The Universal One-Loop Effective Action

And How To Use It

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Outline

Motivation

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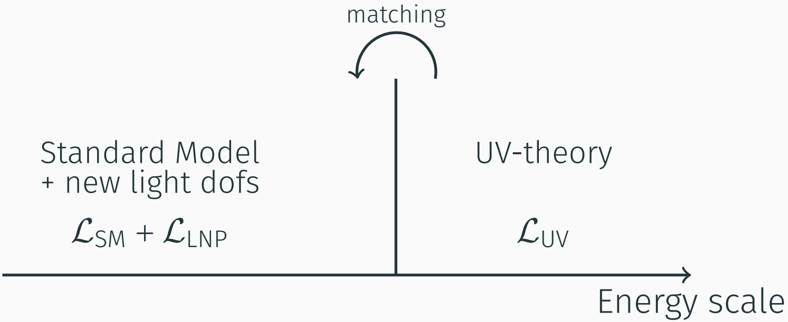
- No preferred BSM-model
- SMEFT: general parameterization of heavy New Physics
- No room for new light degrees of freedom
- Automation of matching desirable



Goal

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All of this should happen at the push of a button!

UOLEA

Functional matching

- Functional matching: Impose $\Gamma_{\text{L,UV}} = \Gamma_{\text{EFT}}$
- Starting point: generating functional

$$Z_{\text{UV}}[J_{\Phi}, J_{\phi}] = \int \mathcal{D}\Phi\mathcal{D}\phi \exp\left[iS_{\text{UV}}[\Phi, \phi] + iJ_{\Phi,x}\Phi_x + iJ_{\phi,x}\phi_x\right]$$

- Only light external particles $\Rightarrow J_{\Phi} = 0$
- Generator of 1LPI Green's functions

$$\Gamma_{\text{L,UV}}[\phi_b] = -i \log Z_{\text{UV}}[J_{\Phi} = 0, J_{\phi}] - \phi_{b,x}J_{\phi,x}$$

- Background field method:

$$\phi = \phi_b + \delta\phi, \qquad \frac{\delta S_{\text{UV}}}{\delta\phi}[\Phi_b, \phi_b] + J_{\phi} = 0$$

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Functional matching

- Expand around background fields

$$
\begin{aligned}
S_{\text{UV}}[\Phi,\phi]+J_{\phi,x}\phi_x =& S_{\text{UV}}[\Phi_b,\phi_b]+J_{\phi,x}\phi_{b,x} \\
&+\frac{1}{2}\begin{pmatrix}\delta\Phi_x^T & \delta\phi_x^T\end{pmatrix}\begin{pmatrix}\frac{\delta^2 S_{\text{UV}}}{\delta\Phi\delta\Phi}[\Phi_b,\phi_b]_{xy} & \frac{\delta^2 S_{\text{UV}}}{\delta\Phi\delta\phi}[\Phi_b,\phi_b]_{xy} \\ \frac{\delta^2 S_{\text{UV}}}{\delta\phi\delta\Phi}[\Phi_b,\phi_b]_{xy} & \frac{\delta^2 S_{\text{UV}}}{\delta\phi\delta\phi}[\Phi_b,\phi_b]_{xy}\end{pmatrix}\begin{pmatrix}\delta\Phi_y \\ \delta\phi_y\end{pmatrix} \\
&+\cdots
\end{aligned}
$$

- Tree-level: $\Gamma^{\text{tree}}_{\text{L,UV}}[\phi_b] = S_{\text{UV}}[\Phi_b,\phi_b]$
- One-loop: $\Gamma^{\text{1-loop}}_{\text{L,UV}}[\phi_b] = ic_s \log\det Q_{\text{UV}}$

$$
Q_{\text{UV}} = \begin{pmatrix}-\frac{\delta^2 S_{\text{UV}}}{\delta\Phi\delta\Phi}[\Phi_b,\phi_b] & -\frac{\delta^2 S_{\text{UV}}}{\delta\Phi\delta\phi}[\Phi_b,\phi_b] \\ -\frac{\delta^2 S_{\text{UV}}}{\delta\phi\delta\Phi}[\Phi_b,\phi_b] & -\frac{\delta^2 S_{\text{UV}}}{\delta\phi\delta\phi}[\Phi_b,\phi_b]\end{pmatrix}
$$

Functional matching

- Similar approach to EFT part, with $S_{\text{EFT}}[\phi] = S_{\text{EFT}}^{\text{tree}}[\phi] + S_{\text{EFT}}^{\text{1-loop}}[\phi]$
- $\Gamma_{\text{EFT}}^{\text{tree}}[\phi_b] = S_{\text{EFT}}^{\text{tree}}[\phi_b]$
- $\Gamma_{\text{EFT}}^{\text{1-loop}}[\phi_b] = S_{\text{EFT}}^{\text{1-loop}}[\phi_b] + ic_s \log\det\left(-\frac{\delta^2 S_{\text{EFT}}^{\text{tree}}}{\delta\phi\delta\phi}[\phi_b]\right)$
- Tree-level matching yields $S_{\text{EFT}}^{\text{tree}}[\phi_b] = S_{\text{UV}}[\Phi_b, \phi_b]$
- Note: $\Phi_b = \Phi_b[\phi_b] = \sum_{n=0}^{\infty} \frac{1}{M^n} F_n[\phi_b]$

Functional Matching

- For one-loop part introduce

$$X_{\sigma\rho} = -\frac{\delta^2 S_{\text{UV,int}}}{\delta\sigma\delta\rho}[\Phi_b, \phi_b],$$
$$\Delta_\rho = -P^2 + M_\rho^2 + X_{\rho\rho},$$
$$P_\mu = iD_\mu$$

- Matching condition yields

$$S_{\text{EFT}}^{\text{1-loop}}[\phi_b] = ic_s \,\text{Tr}\log\left(\Delta_\Phi - X_{\Phi\phi}\Delta_\phi^{-1}X_{\phi\Phi}\right)$$

- Evaluate using a Covariant Derivative Expansion

$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}} = c_s \sum_{ij\dots} f_{ij\dots}\,\text{tr}\left(\mathcal{O}_{ij\dots}\right)$$

Complications due to statistics

- How to treat $\delta\psi \frac{\delta^2 S}{\delta\psi\delta\phi}\delta\phi$?
- What about $\delta\theta \frac{\delta^2 S}{\delta\theta\delta\phi}\delta\phi$ for $\theta \in \mathbb{R}$ and $\phi \in \mathbb{C}$?

Diagonalizations necessary

Unified framework

	Fermion	Boson*
H	$\Xi = \begin{pmatrix}\Psi & \Psi^c & \Lambda\end{pmatrix}^T$	$\Phi = \begin{pmatrix}\Sigma & \Sigma^* & \Theta\end{pmatrix}^T$
L	$\xi = \begin{pmatrix}\psi & \psi^c & \lambda\end{pmatrix}^T$	$\phi = \begin{pmatrix}\sigma & \sigma^* & \theta\end{pmatrix}^T$

*Possible to use result for massive vector bosons in Feynman gauge (and to some extent for gauge bosons)

- One-loop result

$$\mathcal{L} = \frac{1}{16\pi^2} \sum_{\alpha} \sum_{ij\ldots} F^{\alpha}(M_i, M_j, \ldots) \mathcal{O}^{\alpha}_{ij\ldots},$$

- Here $F^{\alpha}(M_i, M_j, \ldots)$ are **universal** coefficients that are **calculated once and for all**

  Example:

  $F(M_i, M_j) = -\frac{1}{6}\tilde{\mathcal{I}}[q^2]^{11}_{ij} = \frac{M_j^4 - M_i^4}{16(M_i^2 - M_j^2)} + \frac{\log(M_i^2/\mu^2) M_i^4 - \log(M_j^2/\mu^2) M_j^4}{24(M_i^2 - M_j^2)} + \frac{M_j^4 - M_i^4}{12(M_i^2 - M_j^2)\epsilon}$

- Theory dependence in $\mathcal{O}^{\alpha}_{ij\ldots}$, e.g.

$$(\mathsf{X}_{\Xi\Xi})_{ij} \gamma^{\mu} (\mathsf{X}_{\Xi\Xi})_{jk} (\mathsf{X}_{\Xi\Xi})_{kl} \gamma_{\mu} (\mathsf{X}_{\Xi\Xi})_{li}$$

$$[P_{\mu}, (\mathsf{X}_{\Xi\Xi})_{ij}][P^{\mu}, (\mathsf{X}_{\Xi\Xi})_{ji}]$$

- Matching amounts to calculating derivatives and sums (and a lot of algebra)

Status

Status

	Φ	V^μ	ψ	v^μ	c
Φ	✓	✓	✓	X	X
V^μ	✓	✓	✓	X	X
ψ	✓	✓	✓	✓/X	X
v^μ	X	X	X	X	X
c	X	X	X	X	X

Known:

- Scalars and massive vectors [1706.07765]
- Unified framework: Fermions, scalars, massive vectors
- Regularization translation DRED-DREG [1806.05171]

Missing:

- gauge bosons (scalar-gauge-boson couplings)
- Higher dimensional operators with derivative couplings in ‘full’ theory
- Ghosts

How to use the UOLEA

Example: Integrating out the gluino from the MSSM

$$\mathcal{L}_{\text{EFT}} \supset c(\tilde{t}^*_{Li}\tilde{t}_{Li})(\tilde{t}^*_{Lj}\tilde{t}_{Lk})(\tilde{t}^*_{Lk}\tilde{t}_{Lj})$$

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Relevant interactions

$$\mathcal{L}_{\text{int}} = -\sqrt{2}g_3\left(\bar{t}P_R\tilde{g}^a T^a\tilde{t}_L - \bar{t}P_L\tilde{g}^a T^a\tilde{t}_R + \tilde{t}^*_L(\tilde{g}^a)^T T^a \mathcal{C}P_L t - \tilde{t}^*_R(\tilde{g}^a)^T T^a \mathcal{C}P_R t\right)$$

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Relevant UOLEA operators

$$\frac{1}{\kappa}\mathcal{L}_{\text{UOLEA}} \supset$$

$$\text{tr}\{-\frac{1}{2}m^2_{\tilde{g}}\tilde{\mathcal{I}}[q^4]^{33}_{\tilde{g}0}g_{\mu\nu\rho\sigma}(\mathsf{X}_{\Xi\xi})^a_i\gamma^\mu(\mathsf{X}_{\xi\Xi})^b_i(\mathsf{X}_{\Xi\xi})^b_j\gamma^\nu(\mathsf{X}_{\xi\Xi})^c_j\gamma^\rho(\mathsf{X}_{\Xi\xi})^c_k\gamma^\sigma(\mathsf{X}_{\xi\Xi})^a_k$$
$$-\frac{1}{6}\tilde{\mathcal{I}}[q^6]^{33}_{\tilde{g}0}g_{\mu\nu\rho\sigma\kappa\lambda}(\mathsf{X}_{\Xi\xi})^a_i\gamma^\mu(\mathsf{X}_{\xi\Xi})^b_i\gamma^\nu(\mathsf{X}_{\Xi\xi})^b_j\gamma^\rho(\mathsf{X}_{\xi\Xi})^c_j\gamma^\sigma(\mathsf{X}_{\Xi\xi})^c_k\gamma^\kappa(\mathsf{X}_{\xi\Xi})^a_k\gamma^\lambda\}$$

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Relevant interactions

$$\mathcal{L}_{\text{int}} = -\sqrt{2}g_3\left(\bar{t}P_R\tilde{g}^a T^a\tilde{t}_L - \bar{t}P_L\tilde{g}^a T^a\tilde{t}_R + \tilde{t}^*_L(\tilde{g}^a)^T T^a \mathcal{C}P_L t - \tilde{t}^*_R(\tilde{g}^a)^T T^a\mathcal{C}P_R t\right)$$

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$$\text{tr}\{-\frac{1}{2}m^2_{\tilde{g}}\tilde{\mathcal{I}}[q^4]^{33}_{\tilde{g}0}g_{\mu\nu\rho\sigma}(\mathrm{X}_{\Xi\xi})^a_i\gamma^\mu(\mathrm{X}_{\xi\Xi})^b_i(\mathrm{X}_{\Xi\xi})^b_j\gamma^\nu(\mathrm{X}_{\xi\Xi})^c_j\gamma^\rho(\mathrm{X}_{\Xi\xi})^c_k\gamma^\sigma(\mathrm{X}_{\xi\Xi})^a_k$$

$$-\frac{1}{6}\tilde{\mathcal{I}}[q^6]^{33}_{\tilde{g}0}g_{\mu\nu\rho\sigma\kappa\lambda}(\mathrm{X}_{\Xi\xi})^a_i\gamma^\mu(\mathrm{X}_{\xi\Xi})^b_i\gamma^\nu(\mathrm{X}_{\Xi\xi})^b_j\gamma^\rho(\mathrm{X}_{\xi\Xi})^c_j\gamma^\sigma(\mathrm{X}_{\Xi\xi})^c_k\gamma^\kappa(\mathrm{X}_{\xi\Xi})^a_k\gamma^\lambda\}$$

$\mathrm{X}_{\Xi\xi}$ and $\mathrm{X}_{\xi\Xi}$ calculated from $\mathcal{L}_{\text{int}}$

$$(\mathrm{X}_{\Xi\xi})^a_{i\alpha\beta} = -\sqrt{2}g_3\left(T^a_{ji}\left[\tilde{t}^*_{Lj}(P_L)_{\alpha\beta} - \tilde{t}^*_{Rj}(P_R)_{\alpha\beta}\right] \quad T^a_{ij}\left[(P_R)_{\alpha\beta}\tilde{t}_{Lj} - (P_L)_{\alpha\beta}\tilde{t}_{Rj}\right]\right),$$

$$(\mathrm{X}_{\xi\Xi})^a_{i\alpha\beta} = -\sqrt{2}g_3\begin{pmatrix} T^a_{ij}\left[(P_R)_{\alpha\beta}\tilde{t}_{Lj} - (P_L)_{\alpha\beta}\tilde{t}_{Rj}\right] \\ T^a_{ji}\left[\tilde{t}^*_{Lj}(P_L)_{\alpha\beta} - \tilde{t}^*_{Rj}(P_R)_{\alpha\beta}\right]\end{pmatrix}$$

Example: Integrating out the gluino from the MSSM

Insert and calculate...

$$c = -\frac{2}{3}d(d+2)g_3^6 m_{\tilde{g}}^2 \tilde{\mathcal{I}}[q^4]_{\tilde{g}0}^{33} - \frac{2}{9}d(d^2+6d+8)g_3^6 \tilde{\mathcal{I}}[q^6]_{\tilde{g}0}^{33}$$

Computational step is simple but tedious

Basic steps

1. Given the Lagrangian specify which fields are heavy
2. Determine which operators are of interest
3. Calculate appropriate derivatives **Can be automated**
4. Plug into formula and do algebra **Can be automated**

Conclusion and Outlook

Conclusion and Outlook

- The UOLEA is a promising result for automation of one-loop matching
- Now the actual automation part has to be performed