Technische Universität München

Global Fit of Flavour Observables

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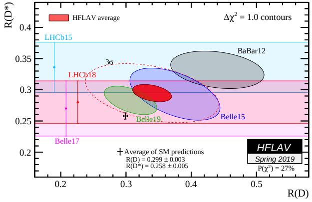
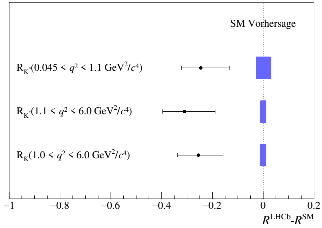
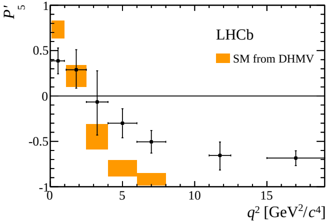
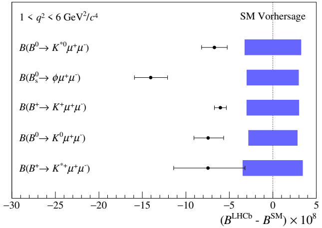
Motivation

B Anomalies

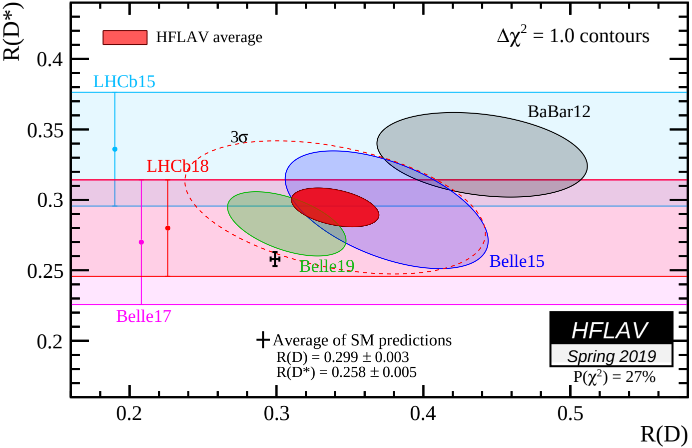

HFLAV average
Δχ² = 1.0 contours
LHCb15
BaBar12
3σ
LHCb18
Belle19
Belle15
Belle17
Average of SM predictions
R(D) = 0.299 ± 0.003
R(D*) = 0.258 ± 0.005
HFLAV
Spring 2019
P(χ²) = 27%
R(D*)
R(D)


SM Vorhersage
R_K*(0.045 < q² < 1.1 GeV²/c⁴)
R_K*(1.1 < q² < 6.0 GeV²/c⁴)
R_K(1.0 < q² < 6.0 GeV²/c⁴)
R^LHCb-R^SM


P′5
LHCb
SM from DHMV
q² [GeV²/c⁴]

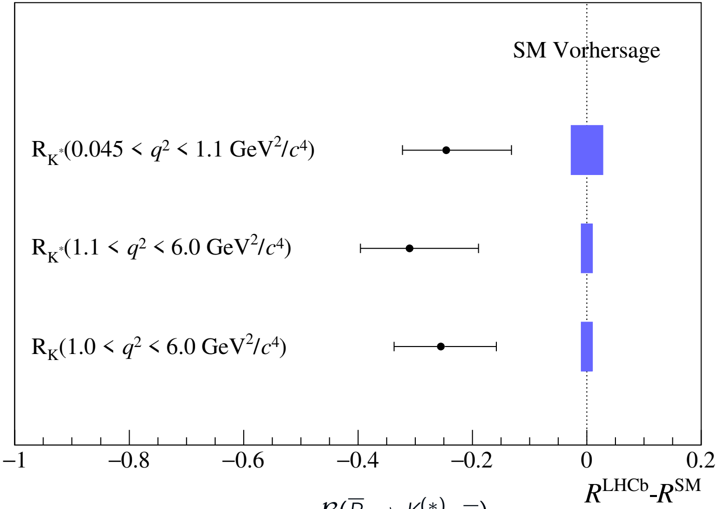

1 < q² < 6 GeV²/c⁴
SM Vorhersage
B(B⁰→ K*⁰μ⁺μ⁻)
B(B⁰s→ φμ⁺μ⁻)
B(B⁺→ K⁺μ⁺μ⁻)
B(B⁰→ K⁰μ⁺μ⁻)
B(B⁺→ K*⁺μ⁺μ⁻)
(B^LHCb - B^SM) × 10⁸

B Anomalies

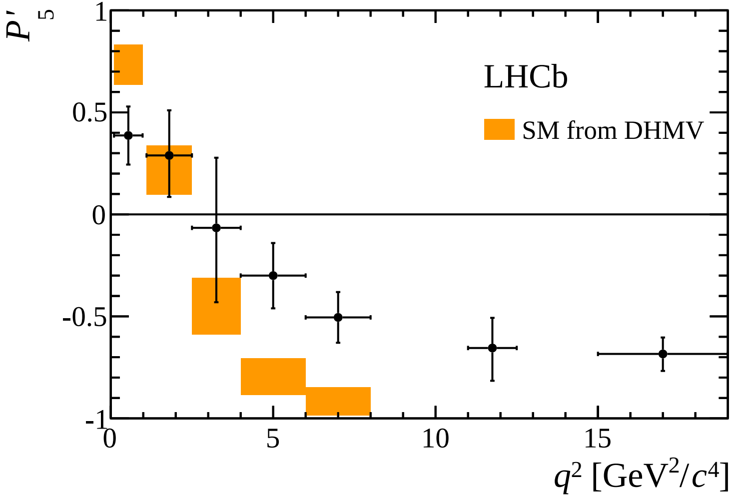


$$R_{D^{(*)}} \equiv \frac{\mathcal{B}(\overline{B} \to D^{(*)}\tau\overline{\nu})}{\mathcal{B}(\overline{B} \to D^{(*)}\{e,\mu\}\overline{\nu})}$$

B Anomalies

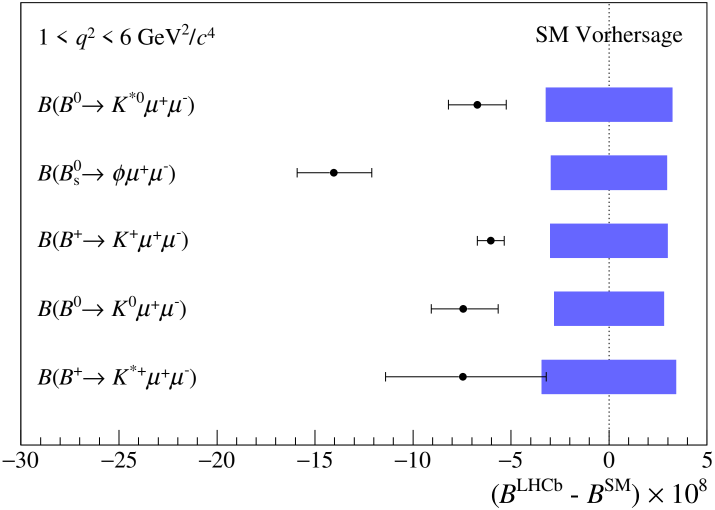


$$R_{K^{(*)}} \equiv \frac{\mathcal{B}(\overline{B} \to K^{(*)} \mu \overline{\nu})}{\mathcal{B}(\overline{B} \to D^{(*)} e \overline{\nu})}$$



$$\frac{1}{\Gamma}\frac{d^4\Gamma(\overline{B}\to\overline{K}^*\mu^+\mu^-)}{dq^2\,d^3\Omega} = \#\times P_5'(q^2)\times f(\Omega) + \text{other terms}$$

B Anomalies



1 < q² < 6 GeV²/c⁴
SM Vorhersage
B(B⁰→ K*⁰μ⁺μ⁻)
B(B⁰ₛ→ φμ⁺μ⁻)
B(B⁺→ K⁺μ⁺μ⁻)
B(B⁰→ K⁰μ⁺μ⁻)
B(B⁺→ K*⁺μ⁺μ⁻)
−30
−25
−20
−15
−10
−5
0
5
(B^LHCb - B^SM) × 10⁸

Global fits

Global fits in various flavours

New Physics fits in individual *flavour sectors* within their respective weak effective (field) theories (WETs):

$b \to s\mu^+\mu^-$ [Munich; Barcelona/Paris; Rome; Lyon/Mainz]

- weak hamiltonian with most-general $[\bar{s}b]\,[\bar{\ell}\ell]$ operators up to dim-6 10×3 WCs (10×6 with LFV)
- restricted to SM operators and coefficients for $[\bar{q}b]\,[\bar{s}q]$ operators up to dim-6

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$b \to c\ell^-\bar{\nu}$ [e.g. Jung,Straub '18; Blanke et al. '19; Murgui et al. '19]

- full basis of operators up to dim-6 with left-handed neutrinos 5×3 WCs

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$b \to u\ell^-\bar{\nu}$ [e.g. Feldmann,Müller,DvD '15]

- full basis of operators up to dim-6 with left-handed neutrinos 5×3 WCs

What about *the one global fit*?

it's complicated

idea is (relatively) recent and "simple"

1. select a NP model
2. generate SMEFT parameter point
3. match to WET/LEFT and run to $\mu \simeq m_b$ [e.g. `wilson`]
4. calculate likelihood [e.q. EOS; `flavio`]

reality

- one implementation of a (close to global) flavour fit using open source tools: `smelli` [Aebischer, Kumar, Stangl, Straub '18]
- to handle simultaneously a large number of nuisance parameters is technically challenging / bordering on the impossible
  - modifies statistical approach; neither frequentist nor Bayesian
  - discussed in the following …
  - presently: best we can do!

global likelihood $\mathcal{L}$ decomposed as:

$$\mathcal{L} = \prod_{C1} \mathcal{L}(\vec{C}) \times \prod_{C2} \mathcal{L}(\vec{C}, \vec{\theta})$$

to categories of likelihood

C1 exp. uncertainties $\gg$ theoretical uncertainties

C2 theoretical uncertainties taken into account

nuisance parameters $\vec{\theta}$

- parameters relevant only for a subset of observables
- control theory uncertainties

example: parameters of $B \to D$ form factors are dominantly relevant in $B \to D\mu\bar{\nu}$ processes only for category C2:

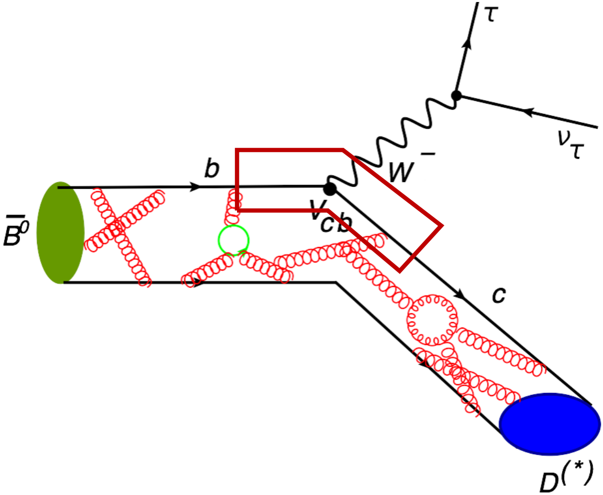
- assume (multivariate) gaussian exp. errors → covariance Σ_e
- assume (multivariate) gaussian theor. unc. → covariance Σ_t
- compute Σ_t in the SM from nuisance parameters $\vec{\theta}$

approximate:

$$\begin{aligned} -2\ln\mathcal{L}_{C2}(\vec{C},\vec{\theta}) &= -2\ln\mathcal{L}_{C2}(\vec{C}) \\ &= \left[\vec{o}_e - \vec{o}_t(\vec{C})\right]^T \left[\Sigma_e + \Sigma_t\right]^{-1} \left[\vec{o}_e - \vec{o}_t(\vec{C})\right] \end{aligned}$$

Aside: theory uncertainties and nuisance parameters (1)

Matrix elements of local operators $\bar{c}\,\Gamma b$ (and $\bar{s}\,\Gamma b$) parametrised through form factors



[Wu 2015]

Aside: theory uncertainties and nuisance parameters (1)

Matrix elements of local operators $\bar{c}\,\Gamma b$ (and $\bar{s}\,\Gamma b$) parametrised through form factors

- functions of momentum transfer (q^2)
- 3 independent functions in e.g. $\overline{B} \to D$ or $B \to K$
- 7 independent functions in e.g. $\overline{B} \to D^*$ or $B \to K^*$
- low-energy QCD effects prohibit diect calculation
  - numerical simulation (lattice QCD) [e.g. HPQCD '15, FNAL/MILC '15]
  - or non-perturbative methods (Light-Cone Sum Rules) [Gubernari, Kokulu, DvD '18]

parameter budget

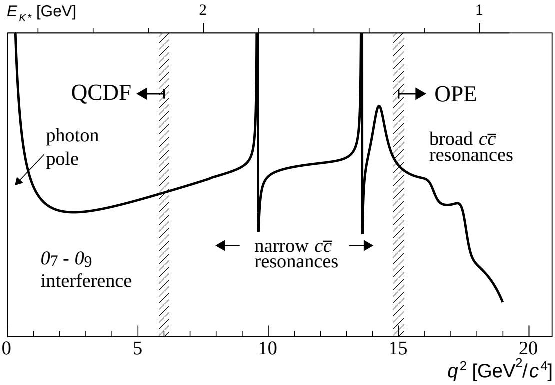
- roughly 3 parameters per form factor

→ 30 nuisance parameters for $\overline{B} \to D^{(*)}\mu\overline{\nu}$

→ 30+ nuisance parameters for $B \to K^{(*)}\mu^+\mu^-$

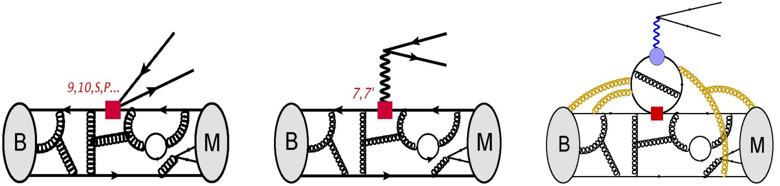
Hadronic Matrix Elements: Non-Local Effects (1)

$B \to K^* \mu^+ \mu^-$ landscape:



[sketch from Blake, Gershon, Hiller 2015]

Hadronic Matrix Elements: Non-Local Effects (2)



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

non-local: $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \, \langle \overline{M}_\lambda(k) | \, T\{ \mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i \mathcal{O}_i(0) \} \, | \overline{B}(q+k) \rangle$

- first approach to a systematic parametrization [Bobeth,Chrzaszcz,DvD,Virto '17]
- need ~ 3 parameters per non-local matrix element
- now total of 60 parameters for $B \to K^{(*)} \mu^+ \mu^-$

Issues

Issues

my (subjective) list of issues with the global fit (two universal, one `smelli` specific), ordered from least to most severe:

- dilution of the anomalies in statistical tests — universal
- NP-dependence of the theory uncertainties — `smelli` spec.
- NP-dependence of the measurements — universal

Dilution

- presently, hints of NP only show up in a tiny corner of flavour processes
  - rare $b \to s\mu^+\mu^-$ decays
  - $b \to c\tau\overline{\nu}$
  - $s \to d\{q\overline{q}, G\}$ $(\varepsilon'/\varepsilon)$

obliged by TUM contract to list this; hi Andrzej!

- a truly global fit would include many more measurements which are fully compatible with the SM
  - $\mathcal{O}(100)$ of observabkes
  - expect a few 2σ outliers, even some 3σ outliers
  - underestimate the statistical significance of the anomalies

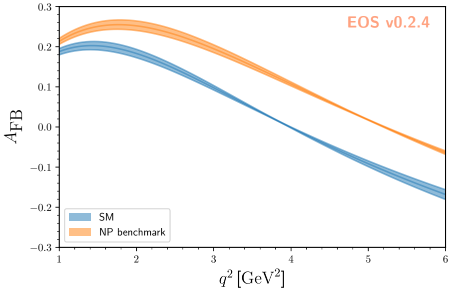
NP-dependence of the theory uncertainties

$$-2\ln\mathcal{L}_{C2}(\vec{C},\vec{\theta}) = -2\ln\mathcal{L}_{C2}(\vec{C}) = \left[\vec{o}_e - \vec{o}_t(\vec{C})\right]^T [\Sigma_e + \Sigma_t]^{-1} \left[\vec{o}_e - \vec{o}_t(\vec{C})\right]$$

- assuming that the theory uncertainties in the SM reflect the theory uncertainties in every NP point can be problematic

- $A_{FB}(q^2)$ in $B \to K^* \mu^+ \mu^-$ features zero-crossing in the SM

- absolute theory uncertainty massively reduced in bins surounding the zero crossing



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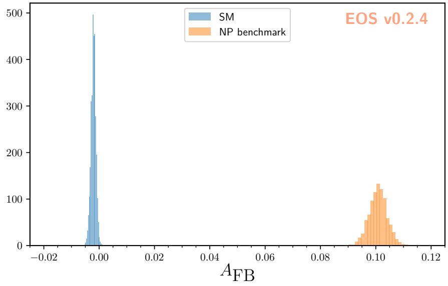


- bin $3.5\text{GeV}^2 \leq q^2 \leq 4.0\text{GeV}^2$

$$\sigma_{\text{NP}} \geq 3 \times \sigma_{\text{SM}}$$

- overestimate range of allowed WCs

NP-dependence of the measurements (1)

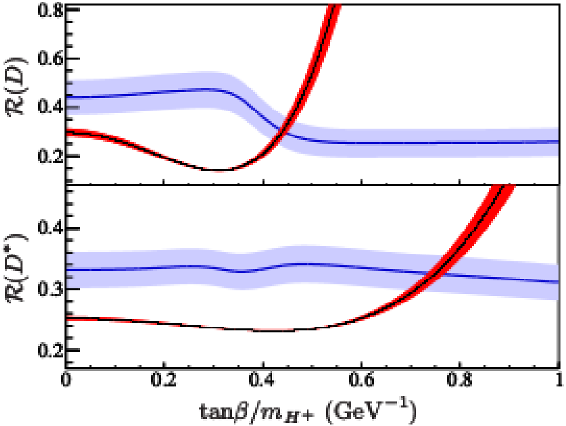
- $R(D^{(*)})$ measurements based on historgram template fits
  - used for $\tau \to \mu\bar{\nu}\nu$ reconstruction by BaBar / Belle / LHC
  - after subtracting backgrounds, determines one relative yield n_τ based on SM shape of the $B \to D^{(*)}\tau(\to \mu\nu\bar{\nu})\bar{\nu}$ mode
  - obtain LFU ratio as

$$R(D) \sim \frac{n_\tau}{1 - n_\tau} \times \text{efficiency corrections}$$

aside: electromagnetic corrections

- recent theory analysis of soft-photon effects [de Boer et al. '18]
- triggered sensitivity study by LHCb members, based on the present LHCb setup [Cali et al '19]
  - find bias up to 8% depending on max. radiated energy

- NP would distort template shapes (in particular: scalar/tensor couplings!)

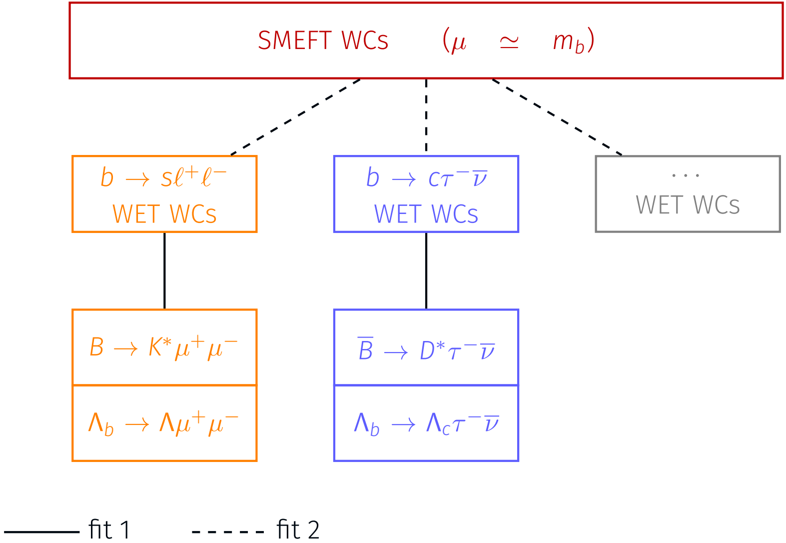


[BaBar '13]

similar plots and statements in Belle and LHCb measurements of $R(D^{(*)})$

What’s the alternative ?

Staged approach



Benefits

- each WET WC fit can be performed separately from each other
  - not quite “once and for all” [(C) I. Brivio]
    but no need to re-fit WET WCs in every SMEFT analysis
  - small overlap of nuisance parameters (e.g. $B \to \pi$ form factors in $b \to u\tau\overline{\nu}$ and $b \to d\ell^+\ell^-$)
  - Bayesian parlance: use WET posteriors as priors for SMEFT fit
- each WET WC fit can be individually checked for consistency
  - do mesonic and baryonic modes agree?
  - do modes related by $SU(3)_F$ agree?
- each WET WC posterior can be stored as random variates
  - SMEFT prior would be implemented as an unbinned likelihood

Summary & Outlook

Summary

flavour and SMEFT

- flavour anomalies are a potential sign of NP
- interpretation of the anomalies within the SMEFT is crucial to understand their possible NP origins

global SMEFT fits to flavour constraints

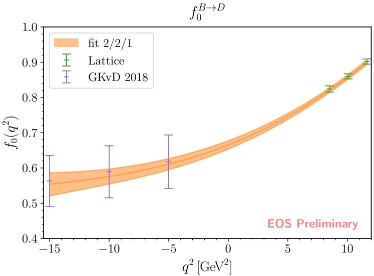
- `smelli` is a good start to explore the flavour constraints on the SMEFT parameter space

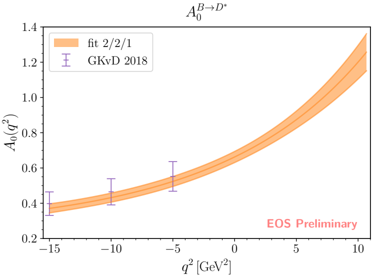
my preferred alternative / cross check

- staged approach with individual fits per sector
- can be implemented within `smelli` / `wilson` tool chain!

$B \to D^{(*)}$ form factor

[Bordone, Jung, DvD to appear]

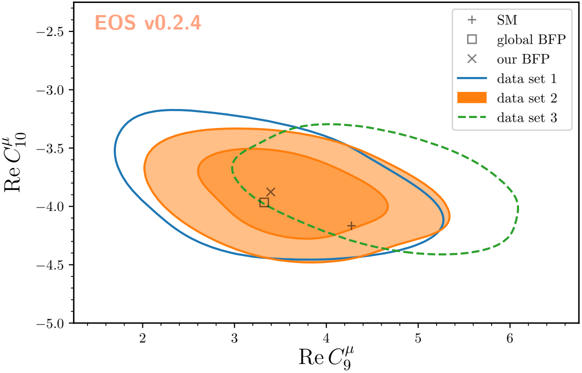




- first pure theory determination form factors at order $1/m_c^2$,
- includes form factors for full basis of dim-6 operators
- covers entire semileptonic phase space $0 \leq q^2 \leq\sim 11\,\text{GeV}^2$

$\Lambda_b \to \Lambda\mu^+\mu^-$

[Blake, Meinel, DvD to appear]



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