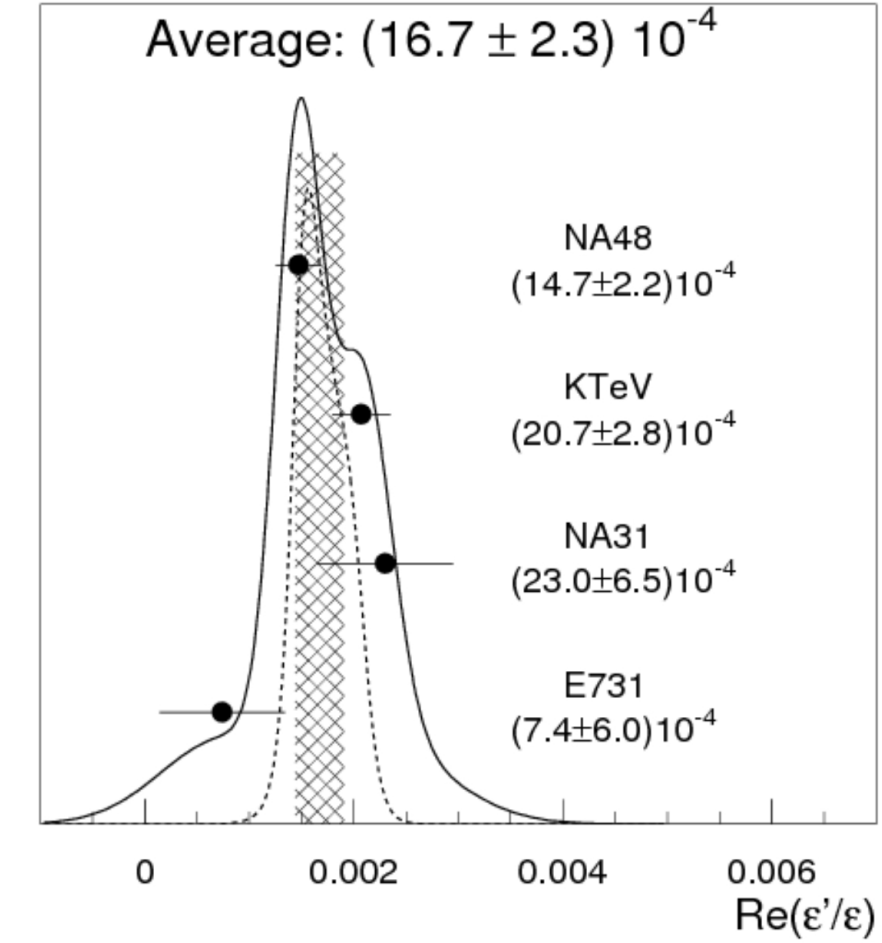
Martin Gorbahn
(University of Liverpool)

Exploring new physics with flavour, 4/July/26



Direct CP violation in 1999

- $\eta_{ij} = \dfrac{\langle \pi^i \pi^j | K_L \rangle}{\langle \pi^i \pi^j | K_S \rangle}$
- $\epsilon_K = (2\eta_{+-} + \eta_{00})/3$
- $\epsilon' = (\eta_{+-} - \eta_{00})/3$
- New KTeV result motivated theory anlaysis: Nierste '99 $\epsilon'/\epsilon = \left(7.7^{+6.0}_{-3.5}\right) \times 10^{-4}$ Bosch et al. '99
- Since ϵ' is direct CP violating, it should discriminate NP models



Why not spontaneous (P &) CP violation

- $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
- 2 Triplets and one Bi-Dublet:
  $\langle \Delta_R \rangle \propto \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$, $\langle \Delta_L \rangle \propto \begin{pmatrix} 0 & 0 \\ v_L e^{i\delta} & 0 \end{pmatrix}$, $\langle \Phi \rangle \propto \begin{pmatrix} k_1 & 0 \\ 0 & k_2 e^{i\alpha} \end{pmatrix}$
- Real couplings, Higgs potential gives complex vevs

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- Real couplings, Higgs potential gives complex vevs
- No go theorem: Model does not decouple to Standard Model
  Barenboim, MG, Uli, Raidal '01
  - Left with light flavour violating neutral Higgs
  - No Pheno: immediatly excluded by tree-level $\Delta S = 2$

Questions and Lessons

- Meson mixing is often a strong constraint
  - Are there symmetry motivated scenarios?
- Deriving Feynman Rules quickly becomes ugly
  - Can we do this for all models?
- Maybe right-handed currents could be useful
- Can we generalise the no go theorem?

Even if anomalies come and go, we learn some physics along the way



Nov 2000 CERN mashup



MSSM → THDM

When I moved to Karlsruhe, we all hoped for an anomaly in $B_s \to \mu\mu$ and hopeful about low energy supersymmetry

- MSSM at infinite $\tan\beta$: Only one Higgs has a vev (SM like Higgs)
- 2nd Higgs receives flavour changing coupling through A-term loops
- Approximate Peccei-Quinn charge conservation (for H_d and b_R) $(\bar{b}_R s_L)(\bar{b}_R s_L)$ requires $\lambda_5 (H_u \cdot H_d)^2$
- λ_5 and other FCNC generated by matching MSSM to THDM

  Gorbahn, Jäger, Nierste, Trine '09
- Has Loop additional loop suppression for ΔM_s and allows large effects in $B_s \to \mu\mu$.

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Was there a tension in $B_s \to \mu\mu$?

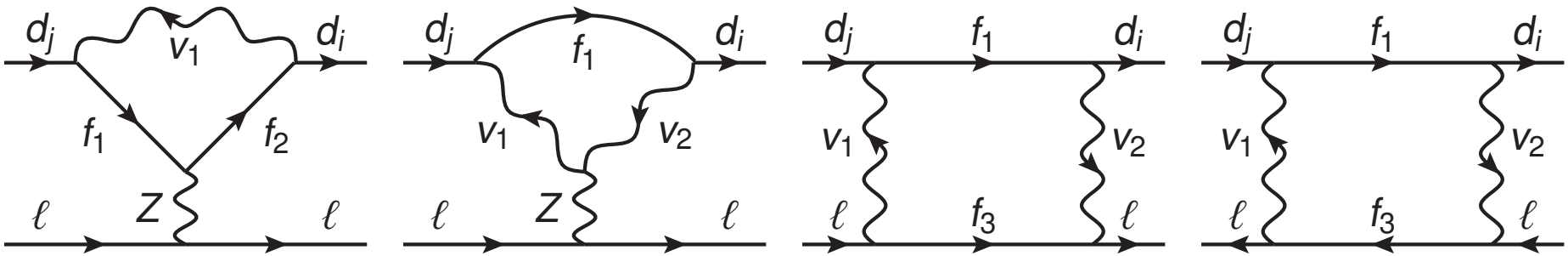

Sind wir nicht alle
ein bissschen Bluna?
Bluna

Calculating without model

Aim: 1-loop results for all renormalisable models:

$$\delta\mathcal{L}_{eff} \supset \frac{C^{sd\nu}_{\sigma\sigma'}}{16\pi^2}\,(\bar{s}\gamma^\mu P_\sigma d)\left(\bar{\nu}\gamma_\mu P_{\sigma'}\nu\right) + \frac{D^{sd}_{\sigma}}{16\pi^2}(\bar{s}\sigma^{\mu\nu}P_\sigma d)F_{\mu\nu} + \text{h.c.}\,.$$

and also $(\bar{s}\Gamma d)^2$



- How to calculate without specifying the theory
- Renorm. theory $\leftrightarrow$ Tree Unitarity [Cornwall et.al. 73/74]
- $0 = \sum_{v_5} g_{v_1 v_2 v_5} g_{v_3 v_4 \bar{v}_5} + g_{v_2 v_3 v_5} g_{v_1 v_4 \bar{v}_5} + g_{v_3 v_1 v_5} g_{v_2 v_4 \bar{v}_5}$
- Idea: Combine generic Lagrangian with sum rules
- [1903.05116] derives relevant sum rules

Generic Vector Interactions

$$\mathcal{L}_3^V = g^{L/R}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{\psi}_{f_1} \gamma^\mu P_{L/R} \psi_{f_2} + \tfrac{i}{6} g^{abc}_{v_1 v_2 v_3} \Big(V_{v_1,\mu} V_{v_2,\nu} \partial^{[\mu} V^{\nu]}_{v_3} + \dots \Big).$$

In SM, for $K \to \pi\nu\bar{\nu}$ we would need the following:

- $g^L_{W^+ \bar{u}_j d_k} = \dfrac{e}{s_w \sqrt{2}} V_{jk}, \quad y^L_{G^+ \bar{u}_j d_k} = \dfrac{m_{uj}}{M_W} \dfrac{e}{s_w \sqrt{2}} V_{jk}$
- $g^L_{Z \bar{f}_j f_k} = \dfrac{2e}{s_{2w}} \left(T_3^f - Q_f s_w^2 \right) \delta_{jk}, \quad g^R_{Z \bar{f}_j f_k} = -\dfrac{2e}{s_{2w}} Q_f s_w^2 \delta_{jk}$
- $g_{ZW^+W^-} = \dfrac{e}{t_w}, \; g_{ZW^+G^-} = -t_w^2 \dfrac{e}{t_w}, g_{ZG^+G^-} = \left(1 - \dfrac{1}{2c_W^2}\right) \dfrac{e}{t_w}$

Eliminate (product of) couplings using $s\langle T\left\{\bar{u}_v(\dots)_{\text{ph}}\right\}\rangle = 0$. E.g. we can combine Z/$\gamma$-Penguin and Boxes using:

$$\sum_Z g^\sigma_{Z\bar{\ell}\ell} g_{Z v_2 \bar{v}_1} = -\delta_{\bar{v}_1 v_2} g^\sigma_{\gamma\bar{\ell}\ell} g_{\gamma v_2 \bar{v}_1} - \sum_{f_3} \left(g^\sigma_{\bar{v}_1 \bar{\ell} f_3} g^\sigma_{v_2 \bar{f}_3 \ell} - g^\sigma_{v_2 \bar{\ell} f_3} g^\sigma_{\bar{v}_1 \bar{f}_3 \ell} \right)$$

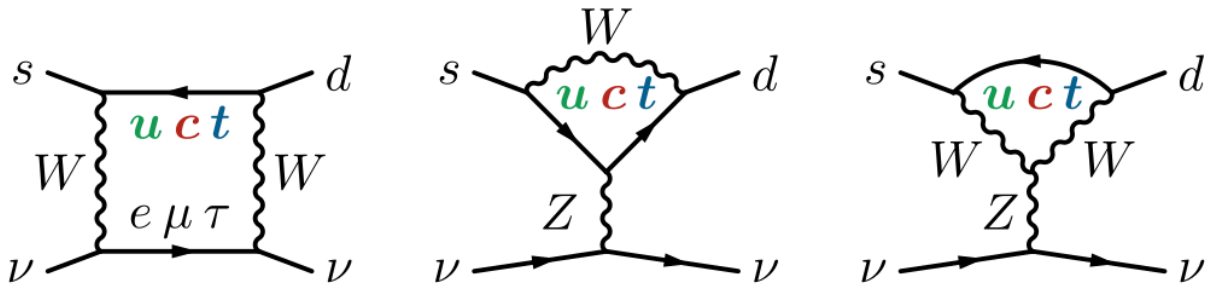
Gauge independent result for $K \to \pi\nu\bar{\nu}$

$$
\begin{aligned}
C_{L\sigma}^{sd\nu} = & \sum_{v_1 v_2 f_1 f_3} \frac{g^L_{\bar{v}_2 \bar{s} f_1} g^L_{v_1 \bar{f}_1 d}}{M^2_{v_1}} g^{\sigma}_{v_2 \bar{\nu} f_3} g^{\sigma}_{\bar{v}_1 \bar{f}_3 \nu} F_V^{\sigma, B' Z}(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_3}_{v_1}) \\
& + \sum_{Z v_1 v_2 f_1 f_2} \frac{g^{\sigma}_{Z \bar{\nu} \nu} g^L_{v_1 \bar{f}_1 d} g^L_{\bar{v}_2 \bar{s} f_2}}{M^2_Z} \Bigg\{ \delta_{f_1 f_2} g_{Z \bar{v}_1 v_2} F^Z_{V''}(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}) \\
& \qquad + \delta_{v_1 v_2} \left[g^L_{Z \bar{f}_2 f_1} F^Z_V(x^{f_1}_{v_1}, x^{f_2}_{v_1}) + g^R_{Z \bar{f}_2 f_1} F^Z_{V'}(x^{f_1}_{v_1}, x^{f_2}_{v_1}) \right] \Bigg\},
\end{aligned}
$$

Extends the Penguin Box Coefficients to generic theories
($X_t \leftrightarrow F_V^{\sigma, B' Z}(0, x^t_W, 1, 0)$ & $F^Z_{V^{(\prime)}}(x, x) = F^Z_{V''}(x, y, 1) = 0$)

- Full results includes also scalars and fermion flow in opposite direction in 2104.10930 and on `https://wellput.github.io/`.

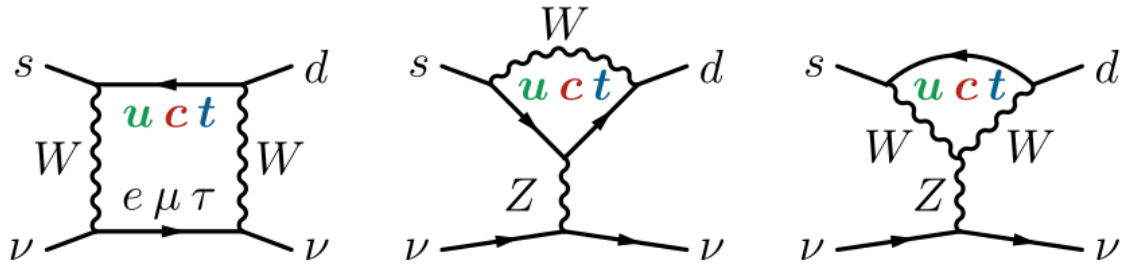
$K \to \pi\bar{\nu}\nu$: Light quark contribution



$$\sum_i \lambda_i F(x_i) = \lambda_t \left(\boldsymbol{F(x_t)} - \boldsymbol{F(x_u)}\right) + \lambda_c \left(\boldsymbol{F(x_c)} - \boldsymbol{F(x_u)}\right)$$

- Charm: $\lambda_c \boldsymbol{F(x_c)} \propto \lambda m_c^2/M_W^2 \log(m_c/M_W)$
  from operator mixing in EFT (NNLO@QCD and NLO@EW)
  Buras, MG, Haisch, Nierste '05 Buras, MG, Haisch, Nierste '06 Brod, MG '08
- Up: $\lambda_c \boldsymbol{F(x_u)} \propto \lambda \Lambda_{\text{QCD}}^2/M_W^2$ from χPT and lattice
  small, 10% of total charm contribution Lunghi Soni; Christ et al.
- Apart from small up quark contribution:
  Only $Q_\nu = (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ with matrix element from K_{l3}
- Similarity to calc of $\eta_u u$ in $\epsilon_K \to$ Joachim's talk

Branching Ratios



$K_L \to \pi^0 \nu\bar{\nu}$ CP violating mode (1% from CP-conserving amplitude $\times \epsilon_K$)

➜ only top-quark contribution: short-dist. uncertainty of $\mathcal{O}(2\%)$

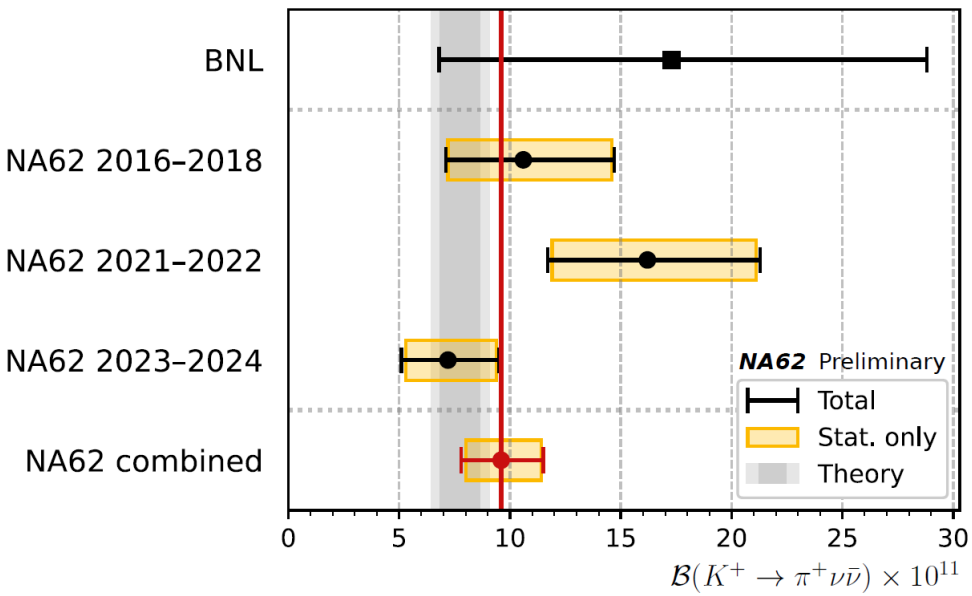
$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})^{\mathrm{SM}} = \kappa_L r_{\epsilon_K} \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t \right)^2$$

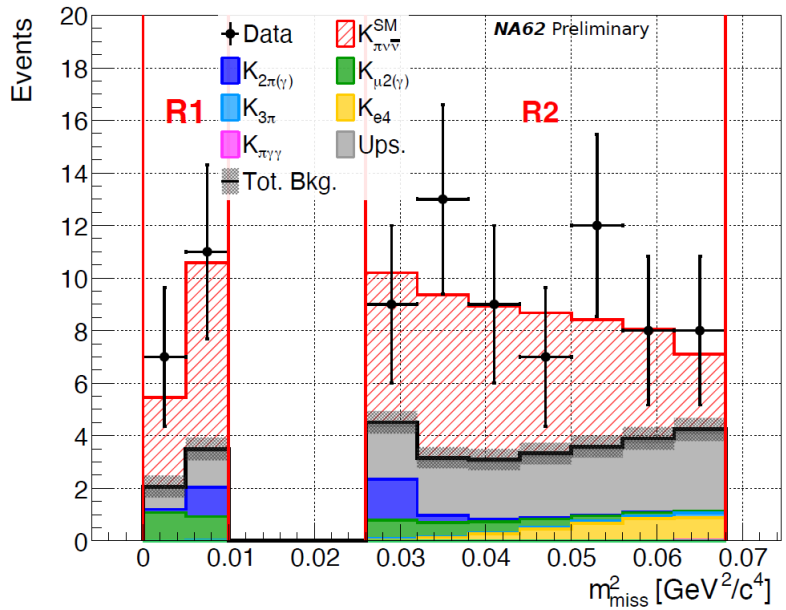
$K^+ \to \pi^+ \nu\bar{\nu}$ CP conserving mode

➜ charm contributes: $\mathcal{O}(3\%)$ long-distance theory uncertainties

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})^{\mathrm{SM}} = \kappa_+ (1 + \Delta_{\mathrm{EM}}) \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t \right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda} (P_c + \delta P_{cu}) + \left(\frac{\mathrm{Re}\lambda_t}{\lambda^5} X_t \right) \right)^2 \right]$$

Experimental Measurement





$K \to \pi\nu\bar{\nu}$ SM prediction

$$10^{11}\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu})^{\mathrm{SM}} = 8.55 \pm 0.40_{\mathrm{CKM}} \pm 0.19_{\mathrm{LD}} \pm 0.13_{\mathrm{SD}} \pm 0.07_{\mathrm{para}}$$

$$10^{11}\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu})^{\mathrm{SM}} = 3.15 \pm 0.22_{\mathrm{CKM}} \pm 0.04_{\mathrm{SD}} \pm 0.02_{\mathrm{LD}} \pm 0.02_{\mathrm{para}}$$

$\lambda = 0.22497 \pm 0.00070$, $A = 0.839 \pm 0.011$, $\bar{\rho} = 0.1581 \pm 0.0092$,
$\bar{\eta} = 0.3548 \pm 0.0072$ (corresponding to $|V_{cb}| = 0.0425 \pm 0.0006$) PDG-UTfit 24

NA62 [2605.02415] and KOTO measuments

$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu})^{\mathrm{NA62}} = (9.6^{+1.9}_{-1.8}) \times 10^{-11}$$

$$\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu})^{\mathrm{KOTO}} < 3.0 \times 10^{-9} \qquad @90\%\ \mathrm{CL}$$

Ratios of rates as clean SM tests (essentially CKM-parameter free – up to $\mathcal{O}(3\%)$)

$$R_S = \frac{\mathrm{BR}(K_S \to \mu\mu)_{\ell=0}}{\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu})} \sim 1.55 \times 10^{-2} \frac{Y(x_t)}{X(x_t)}$$

Buras, Venturini 21

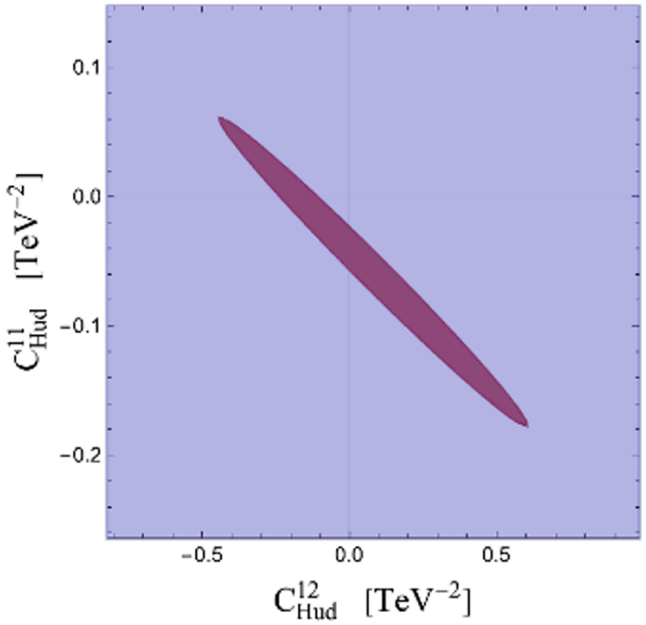
Do not give up

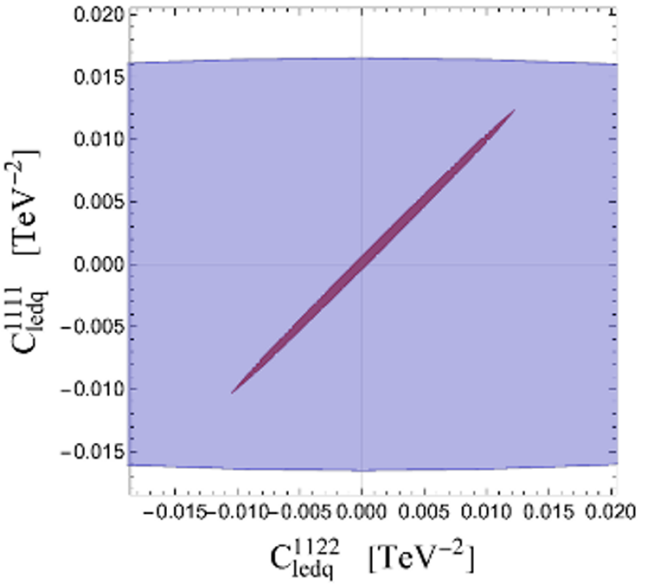
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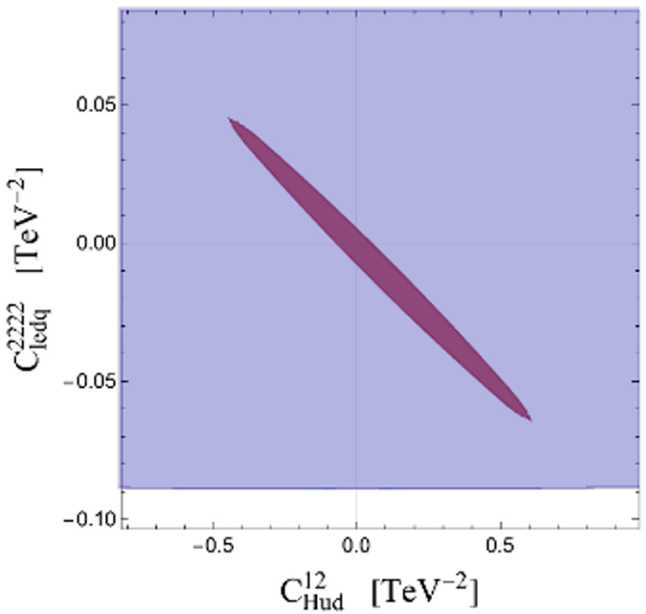
- $\Gamma(K_{\mu 2})/\Gamma(\pi_{\mu 2}) \to |V_{us}/V_{ud}| f_K/f_\pi$
- $\Gamma(K_{\ell 3}) \to f_K |V_{us}|$
- Unitarity: $|V_{ud}^2| + |V_{us}^2| + |V_{ub}^2| = 1$
- $0^+ \to 0^+$ nuclear β decays using Hardy,Towner'20: $|V_{ud}|$
- or $|V_{ud}| \tau_n (1 + 3\lambda^2)(1 + \Delta_f)(1 + \Delta_R) = 5283.321(5) s$
  - λ, Δ_f & Δ_R ratio of (axial-)vector coupling, phase space & radiative corrections

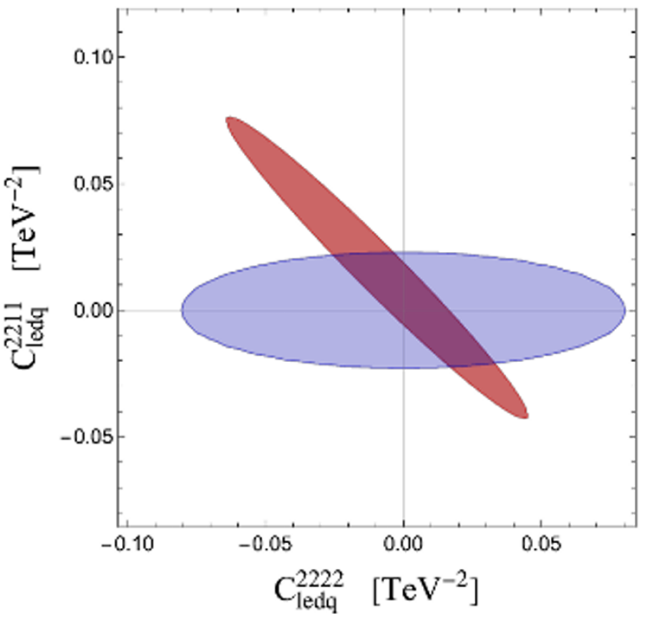
Example: SMEFT with right handed currents

- Consider: $\mathcal{L} \supset C^{ij}_{Hud} Q^{ij}_{Hud} + C^{prst}_{ledq} Q^{prst}_{ledq}$
  - 2 dimension 6 SMEFT operators – think $C = X_{NP}/(1\,TeV)^2$
- These operators lead to right-handed charged current interactions
  - $Q^{ij}_{Hud} = \left(\tilde{H}^\dagger D_\mu H\right)\left(\bar{u}^i_R \gamma^\mu d^j_R\right) + \text{h.c.} \;\rightarrow\; \frac{g v^2}{2\sqrt{2}} W^+_\mu \bar{u}^i_R \gamma^\mu d^j_R + \text{h.c.}$
  - $Q^{prst}_{ledq} = (\bar{l}_p e_r)(\bar{d}_s q_t) = (\bar{\nu}_{Lp} e_{Rr})(\bar{d}_{Rs} u_{Lt}) + (\bar{e}_{Lp} e_{Rr})(\bar{d}_{Rs} d_{Lt})$
  - Flavour index in the down basis, $u_{L2} = V^*_{ud} u_L + V^*_{cd} c_L + \ldots$
- That compete with SM $G_F V_{ud} (\bar{\nu}_L \gamma_\mu e_L)(\bar{d}_L \gamma^\mu u_L)$



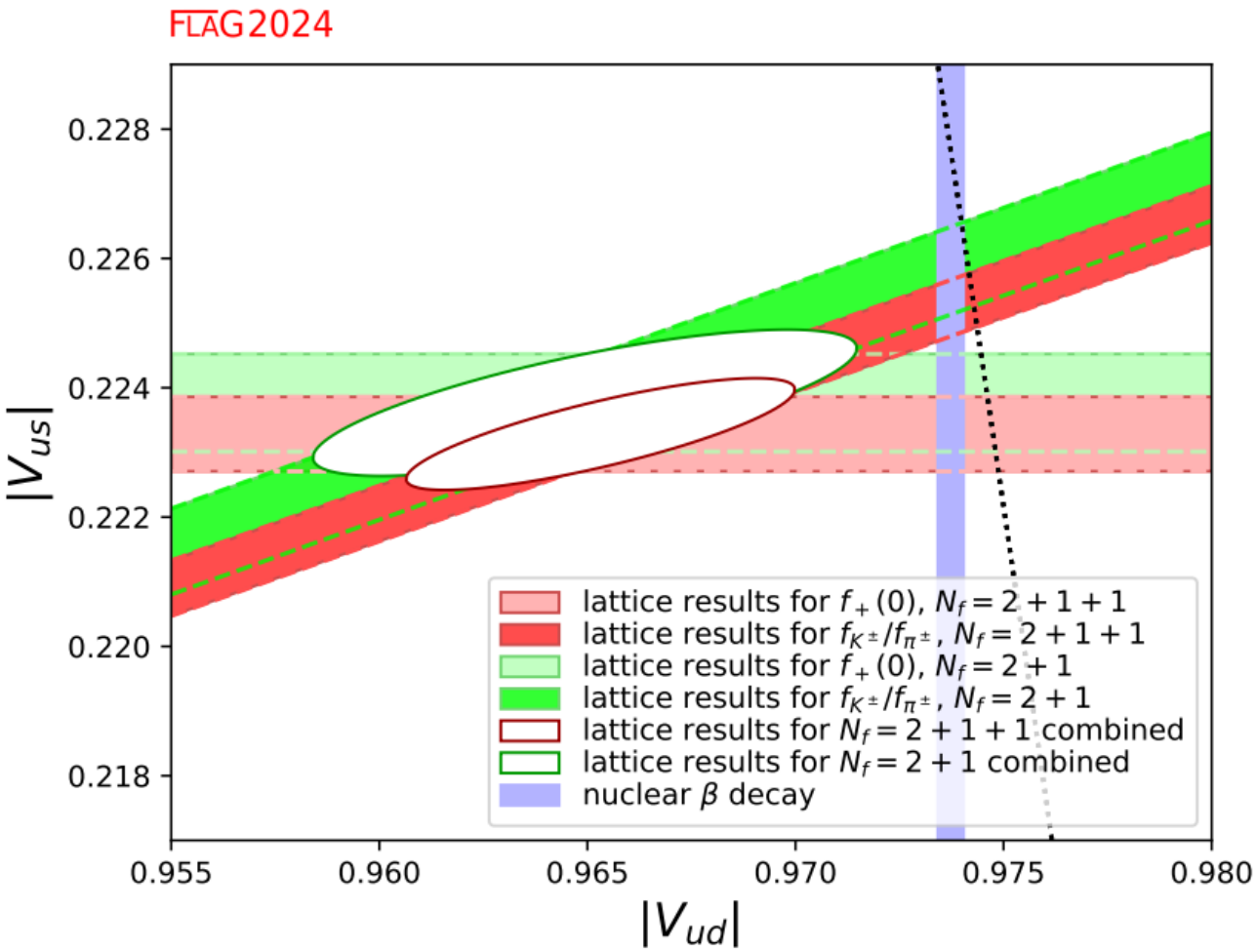






Collider (blue) and Charged Current (red) constraints Cirigliano et.al.,'24

Lattice input FLAG 2024



- Standard Model improvements: Francesco's talk

Take home message

- Models come

Take home message

- Models come
- and anomalies go

Take home message

- Models come
- and anomalies go
- Uli stays the same