

# $h \rightarrow b\bar{b}$ AT NLO IN SMEFT

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# MOTIVATION FOR $h \rightarrow b\bar{b}$ AT NLO IN SMEFT

## 1) Phenomenology:

- $h \rightarrow b\bar{b}$  largest Higgs branching fraction
- can measure Higgs SM Yukawa at (sub)percent level at Higgs factory
- $\Rightarrow$  NLO SMEFT calculation sets long-term baseline for analysis in EFT

## 2) SMEFT development:

- reveals many non-trivial features of SMEFT at NLO in (relatively) simple setting
- analytic results useful for benchmarking automated codes for NLO SMEFT

Some things we dealt with in full NLO calculation:

[Jonathan Cullen, B.P., Darren Scott: [arXiv:1904.06358](https://arxiv.org/abs/1904.06358) ]

- renormalize  $e, M_W, M_Z, m_b, C_i$ , plus external  $b$ -quark,  $h$ -boson fields (45  $C_i$  appear at NLO, checks 100s of entries in 1-loop anom. dims. [[Alonso, Jenkins, Manohar, Trott](#)])
- gauge invariance: tadpoles and gauge fixing in SMEFT
- find appropriate renormalization scheme for combining EW and QCD corrections
- find appropriate method for estimating higher-orders from scale variations

# OUTLINE OF NLO CALCULATION

Basic outline:

- specify input parameters and renormalization scheme
- write down LO and UV counterterm amplitudes for  $h \rightarrow b\bar{b}$
- calculate one-loop  $h \rightarrow b\bar{b}$  matrix elements and various two-point functions for counterterms
- calculate real emissions of photons, gluons, add together with UV-renormalized virtual corrections to get IR finite answer

In general, every piece of calculation gets dim-4 (SM) and dim-6 (SMEFT) contributions, dim-8 terms are dropped

# INPUT PARAMETERS I: OUR SET-UP

We use mass basis Lagrangian parameters:

$$\alpha_s, \alpha, m_f, m_H, M_W, M_Z, V_{ij}, C_i$$

- $C_i$  are Wilson coefficients of dim-6 operators and renormalized in the  $\overline{\text{MS}}$  scheme; notation:  $\mathcal{L}^{(6)} = \sum_i C_i(\mu) Q_i(\mu)$
- $M_W, M_Z$  renormalized in on-shell scheme.
- will discuss renormalization scheme for  $e$  and  $m_b$  later
- $M_H, m_t, m_\tau$  kept non-zero but not renormalized at NLO. all other  $m_f = 0$ .
- we approximate  $V_{ij} = \delta_{ij}$
- common to trade  $\alpha$  for  $G_F$  (possible, although we prefer not to)

# INPUT PARAMETERS II

- many non-trivial SMEFT effects are involved in rotation to mass basis
- example 1: Higgs vev  $\langle H^\dagger H \rangle \equiv v_T^2/2$

$$\frac{1}{v_T} = \frac{1}{\hat{v}_T} \left( 1 + \hat{v}_T^2 \frac{\hat{c}_w}{\hat{s}_w} \left[ C_{HWB} + \frac{\hat{c}_w}{4\hat{s}_w} C_{HD} \right] \right); \quad \hat{v}_T = 2M_W \hat{s}_w / e; \quad \hat{c}_w = \frac{M_W}{M_Z}$$

- example 2: covariant derivative in mass basis

$$\begin{aligned} D_\mu = & \partial_\mu - i \frac{e}{\hat{s}_w} \left[ 1 + \frac{\hat{c}_w^2 \hat{v}_T^2}{4\hat{s}_w^2} C_{HD} + \frac{\hat{c}_w \hat{v}_T^2}{\hat{s}_w} C_{HWB} \right] (\mathcal{W}_\mu^+ \tau^+ + \mathcal{W}_\mu^- \tau^-) \\ & - i \left[ \frac{e}{\hat{c}_w \hat{s}_w} \left( 1 + \frac{(2\hat{c}_w^2 - 1)\hat{v}_T^2}{4\hat{s}_w^2} C_{HD} + \frac{\hat{c}_w \hat{v}_T^2}{\hat{s}_w} C_{HWB} \right) (\tau^3 - \hat{s}_w^2 Q) \right. \\ & \left. + e \left( \frac{\hat{c}_w \hat{v}_T^2}{2\hat{s}_w} C_{HD} + \hat{v}_T^2 C_{HWB} \right) Q \right] \mathcal{Z}_\mu - ieQ \mathcal{A}_\mu, \end{aligned}$$

- gauge fixing is also much more involved than SM

# UV COUNTERTERMS I: THE LO AMPLITUDE

- LO decay amplitude

$$i\mathcal{M}^{(0)}(h \rightarrow b\bar{b}) = -i\bar{u}(p_b) \left( \mathcal{M}_L^{(0)} P_L + \mathcal{M}_L^{(0)*} P_R \right) v(p_{\bar{b}})$$

- split into dim-4 and dim-6 contributions

$$\mathcal{M}_L^{(0)} = \mathcal{M}_L^{(4,0)} + \mathcal{M}_L^{(6,0)}$$

- Explicit results

$$\mathcal{M}_L^{(4,0)} = \frac{m_b}{\hat{v}_T},$$

$$\mathcal{M}_L^{(6,0)} = m_b \hat{v}_T \left[ C_{H\Box} - \frac{C_{HD}}{4} \left( 1 - \frac{\hat{c}_w^2}{\hat{s}_w^2} \right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} - \frac{\hat{v}_T}{m_b} \frac{C_{bH}^*}{\sqrt{2}} \right]$$

- note: only  $Q_{bH} \sim (H^\dagger H)(\bar{b}_L b_R H) + \text{h.c.}$  contributes to LO Feynman diagram
- other  $C_i$  are from rotation to mass basis

$$Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H), \quad Q_{HWB} = H^\dagger \sigma^I H W_{\mu\nu}^I B^{\mu\nu}, \quad Q_{HD} = (H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$$

# UV COUNTERTERMS II: BARE AND RENORMALIZED PARAMETERS

- replace fields and parameters in bare LO amplitude by renormalized ones
  - wavefunction renormalization

$$h^{(0)} = \sqrt{Z_h} h = \left(1 + \frac{1}{2} \delta Z_h\right) h,$$

$$b_L^{(0)} = \sqrt{Z_b^L} b_L = \left(1 + \frac{1}{2} \delta Z_b^L\right) b_L,$$

$$b_R^{(0)} = \sqrt{Z_b^R} b_R = \left(1 + \frac{1}{2} \delta Z_b^R\right) b_R$$

- masses, electric charge, and Wilson coefficients

$$M^{(0)} = M + \delta M, \quad e^{(0)} = e + \delta e, \quad C_i^{(0)} = C_i + \delta C_i$$

- expand to linear order in counterterms, separating dim-4 and dim-6 contributions
- all counterterms except  $C_i$  obtained by calculating two-point functions in mass basis

# UV COUNTERTERMS III: COUNTERTERM AMPLITUDE

dimension-4 counterterm is

$$\delta\mathcal{M}_L^{(4)} = \frac{m_b}{\hat{v}_T} \left( \frac{\delta m_b^{(4)}}{m_b} - \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*} \right)$$

dimension-6 counterterm is

$$\begin{aligned} \delta\mathcal{M}_L^{(6)} = & \frac{m_b}{\hat{v}_T} \left( \frac{\delta m_b^{(6)}}{m_b} - \frac{\delta\hat{v}_T^{(6)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*} \right) \\ & + \mathcal{M}_L^{(6,0)} \left( \frac{\delta m_b^{(4)}}{m_b} + \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*} \right) \\ & - \frac{\hat{v}_T^2}{\sqrt{2}} C_{bH}^* \left( \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} - \frac{\delta m_b^{(4)}}{m_b} \right) + m_b \hat{v}_T \left[ C_{HWB} + \frac{\hat{c}_w}{2\hat{s}_w} C_{HD} \right] \delta \left( \frac{\hat{c}_w}{\hat{s}_w} \right)^{(4)} \\ & + m_b \hat{v}_T \left( \delta C_{H\Box} - \frac{\delta C_{HD}}{4} \left( 1 - \frac{\hat{c}_w^2}{\hat{s}_w^2} \right) + \frac{\hat{c}_w}{\hat{s}_w} \delta C_{HWB} - \frac{\hat{v}_T}{m_b} \frac{\delta C_{bH}^*}{\sqrt{2}} \right) \end{aligned}$$

where

$$\frac{\delta\hat{v}_T}{\hat{v}_T} \equiv \frac{\delta M_W}{M_W} + \frac{\delta\hat{s}_w}{\hat{s}_w} - \frac{\delta e}{e}$$

and

$$\frac{\delta\hat{s}_w}{\hat{s}_w} = -\frac{\hat{c}_w^2}{\hat{s}_w^2} \left( \frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z} \right), \quad \delta \left( \frac{\hat{c}_w}{\hat{s}_w} \right)^{(4)} = -\frac{1}{\hat{c}_w \hat{s}_w} \left( \frac{\delta\hat{s}_w^{(4)}}{\hat{s}_w} \right)$$

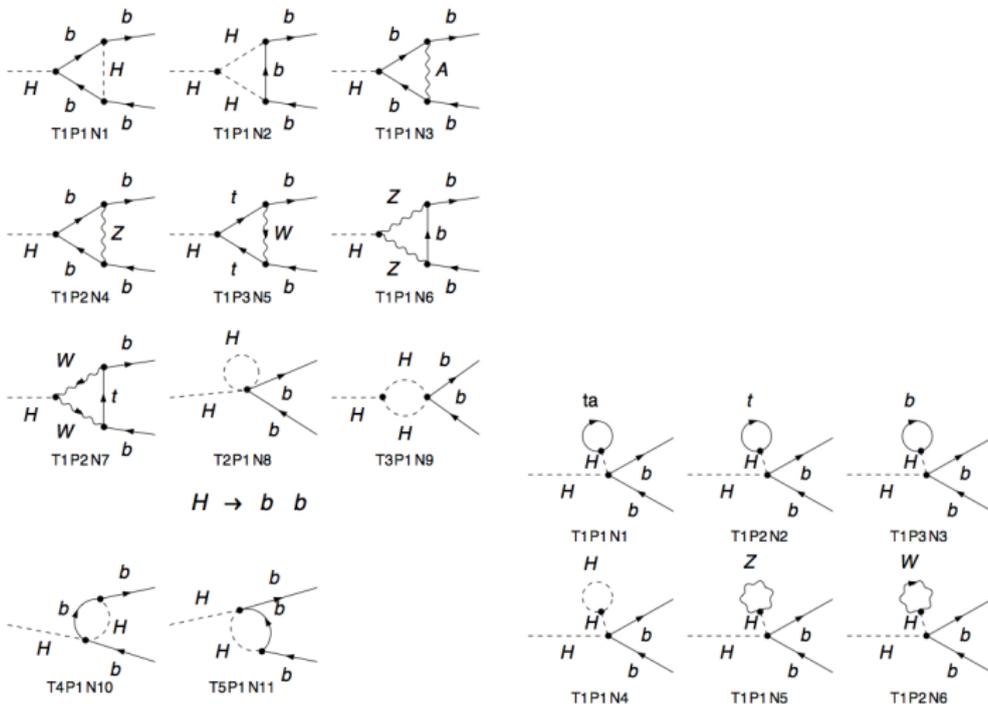
# (AUTOMATED) ONE-LOOP CALCULATION

- one-loop  $h \rightarrow b\bar{b}$  matrix elements and two-point functions for counterterms involve many Feynman diagrams
- we used normal chain of automation: Feynrules (in-house model file, including gauge fixing and ghosts), Feynarts, FormCalc, Package X

All loop integrals obtained analytically in terms of Passarino-Veltmann integrals and also given in terms of standard functions in electronic files with paper

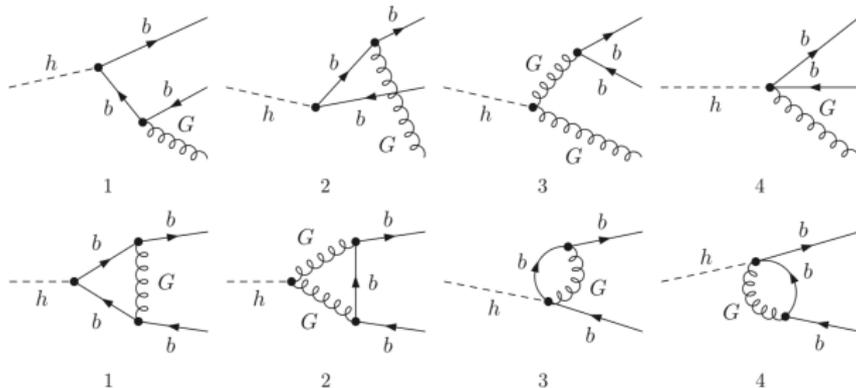
# SOME EXAMPLE ONE-LOOP DIAGRAMS

Involving SM and  $Q_{bH} \sim H^\dagger H \bar{b}_L b_R H + \text{h.c.}$

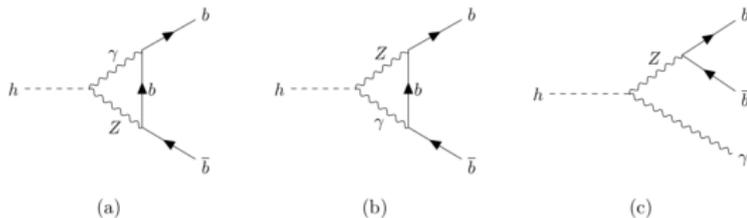


# QCD-QED CORRECTIONS

- QCD corrections by far simplest to calculate [Gauld, B.P., Scott '16]
- UV-renormalized one-loop amplitudes have IR divergences canceled by real emissions



- most corrections involving photons can be obtained analogously, exception is graphs involving  $h\gamma Z$  vertex



# ANALYTIC STRUCTURE OF $h\gamma Z$ CORRECTIONS

Performing phase-space integrals one finds:

$$\Gamma_{h\gamma Z} \propto v_b \left[ 2(C_{HB} - C_{HW})\hat{c}_w\hat{s}_w + C_{HWB}(\hat{c}_w^2 - \hat{s}_w^2) \right] F_{h\gamma Z} \left( \frac{M_Z^2}{m_H^2}, \frac{\mu^2}{m_H^2}, \frac{m_b^2}{m_H^2} \right)$$

$$\begin{aligned} F_{h\gamma Z}(z, \hat{\mu}^2, b) = & \frac{3}{4}\beta(8z - 5) - \beta^3 \left( \frac{39}{4} + \frac{z}{b} \right) - \frac{4}{3}\beta^2\pi^2\bar{z} + \frac{4}{3}\pi^2 z\bar{z} + 6\beta \left( \beta^2 - \frac{2}{3}z \right. \\ & \left. + \frac{(2b - \beta^2)z^2}{12b^2} \right) \ln(b) + 2(\beta^2 - z)\bar{z} \ln(x_z)^2 - 4\beta_z z\bar{z} \ln(x_{\beta_z}) \\ & + \ln(x) \left( -\frac{1}{8} (15 + 7\beta^4 + 8z(4z - 7) + \beta^2(2 + 8z)) + 2(z - \beta^2)\bar{z} \ln(x_z) \right. \\ & \left. + 4(\beta^2 - z)\bar{z} \ln(1 - xx_z) + 2(\beta^2 - z)\bar{z} \ln(x_{\beta_z}) \right) \\ & + \ln(x_z) \left( \frac{\beta\beta_z z (\beta^2(2b + z) - 2bz)}{2b^2} + 2(z - \beta^2)\bar{z} \ln(x_{\beta_z}) \right) \\ & + 4\beta_z z\bar{z} \ln(\bar{z}) + \frac{\beta^3(\beta^2 + 2b)z^2 \ln(z)}{2b^2} - 6\beta^3 \ln(\hat{\mu}^2) \\ & + 4(\beta^2 - z)\bar{z} \left( \text{Li}_2 \left( \frac{x}{x_z} \right) + \text{Li}_2(xx_z) \right) \end{aligned}$$

where

$$\beta = \sqrt{1 - 4b}, \quad \beta_z = \sqrt{1 - \frac{4b}{z}}, \quad x = \frac{1 - \beta}{1 + \beta}, \quad x_z = \frac{1 - \beta_z}{1 + \beta_z}, \quad x_{\beta_z} = \frac{\beta - \beta_z}{\beta + \beta_z}, \quad \bar{z} = 1 - z$$

# CROSS-CHECKS AND FEATURES

Results involve 45 different Wilson coefficients (generally complex). Cross-checks:

- all UV and IR poles cancel (and  $\mu$ -dependence consistent with RG eqns)
- SM results reproduced from dim. 4 terms
- all results calculated in unitary and Feynman gauge with full agreement

Interesting features:

- structure of wave-function renormalization of  $b$ -quark field
- Higgs- $Z$  mixing
- Ward identities and electric charge renormalization
- structure of tadpole contributions

# FERMION W.F. RENORMALIZATION IN SMEFT

- Decompose two-point function of fermion  $f$  as

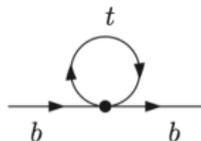
$$\Gamma^f(p) = i(\not{p} - m_f) + i \left[ \not{p} \left( P_L \Sigma_f^L(p^2) + P_R \Sigma_f^R(p^2) \right) + m_f \left( \Sigma_f^S(p^2) P_L + \Sigma_f^{S*}(p^2) P_R \right) \right]$$

- In on-shell renormalization scheme

$$\delta Z_f^L = -\widetilde{\text{Re}} \Sigma_f^L(m_f^2) + \Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2) - m_f^2 \frac{\partial}{\partial p^2} \widetilde{\text{Re}} \left[ \Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2) \right] \Big|_{p^2=m_f^2},$$

$$\delta Z_f^R = -\widetilde{\text{Re}} \Sigma_f^{f,R}(m_f^2) - m_f^2 \frac{\partial}{\partial p^2} \widetilde{\text{Re}} \left[ \Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2) \right] \Big|_{p^2=m_f^2}$$

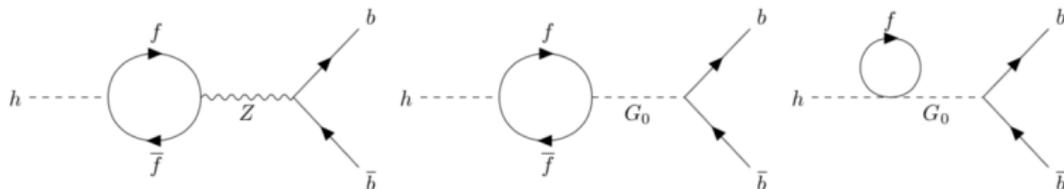
- $\Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2)$  vanishes in SM, but is proportional  $\text{Im}(C_i)$  in SMEFT.
- appears in many places in renormalization of amplitude – example:



$$Z_b^L = \frac{1}{\epsilon} \left[ -\frac{m_t^3}{m_b} \left( (2N_c + 1) \left( C_{qtqb}^{(1)} - C_{qtqb}^{(1)*} \right) + c_{F,3} \left( C_{qtqb}^{(8)} - C_{qtqb}^{(8)*} \right) \right) \right] + \text{finite}$$

# HIGGS-Z (GOLDSTONE) MIXING

- unlike SM, in SMEFT Higgs can mix into  $Z$  and neutral Goldstone boson  $G_0$



- $h$ - $G_0$  mixing is that between real and imaginary parts of doublet:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2}i\phi^+(x) \\ [1 + C_{H,\text{kin.}}] h(x) + i \left[ 1 - \frac{\hat{v}_T^2}{4} C_{HD} \right] \phi^0(x) + v_T \end{pmatrix}$$

- mixing is therefore proportional to imaginary parts of Wilson coefficients and reads

$$\eta_5 = \frac{\sqrt{2}}{\hat{v}_T} \text{Im} [N_c m_b C_{bH} - N_c m_t C_{tH} + m_\tau C_{\tau H} + \dots]$$

- this term exactly cancels one appearing in renormalization of  $Q_{bH}$  (i.e. that in  $\hat{C}_{bH}$  calculated in [\[Jenkins, Manohar, Trott '13\]](#))

# ELECTRIC CHARGE RENORMALIZATION

**SM:**  $ff\gamma$  vertex related to two-point fcn through Ward identities:

- result

$$\frac{\delta e^{(4)}}{e} = \frac{1}{2} \frac{\partial \Sigma_T^{AA(4)}(k^2)}{\partial k^2} \Big|_{k^2=0} - \frac{(v_f^{(4)} - a_f^{(4)}) \Sigma_T^{AZ(4)}(0)}{Q_f M_Z^2}$$

- $v_f^{(4)} - a_f^{(4)} = -Q_f \hat{s}_w / \hat{c}_w \Rightarrow \delta e^{(4)}$  independent of fermion charge

**SMEFT:** we find by renormalizing  $ff\gamma$  vertex directly (not using Ward identities)

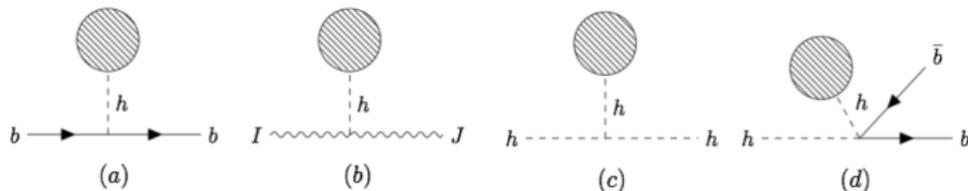
- result

$$\frac{\delta e^{(6)}}{e} = \frac{1}{2} \frac{\partial \Sigma_T^{AA(6)}(k^2)}{\partial k^2} \Big|_{k^2=0} + \frac{1}{M_Z^2} \left( \frac{\hat{c}_w}{\hat{s}_w} \Sigma_T^{AZ(6)}(0) - \frac{\hat{v}_T^2}{4\hat{c}_w \hat{s}_w} C_{HD} \Sigma_T^{AZ(4)}(0) \right)$$

- $v_f^{(6)} - a_f^{(6)} = 2C_{Hf} \hat{v}_T^2 / 4\hat{c}_w \hat{s}_w \Rightarrow$  Naive application of SM Ward identity is wrong, due to operators of form  $Q_{Hf} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{f} \gamma^\mu f)$

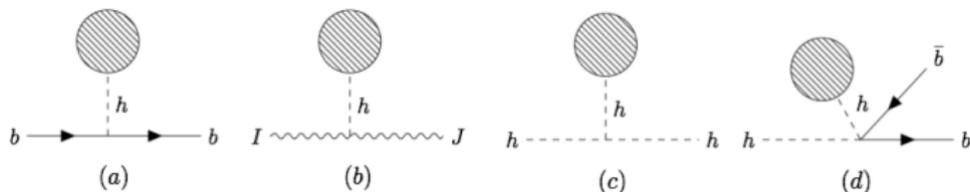
# TADPOLES I

- we used FJ tadpole scheme [Fleischer, Jegerlehner '80]
- discussion in [Denner, Jenniches, Lang, Sturm '16] shows FJ scheme implemented by simply calculating all tadpole contributions to  $n$ -point functions
- tadpoles needed for  $h \rightarrow b\bar{b}$  in SMEFT



- we calculated tadpoles in Feynman and unitary gauge and found expected results:
  - (1) tadpoles cancel in on-shell scheme
  - (2) mass and electric charge counterterms, and matrix elements +wavefunction renormalization separately gauge invariant after adding tadpoles
- structure of tadpoles contributions in SMEFT richer than in SM

# TADPOLES II



$$IJ \in \{\gamma\gamma, \gamma Z, WW, ZZ\}$$

- (a) contributes to  $\delta m_b$  in SM and SMEFT, but also to  $\delta Z_b^L$  in SMEFT

$$\delta Z_{b,\text{tad.}}^L = -\frac{i\sqrt{2}\hat{v}_T^2}{m_H^2 m_b} \text{Im}(C_{bH}) T^{(4)}$$

- (b) contributes to  $\delta M_W, \delta M_Z$  in SM and SMEFT, also to  $\delta e$  in SMEFT ( $IJ = \gamma\gamma$ )

$$\frac{\delta e^{\text{cl.4,(6)}}}{e} = \frac{1}{16\pi^2} \left[ c_{h\gamma\gamma} A_0(m_H^2) + 4\hat{c}_w \hat{s}_w C_{HWB} (4M_W^2 - 3A_0(M_W^2)) \right] - 2c_{h\gamma\gamma} \frac{\hat{v}_T}{m_H^2} T_{\text{un.}}^{(4)}$$

- (c) contributes to  $\delta Z_h$  in SMEFT (through  $C_{HD}$  and  $C_{H\Box}$ ), but not in SM
- (d) contributes to  $h \rightarrow b\bar{b}$  matrix element in SMEFT, but not in SM

To quote meaningful results, need to

- fix a renormalization scheme (preferably one where radiative corrections minimal)
- assign an uncertainty to uncalculated higher orders (usually through scale variations)

# ENHANCED NLO CORRECTIONS I: QCD CORRECTIONS

- QCD/QED corrections generate  $\ln m_b/m_H$  terms when  $\mu = m_H$ :

$$\frac{\Gamma_{g,\gamma}^{(1)}}{\Gamma^{(4,0)}} \approx \ln^2 \left( \frac{m_b^2}{m_H^2} \right) \frac{\hat{v}_T^2}{\pi} (C_F \alpha_s C_{HG} + Q_b^2 \alpha c_{h\gamma\gamma})$$

$$+ c_{m_b} \ln \left( \frac{m_b^2}{m_H^2} \right) \frac{3}{2} \left( \frac{C_F \alpha_s + Q_b^2 \alpha}{\pi} \right) \left[ 1 + 2\hat{v}_T^2 \left( C_{H\Box} - \frac{C_{HD}}{4} \left( 1 - \frac{\hat{c}_w^2}{\hat{s}_w^2} \right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} - \frac{\hat{v}_T}{m_b} \frac{C_{bH}}{2\sqrt{2}} \right) \right]$$

- double logs of IR origin remain and are in fact largest NLO correction
  - $Q_{HG} = H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$  gives  $hgg$  vertex strongly constrained from Higgs production
- $c_{m_b} = 1$  in on-shell scheme,  $c_{m_b} = 0$  ( $\overline{\text{MS}}$  scheme) for  $m_b$ .
- $\Rightarrow$  **QCD/QED prefers  $\overline{\text{MS}}$  scheme for  $m_b$**  (running mass resums single UV logs)

# ENHANCED NLO CORRECTIONS II: TADPOLES

- QCD and QED corrections prefer  $\overline{\text{MS}}$  scheme for  $m_b$  and  $e$
- however, in  $\overline{\text{MS}}$  scheme tadpoles don't cancel in decay rate and contribute  $m_t^4$  enhanced corrections. Example, in SM in  $m_t \rightarrow \infty$  limit

$$\overline{\text{MS}} \text{ scheme: } \frac{\overline{\Gamma}_t^{(4,1)}}{\Gamma^{(4,0)}} \approx -\frac{N_c}{2\pi^2} \frac{m_t^4}{\hat{v}_T^2 m_H^2} \approx -15\%$$

$$\text{on-shell scheme: } \frac{[\Gamma_t]^{\text{O.S.}(4,1)}}{\Gamma^{(4,0)}} = \frac{m_t^2}{16\pi^2 \hat{v}_T^2} \left( -6 + N_c \frac{7 - 10\hat{c}_w^2}{3\hat{s}_w^2} \right) \approx -3\%$$

- similar behaviour in SMEFT contributions to decay rate
- $\Rightarrow$  **EW corrections prefer on-shell scheme for  $m_b$  and  $e$  to avoid large corrections from top loops (and heavy EW bosons)**

Combining EW and QCD corrections is a non-trivial problem. Would like to calculate QCD in  $\overline{\text{MS}}$  scheme, but EW in on-shell scheme...

# DECOUPLING RELATIONS I

- decoupling relations connect  $\overline{MS}$  parameters in SM, with those in low-energy theory where top and heavy bosons integrated out:

$$\overline{m}_b(\mu) = \zeta_b(\mu, m_t, m_H, M_W, M_Z) \overline{m}_b^{(\ell)}(\mu)$$

- decoupling constants  $\zeta_i$  contain contributions from heavy particles.
- $\zeta_b$  calculated by relating on-shell mass with  $\overline{MS}$  masses in SM and low-energy theories:

$$m_b = z_b^{-1}(\mu, m_b, m_t, m_H, M_W, M_Z) \overline{m}_b(\mu) = \left[ z_b^{(\ell)}(\mu, m_b) \right]^{-1} \overline{m}_b^{(\ell)}(\mu)$$

$$\Rightarrow \zeta_b(\mu, m_t, m_H, M_W, M_Z) = \frac{z_b(\mu, m_b, m_t, m_H, M_W, M_Z)}{z_b^{(\ell)}(\mu, m_b)} \Bigg|_{m_b \rightarrow 0}$$

- works analogously for electric charge. The connection between low energy parameters and experiment are:

from  $B$ -physics:  $\overline{m}_b^{(\ell)}(\overline{m}_b^{(\ell)}) \approx 4.2 \text{ GeV}$

from LEP:  $\overline{\alpha}^{(\ell)}(M_Z) = \alpha(M_Z) \left( 1 + \frac{100\alpha}{27\pi} \right), \quad \alpha(M_Z) \approx 1/129, \alpha \approx 1/137$

# DECOUPLING RELATIONS II

- dim.4 contributions to  $\zeta_i$  well known, we calculated dim.6 corrs. Example:

$$\zeta_e^{(4,1)} = \frac{\alpha}{\pi} \left[ -\frac{1}{12} - \frac{7}{8} \ln \left( \frac{\mu^2}{M_W^2} \right) + \frac{N_c}{6} Q_t^2 \ln \left( \frac{\mu^2}{m_t^2} \right) \right]$$

$$\zeta_e^{(6,1)} = \frac{\alpha}{\pi} \left[ \sqrt{2} \hat{v}_T m_t N_c Q_t \left( \hat{c}_w \frac{\text{Re}(C_{tB})}{e} + \hat{s}_w \frac{\text{Re}(C_{tW})}{e} \right) \ln \left( \frac{\mu^2}{m_t^2} \right) + 9 \frac{C_W}{e} \hat{s}_w M_W^2 \ln \left( \frac{\mu^2}{M_W^2} \right) \right] + \frac{\delta e^{\text{cl.4(6)}}}{e} \Big|_{\text{fin.}, m_b \rightarrow 0}$$

- relation between NLO decay rate using low-energy parameters vs. SM params:

$$\bar{\Gamma}_\ell^{(4,1)} = \bar{\Gamma}^{(4,1)} + 2\bar{\Gamma}^{(4,0)} \left( \zeta_b^{(4,1)} + \zeta_e^{(4,1)} \right),$$

$$\bar{\Gamma}_\ell^{(6,1)} = \bar{\Gamma}^{(6,1)} + 2\bar{\Gamma}^{(4,0)} \left( \zeta_b^{(6,1)} + \zeta_e^{(6,1)} \right) + 2\bar{\Gamma}^{(6,0)} \zeta_b^{(4,1)} + \sqrt{2} C_{bH} \frac{(\bar{v}^{(\ell)})^3}{\bar{m}_b^{(\ell)}} \bar{\Gamma}^{(4,0)} \left( \zeta_b^{(4,1)} + \zeta_e^{(4,1)} \right)$$

- illustrative results: QCD-QED corrections and EW corrections in  $m_t \rightarrow \infty$  limit:

$$\bar{\Gamma}_{\ell,g,\gamma} = \bar{\Gamma}_{g,\gamma}, \quad \bar{\Gamma}_{\ell,t} = [\Gamma_t]^{\text{O.S.}}$$

- interpretation: QCD-QED corrections in  $\overline{\text{MS}}$  scheme (UV logs resummed), heavy-particle EW corrections in on-shell (no large tadpoles)

# TOWARDS NUMERICS: INPUTS

- choose  $\mu = m_H$  in Wilson coefficients and  $\overline{\text{MS}}$  parameters
- numerical values of parameters are  $(\bar{v}^{(\ell)}(m_H) \equiv 2M_W \hat{s}_w / \bar{e}^{(\ell)}(m_H))$

$m_H$	125 GeV	$\bar{m}_b^{(\ell)}(m_H)$	3.0 GeV
$m_t$	173 GeV	$\bar{e}^{(\ell)}(m_H)$	$\sqrt{4\pi/128}$
$M_W$	80.4 GeV	$\bar{v}^{(\ell)}(m_H)$	240 GeV
$M_Z$	91.2 GeV	$\alpha_s(m_H)$	0.1

- use dimensionless coefficients  $\tilde{C}_i(m_H) \equiv \Lambda_{\text{NP}}^2 C_i(m_H)$ , then dim.6 contributions suppressed by  $v^2/\Lambda^2$  (not necessary to specify  $\Lambda$ )

# THE DECAY RATE AND MFV

- The LO decay rates in SM and SMEFT read:

$$\Gamma^{(4,0)} = \frac{N_c m_H m_b^2}{8\pi \hat{v}_T^2},$$

$$\Gamma^{(6,0)} = 2\Gamma^{(4,0)} \left[ C_{H\Box} - \frac{C_{HD}}{4} \left( 1 - \frac{\hat{c}_w^2}{\hat{s}_w^2} \right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} - \frac{\hat{v}_T}{m_b} \frac{C_{bH}}{\sqrt{2}} \right] \hat{v}_T^2$$

- $C_{bH}$  contributions enhanced by  $v/m_b$  compared to SM, because  $Q_{bH} \sim (H^\dagger H)(\bar{b}_L b_R H) + \text{h.c.}$  has chirality flip but no Yukawa
- in MFV (flavour violation through Yukawas),  $C_{bH} \sim m_b$ , so no enhancement
- happens with 6 other coeffs:  $C_{bG}$ ,  $C_{bW}$ ,  $C_{bB}$ ,  $C_{Htb}$ ,  $C_{qtqb}^{(1)}$  and  $C_{qtqb}^{(8)}$
- Higgs is thus useful for testing MFV, but in numerics will factor out  $m_b$  factors so that no operator gets  $1/m_b$  enhancement

# NUMERICAL RESULTS AT $\mu = m_H$

- define results in units of LO decay rate:

$$\Delta^{\text{LO}}(\mu) \equiv \frac{\bar{\Gamma}_\ell^{(4,0)}(\mu) + \bar{\Gamma}_\ell^{(6,0)}(\mu)}{\bar{\Gamma}_\ell^{(4,0)}(m_H)},$$

$$\Delta^{\text{NLO}}(\mu) \equiv \Delta^{\text{LO}}(\mu) + \frac{\bar{\Gamma}_\ell^{(4,1)}(\mu) + \bar{\Gamma}_\ell^{(6,1)}(\mu)}{\bar{\Gamma}_\ell^{(4,0)}(m_H)}$$

- results at  $\mu = m_H$

$$\Delta^{\text{LO}}(m_H) = 1 + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\text{NP}}^2} \left[ 3.74 \tilde{C}_{HWB} + 2.00 \tilde{C}_{H\Box} - 1.41 \frac{\bar{v}^{(\ell)}}{\bar{m}_b^{(\ell)}} \tilde{C}_{bH} + 1.24 \tilde{C}_{HD} \right]$$

$$\begin{aligned} \Delta^{\text{NLO}}(m_H) = & 1.13 + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\text{NP}}^2} \left\{ 4.16 \tilde{C}_{HWB} + 2.40 \tilde{C}_{H\Box} - 1.73 \frac{\bar{v}^{(\ell)}}{\bar{m}_b^{(\ell)}} \tilde{C}_{bH} + 1.33 \tilde{C}_{HD} \right. \\ & + 2.75 \tilde{C}_{HG} - 0.12 \tilde{C}_{Hq}^{(3)} + \left( -7.9 \tilde{C}_{Ht} + 5.8 \tilde{C}_{Hq}^{(1)} + 3.1 \frac{\bar{v}^{(\ell)}}{\bar{m}_b^{(\ell)}} \tilde{C}_{qtqb}^{(1)} - 3.1 \tilde{C}_{tH} + 2.7 \tilde{C}_{HW} \right. \\ & \left. \left. + 2.4 \tilde{C}_H - 1.9 \frac{\bar{v}^{(\ell)}}{\bar{e}^{(\ell)} \bar{m}_b^{(\ell)}} \tilde{C}_{bW} - 1.3 \tilde{C}_{qb}^{(8)} - 1.3 \frac{\tilde{C}_{tW}}{\bar{e}^{(\ell)}} - 1.0 \tilde{C}_{qb}^{(1)} \right) \times 10^{-2} + \dots \right. \end{aligned}$$

- $C_{HG}$  by far largest NLO correction. In general, corrections involving QCD or top-loops largest.

# CORRECTIONS TO LO RESULTS

	SM	$\tilde{C}_{HWB}$	$\tilde{C}_{H\Box}$	$\tilde{C}_{bH}$	$\tilde{C}_{HD}$
NLO QCD-QED	18.2%	17.9%	18.2%	18.2%	18.2%
NLO large- $m_t$	-3.1%	-4.6%	3.2%	3.5%	-9.0%
NLO remainder	-2.2%	-1.9%	-1.2 %	0.6%	-2.0%
NLO correction	12.9%	11.3%	20.2%	22.3%	7.1%

**TABLE:** Size of NLO corrections to different terms in LO decay rate, split into QCD-QED, large  $m_t$ , and remaining components.

- applying SM  $K$ -factor to dim.6 operators bad approximation once EW corrs included!!

# SCALE UNCERTAINTIES I

- can estimate uncalculated, higher-order corrections through scale variations
- the  $C_i$  are unknown. Therefore, use RG eqns to express  $C_i(\mu_C)$  in terms of  $C_i(m_H)$
- in practice will use  $\mu = m_H/2, 2m_H$ , so need only fixed-order solutions: ( $\dot{C} = \frac{dC}{d \ln \mu}$ )

$$C_i(\mu_C) = C_i(m_H) + \ln\left(\frac{\mu_C}{m_H}\right) \dot{C}_i(m_H),$$

$$\bar{m}_b^{(\ell)}(\mu_R) = \bar{m}_b^{(\ell)}(m_H) \left[ 1 + \gamma_b(m_H) \ln\left(\frac{\mu_R}{m_H}\right) \right],$$

$$\bar{\alpha}^{(\ell)}(\mu_R) = \bar{\alpha}^{(\ell)}(m_H) \left[ 1 + 2\gamma_e(m_H) \ln\left(\frac{\mu_R}{m_H}\right) \right],$$

- **note:** it is possible (and preferable) to vary  $\mu_C$  and  $\mu_R$  independently in order to get a conservative uncertainty estimate, by evaluating

$$\bar{\Gamma}_\ell^{(6,0)}(\mu_R, \mu_C) = \bar{\Gamma}_\ell^{(6,0)}(\mu_C) \Big|_{\bar{p}(\mu_C) \rightarrow \bar{p}(\mu_R)},$$

$$\bar{\Gamma}_\ell^{(6,1)}(\mu_R, \mu_C) = \left\{ \bar{\Gamma}_\ell^{(6,1)}(\mu_C) + 2 \left[ \ln\left(\frac{\mu_C}{m_H}\right) - \ln\left(\frac{\mu_R}{m_H}\right) \right] \left( \gamma_b(\mu_C) \bar{\Gamma}_\ell^{(6,0)}(\mu_C) \right. \right. \\ \left. \left. + \frac{C_{bH}(\mu_C)}{\sqrt{2}} \frac{(\bar{v}^{(\ell)})^3(\mu_C)}{\bar{m}_b^{(\ell)}(\mu_C)} \bar{\Gamma}_\ell^{(4,0)}(\mu_C) [\gamma_b(\mu_C) + \gamma_e(\mu_C)] \right) \right\} \Big|_{\bar{p}(\mu_C) \rightarrow \bar{p}(\mu_R)}$$

$$\bar{p}(\mu) \in \{ \bar{\alpha}^{(\ell)}(\mu), \bar{m}_b^{(\ell)}(\mu), \alpha_s(\mu) \}$$

# SCALE UNCERTAINTIES II

- result from varying  $\mu_C, \mu_R$  independently by factor of 2 and adding in quadrature:

$$\Delta^{\text{LO}}(m_H, m_H) = (1 \pm 0.08) + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\text{NP}}^2} \left\{ \begin{aligned} & (3.74 \pm 0.36) \tilde{C}_{H\text{WB}} + (2.00 \pm 0.21) \tilde{C}_{H\Box} - (1.41 \pm 0.07) \frac{\bar{v}^{(\ell)}}{\bar{m}_b^{(\ell)}} \tilde{C}_{bH} + (1.24 \pm 0.14) \tilde{C}_{HD} \\ & \pm 0.35 \tilde{C}_{HG} \pm 0.19 \tilde{C}_{Hq}^{(1)} \pm 0.18 \tilde{C}_{Ht} \pm 0.11 \tilde{C}_{Hq}^{(3)} + \dots \end{aligned} \right\}$$

$$\Delta^{\text{NLO}}(m_H, m_H) = 1.13_{-0.04}^{+0.01} + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\text{NP}}^2} \left\{ \begin{aligned} & (4.16_{-0.14}^{+0.05}) \tilde{C}_{H\text{WB}} + (2.40_{-0.09}^{+0.04}) \tilde{C}_{H\Box} \\ & + (-1.73_{-0.03}^{+0.04}) \frac{\bar{v}^{(\ell)}}{\bar{m}_b^{(\ell)}} \tilde{C}_{bH} + (1.33_{-0.04}^{+0.01}) \tilde{C}_{HD} + (2.75_{-0.48}^{+0.49}) \tilde{C}_{HG} \\ & + (-0.12_{-0.01}^{+0.04}) \tilde{C}_{Hq}^{(3)} + (-0.08_{-0.01}^{+0.05}) \tilde{C}_{Ht} + (0.06_{-0.05}^{+0.00}) \tilde{C}_{Hq}^{(1)} + (0.00_{-0.04}^{+0.07}) \frac{\tilde{C}_{tG}}{g_s} + \dots \end{aligned} \right\}$$

- in general, scale uncertainties in LO result overlap with NLO one, and scale uncertainties decrease between LO and NLO
- exception is  $C_{HG}$ , which gives large corrections unrelated to RG eqs.
- scale variation of  $C_{HG}$  gives rise to  $C_{tG}$ , which is a significant contribution although it is two-loop order

# CORRELATED SCALE VARIATIONS

- can also **correlate**  $\mu_C = \mu_R = \mu$  and vary them simultaneously. In units of  $\bar{v}^2/\Lambda_{\text{NP}}^2$ , for operators appearing at LO, this yields

$$(3.74 \pm 0.20)\tilde{C}_{HWB} + (2.00 \pm 0.06)\tilde{C}_{H\Box} - (1.41 \pm 0.08)\frac{\bar{v}^{(\ell)}}{\bar{m}_b^{(\ell)}}\tilde{C}_{bH} + (1.24 \pm 0.02)\tilde{C}_{HD}$$

- **quadrature** result in same units:

$$(3.74 \pm 0.36)\tilde{C}_{HWB} + (2.00 \pm 0.21)\tilde{C}_{H\Box} - (1.41 \pm 0.07)\frac{\bar{v}^{(\ell)}}{\bar{m}_b^{(\ell)}}\tilde{C}_{bH} + (1.24 \pm 0.14)\tilde{C}_{HD}$$

- **correlated  $\mu$  variations vastly underestimate size of NLO corrections!!**

- calculated full NLO corrections to  $h \rightarrow bb$  in SMEFT
- forms basis for serious phenomenological analysis of  $h \rightarrow bb$  in SMEFT
- calculation involved many non-trivial conceptual issues in SMEFT, and can be used as a template for future calculations combining EW and QCD corrections