$B\pi$ excited-state contamination in B-meson observables

Oliver Bär

in collaboration with

Antoine Gerardin, Simon Kuberski, Rainer Sommer

Exploring new physics with flavour

KIT

2 - 3 July 2026

Outline

- Motivation: Nucleon-pion-state contamination in nucleon form factors
- …
- …
-

MINERvA: Axial form factor of the nucleon

MINERvA experiment 2022: First measurements of neutrino-proton scattering

Cai *et al*, Nature Vol 614 (2023)

$$\overline{\nu}_\mu \, p \quad \rightarrow \quad \mu^+ \, n$$

direct handle on the proton's axial form factor !

Previous measurements used nucleon bound states (e.g. Deuterium) involves "nuclear physics corrections"

Form factor (ff) decomposition: $\langle N(p')|A_\mu|N(p)\rangle$

axial vector current

$G_{\mathrm{A}}(Q^2)$ axial ff

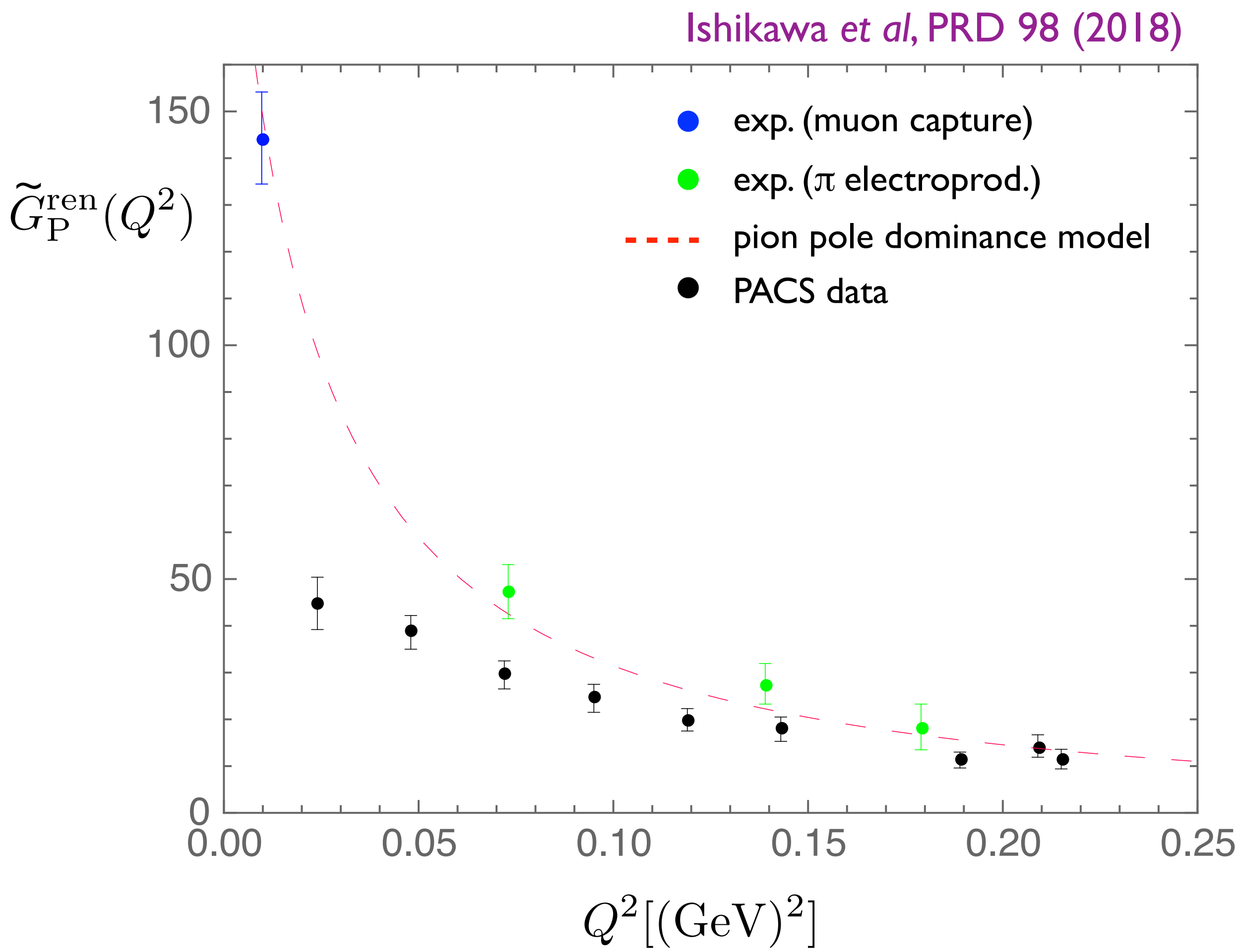
$\tilde{G}_{\mathrm{P}}(Q^2)$ induced pseudo scalar ff

$(p' - p)^2 = -Q^2$ momentum transfer

Nucleon axial radius $r_A^2 \equiv - \dfrac{6}{G_{\mathrm{A}}(0)} \dfrac{dG_{\mathrm{A}}}{dQ^2}\Big|_{Q^2=0}$ $\Rightarrow$ $r_A = 0.73 \pm 0.17\,\mathrm{fm}$ MINERvA

Induced pseudo scalar form factor - lattice data

Data by the PACS collaboration



The form factors can be computed in lattice QCD using standard techniques

Some lattice parameters:

$$a \approx 0.085\,\mathrm{fm}$$

$$M_\pi \approx 146\,\mathrm{MeV}$$

$$L \approx 8.1\,\mathrm{fm}$$

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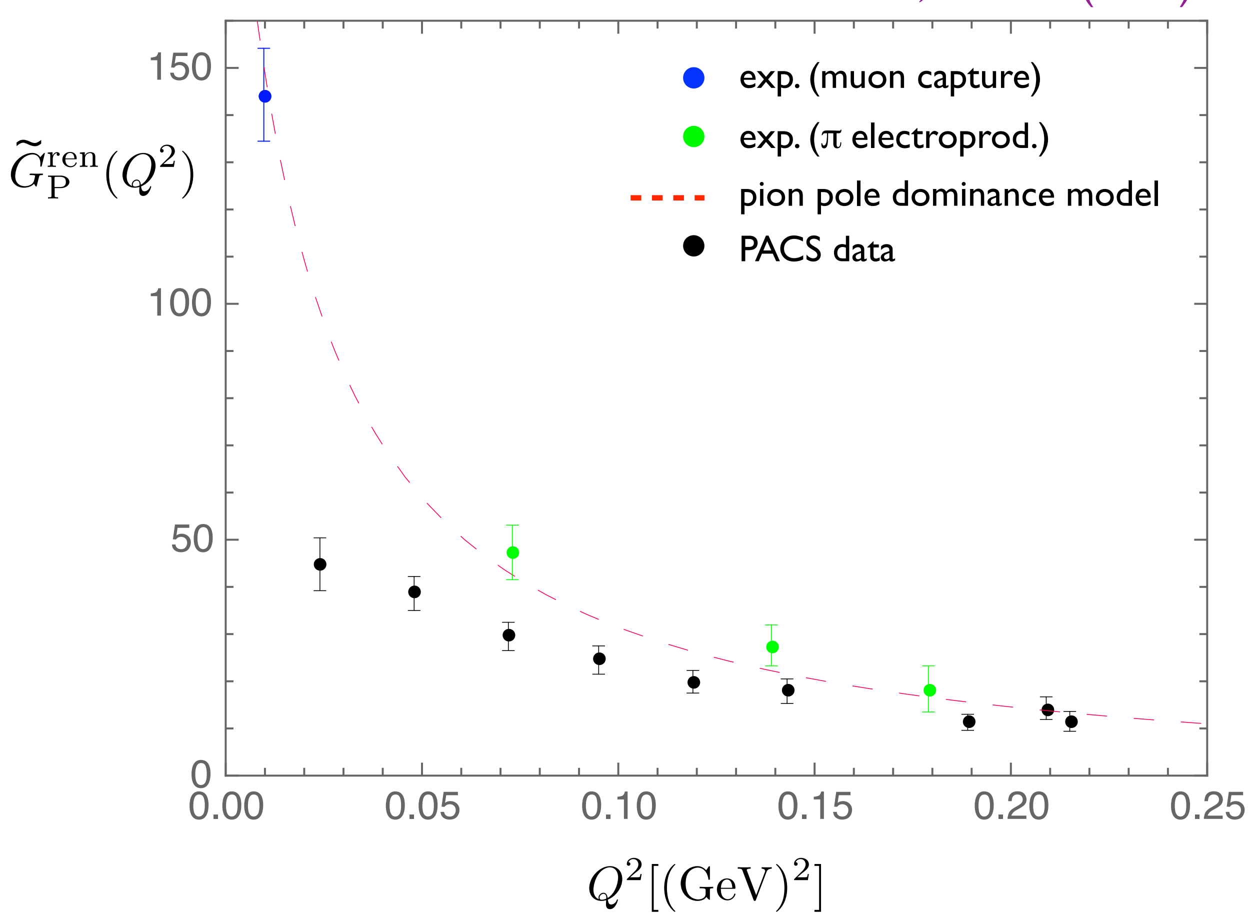
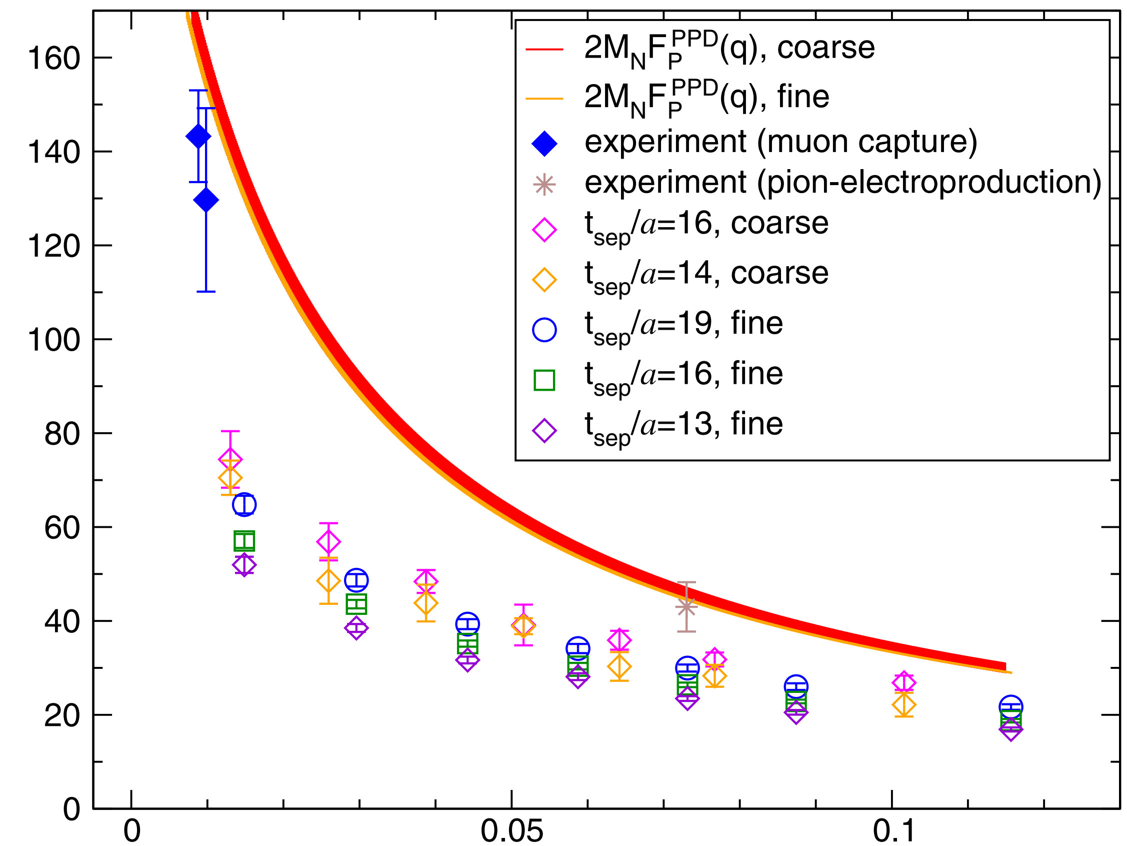
Ishikawa *et al*, PRD 98 (2018)



Tsuji *et al*, PRD 109 (2024)



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Year 2018

$$a \approx 0.085\,\mathrm{fm}\,,\ \ 0.063\,\mathrm{fm}$$
$$M_\pi \approx 135\,\mathrm{MeV}$$
$$L \approx 10.8\,\mathrm{fm}$$

Year 2024

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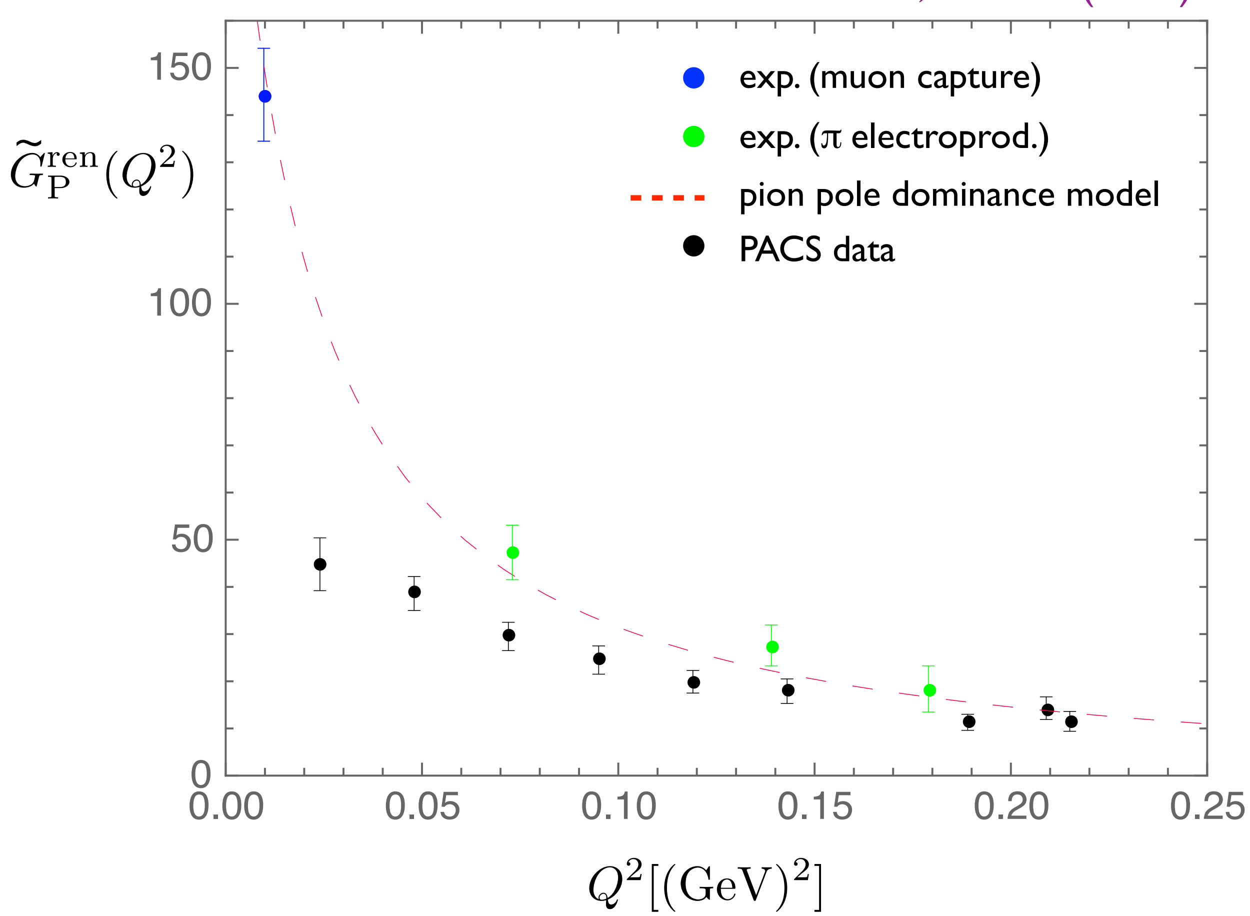

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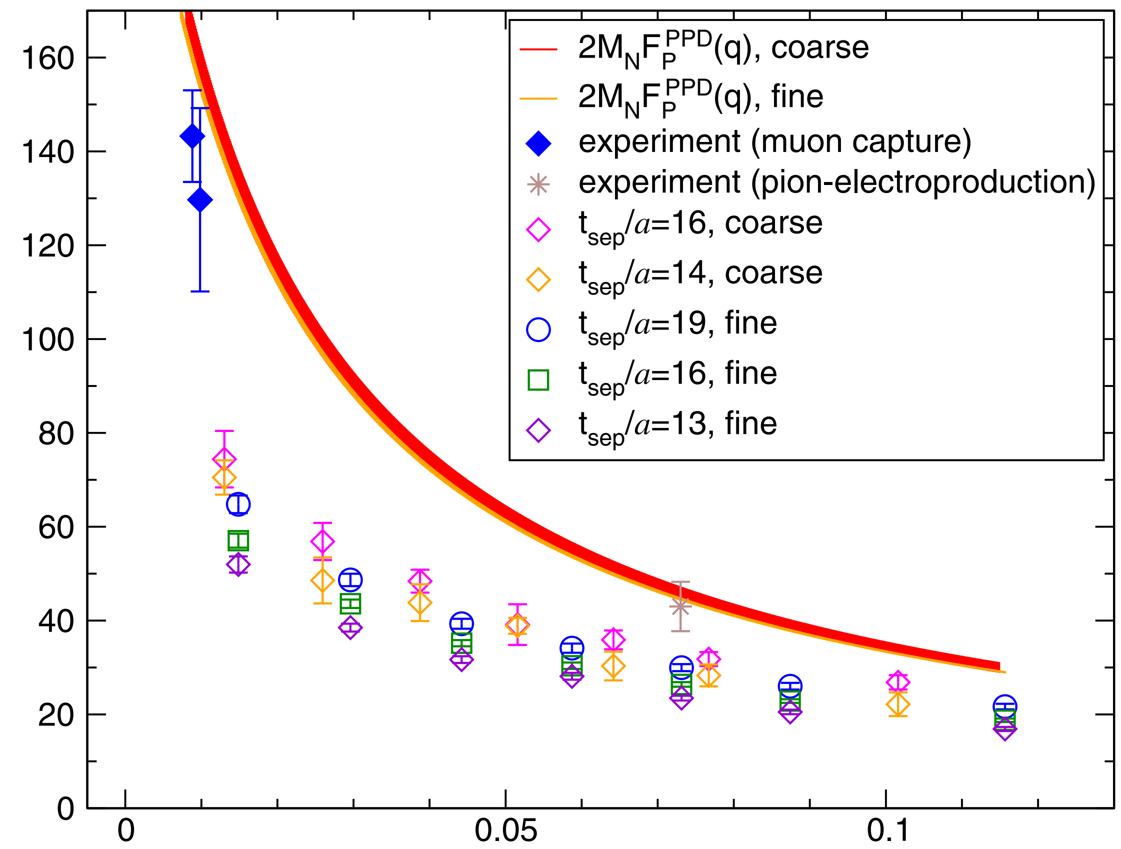

The lattice data underestimate the ff,
in particular for small Q^2

Some lattice parameters:

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Year 2018

$$a \approx 0.085\,\mathrm{fm}\,,\ \ 0.063\,\mathrm{fm}$$
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Year 2024

Induced pseudo scalar form factor - lattice data

Origin of the underestimation:
(wide agreement in the lattice community, after many years of discussion …)

(Large) excited-state contamination due to a two-particle nucleon-pion $(N\pi)$ state

Based on many studies

- employing lattice simulations   Jang *et al*, PRD 109 (2024)
  Barca, Bali and Collins, PRD 107 (2023)
  Bali *et al*, JHEP 05 (2020)
  Jang *et al*, PRL 124 (2020)
  ….
- using Chiral Perturbation Theory (ChPT)   OB, PRD 99 (2019)

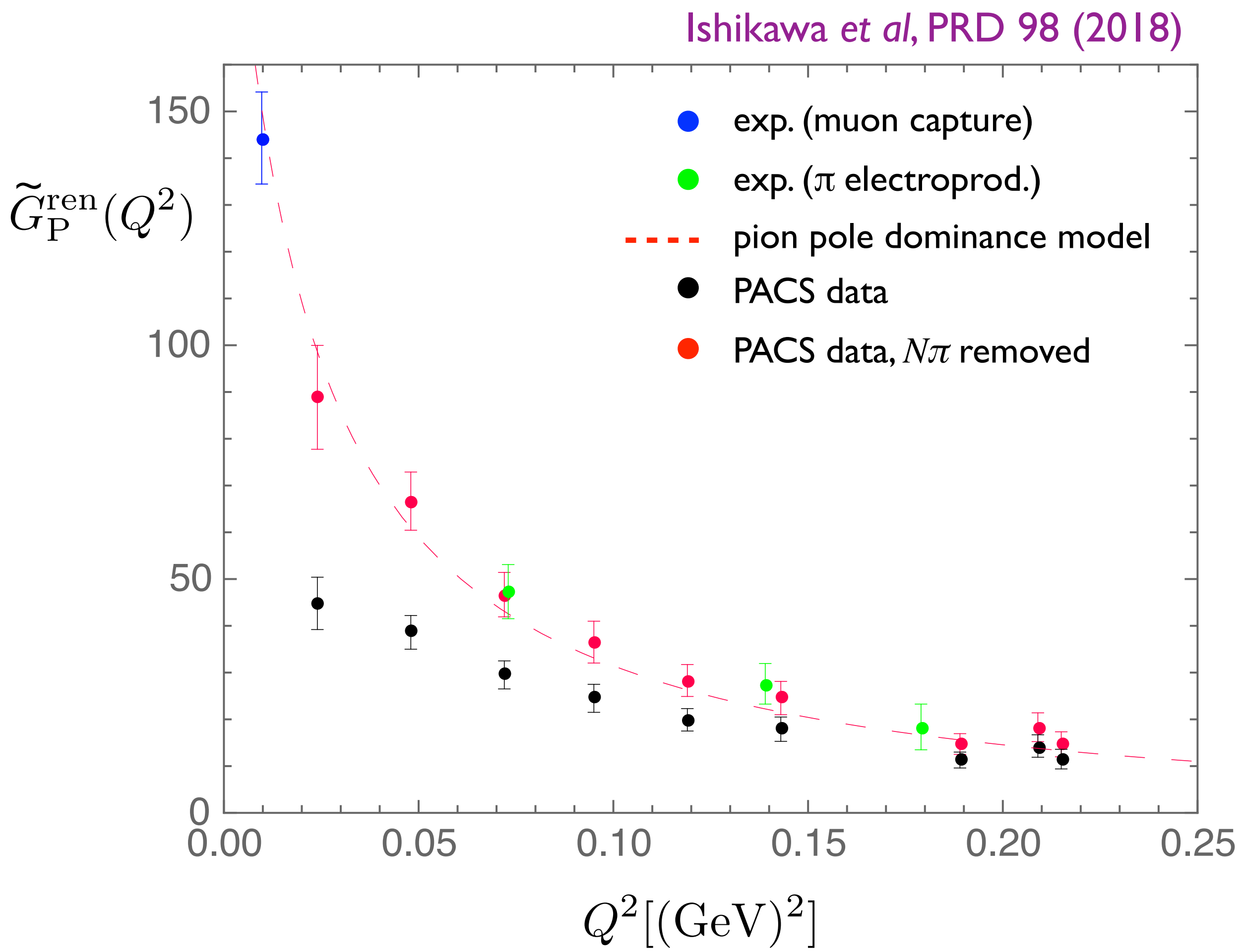
ChPT ➡ predicts a large $N\pi$-state contamination

➡ predicts an underestimation

➡ provides a correction formula to "remove" the $N\pi$-state contamination from the lattice data

Induced pseudoscalar form factor - lattice data

Data by the PACS collaboration



The "ChPT corrected" data agree much better with the experimental data / ppd model

Important question

Question: Are there more observables similarly affected ?

Answer: Yes, the form factor relevant for B to π decay

➥ ChPT predicts a large $\mathrm{B}\pi$-state contamination

A. Broll, OB and R. Sommer
Eur. Phys. J C 83 (2023) 757

New results:
- Large Bπ-state contamination is confirmed by lattice results
- Strategy to deal with it

OB, A. Gerardin, S. Kuberski and R. Sommer
in preparation

Outline

- Motivation: Nucleon-pion-state contamination in nucleon form factors
- Lattice basics: Excited-state contamination in B-meson 2pt function
- ChPT and the Bπ excited-state contamination in
  - B-meson decay constant
  - Vector current form factor for $B \to \pi l \overline{\nu}_l$
- Lattice results for the excited-state contamination in the form factor
- Outlook

Introduction: B-meson 2-pt function

- Consider the B-meson 2-pt function $C_2(t) = \sum_{\vec{x}} \langle B(\vec{x},t) B^\dagger(\vec{0},0)\rangle$
  - Σ_x : projection to zero momentum
  - B : interpolating field, quantum numbers of the B-meson

- Spectral decomposition
finite spatial volume ⇒ discrete spectrum

→ $$C_2(t) = b_0 e^{-M_B t} + \underbrace{b_1 e^{-E_1 t} + b_2 e^{-E_2 t} + \ldots}_{\text{excited-state contribution}}$$

$b_0 \propto |\langle 0|B(\vec{0},0)|B(\vec{p}=0)\rangle|^2$

$$M_{\text{eff}}(t) = -\partial_t \ln C_2(t)$$
effective mass

→ $$M_{\text{eff}}(t) = M_B + \frac{b_1}{b_0} e^{-\Delta E_1 t} + \frac{b_2}{b_0} e^{-\Delta E_2 t} + \ldots$$

$$\Delta E_k = E_k - M_B$$

⇒ time separation needs to be sufficiently large for small excited-state corrections

Note: analogous statement for 3-pt functions with more than one time separation

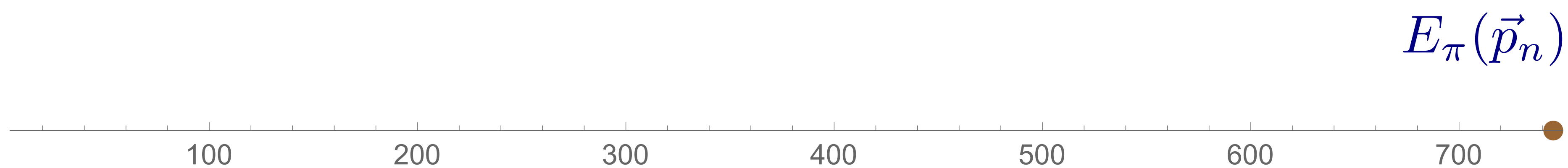
Bπ and B*π states[1]

Consider static B and B*-mesons:

$$\Delta E_n \approx E_\pi(\vec{p}_n) \qquad \vec{p}_n = \frac{2\pi}{L}\vec{n}$$

$$M_\pi L = 4$$

$$M_\pi = 400\,\mathrm{MeV}$$



[1] For simplicity I often won't distinguish between B and B* in the following …

$B\pi$ and $B^*\pi$ states[1]

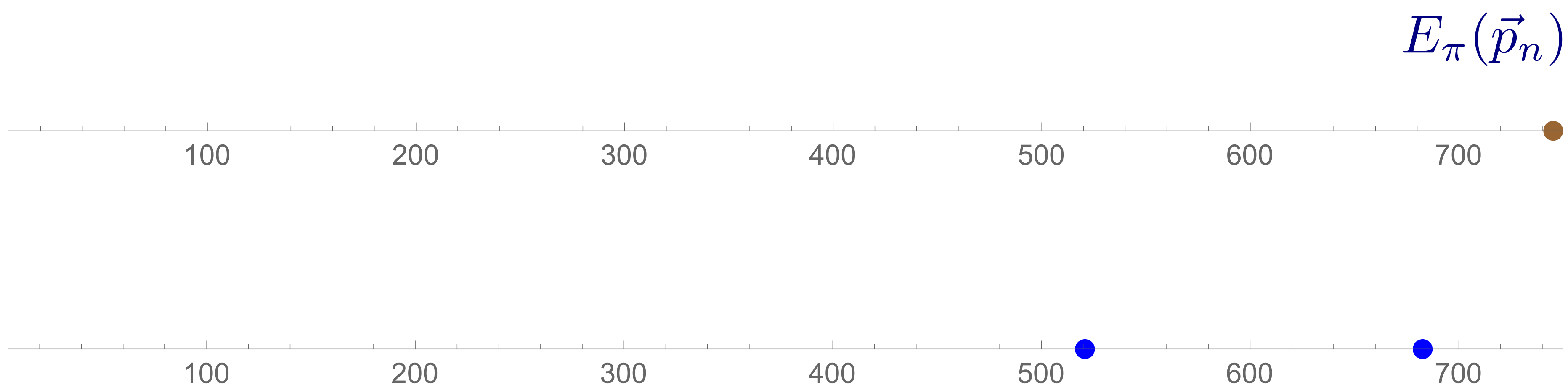
Consider static B and B*-mesons:

$$\Delta E_n \approx E_\pi(\vec{p}_n) \qquad \vec{p}_n = \frac{2\pi}{L}\vec{n}$$

$M_\pi L = 4$

$M_\pi = 400\,\mathrm{MeV}$

$M_\pi = 280\,\mathrm{MeV}$



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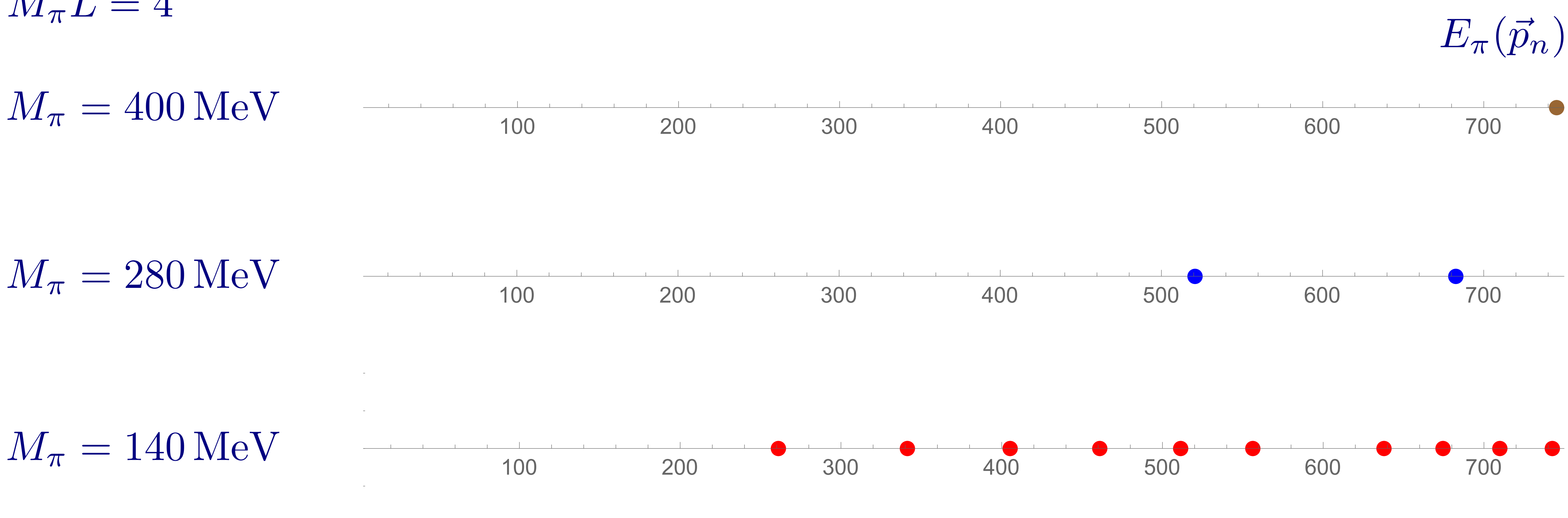


The number of low-lying B*π states increases rapidly with decreasing pion mass !

[1] For simplicity I often won't distinguish between B and B* in the following …

Bπ and B*π states[1]

Consider static B and B*-mesons:

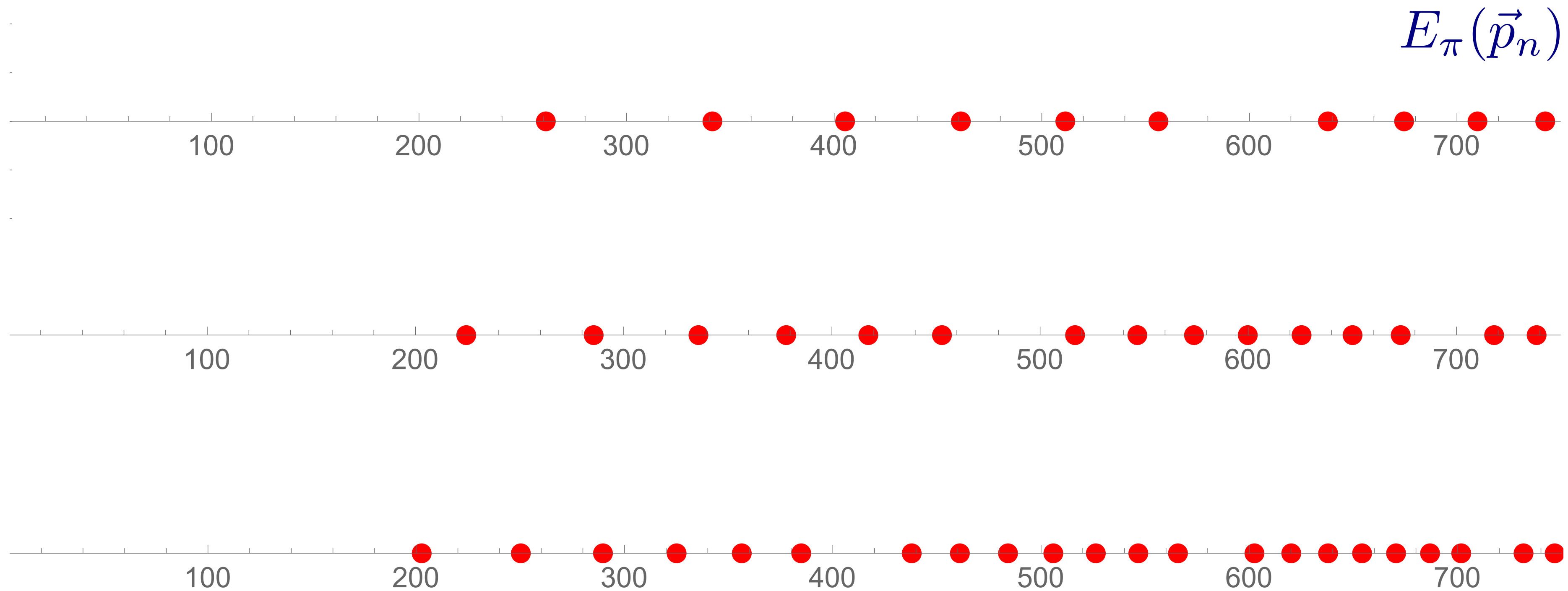
$$\Delta E_n \approx E_\pi(\vec{p}_n) \qquad \vec{p}_n = \frac{2\pi}{L}\vec{n}$$

$M_\pi = 140\,\mathrm{MeV}$

$M_\pi L = 4$

$M_\pi L = 5$

$M_\pi L = 6$



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Bπ and B*π states[1]

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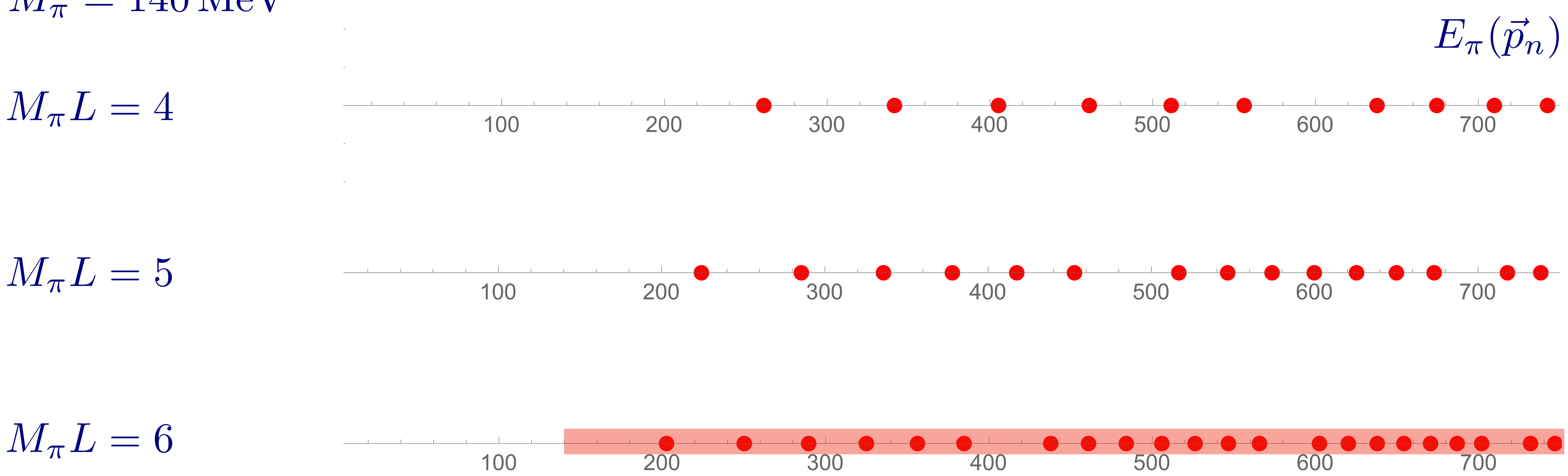


Infinite volume: continuous 2-particle spectrum, threshold = $M_B + M_\pi$

[1] I often won't distinguish between B and B* in the following …

Bπ excited states

$$M_{\mathrm{eff}}(t) \approx M_B + \frac{b_1}{b_0} e^{-E_\pi(\vec{p}_1)} + \frac{b_2}{b_0} e^{-E_\pi(\vec{p}_2)} + \dots$$

- Expectation: Many Bπ states contribute to the correlator and the effective mass
- Two questions
    1. How big is their impact (= how big is b_k/b_0) ?
    2. If non-negligible: How to deal with them ?
- Concerning 1. → Use Chiral Perturbation theory to get estimates

Chiral perturbation theory (ChPT)

- Well-established and widely used:

ChPT: low-energy effective theory of QCD

Weinberg 1979
Gasser, Leutwyler 1984
…

- Many applications in lattice QCD
  - Pion mass dependence of observables
  - Finite-volume (FV) effects due to pions
  - Taste-breaking effects with staggered fermions
  - …
- In the following: $B\pi$-state contamination in B-meson observables
  Main idea:
  - Compute correlation functions for large time separations in Heavy-Meson (HM) ChPT
  - Extract the $B\pi$-state contamination

M. Wise (1992), Burdman, Donoghue (1992), …

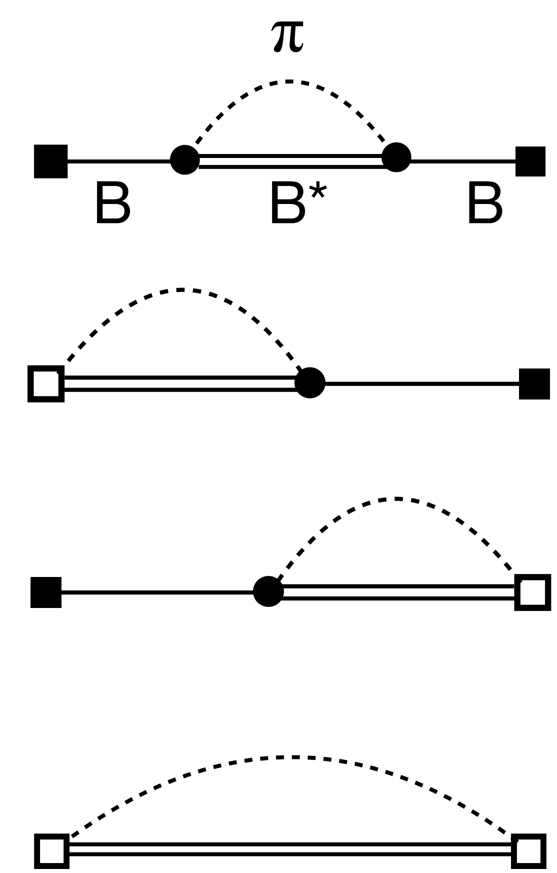
Example: Bπ contribution in HMChPT - 2pt function

- Example: Feynman diagrams for the leading Bπ contribution in the 2-pt function

  ➥ Extract contribution with exponential

$$\exp\Big[-\left(M_B + E_{\pi,\vec{p}}\right)|t|\Big] \qquad \vec{p} = \vec{p}_\pi = -\vec{p}_B$$

ignore the others



- Obtain results for relative Bπ contribution:

$$\frac{C_2^{B\pi}(t)}{C_2^{B}(t)} \equiv \Delta C_2^{B\pi}(t) = \sum_{\vec{p}} c_{2\mathrm{pt}}(\vec{p})\, e^{-E_{\pi,\vec{p}}\,t}$$

non-trivial result
of the ChPT calculation

Recall Introduction

$$M_{\mathrm{eff}}(t) \approx M_B + \frac{b_1}{b_0} e^{-E_\pi(\vec{p}_1)} + \frac{b_2}{b_0} e^{-E_\pi(\vec{p}_2)} + \ldots$$

Example: Bπ contribution in HMChPT - 2pt function

$$\Delta C_2^{B\pi}(t) = \sum_{\vec{p}} c_{2\mathrm{pt}}(\vec{p})\, e^{-E_{\pi,\vec{p}}\, t}$$

$$c_{2\mathrm{pt}}(\vec{p}) \;=\; \frac{3}{8(fL)^2 E_{\pi,\vec{p}} L}\, \frac{p^2}{E_{\pi,\vec{p}}^2} \left(\underbrace{g}_{\text{LO}} + \underbrace{\beta_1 E_{\pi,\vec{p}}}_{\text{NLO}}\right)^2$$

Comments

- expected $1/L^3$ dependence (2-particle state!)
- Σ_{p} → "tower of states" (recall: 1-loop diagrams)
- infinite volume limit can be taken (→ non-zero, involves modified Bessel and Struve functions)
- depends on Low-Energy-Constants LECs): f, g (LO) and β_1 (NLO)

Estimates: $f \approx f_\pi \approx 93\,\mathrm{MeV}$ PDG

$g \approx 0.49$ Lattice: A. Gerardin *et al.* (2022)

$\beta_1 \approx 0.14(4)\,\mathrm{GeV}^{-1}$ Lattice: B. Colquhoun *et al.* (2022)

⇒ ChPT *prediction* for the Bπ contamination in effective mass and decay constant

Bπ contribution in the effective decay constant

Example $\langle 0|A_4^{\mathrm{RGI}}(0)|B(\vec{p}=0)\rangle \equiv \hat{f}$ heavy light decay constant

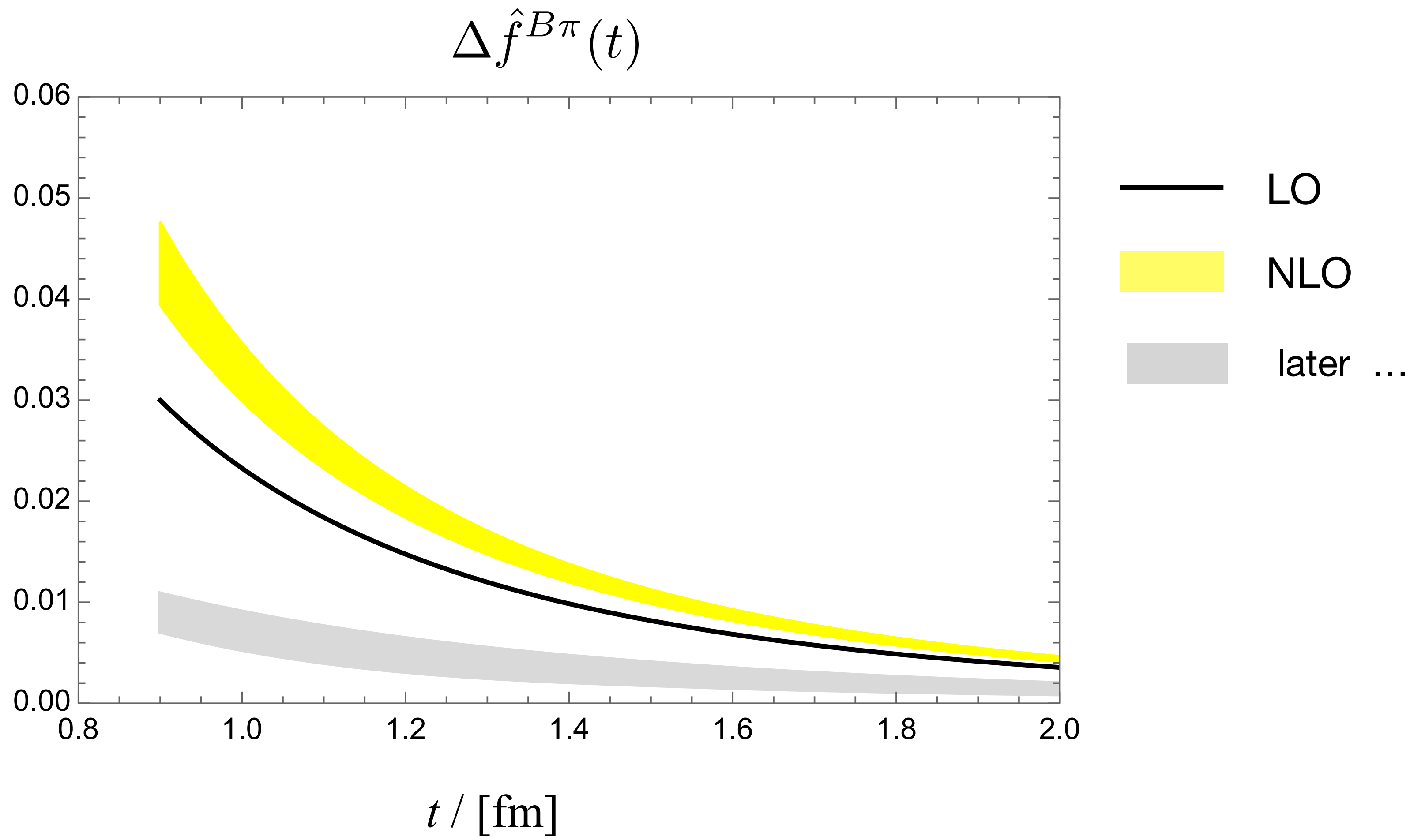
$\hat{f} = f_B \sqrt{M_B} \,/\, C_{\mathrm{PS}}$

Estimator
not unique

$$\hat{f}_{\mathrm{eff}}(t) = \sqrt{2}\,\sqrt{C_2(t)}\,e^{\frac{1}{2}M_B^{\mathrm{eff}}(t)\,t}$$

$$= \hat{f}\Big(1 + \Delta\hat{f}^{B\pi}(t)\Big) \xrightarrow{t\,\to\,\infty} \hat{f}$$

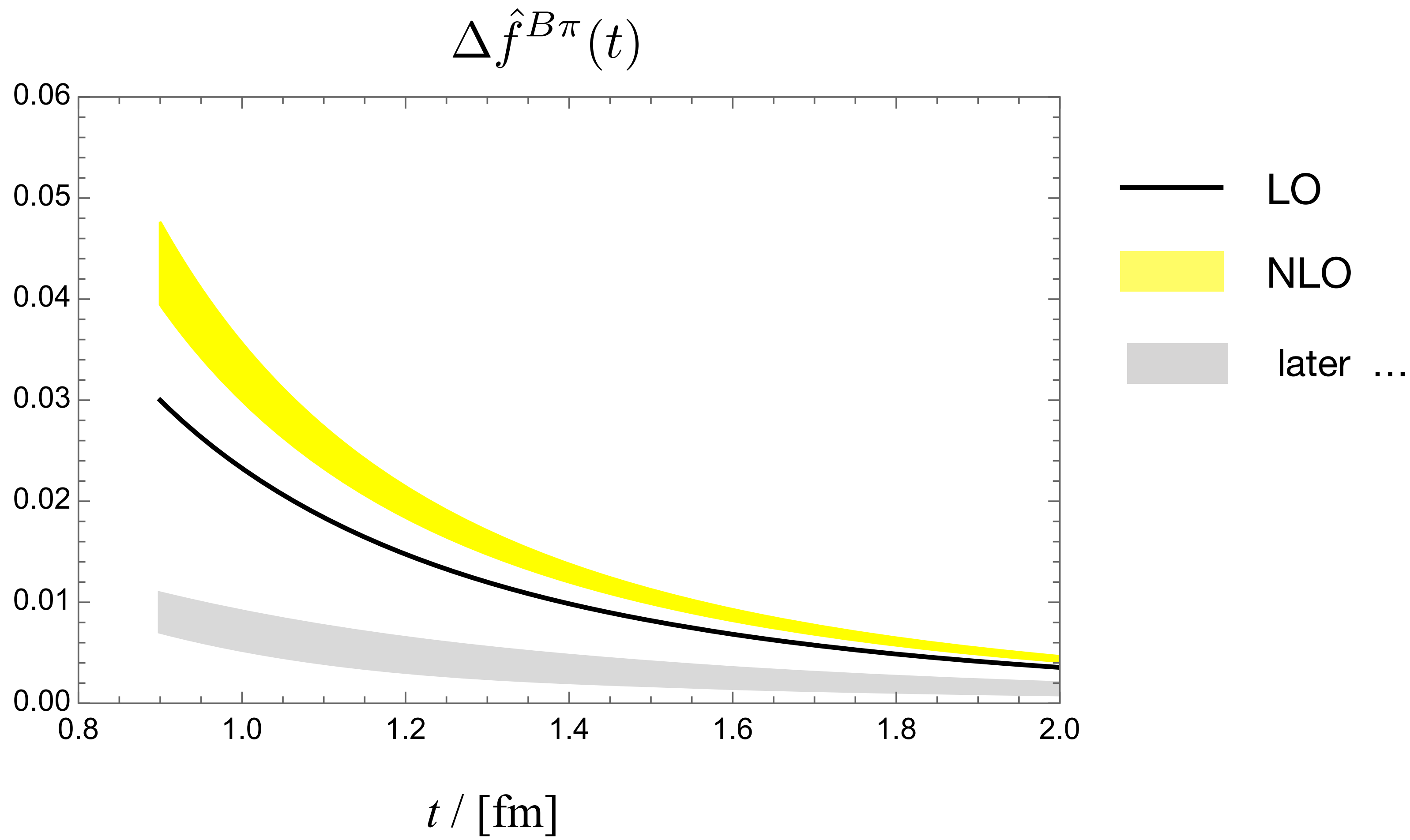
Bπ contribution in the effective decay constant



$t = 1.3$ fm: Bπ-state contamination leads to an *overestimation* of ≈ 2%

small effect but relevant for % precision

Bπ contribution in the effective decay constant



$t = 1.3$ fm: Bπ-state contamination leads to an *overestimation* of ≈ 2%

small effect but relevant for % precision

Analogous results for B-meson mass, BB*π coupling g_π → Bπ contamination ≈ a few percent

A. Broll, OB and R. Sommer
Eur. Phys. J C 83 (2023) 757

Semileptonic B decay

- Semileptonic B decay $B \to \pi l \overline{\nu}_l$

➥ matrix element $\langle \pi(p_\pi) \,|\, V^\mu \,|\, B(p_B) \rangle$

$p_\pi^k \, h_\perp(E_\pi) \qquad \mu = k$

$h_{||}(E_\pi) \qquad \mu = 0$

$\vec{p}_B = 0$

heavy-light vector current

form factor decomposition

related to f_+ and f_0

- Extract form factors from suitably defined ratio of 3pt and 2pt functions

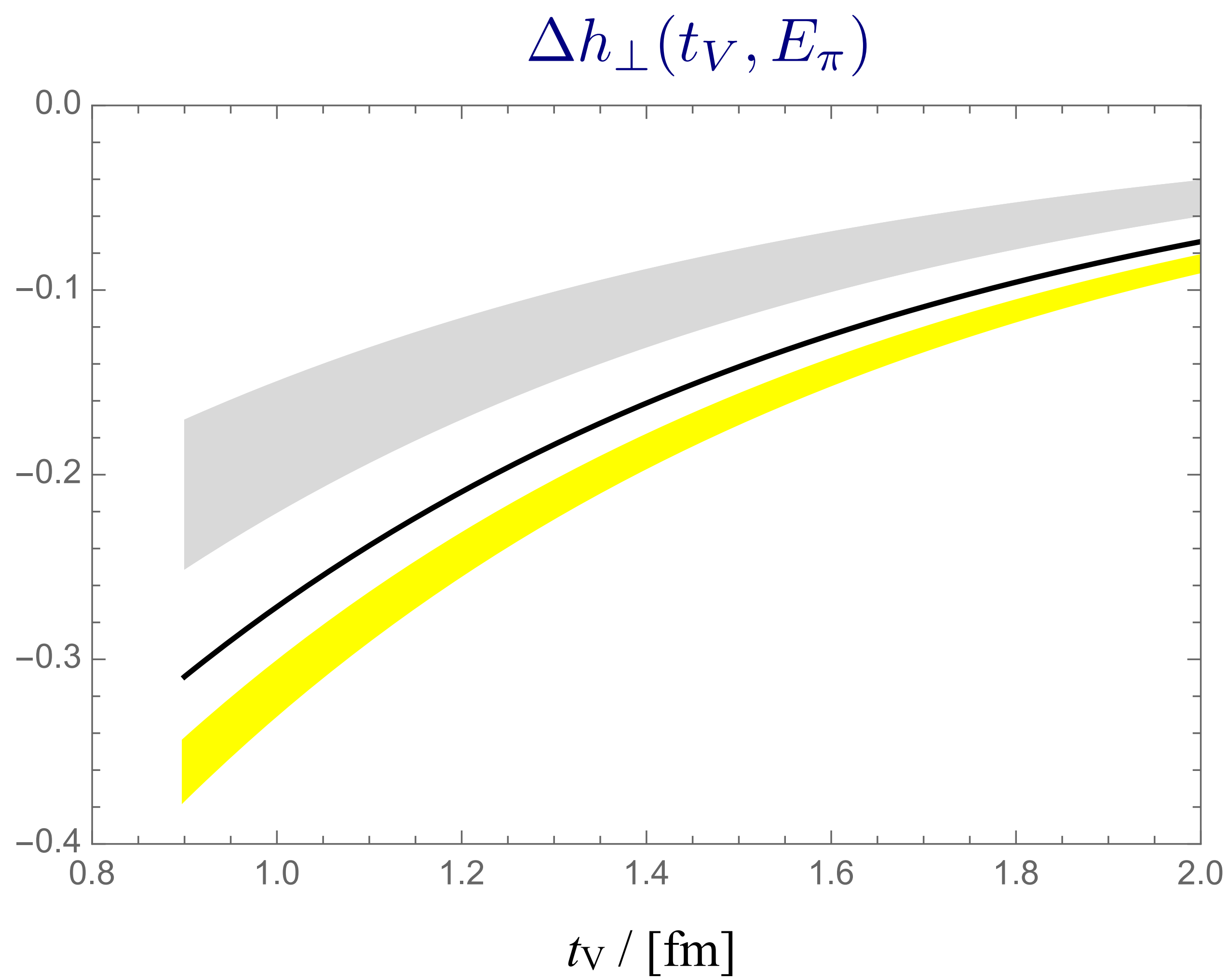
➥ effective form factors $h_\perp^{\text{eff}}(t_V, E_\pi) \; = \; h_\perp(E_\pi) \Big[1 + \Delta h_\perp(t_V, E_\pi) \Big]$

current insertion time

Bπ-state contamination

stemming from $\langle \pi \,|\, V^k \,|\, B^* \pi \rangle$

Bπ contamination $\Delta h_\perp$



$$|\vec{p}| = \frac{2\pi}{L} \qquad M_\pi L = 4$$

$$\Rightarrow \quad E_{\pi,\vec{p}} \approx 260\,\mathrm{MeV}$$

The B*π contamination in the 3-pt function …

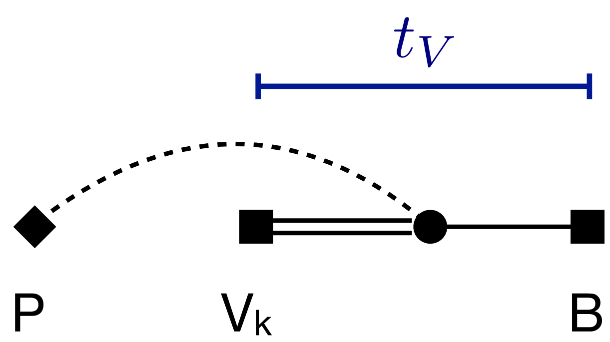
- leads to an *underestimation of ~ 20%* of the form factor
- Why so much bigger than in the 2-pt function ?

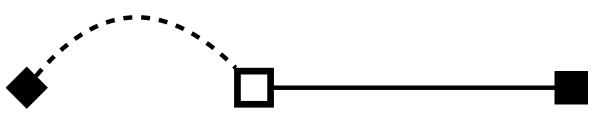
much larger at physical pion mass

Why is the Bπ contamination in $\Delta h_\perp$ so big?

Feynman diagrams for

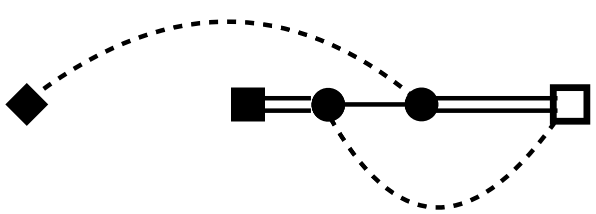
$C_3^B(t, t_V)$

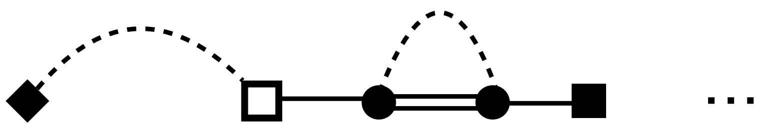




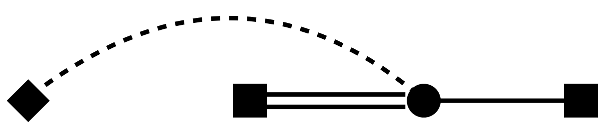
$C_3^{B\pi}(t, t_V)$

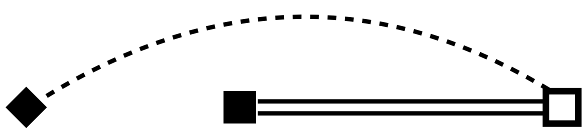
loop diagrams





tree diagrams

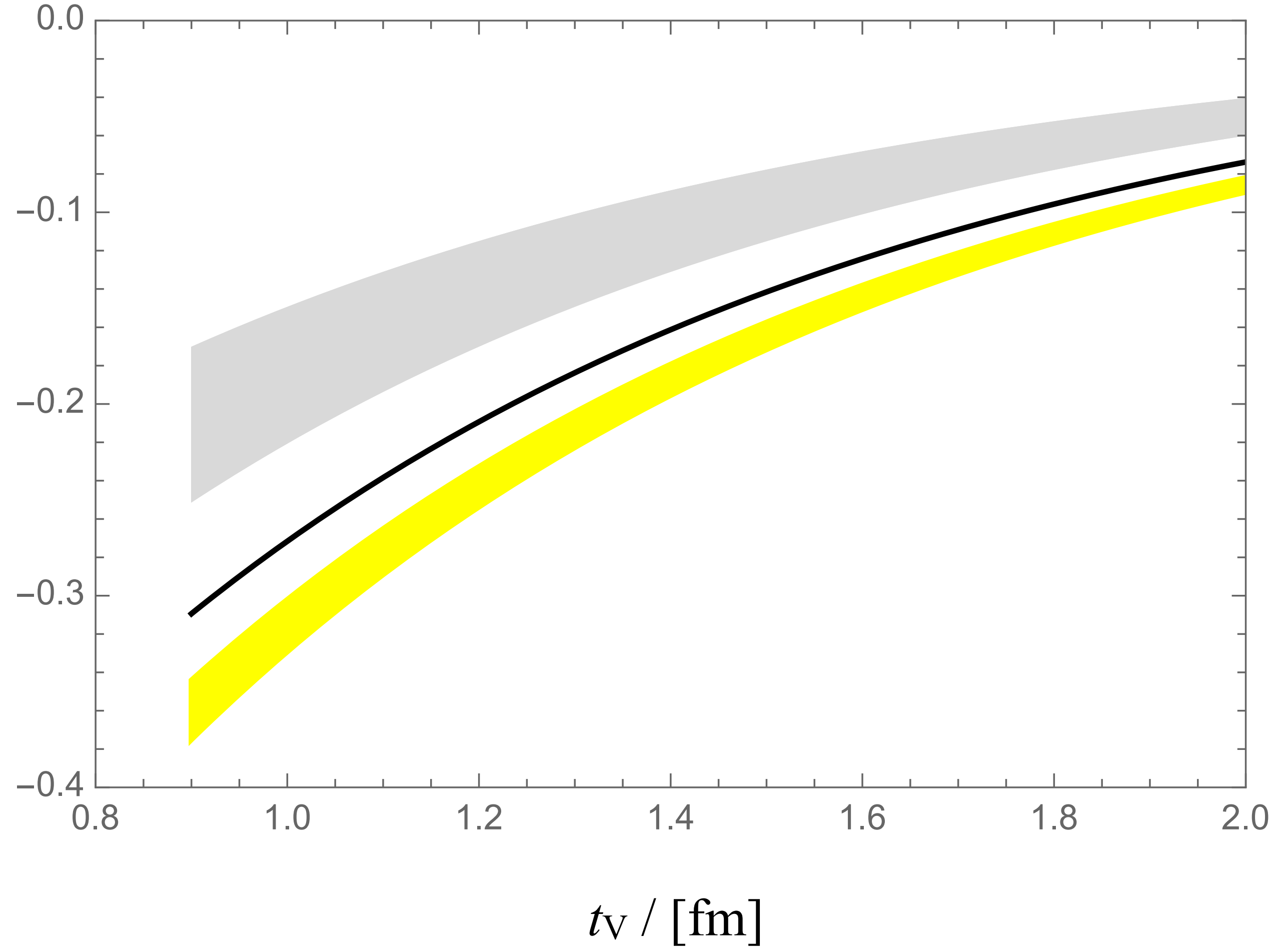




$$\Delta h_\perp^{B\pi} = \Delta h_\perp^{B\pi,\mathrm{tree}} + \Delta h_\perp^{B\pi,\mathrm{loop}} \approx \Delta h_\perp^{B\pi,\mathrm{tree}}$$

Bπ contamination $\Delta h_\perp$

$\Delta h_\perp(t_V, E_\pi)$



$|\vec{p}| = \frac{2\pi}{L} \qquad M_\pi L = 4$

$\Rightarrow \quad E_{\pi,\vec{p}} \approx 260\,\mathrm{MeV}$

$$\Delta h_\perp^{B\pi}(t_V, E_{\pi,\vec{p}}) \approx \Delta h_\perp^{B\pi,\mathrm{tree}}(t_V, E_{\pi,\vec{p}}) = -1 \times e^{-E_{\pi,\vec{p}}\, t_V} \quad + \text{ NLO}$$

Note:

- no factor $1/L^3$ $\Rightarrow$ sometimes called "*volume enhanced*" contribution
- no sum over pion momenta (i.e. no tower of states), *one fixed pion momentum only*
- exactly the same result as in $\tilde{G}_\mathrm{P}$ in the nucleon sector (recall *Motivation*)

Questions

- HMChPT predicts a non-negligible and in one case a significant $B\pi$ contamination
  - How reliable are these NLO ChPT results?
  - How to deal with the $B\pi$ contamination ?
- Work in progress (almost finished):
  Lattice study including two-hadron ($B\pi$) interpolating fields (“GEVP”)
  - confirms the HMChPT predictions
  - demonstrates that the dominant $B\pi$ contamination in the form factor can be subtracted

Lattice study: Methodology

Compute N x N matrix of 2-pt correlation functions

$$C_{ij}^{(2)}(t) = \sum_{\mathbf{x}} \langle \overline{\mathcal{O}}_i(\mathbf{x},t)\mathcal{O}_j(0)\rangle = \sum_{n=1}^{\infty} \psi_{ni}^* \psi_{nj} e^{-E_n t}, \quad i,j = 1,...N$$

with N interpolating fields O_j, including

- standard single-hadron interpolators
- two-hadron ($B^*\pi$) interpolators

overlap factors $\psi_{nj} = \langle 0|\mathcal{O}_j|B_n\rangle$.

one or two hadron eigenstate

Analogously vector of 3-pt correlation functions with the vector current

$$C_{\mu,j}^{(3)}(t_\pi, t_h; \mathbf{p}) = \sum_{n,m=1}^{\infty} \frac{\langle 0|\hat{\mathcal{O}}_\pi|\pi_m(\mathbf{p})\rangle}{2E_{\pi_m}} \mathcal{M}_{mn}^{\mu}(\mathbf{p})\, \psi_{nj}\, e^{-E_m t_\pi} e^{-E_n t_h}$$

with matrix elements $\mathcal{M}_{mn}^{\mu}(\mathbf{p}) = \langle \pi_m(\mathbf{p})|V^\mu|B_n\rangle_{\mathrm{HQET}}$

Lattice study: Methodology

- Performing a standard *generalised eigenvalue problem* (*GEVP*) analysis we obtain estimates for
  - the lowest N energy levels E_n
  - the overlap factors $\psi_{nj} = \langle 0|\mathcal{O}_j|B_n\rangle.$
  - the matrix elements $\mathcal{M}^{\mu}_{mn}(\mathbf{p}) = \langle \pi_m(\mathbf{p})|V^{\mu}|B_n\rangle_{\mathrm{HQET}}$

M. Lüscher, U.Wolff (1990)
B. Blossier et. al. (2009)
…

- In practice we used N = 5 or 6
  - 1 or 2 single B-meson interpolators
  - 4 $B^*\pi$ interpolators with the smallest non-vanishing spatial momenta

Lattice study: Gauge ensembles

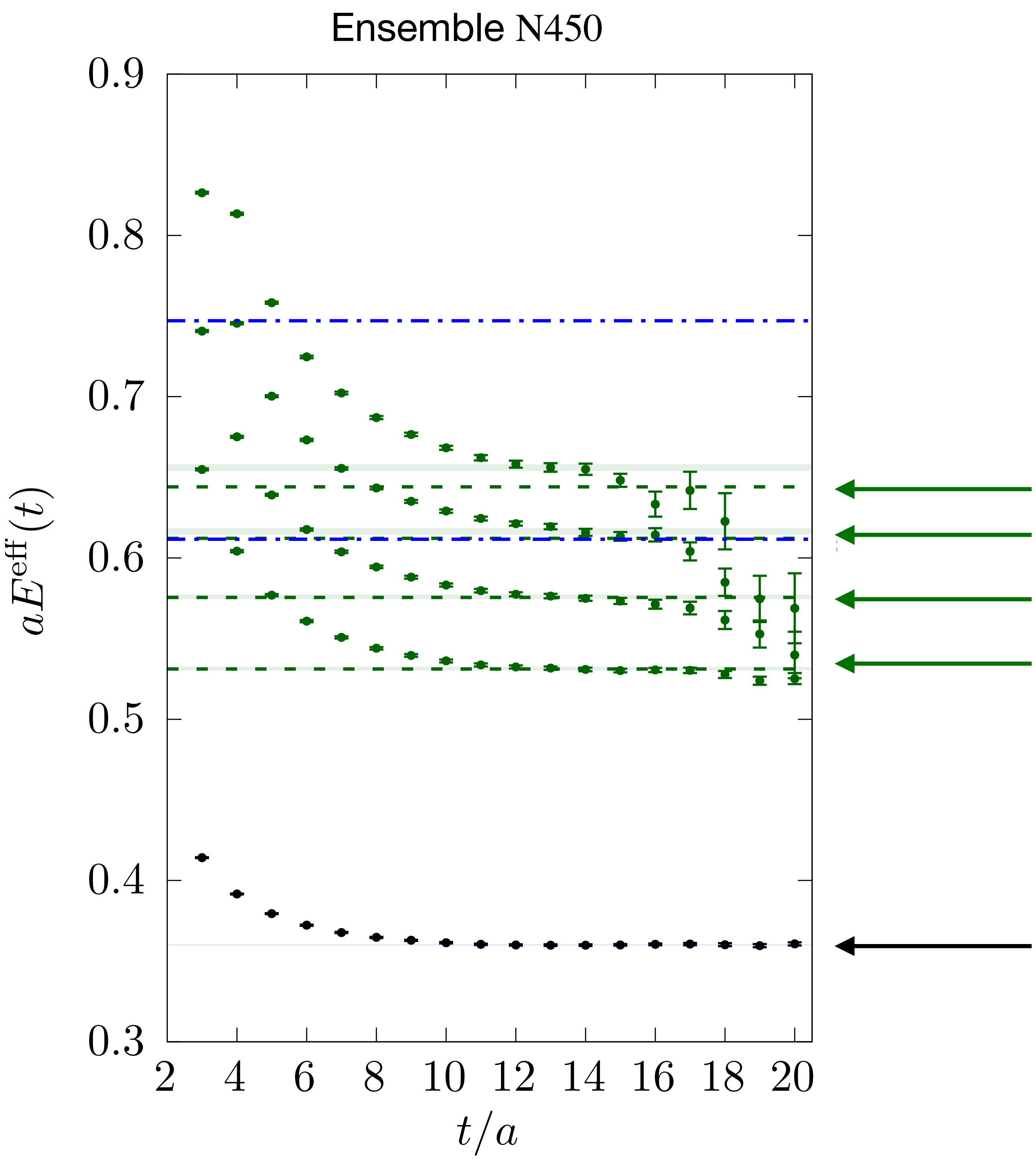
Use of CLS gauge ensembles:

M. Bruno et. al. (2015)
…

Id	a [fm]	L/a	V [fm]	BC	m_π [MeV]	Distillation	Stoch. ATA	$m_\pi L$
B451	0.076	32	2.4	periodic	417	yes	yes	5.2
N450	0.076	48	3.6	periodic	280	yes	yes	5.2
D451	0.076	64	4.9	periodic	215	no	yes	5.3

In the following: results for N450 only

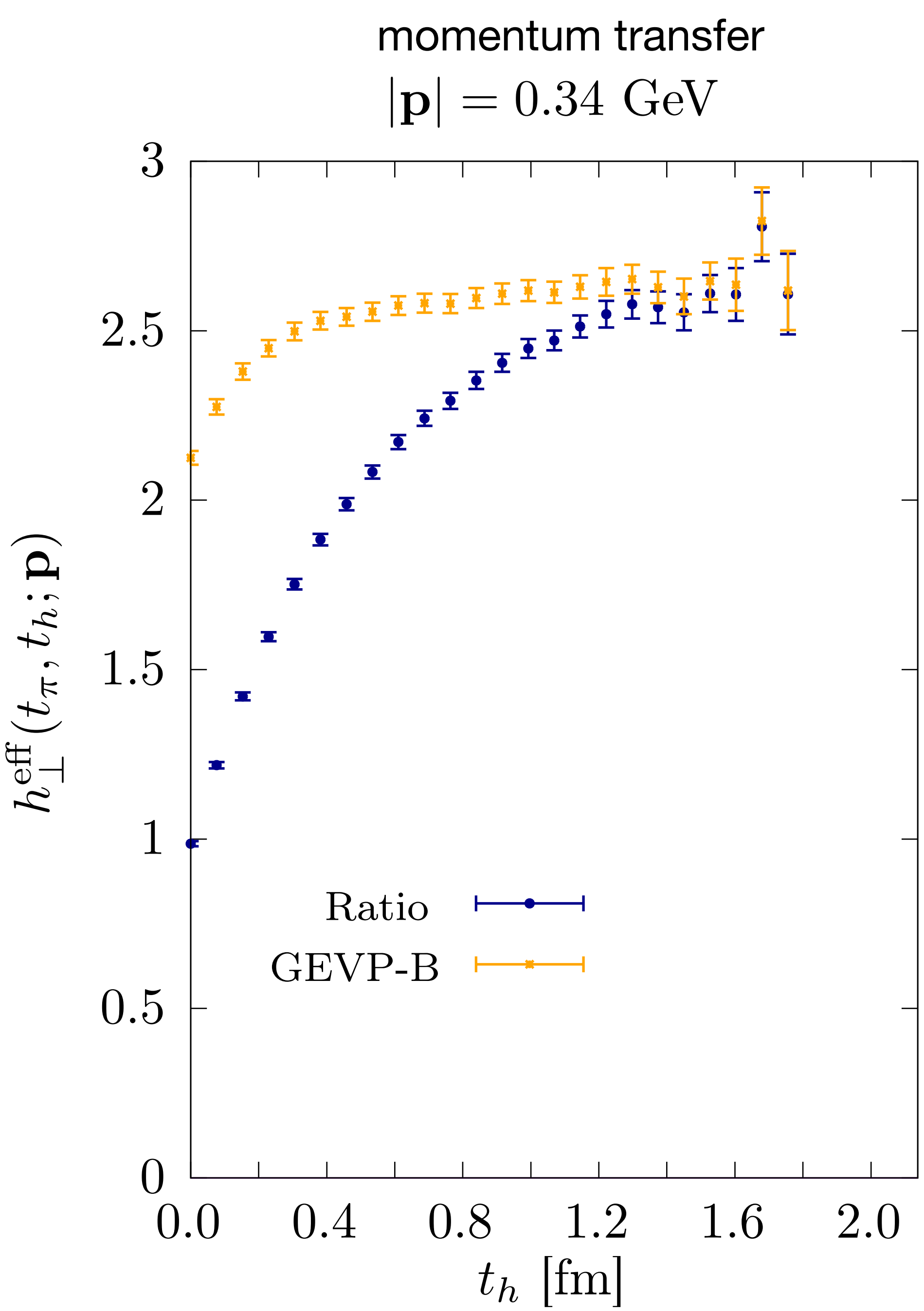
Results: Heavy-light meson spectrum



expected $B^*(p)\pi(-p)$ energies assuming no interaction

single B-meson ground state

Results: form factor $h_\perp$

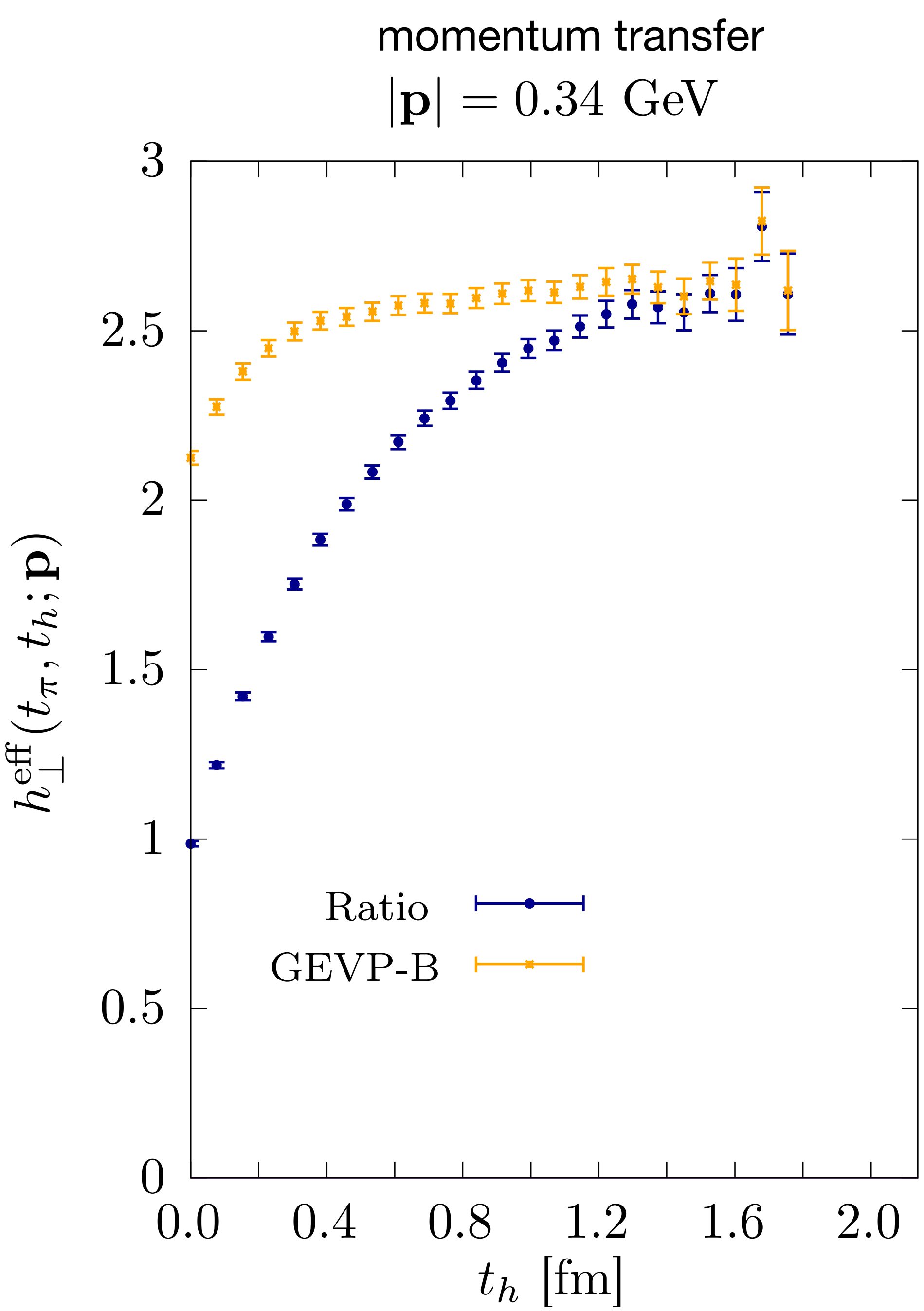


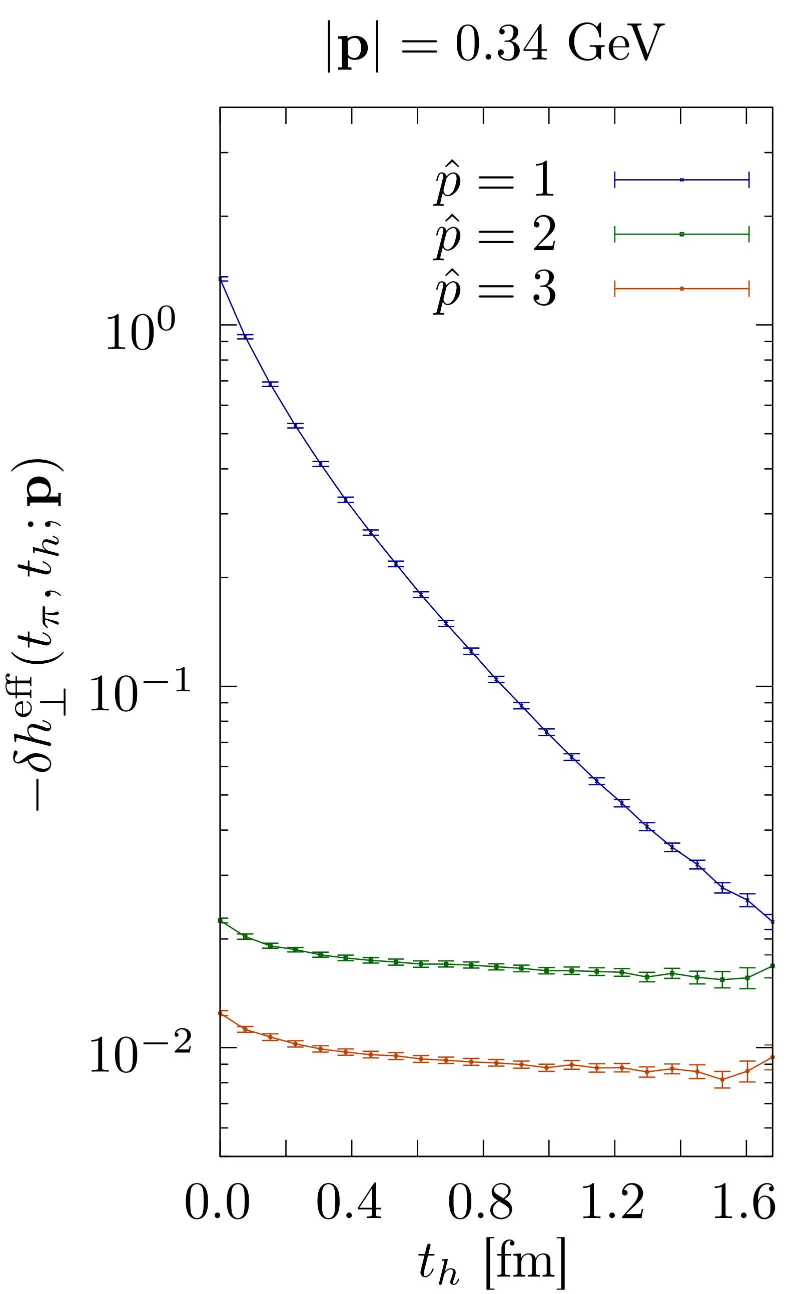
effective form factor from:

GEVP ($B^*\pi$ contamination removed)

simple ratio ($B^*\pi$ present)

Results: form factor $h_\perp$





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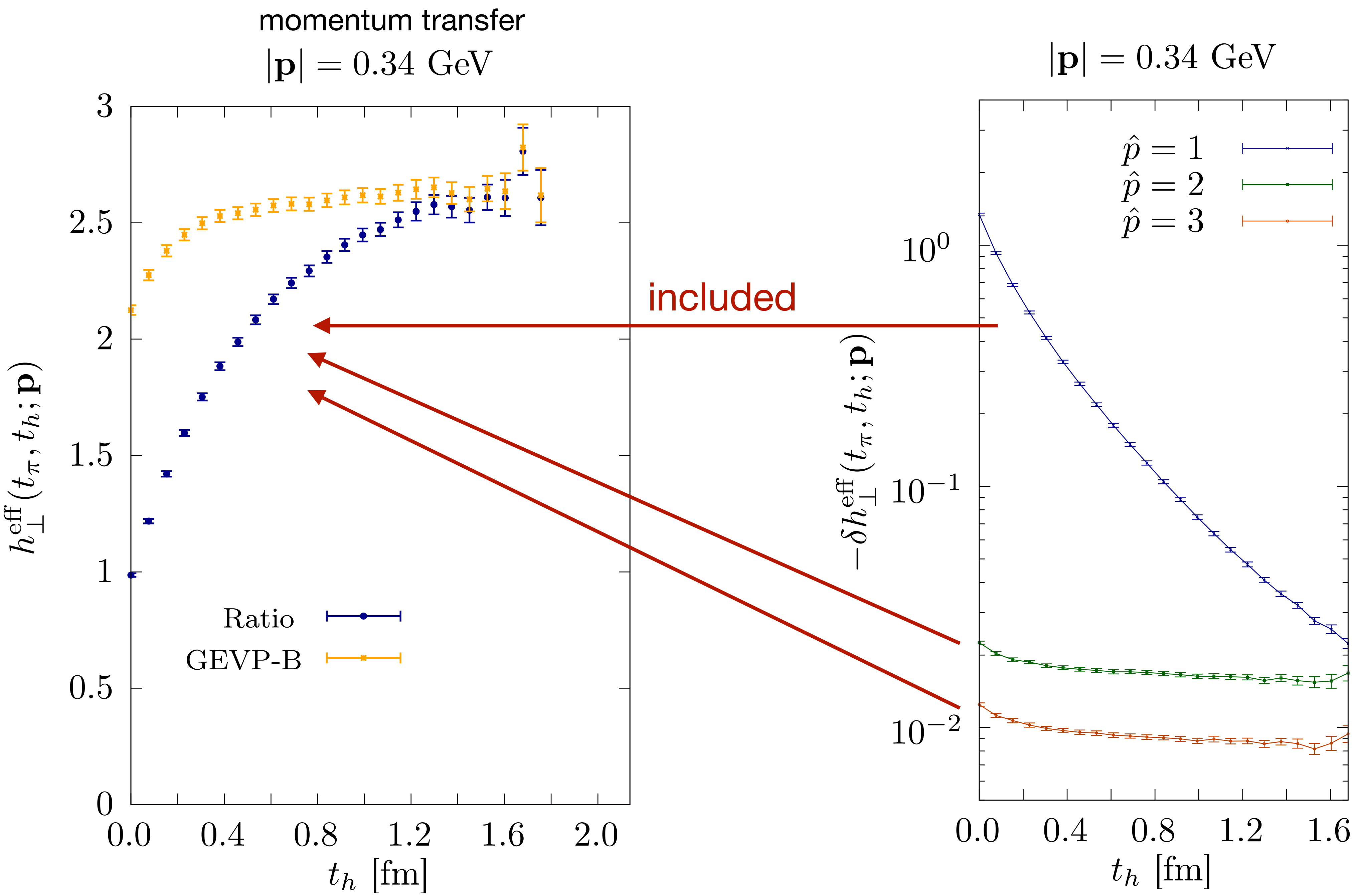
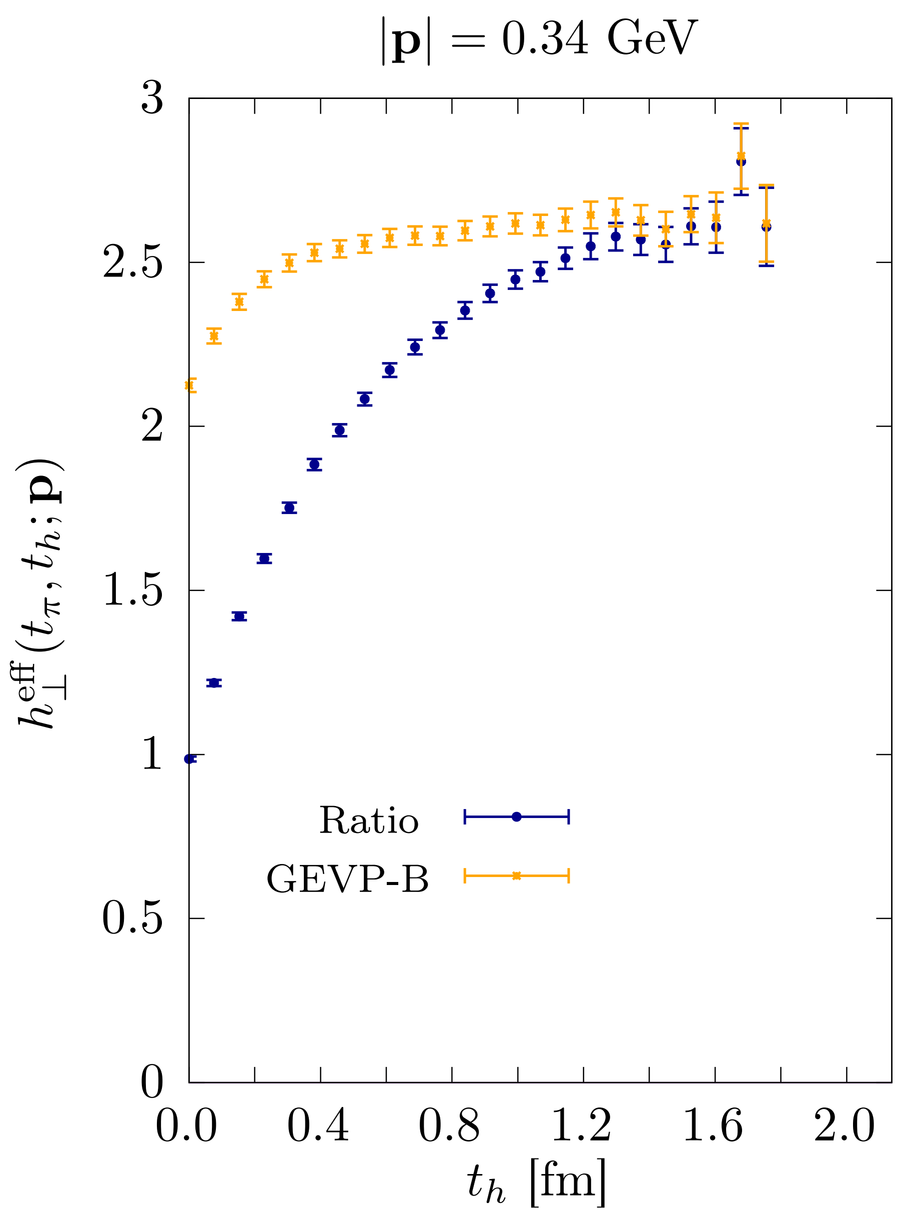
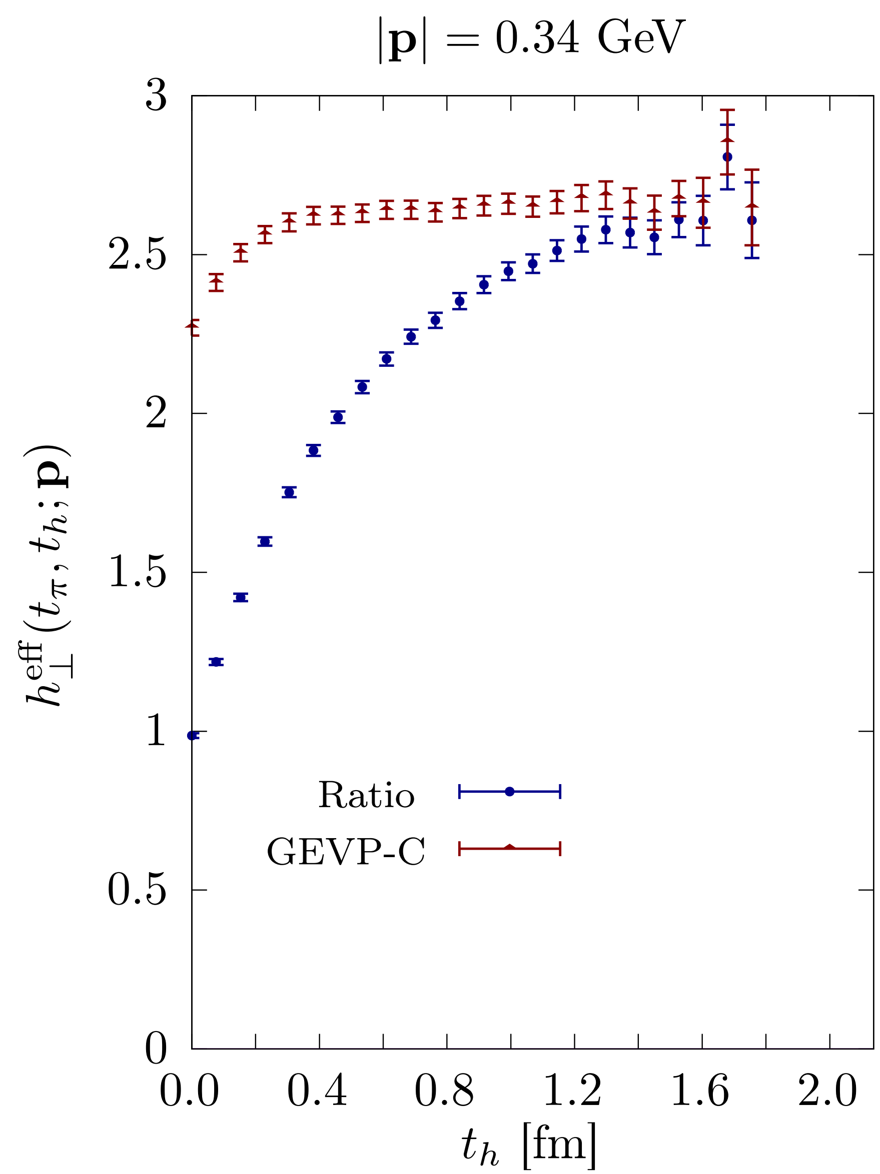


The GEVP confirms: The dominant excited-state contamination stems from one single $B^*\pi$ state !

Results: form factor $h_\perp$

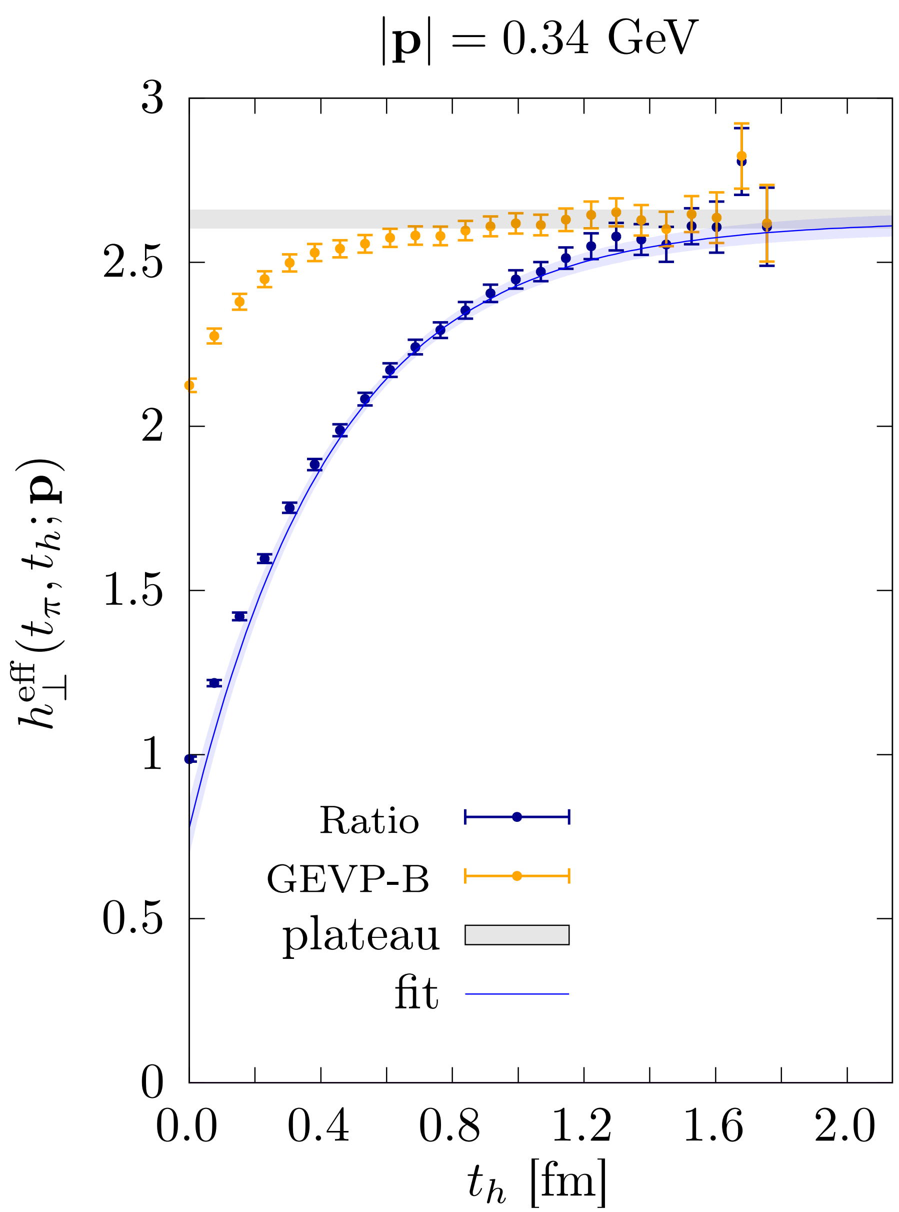




GEVP with the dominant $B^*\pi$ state only

essentially no difference

Removing the $B^*\pi$ state without the GEVP



HMChPT inspired *fit ansatz* for the ratio data:

$$h_\perp^{\mathrm{eff}}(t_h; \mathbf{p}) = h_\perp(\mathbf{p}) \times \left(1 + Z e^{-E_{B\pi} t_h}\right)$$

free fit parameter $\quad m_B + E_\pi \quad$ input !

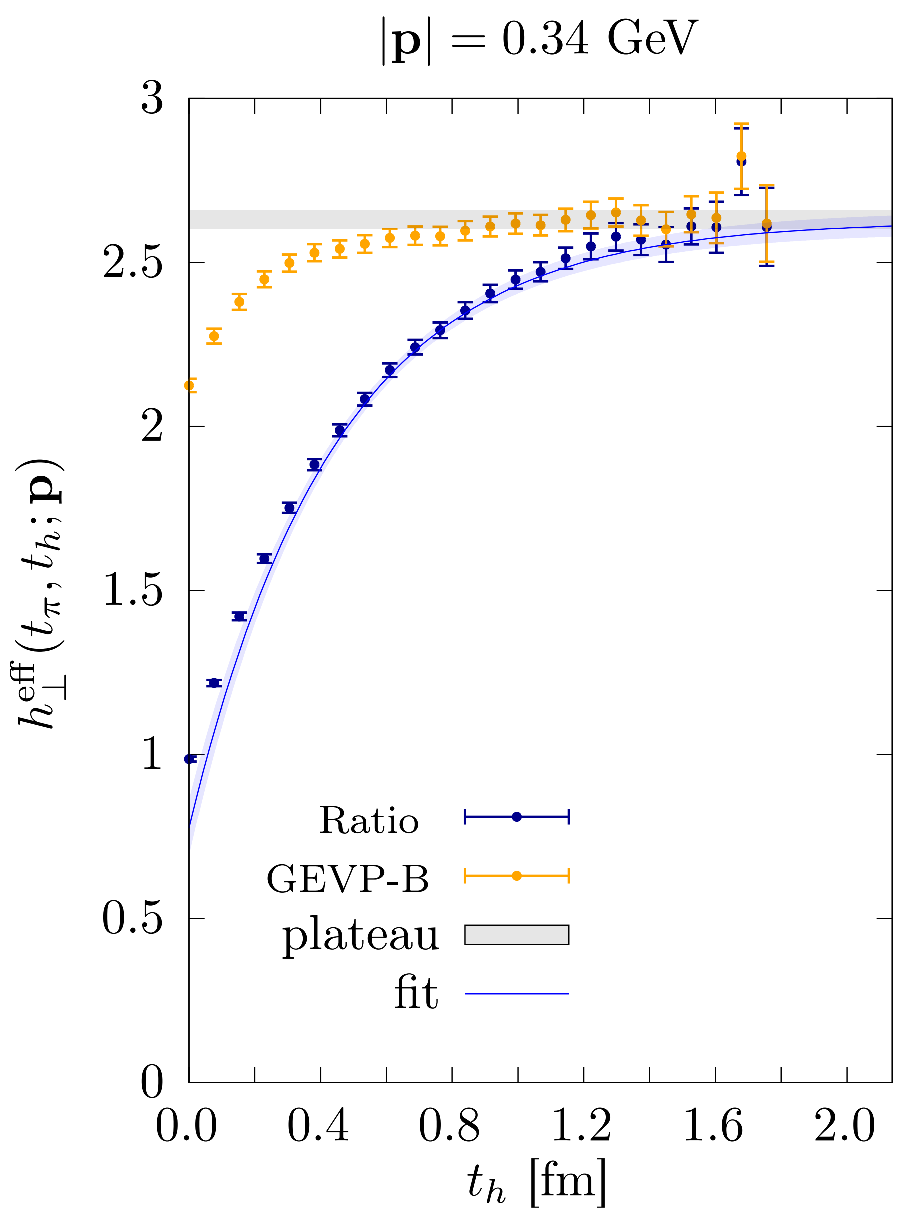
$\Rightarrow$ leads to the same result for the form factor

as a plateau fit to the GEVP data

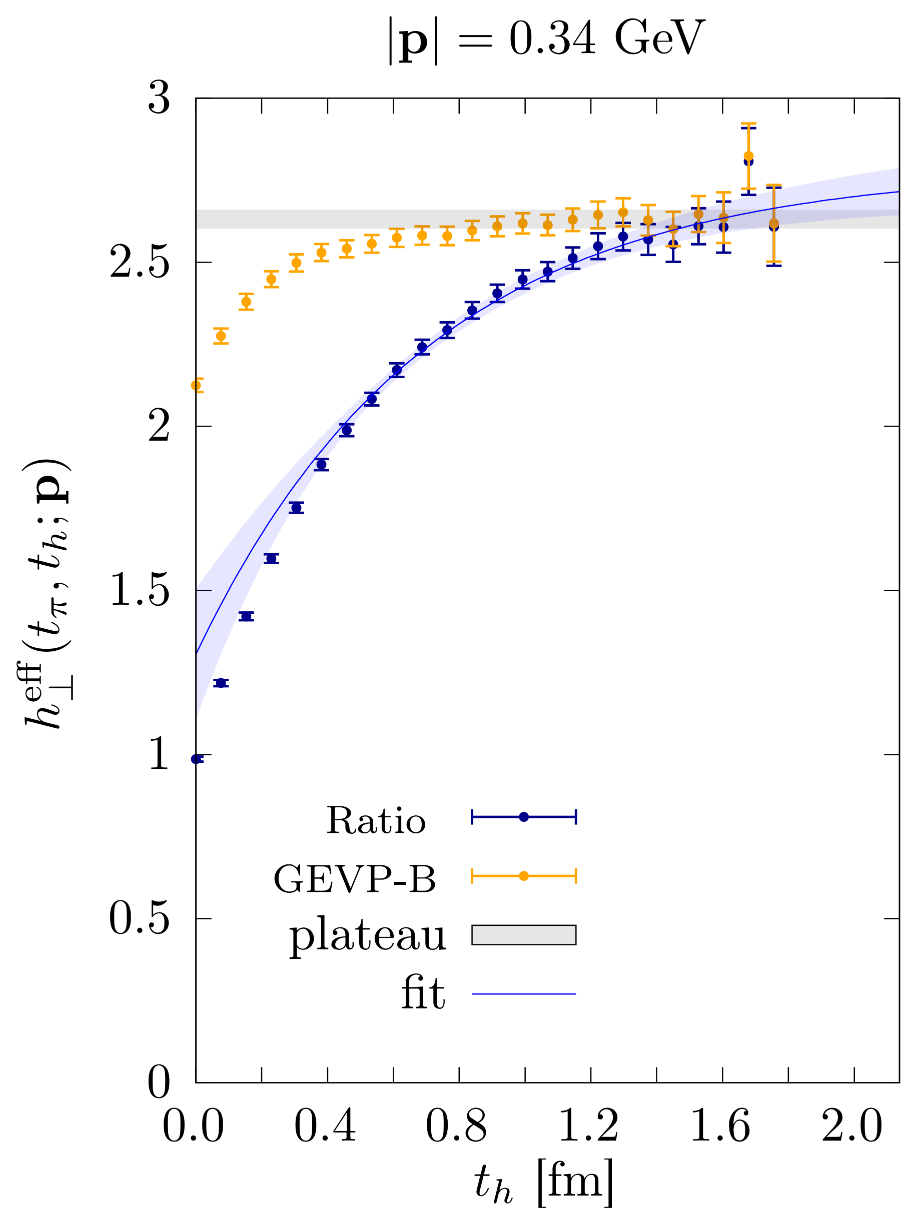
This is numerically much cheaper

Removing the $B^*\pi$ state without the GEVP

Note: It is necessary to fix the two hadron Energy $E_{B\pi}$ to expected value



$E_{B\pi} = m_B + E_\pi$ fixed



$E_{B\pi}$ free fit parameter

$\Rightarrow$ GEVP plateau value not reproduced

Summary / Outlook

- HMChPT predicts a large $B\pi$ contamination in the effective form factor for $h_\perp$
- Dedicated lattice simulations (= GEVP including two-hadron ($B\pi$) interpolating fields) confirm the HMChPT predictions:
  - large underestimation of the form factor
  - dominant contribution stems from one $B\pi$ state
- Viable strategies to deal with it:
  - GEVP with two-hadron interpolator for this dominant $B\pi$ state
  - remove contribution by HMChPT-inspired fit

$\Rightarrow$ reliable form factor results

(work in progress - Alpha collaboration)

Backup slides

Interpolating B-meson fields in HMChPT

- We are interested in correlation functions of interpolating fields for the B-mesons

Quark level: $\overline{q}_r(x)\Gamma Q(x)$ Γ: Clifford algebra element e.g. γ_5 or γ_k

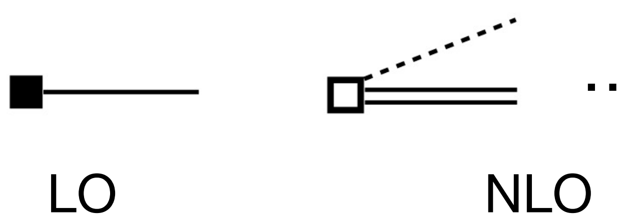
light u,d quark heavy (static) b-quark

- Are mapped to HMChPT as usual: most general term compatible with the symmetries

➥ LO 1 term
NLO 2 terms

local

→ interpolating field for pseudo scalar B-meson



Smeared interpolating B-meson fields in HMChPT

- We are interested in correlation functions of interpolating fields for the B-mesons

Quark level: $\overline{q}_r(x)\Gamma Q(x)$ Γ: Clifford algebra element e.g. γ_5 or γ_k

light u,d quark heavy (static) b-quark

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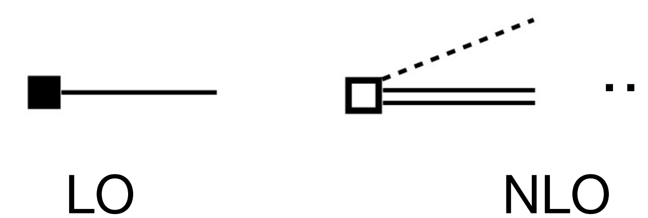
➥ LO 1 term
NLO 2 terms

local → interpolating field for pseudo scalar B-meson



- Holds also for *smeared interpolating fields* with provided

$$R_{\rm smear} \ll \frac{1}{M_\pi}$$

$$\overline{q}_r(x) \;\rightarrow\; \overline{q}_r^{\,\rm sm}(x) = \int d^4y\, \overline{q}_r(y) K(y,x)$$

$R_{\rm smear} = 0.3 \ldots 0.4$ fm seem okay

- Difference between local and smeared interpolators: different values for the LECs

Impact of smearing (?)

Interpolating B-meson field

local

smeared

$\beta_1 \approx 0.14(4)\,\mathrm{GeV}^{-1}$

B. Colquhoun *et al.* [JLQCD], 2022

$\tilde{\beta}_1 \approx$?

Expectation:

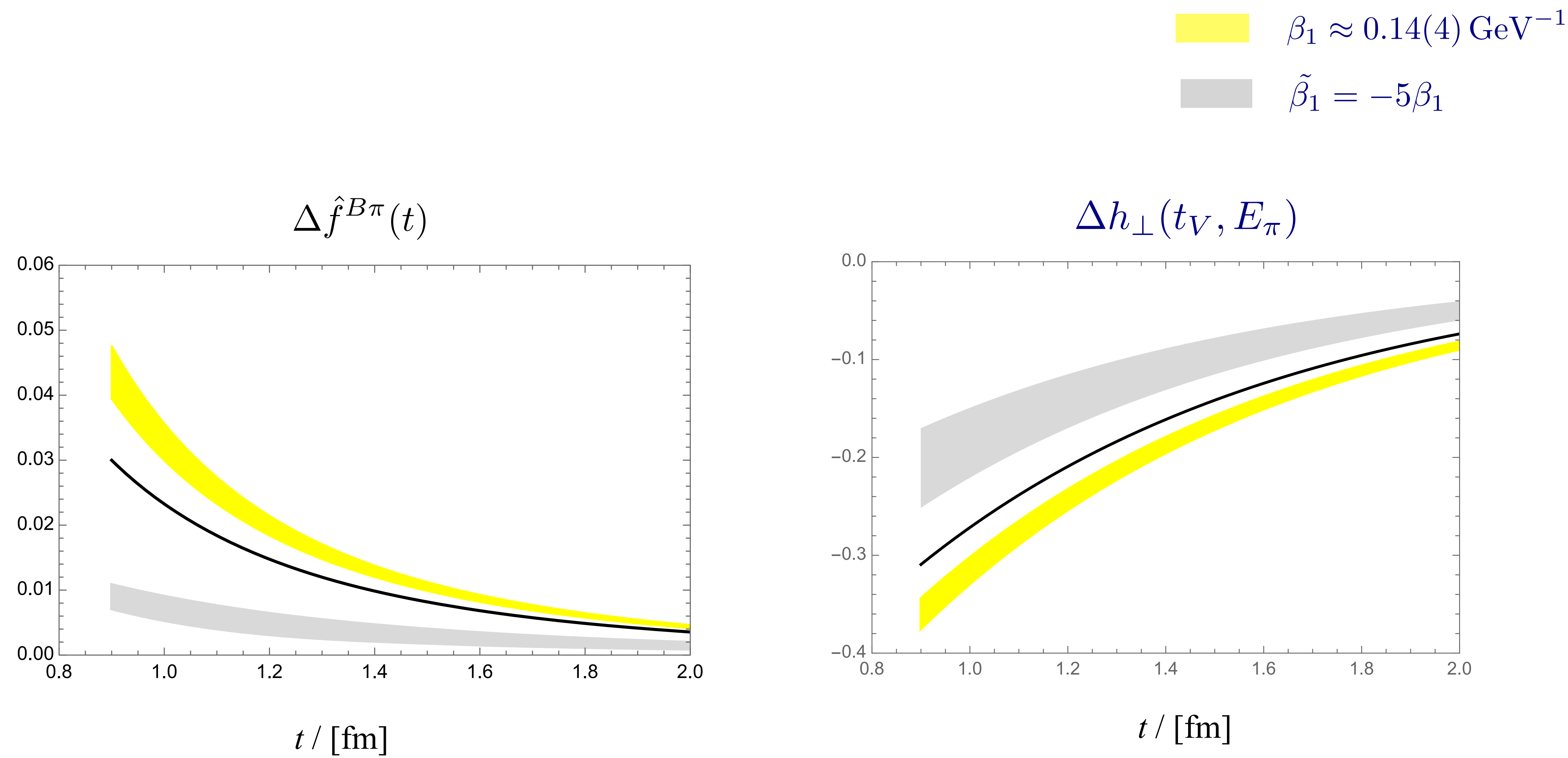
Smearing

increases overlap with the ground state

decreases overlap with the excited states

Justified for Bπ states?

Impact of smearing (?)



Smearing has the potential to significantly reduce the Bπ contamination

But: Is there a smearing procedure that causes $\tilde{\beta}_1 \approx -5\beta_1$?

Lattice determination of LECs β₁ and $\tilde{\beta}_1$

$\mathbf{p}^*$: reference momentum

HM ChPT prediction to NLO:
A. Broll, R. Sommer, OB, (2023)

$$R(\mathbf{p}) \equiv \frac{\langle \pi(\mathbf{p})|V_k|B\rangle}{\langle \pi(\mathbf{p}^*)|V_k|B\rangle} = \frac{1-\beta_1\, E_\pi(\mathbf{p})/g}{1-\beta_1\, E_\pi(\mathbf{p}^*)/g} \times \frac{E_\pi(\mathbf{p}^*)}{E_\pi(\mathbf{p})} \times \frac{\mathbf{p}_k}{(\mathbf{p}^*)_k}$$

➡ extract β_1 from the pion energy dependence

Lattice estimator: $R^{\mathrm{eff}}(\mathbf{p}, t_V)$

suitably defined ratio of 3pt and 2pt functions

Smearing of the vector current: $V_k \to \tilde{V}_k$ $\Rightarrow$ $R(\mathbf{p}) \to \tilde{R}(\mathbf{p})$

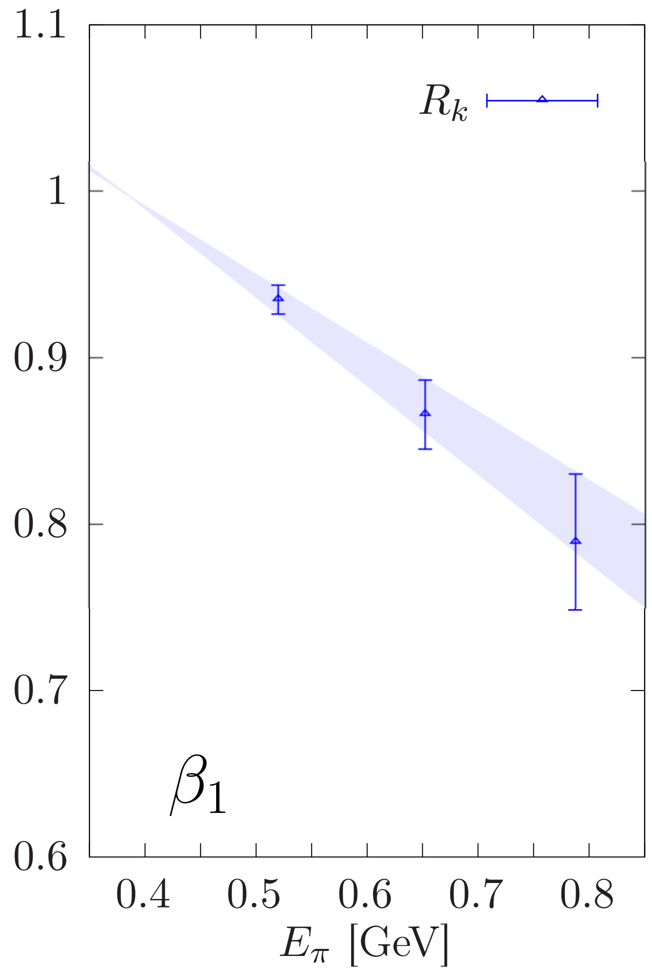
$\beta_1 \to \tilde{\beta}_1$

➡ extract $\tilde{\beta}_1$

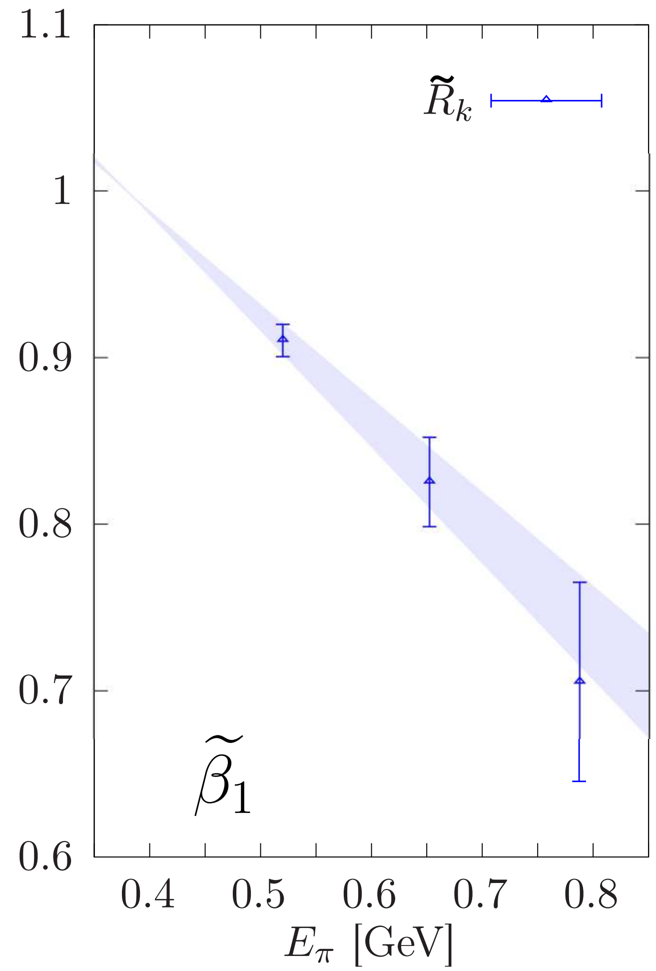
Lattice determination of LECs β_1 and $\tilde{\beta}_1$

Lattice results → contribution by Antoine Gerardin, *Lattice 2024*

local current



smeared current



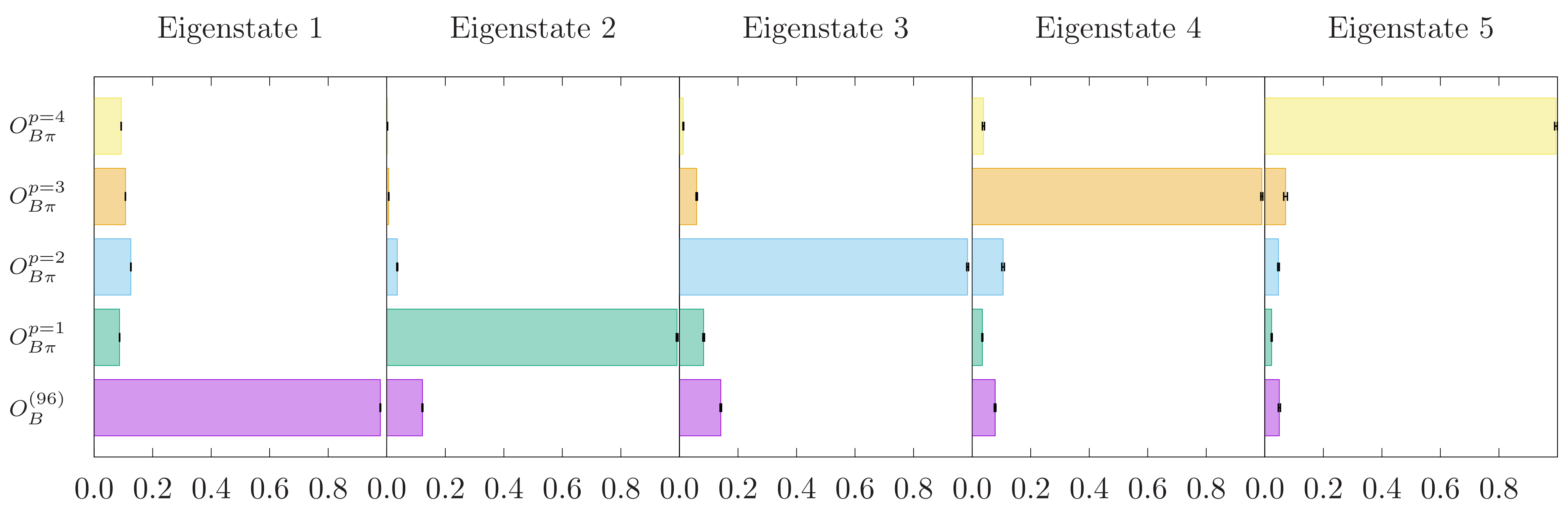
Gaussian smearing

$r_{\text{smear}} \approx 0.45\,\text{fm}$

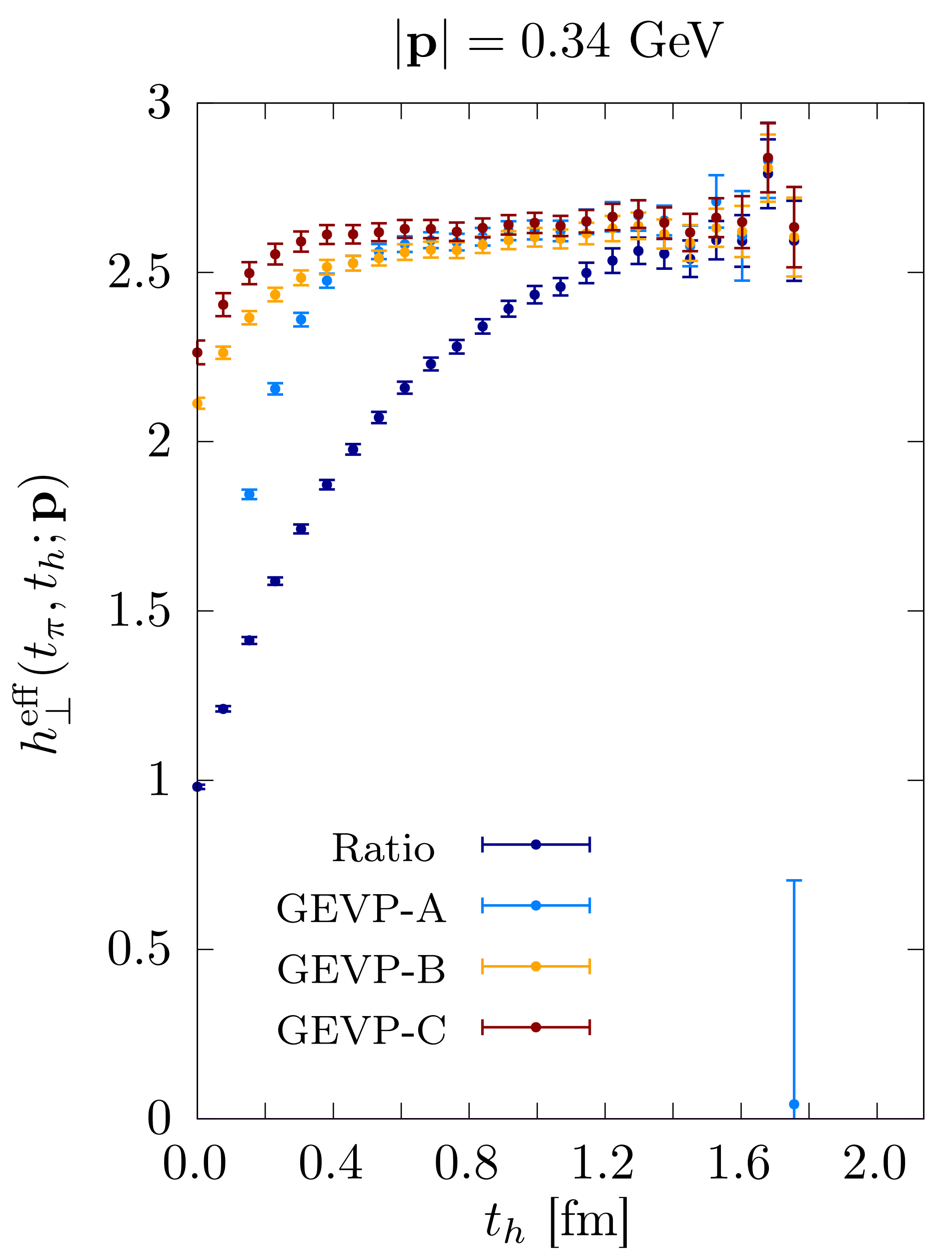
$$\beta_1 \approx 0.20(2)\text{GeV}^{-1} \qquad \tilde{\beta}_1 \approx 0.23(3)\text{GeV}^{-1}$$

$\beta_1 \approx \tilde{\beta}_1$ ⇒ small impact of Gaussian smearing on Bπ excited states !

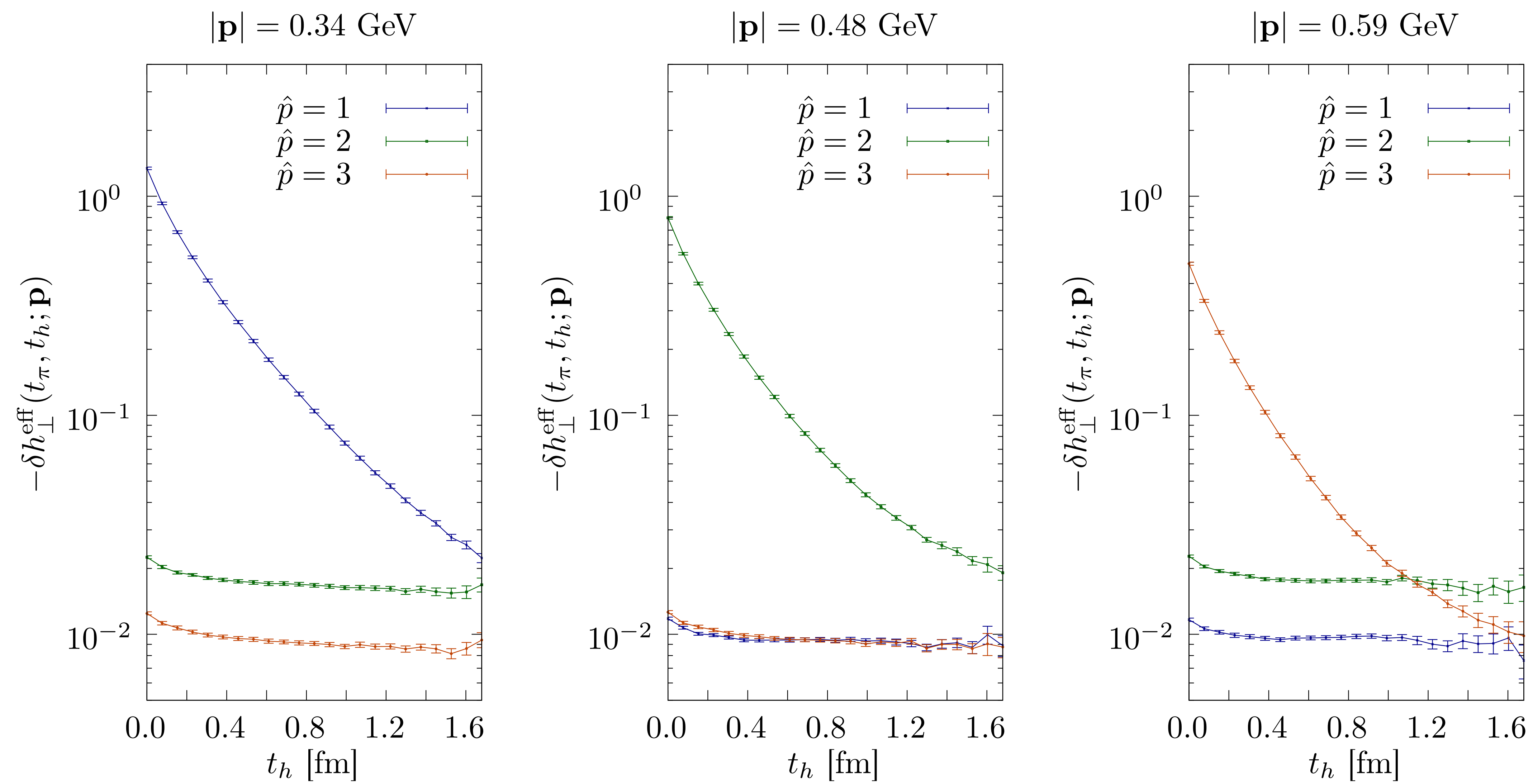
Recall: $\beta_1 \approx 0.14(4)\text{GeV}^{-1}$ by JLQCD



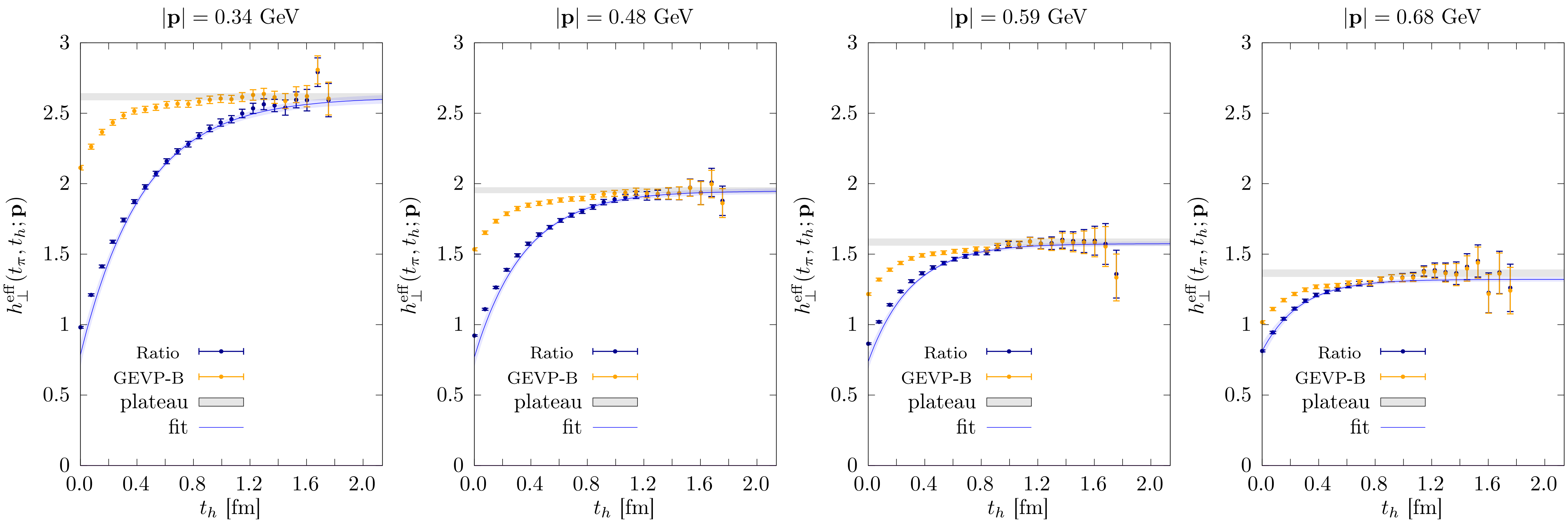
Normalised overlap factors for GEVP-B on N450



Comparison GEVP-A, B and C on N450



Relative contribution of the lowest 3 $B\pi$ states to the excited state contribution $-\delta h_{\perp}$ on N450



Two parameter fit to the form factor using $$h_\perp^{\mathrm{eff}}(t_h;\mathbf{p}) = h_\perp(\mathbf{p}) \times \left(1 + Ze^{-E_{B\pi}t_h}\right)$$

N450 data

QCD with a static b-quark

- Our starting point: QCD with a *heavy static b-quark*

E. Eichten and B. Hill (1990), …

$$\mathcal{L} = \sum_r \overline{q}_r(\gamma_\mu D_\mu + m_l)q_r + \overline{Q}(D_4 + m_b)Q + \mathcal{L}_{\rm gauge}$$

Note: LO in the *heavy quark expansion*, $1/\mathrm{m_b}$ corrections (not here)

- High degree of symmetry
  - Heavy quark spin symmetry ➞ mass degenerate B and B* mesons
  - Local flavor number (LFN) symmetry: $Q(x) \to \exp[i\eta(\vec{x})]\, Q(x)$
  - Chiral symmetry (in the light quark sector)
  - Isospin symmetry (if we assume $\mathrm{m_l = m_u = m_d}$) ➞ 3 mass degenerate pions

- Corresponding chiral effective theory ➞ Heavy Meson (HM) ChPT
  must respect all these symmetries

M. Wise (1992)
Burdman, Donoghue (1992)
…

- Note: We work in continuum QCD (and ignore Lorentz symmetry breaking at finite lattice spacing)

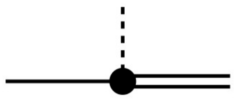
Basics of Heavy Meson (HM) ChPT

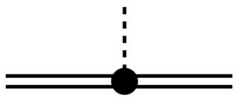
- Heavy quark spin symmetry → multiplet $H = P_+ \left(iB_k^*\gamma_k + iB\gamma^5\right) \qquad P_+ = (1+\gamma_4)/2$

  $\bar{H} = \gamma_4 H^\dagger \gamma_4$

- Relevant interaction given by

$$\mathcal{L}_{\rm int} = i\frac{g}{f}\,\mathrm{tr}\left(H\gamma_5\gamma_\mu\partial_\mu\pi\bar{H}\right) + \ldots \quad \rightarrow$$


$BB^*\pi$


$B^*B^*\pi$

Note:
- one pion derivative
- two LO LECs f: pion decay constant

  g: $BB^*\pi$ coupling

  chiral limit values

Interpolating B-meson fields in HMChPT

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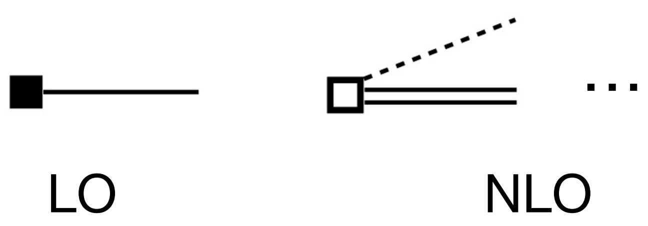
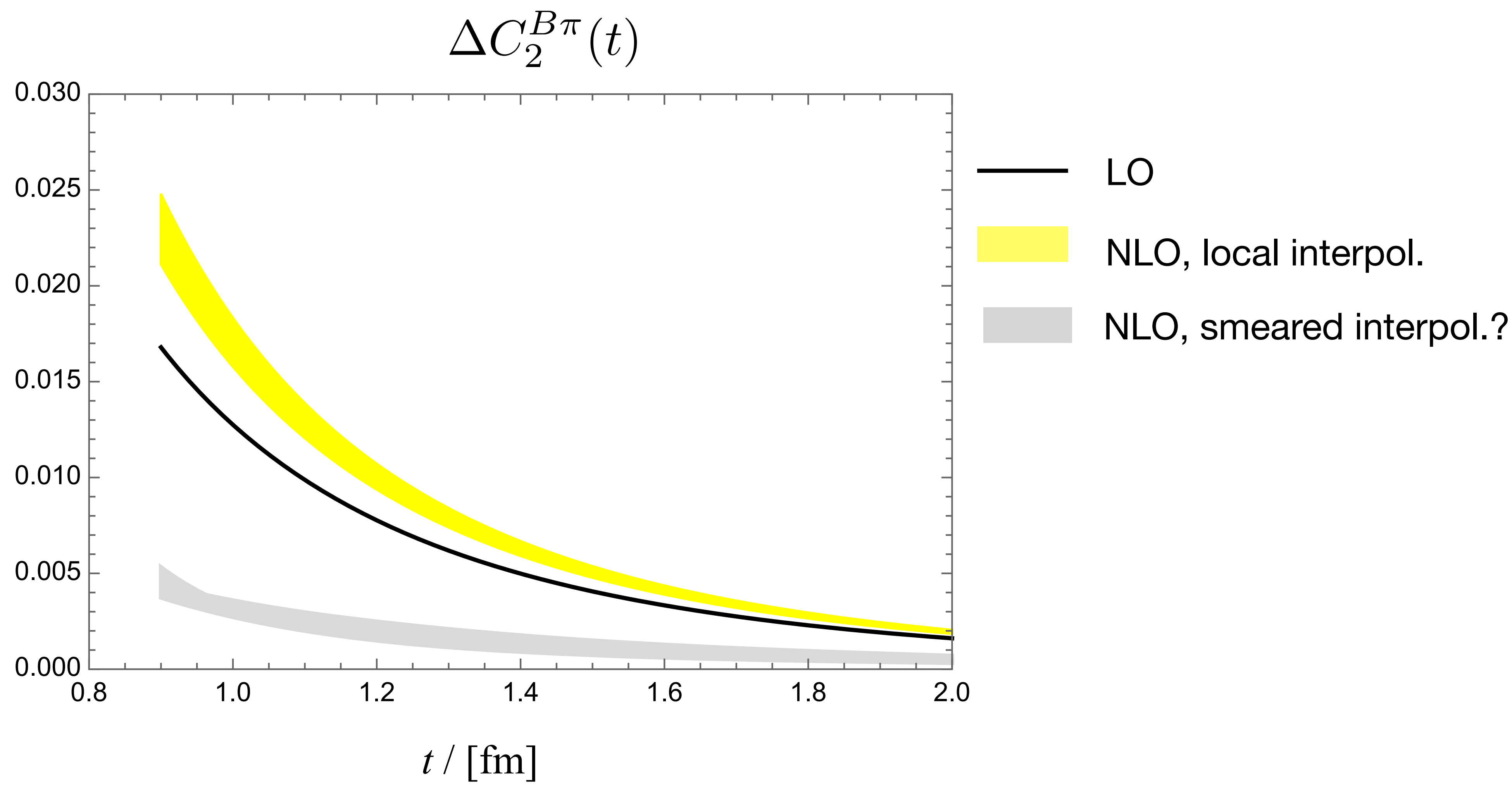
➥ LO 1 term
NLO 2 terms

→ interpolating field for pseudoscalar B-meson



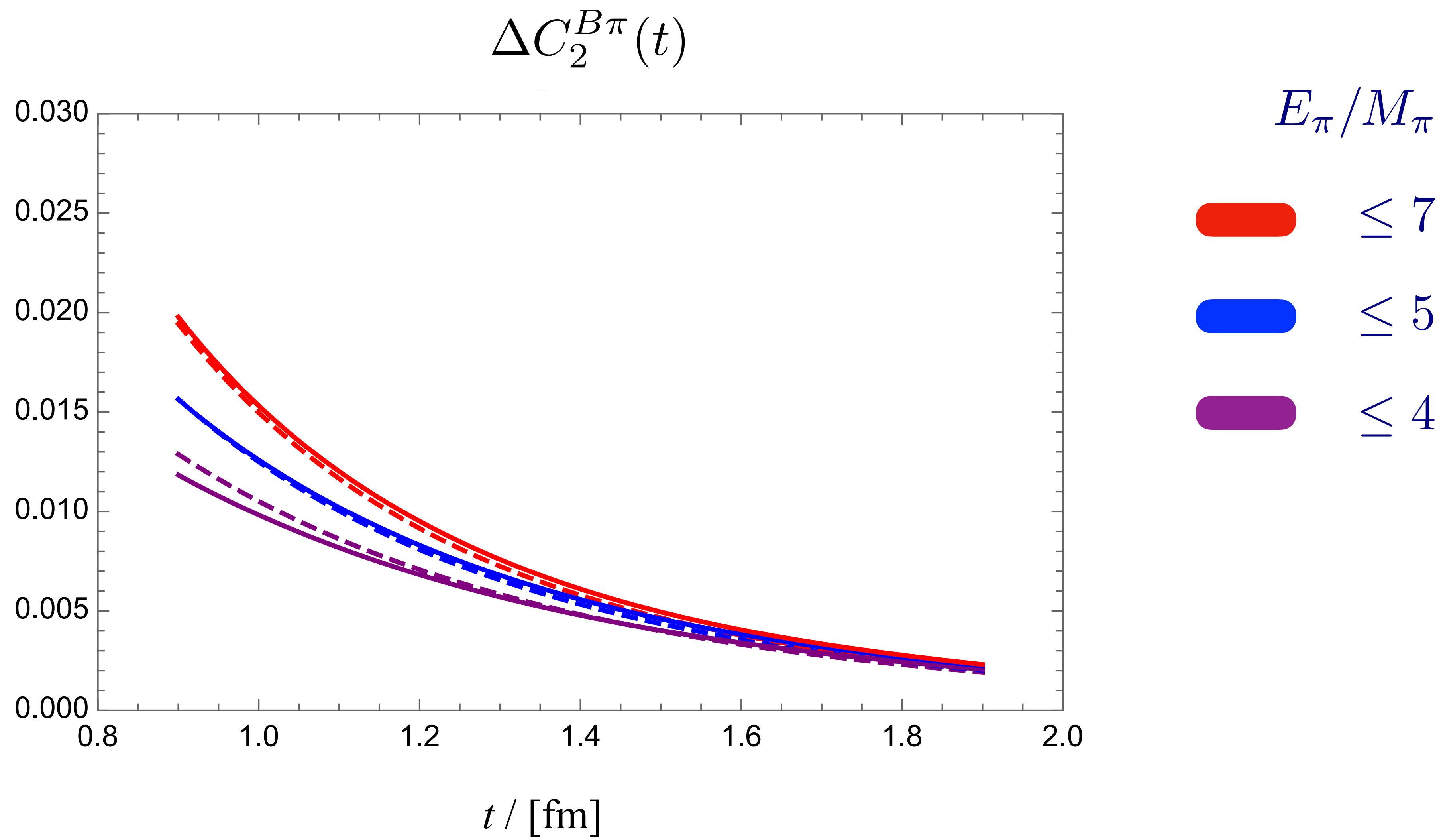
Bπ contribution in the 2pt function



$t = 1.3$ fm: Bπ-state contamination leads to an *overestimation* of ≈ 1%

if local interpolating fields are used

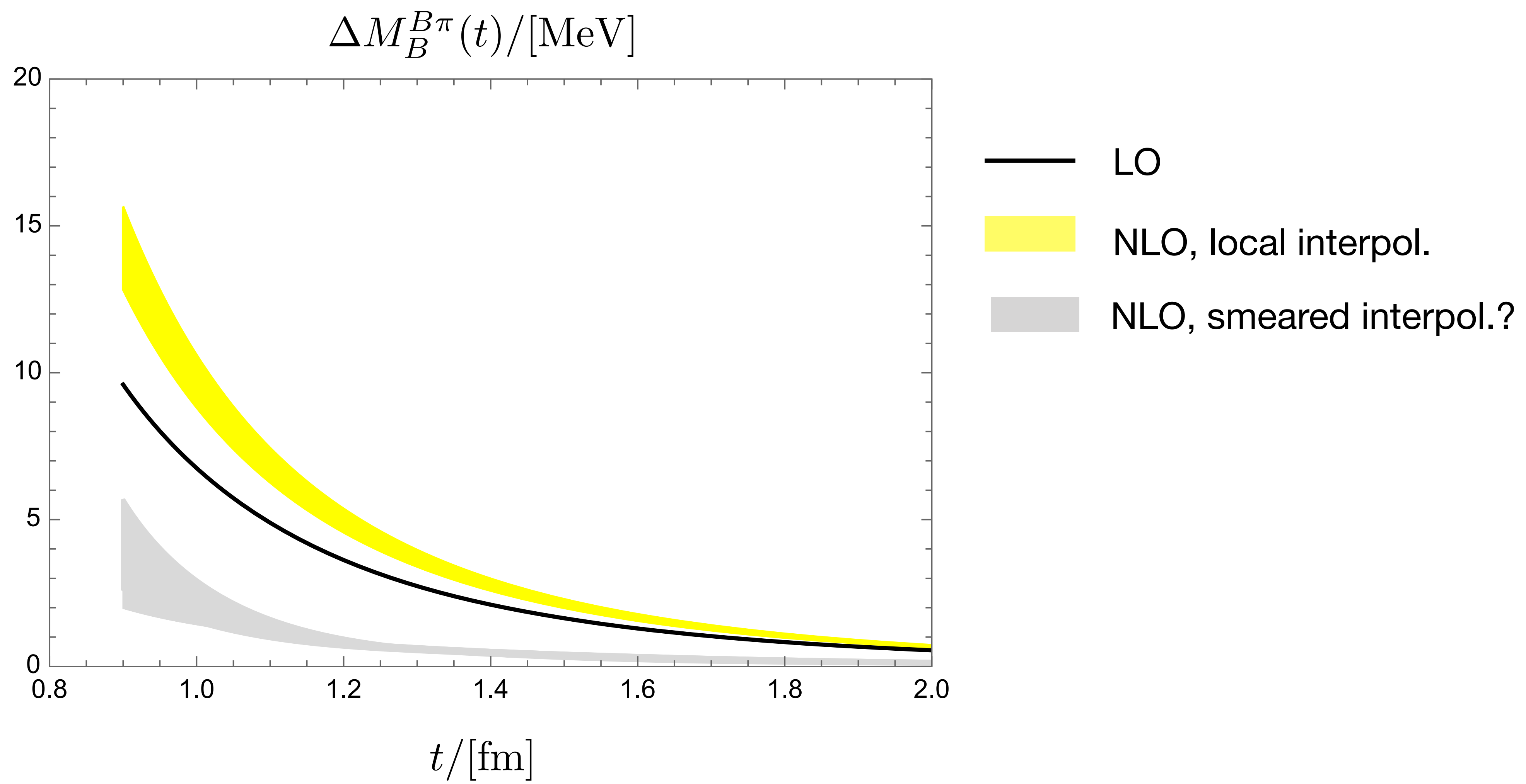
$B\pi$ contribution in the 2pt function



dashed lines: $M_\pi L = 4$

solid lines: infinite volume limit

Bπ contribution in the effective mass



$t = 1.3$ fm: Bπ-state contamination leads to an *overestimation* of ≈ 5 MeV

if local interpolating fields are used

Antoine Gerardin, talk at *Lattice 2024*

Calculation of the LECs : β_1 and $\widetilde{\beta}_1$

- HMχPT prediction (at NLO) for the form factor $h_\perp$ [ref still missing]

$$\frac{\langle \pi(\mathbf{p})|V_k|B\rangle}{\langle \pi(\mathbf{p}^\star)|V_k|B\rangle} = \frac{1-\beta_1/g\,E_\pi(\mathbf{p})}{1-\beta_1/g\,E_\pi(\mathbf{p}^\star)} \times \frac{E_\pi(\mathbf{p}^\star)}{E_\pi(\mathbf{p})} \times \frac{p_k}{(p^\star)_k}$$

$\rightarrow$ extract β_1 from the pion energy dependence

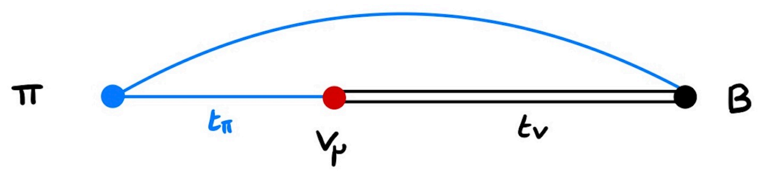
$\rightarrow$ smearing of the vector current ($V_\mu \rightarrow \widetilde{V}_\mu$) : gives access to $\widetilde{\beta}_1$ (LECs for smeared B operators)

[O. Bär, A. Broll, R. Sommer '23]

- Matrix element obtained from 3-point functions in the static limit

$$C^{(3)}_\mu(t_\pi, t_v; \mathbf{p}) = \frac{a^9}{V^2} \sum_{\mathbf{x_f},\mathbf{y},\mathbf{x_i}} \langle \overline{\mathcal{O}}_\pi(\mathbf{x_f}, t_v+t_\pi)\, V_\mu(\mathbf{y}, t_v)\, \mathcal{O}_B(\mathbf{x_i}, 0)\rangle\; e^{-i\mathbf{p}(\mathbf{x_f}-\mathbf{y})}$$

Replace local by smeared vector current : $\widetilde{V}_\mu \longrightarrow \widetilde{C}^{(3)}_\mu$



- Lattice estimator :

$$R^{\mathrm{eff}}(t, t_v; \mathbf{p}) \equiv \frac{E_\pi(\mathbf{p})}{E_\pi(\mathbf{p}^\star)}\,\frac{(p^\star)_k}{p_k} \times \frac{\widetilde{C}^{(3)}_k(t_\pi, t_v; \mathbf{p})}{\widetilde{C}^{(3)}_k(t_\pi, t_v; \mathbf{p}^\star)}\,\frac{C^{(2)}_\pi(t_\pi, \mathbf{p}^\star)}{C^{(2)}_\pi(t_\pi, \mathbf{p})}$$

$\rightarrow$ this estimator is itself affected by excited states : can be used to correct our data

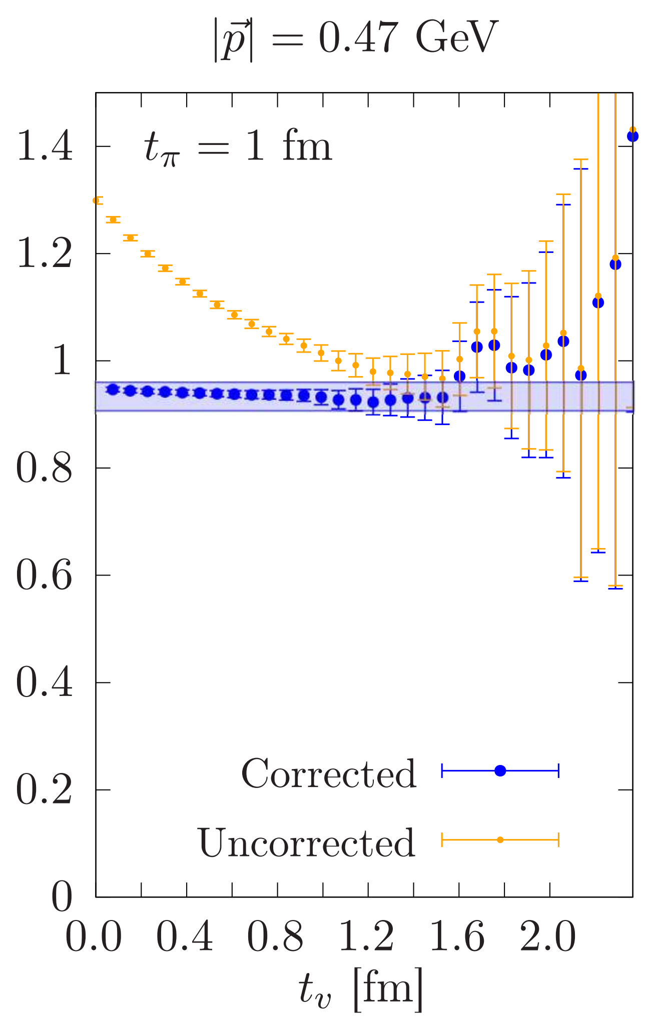
$$1+\delta_{B\pi}(t_v;\mathbf{p}) \;=\; \frac{1+\Delta h_\perp(t_v;\mathbf{p})}{1+\Delta h_\perp(t_v;\mathbf{p}^\star)} \;\approx\; 1 + e^{-(E_\pi(\mathbf{p})-E_\pi(\mathbf{p}^\star))t_v} + \frac{\beta_1+\tilde{\beta}_1}{g}\left(E_\pi(\mathbf{p})e^{-E_\pi(\mathbf{p})t} - E_\pi(\mathbf{p}^\star)e^{-E_\pi(\mathbf{p}^\star)t}\right) + \cdots$$

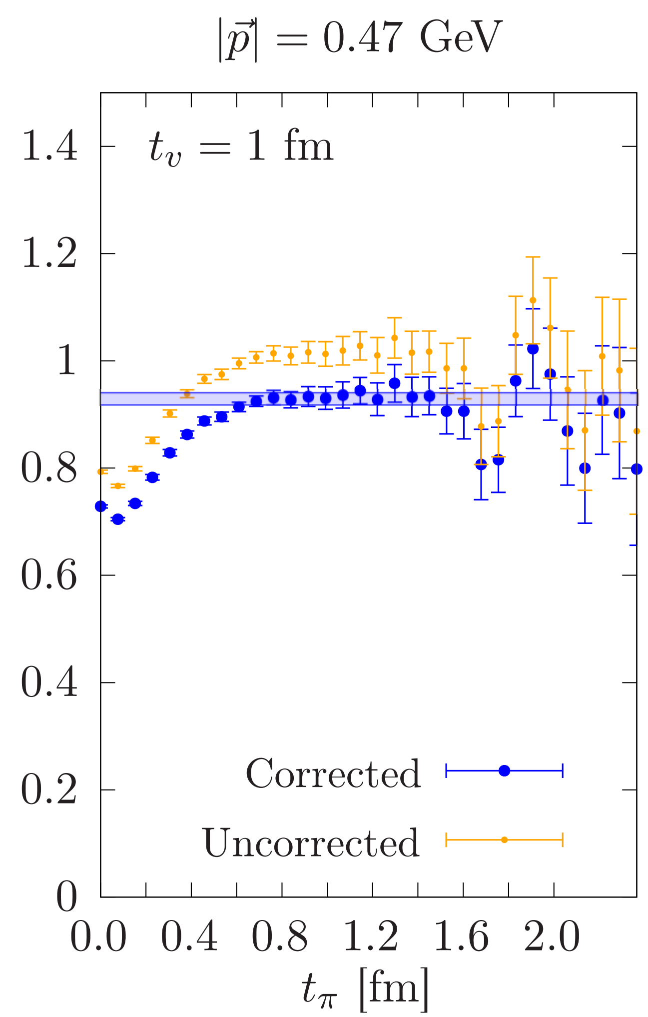
Antoine Gerardin, talk at *Lattice 2024*

Preliminary results : β_1 and $\tilde{\beta}_1$

$$R^{\text{eff}}(t,t_v;\mathbf{p}) \equiv \frac{E_\pi(\mathbf{p})}{E_\pi(\mathbf{p}^\star)}\frac{(p^\star)_k}{p_k} \times \frac{\widetilde{C}_k^{(3)}(t,t_v;\mathbf{p})}{\widetilde{C}_k^{(3)}(t,t_v;\mathbf{p}^\star)}\frac{C_\pi^{(2)}(t-t_v,\mathbf{p}^\star)}{C_\pi^{(2)}(t-t_v,\mathbf{p})}$$

- Plateaus at fixed t_π (left) or at fixed t_v (right)





$\approx 10\%$ correction $(B^*\pi)$

- Repeat the analysis for different values of E_π in the range [0.29 : 0.85] GeV