Flavour Physics and Tests of Lorentz Invariance

Michael Schmelling – MPI for Nuclear Physics

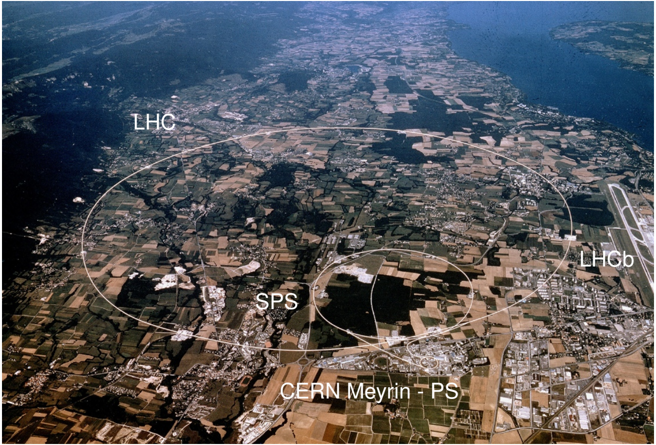
Outline

- LHCb
- B-mixing
- Flat spacetime
- Next steps
- Outlook

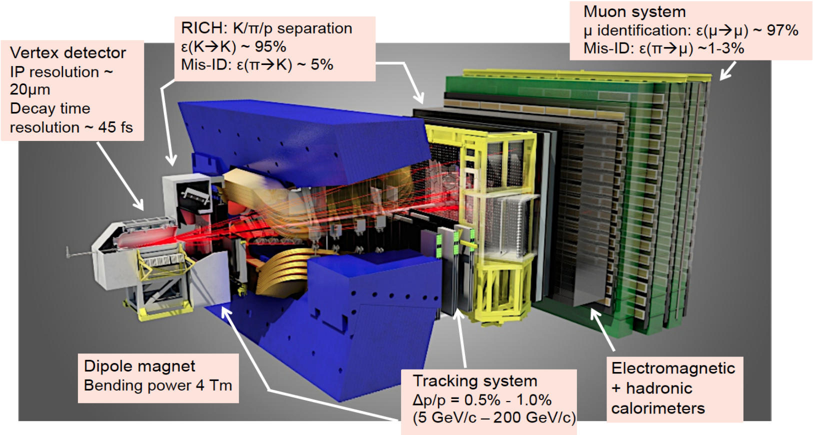
. . . on occasion of . . .

Uli Nierste's birthday

1 LHCb

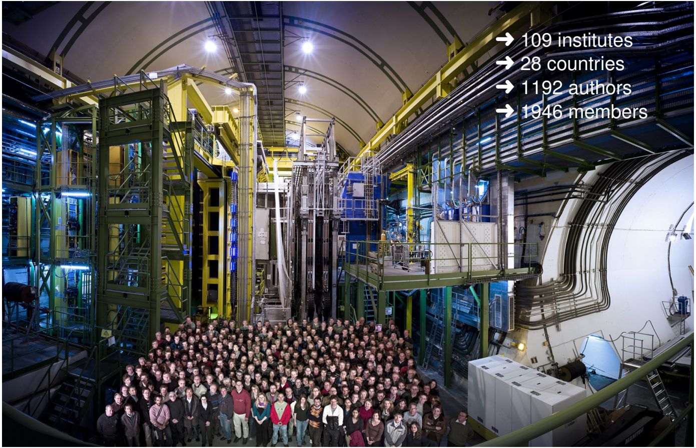


The Run 1 / Run 2 LHCb detector

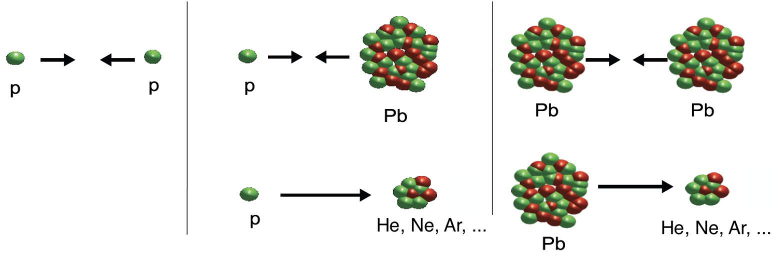


JINST 3 (2008) S08005, IJMPA 30 (2015) 1530022

The LHCb collaboration

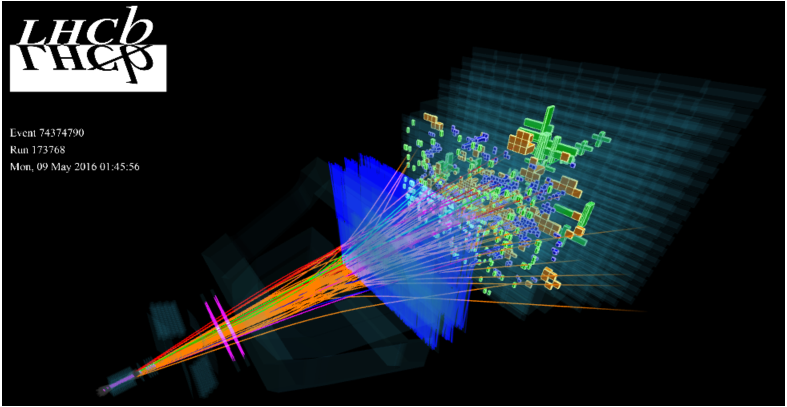


LHCb physics portfolio



- many different beam-target combinations
- colliding-beam and fixed-target mode
- large range of center-of-mass energies O(100 GeV - 10 TeV)
- 37 fb^{-1} integrated pp luminosity

❖ the raw material from which to extract physics



❖ test of Lorentz invariance in B-mixing

PRL 116(2016)241601

- effective Hamiltonian of $B\bar{B}$ flavour eigenstates

$$H = \begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12} & \Gamma_{22} \end{pmatrix}$$

- mass eigenstates which propagate and mix

$$|B_H\rangle = p\sqrt{1+z}|B\rangle - q\sqrt{1-z}|\bar{B}\rangle \quad \text{and} \quad |B_L\rangle = p\sqrt{1-z}|B\rangle + q\sqrt{1+z}|\bar{B}\rangle$$

  - ► with CPT violating parameter z

$$z = \frac{\delta m - i\delta\Gamma/2}{\Delta M + i\Delta\Gamma/2} = \frac{(M_{11}-M_{22}) - i(\Gamma_{11}-\Gamma_{22})/2}{(M_H - M_L) + i(\Gamma_L - \Gamma_H)/2}$$

$z \neq 0$ from Standard Model Extensions (SME)

- Planck-scale physics may break CPT and Lorentz invariance
  - effective field theory at low energy acquires symmetry breaking terms
  - new couplings between SM fields and uniform global tensor fields
  - coupling suppressed by powers of Planck scale
- prediction

$$\delta m - i\delta\Gamma/2 = \beta^{\mu}\Delta a_{\mu}$$

  - $\beta^{\mu} = \gamma(1, \vec{\beta})$: 4-velocity of B-meson
  - Lorentz invariance violated by absolute direction a_{μ}
- experimental considerations
  - large sensitivity to Δa_{μ} due to small $\Delta m_d = 3.3 \cdot 10^{-4}$ eV and $\Delta m_s = 1.2 \cdot 10^{-2}$ eV
  - additional gain from gamma-factor $\gamma \sim 20$

Experimental signatures: lifetime distributions

- qualitative picture for $B_d \bar{B}_d$ mixing with decay to CP eigenstate $J/\psi K_S^0$
  - with no CPV in decay, $\Delta\Gamma = 0$ and mixing phase 2β:

$$\frac{dn}{dt} \propto e^{-(\Gamma_{11}+\Gamma_{22})t/2}\Big(1 - \zeta\, \sin(\Delta m\, t)\sin(2\beta) \\ + \zeta\, \mathrm{Re}(z)\,\big(1-\cos(\Delta m\, t)\big)\cos(2\beta) \\ - \mathrm{Im}(z)\,\big(1-\cos(\Delta m\, t)\big)\,\big(\sin(2\beta)-\sin(\Delta m\, t)\big)\Big)$$

  - $\zeta = +1(-1)$ for an initial $B_d(\bar{B}_d)$
  - lifetime distributions depend on z, i.e. direction and energy of B-mesons
- analogous findings for $B_s \to J/\psi K^+K^-$
  - same signature but more involved math from angular analysis of K^+K^- system

results from both systems ➜

Observables

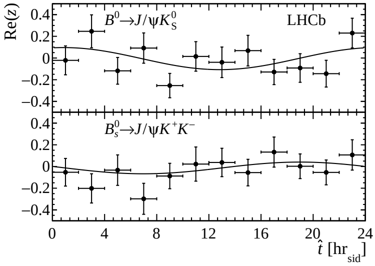
- measurement of Δa_μ in a right handed cartesian coordinate system
  - z-axis along the rotation axis of the earth
  - x-axis from the sun to the vernal equinox
- LHCb measures momenta in a small cone with $\sim 5\,\mathrm{deg}$ opening angle
  - angle between LHCb and z-axis: $\chi = 112.4\,\mathrm{deg}$
  - expectation for B-mesons along the axis of the experiment

$$\mathrm{Re}(z) \approx \frac{\gamma}{\Delta m}\left[\Delta a_0 + \cos\chi \Delta a_z + \sin\chi\left(\Delta a_y \sin(\Omega t) + \Delta a_x \cos(\Omega t)\right)\right]$$

  - constant term plus sidereal modulation
  - observables LHCb is sensitive to

$$\Delta a_{\parallel} = \Delta a_0 + \cos\chi \Delta a_z \quad , \quad \Delta a_x \quad \text{and} \quad \Delta a_y$$

Results



- $B_d \to J/\psi K^0_S$

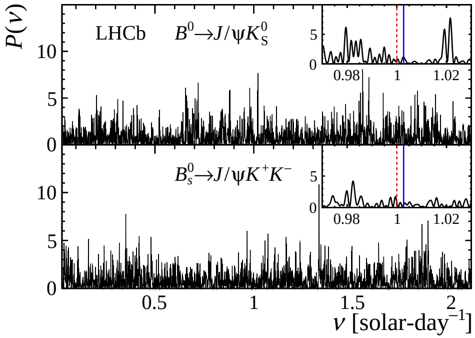
$$\Delta a_\parallel = (-0.10 \pm 0.82 \pm 0.54) \times 10^{-15}\ \text{GeV}$$
$$\Delta a_x = (+1.97 \pm 1.30 \pm 0.29) \times 10^{-15}\ \text{GeV}$$
$$\Delta a_y = (+0.44 \pm 1.26 \pm 0.29) \times 10^{-15}\ \text{GeV}$$

- $B_s \to J/\psi K^+ K^-$

$$\Delta a_\parallel = (-0.89 \pm 1.41 \pm 0.36) \times 10^{-14}\ \text{GeV}$$
$$\Delta a_x = (+1.01 \pm 2.08 \pm 0.71) \times 10^{-14}\ \text{GeV}$$
$$\Delta a_y = (-3.83 \pm 2.09 \pm 0.71) \times 10^{-14}\ \text{GeV}$$

- systematics due to external inputs, production asymmetries, detector effects . . .

Probing arbitrary modulation frequencies



- ❖ Lomb-Scargle periodogram
  - scan frequencies
  - spectrum for pure noise

$$\frac{dn}{dP} = e^{-P}$$

- ► 5-sigma level w/o look-elsewhere effect

$$P(\nu) > 14.38$$

- ► no evidence for signal

inset: zoom to 1/solar day (red) and 1/sidereal day (blue)

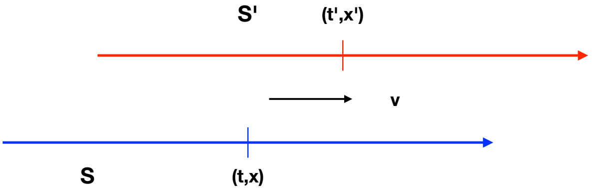
3 Flat spacetime

❖ how fundamental is Lorentz invariance and does it make sense to break it?

- experimental probe of spacetime: motion
- properties of spacetime determine transformations between moving observers
  - Galilei transformations of classical mechanics
  - Lorentz transformations of Special Relativity
- basic assumptions:
  - nature is consistent ➜ can be described mathematically
  - spacetime is a continuum, i.e. all space-time points have well defined coordinates
- goal: determine the logically possible transformation laws
  - the results rely only on phenomenological consistency
  - they are valid independently of whether spacetime is fundamental or emergent

Boost transformations in two-dimensional spacetime

❖ two reference frames moving relative to each other



- the system S' is moving in S with velocity v
- consider an event with coordinates (t', x') for an observer in S'
- determine the coordinates (t, x) of the same event for an observer in S

$$t = T_v(t', x') \quad \text{and} \quad x = X_v(t', x')$$

❖ consider a “flat” spacetime continuum

- transformation between displacements (dt', dx') and (dt, dx)

$$dt = \frac{\partial T_v}{\partial t'} dt' + \frac{\partial T_v}{\partial x'} dx' \quad \text{and} \quad dx = \frac{\partial X_v}{\partial t'} dt' + \frac{\partial X_v}{\partial x'} dx'$$

  - in flat spacetime the transformation is independent of t' and x'
  - the partial derivatives are only functions of v ➜ in matrix notation:

$$\begin{pmatrix} dt \\ dx \end{pmatrix} = \Lambda(v) \begin{pmatrix} dt' \\ dx' \end{pmatrix} \quad \text{with} \quad \Lambda(v) = \begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix}$$

- boundary condition if S and S' use the same units: $\Lambda(0) = 1$

determine the matrix elements ➜

❖ take a point at rest in S': $dx' = 0$ and $dx/dt = v$

$$\begin{pmatrix} dt \\ dx \end{pmatrix} = \begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix} \begin{pmatrix} dt' \\ 0 \end{pmatrix} \quad \rightarrow \quad \frac{dx}{dt} = \frac{a_{10}\,dt'}{a_{00}\,dt'} = v \quad \text{or} \quad a_{10} = v\,a_{00}$$

- transformation, with $a_{00} \equiv \gamma(v)$ pulled in front of the matrix

$$\Lambda(v) = \gamma(v) \begin{pmatrix} 1 & b(v) \\ v & d(v) \end{pmatrix}$$

  - $\gamma(v) = dt/dt'$: dimensionless time-dilation factor
  - $b(v) = a_{10}/\gamma(v)$: function with dimension [time]/[length] i.e. 1/velocity
  - $d(v) = a_{11}/\gamma(v)$: transformation of lengths between S and S'

require that transformations form a group ➜

❖ there exists an inverse transformation with velocity parameter $\bar{v} = f(v)$

$$\Lambda(v)\,\Lambda(\bar{v}) = 1$$

- logical consistency requires

$$f(f(v)) = v \quad \text{general solution:} \quad f(v) = \frac{-v}{1 + \sigma\, v} = \bar{v}$$

  - ► new constant of nature: anisotropy parameter σ with dimension 1/velocity

- determine functions $b(v)$ and $d(v)$ from group property

$$\Lambda(v)\,\Lambda(\bar{v}) = \gamma(v)\gamma(\bar{v}) \begin{pmatrix} 1 + \bar{v}\, b(v) & b(\bar{v}) + b(v)\, d(\bar{v}) \\ v + \bar{v}\, d(v) & v\, b(\bar{v}) + d(v)\, d(\bar{v}) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

contraints from off-diagonal terms ➜

- lower-left off-diagonal element, substituting $\bar{v}$

$$v + \bar{v}\, d(v) = 0 \quad \Rightarrow \quad d(v) = \frac{v}{\bar{v}} = 1 + \sigma\, v$$

- upper-right off-diagonal element, substituting $\bar{v}$ and $d(\bar{v})$

$$b(\bar{v}) + b(v)\, d(\bar{v}) = 0 \quad \Rightarrow \quad b\left(\frac{-v}{1+\sigma\, v}\right) = \frac{-b(v)}{1+\sigma\, v}$$

  - it follows

$$b(v) \propto v \quad \text{with correct dimensions} \quad b(v) = \alpha \frac{v}{c^2} \quad \text{with} \quad \alpha \in \{-1, 0, +1\}$$

  - new constant of nature (for $\alpha \neq 0$): absolute velocity scale c
  - resulting transformation

$$\Lambda(v) = \gamma(v) \begin{pmatrix} 1 & \alpha v / c^2 \\ v & 1 + \sigma\, v \end{pmatrix}$$

- require that spacetime is a metric space with metric g
  - invariance of g

$$\Lambda^T \, g \, \Lambda = g \qquad \text{implies} \qquad |\det \Lambda| = 1$$

- final result, with substitutions $t \to ct$, $v \to v/c = \beta$ and $\sigma = 2\eta/c$

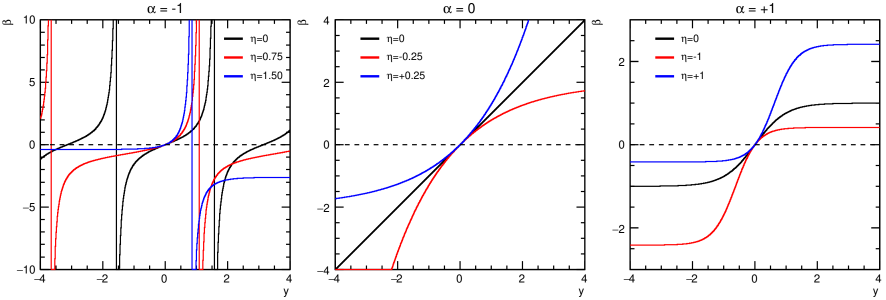
$$\Lambda = \begin{pmatrix} \gamma & \alpha\gamma\beta \\ \gamma\beta & 1 + 2\eta\gamma\beta \end{pmatrix} \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1 + 2\eta\beta - \alpha\beta^2}} \quad \text{and} \quad g = \begin{pmatrix} 1 & \eta \\ \eta & -\alpha \end{pmatrix}$$

- interpretation for $\eta = 0$:
  - $\alpha = -1$: euclidean spacetime with Λ = rotation matrix
  - $\alpha = \ \ 0$: spacetime of classical mechanics with Λ = Galilei transformation
  - $\alpha = +1$: Minkowski spacetime with Λ = Lorentz transformation

consistency check ➜

- limiting case of a sequence of n small boosts $\Delta\beta = y/n$

$$\Lambda(y) = \lim_{n\to\infty} \left(\Lambda(y/n)\right)^n \quad \text{with resulting velocity} \quad \beta(y)$$



only $\alpha = 0$ and $\alpha = 1$ result in a logically consistent monotonic behaviour

Result

❖ findings from 2-dim spacetime

- flat isotropic and metric spacetime has either Galilei- or Lorentz invariance
  - for Galilei invariance space and time decouple
  - Minkowski metric appears the most natural choice
  - experiment has to decide which one is realized in natuer
- general case: flat anisotropic metric spacetime

❖ generalisation to 4-dim spacetime

$$g = \left(\begin{array}{c|ccc} 1 & \eta_x & \eta_y & \eta_z \\ \hline \eta_x & -1 & 0 & 0 \\ \eta_y & 0 & -1 & 0 \\ \eta_z & 0 & 0 & -1 \end{array}\right)$$

➜ space is isotropic but time is not orthogonal to space

4 Next steps

❖ boost transformation Λ that leaves new g invariant

$$\Lambda^T g \Lambda = g \quad \text{with} \quad g = \begin{pmatrix} 1 & \vec{\eta}^T \\ \vec{\eta} & -1 \end{pmatrix}$$

► generalized Lorentz transformation

$$\Lambda(\beta) = \begin{pmatrix} \gamma & \gamma \beta^T + \rho[(\beta^T\beta)\eta - (\eta^T\beta)\beta]^T \\ \gamma\beta & 1 + \gamma\beta\eta^T + \rho\beta[\beta + (\eta^T\beta)\eta]^T \end{pmatrix} \quad \text{with} \quad \rho = \frac{\gamma - 1 + \gamma(\eta^T\beta)}{(\eta^T\beta)^2 + \beta^T\beta}$$

and

$$\beta^T = (\beta_x, \beta_y, \beta_z) \qquad \eta^T = (\eta_x, \eta_y, \eta_z) \qquad \gamma = \frac{1}{\sqrt{1 + 2\eta^T\beta - \beta^T\beta}}$$

- 4-momentum of a particle at rest

$$p_0 = \begin{pmatrix} m \\ \vec{0} \end{pmatrix}$$

- 4-momentum of a particle moving with velocity $\vec{\beta}$

$$p = \Lambda\, p_0 = \gamma \begin{pmatrix} m \\ m\,\vec{\beta} \end{pmatrix} = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$$

- invariant mass

$$p^T\ g\ p = \left(\Lambda\ p_0\right)^T\ g\ \left(\Lambda\ p_0\right) = p_0^T\ \Lambda^T\ g\ \Lambda\ p_0 = p_0^T\ g\ p_0 = m^2$$

exploit $p^T g\ p = m^2$ ➜

- time dilatation factor

$$p = \begin{pmatrix} m\gamma \\ m\gamma\vec{\beta} \end{pmatrix} \Rightarrow m^2 = m^2\gamma^2 \begin{pmatrix} 1 \\ \vec{\beta} \end{pmatrix}^T \begin{pmatrix} 1 & \vec{\eta}^T \\ \vec{\eta} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ \vec{\beta} \end{pmatrix} = m^2\gamma^2(1 + 2\vec{\eta}^T\vec{\beta} - \vec{\beta}^T\vec{\beta})$$

and thus $\gamma = \dfrac{1}{\sqrt{1 + 2(\vec{\eta}^T\vec{\beta}) - \vec{\beta}^T\vec{\beta}}}$

- dispersion relation

$$p = \begin{pmatrix} E \\ \vec{p} \end{pmatrix} \Rightarrow m^2 = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}^T \begin{pmatrix} 1 & \vec{\eta}^T \\ \vec{\eta} & -1 \end{pmatrix} \begin{pmatrix} E \\ \vec{p} \end{pmatrix} = E^2 + 2E\,(\vec{\eta}^T\vec{p}) - (\vec{p}^T\vec{p})$$

and thus $E = \sqrt{m^2 + \vec{p}^T\vec{p} + (\vec{\eta}^T\vec{p})^2} - \vec{\eta}^T\vec{p}$

Electrodynamics

- Lorentz force on a particle with charge q in a static field

$$\frac{d}{dt}p^{\mu} = q\,F^{\mu\nu}u_{\nu} = q\,F^{\mu\nu}u^{\rho}g_{\nu\rho}$$

with $u^{\rho} = \begin{pmatrix} 1 \\ \vec{\beta} \end{pmatrix}$ and $F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$

  - classical electrodynamics: g = Minkowski metric

- Lorentz force for $E = 0$

$$\frac{d\vec{p}}{dt} = q(\vec{\beta} - \vec{\eta}) \times \vec{B}$$

  - particle trajectory in magnet spectrometer depends on metric

5 Outlook

❖ alternative ways to look for New Physics

- introducing extra couplings – as in SME
  - straw-man models to parameterize deviations from the SM
  - attempts to construct observables that probe Planck-scale physics
- allowing for new degrees of freedom in established theories e.g. . . .
  - Planck-scale physics to generate off-diagonal terms in the spacetime metric
  - conceptually simple with extra work to achieve consistency
  - theorists: redo predictions with $g \rightarrow g'$
  - experimentalists: be aware of extra systematics in measurements
- follow both venues to disentangle conspiracies
- for the full picture probe as many systems as possible . . .