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Real-time dynamics of the confining string

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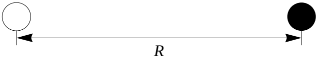
Outline

Introduction

Confining QCD string

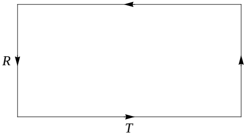
Static test color charges:



Wilson loop operator:

$$W(C) = \mathrm{Tr}\left\{ P \exp\left[i \oint_C A_\mu(x) dx^\mu \right] \right\}.$$

Contour C:



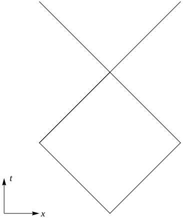
$$\langle W(C) \rangle \underset{T>>R}{\simeq} \exp[-V(R)T].$$

At large distances - *area law* $V(R) = \kappa\, R$ - linear confinement; $\kappa \simeq 1\mathrm{GeV}/\mathrm{fm}$ - string tension.

QCD string

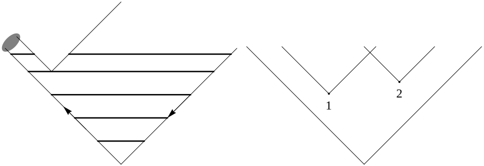
Confinement in high energy scattering - The Lund model

Massless relativistic string - string ends move with the speed of light.



If string doesn't break - motion in one space dimension, so-called yoyo-mode.

Lund model - semiclassical picture of string breaking.



Electric field vanishes between produced pairs. Points 1 and 2 space-like separated.
The string between two breaks should be on mass shell $m_\perp^2 = m^2 + \mathbf{p}_\perp^2 = E^2 - p_z^2$.
Points are space-like separated - breaks happen independently.

Motivation

- Despite all the successes of perturbative QCD, there is a need to understand better the nonperturbative aspects.
- Soft-physics data from the LHC, for example, may challenge the standard fragmentation picture.
- We need a better understanding of parton fragmentation in heavy ion physics.
- Known models for fragmentation could be challenged in ultra high energy cosmic rays.

- We consider confinement due to dual Meissner effect. Magnetic monopoles appear in the so-called Abelian projection.
- The dynamics along the jet axis is assumed to be Abelian and dimensionally reduced. The latter comes from a scaling argument, where the longitudinal scale is much larger than transverse.

Abelian model of the QCD string

The Schwinger model

Massless QED in $1+1$ dimensions, known as the Schwinger model:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\not{D}\psi$$

where, $\not{D} \equiv \gamma^\mu D_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + igA_\mu$.

Shares with QCD many important properties:

- Confinement and screening or color charge,
- Chiral symmetry breaking,
- Axial anomaly,
- θ-vacuum.

The theory is exactly soluble.

Bosonization

In $1+1$ we have the exact bosonization relations:

$$\begin{aligned}\bar{\psi}(x)i\gamma^{\mu}\partial_{\mu}\psi(x) &\rightarrow \frac{1}{2}\partial_{\mu}\phi(x)\partial^{\mu}\phi(x)\\ \bar{\psi}(x)\gamma^{\mu}\psi(x) &\rightarrow -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_{\nu}\phi(x)\\ \bar{\psi}(x)\psi(x) &\rightarrow -c\frac{g}{\sqrt{\pi}}\cos(2\sqrt{\pi}\phi(x)),\end{aligned}$$

where $c = \frac{e^{\gamma}}{2\pi}$ and $\gamma \simeq 0.5772156649$ is the Euler's constant. The Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi + \frac{g}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_{\nu}\phi A_{\mu} - j^{\mu}_{ext}A_{\mu}.$$

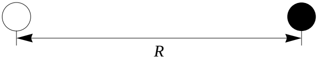
We have added an external current, which we parametrize as $j^{\mu}_{ext}(x) = -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_{\nu}\phi_{ext}(x)$.

Integrating out F_{01}, we get the effective Lagrangian:

$$\boxed{\mathcal{L} = \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - \frac{1}{2}\left(\frac{g}{\sqrt{\pi}}\right)^2(\phi+\phi_{ext})^2.}$$

Confinement and axial anomaly

Consider again:



Including massive fermions with mass M, in the limit $cmM << g^2$ [**Y. Frishman and J. Sonnenschein, Cambridge, UK: Univ. Pr. (2010)**],

$$V(L) = 2\pi^2 cmML + \frac{g\sqrt{\pi}}{2}\left(1 - e^{-\frac{g}{\sqrt{\pi}}L}\right).$$

Axial current in $1+1$ dimensions:

$$j_5^\mu(x) \equiv \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x) = \frac{1}{\sqrt{\pi}}\partial^\mu\phi(x).$$

From equation of motion

$$(\Box + m^2)\phi(x) = 0 \Rightarrow \partial_\mu\partial^\mu\phi(x) = -\frac{g^2}{\pi}\phi(x),$$

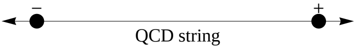
and the fact that $F^{01} = -\frac{g}{\sqrt{\pi}}\phi$, we get the anomaly:

$$\boxed{\partial_\mu j_5^\mu = \frac{g}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}.}$$

Fragmentation functions of jets in vacuum

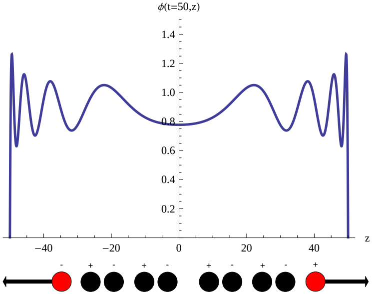
QCD string breaking and particle creation

Consider again:

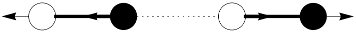


External charge density:

$$j^0_{\text{ext}}(x) = -\delta(z+t)\theta(-z) + \delta(z-t)\theta(z).$$



Breaking different from the semi-classical picture:



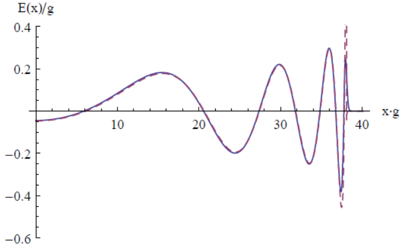
E-field doesn't vanish between the created pair.

Lattice studies

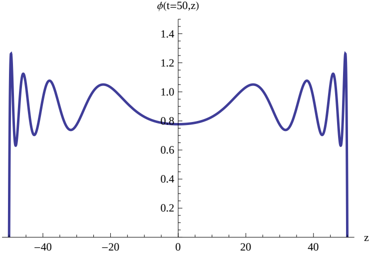
We have neglected the back-reaction to the source.

In [F. Hebenstreit and J. Berges. 2014, arXiv:1406.4273] this was taken into account in their numerical calculations.

Two receding, nearly massless fermions with $g/M = 100$, peak momentum $p/M \simeq 700$ and width $\Delta x \simeq 2/(5g)$, are created by a pulse of the electric field. These fermions are referred to as self-consistent.



Electric field $E(t \simeq 38/g, x)$ for self-consistent fermions (solid line) and external charges (dashed line) for $g/M = 100$, with $E(t, -x) = E(t, x)$.



Fragmentation functions in $e^+e^- \to \mathrm{hadrons}$

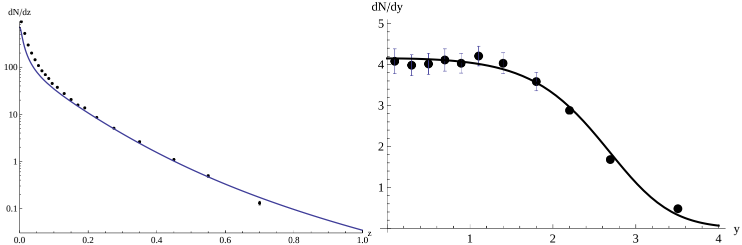
Dynamics in our model is described by the equation

$$(\Box + m^2)\phi(x) = -m^2\phi_{ext}(x) = j(x).$$

Particle creation by a classical source. Momentum distribution

$$\frac{dN}{dp} = \frac{|\tilde{j}(p)|^2}{2E_p},$$

where $E_p \equiv \sqrt{p^2 + m^2}$ and $\tilde{j}(p) = \int d^2x e^{ip\cdot x} j(x)$. Consider a source $j^\mu = \int d\tau \frac{dy^\mu}{d\tau} \delta^{(2)}(x - y(\tau))$, where $y^\mu = (\tau, v\tau)$ and $v = \frac{p_{jet}}{\sqrt{p_{jet}^2 + Q_0^2}}$. We calculate:



Data from OPAL Collaboration for $\sqrt{s} = 201.7$ GeV (left) and TPC/Two Gamma for $\sqrt{s} = 29$ GeV (right).

F. L. and Dmitri E. Kharzeev. Int.J.Mod.Phys.,E21:1250088, 2012, arXiv:1111.0493

where $z = p/p_{jet}$ and $y = \frac{1}{2}\ln\frac{E_p+p}{E_p-p}$. We find $Q_0 \simeq 2$ GeV, $m \simeq 0.6$ GeV. Q_0 is roughly the scale at which pQCD cascade stops.

Jets in medium - Energy loss

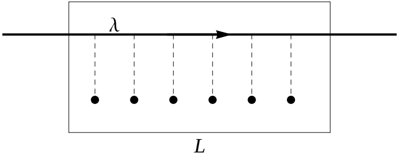
Landau-Pomeranchuk-Migdal (LPM) effect

Assume that a quark with energy E moves in QCD medium of length L.

Multiple scattering within the medium - collisional and radiative energy loss.

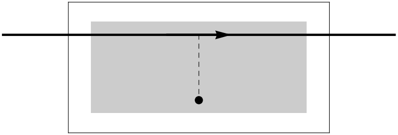
We focus on radiative energy loss [**R. Baier, D. Schiff, and B.G. Zakharov. Ann.Rev.Nucl.Part.Sci., 50:37-69, 2000, hep-ph/0002198**]. Define $l_{coh} = \frac{\omega}{\langle k_\perp^2 \rangle}$

For $l_{coh} \leq \lambda$ - Bethe-Heitler regime:



$$\omega \frac{dI}{d\omega dz}\Big|_{BH} \simeq \frac{\alpha_s}{\pi} N_c \frac{1}{\lambda}.$$

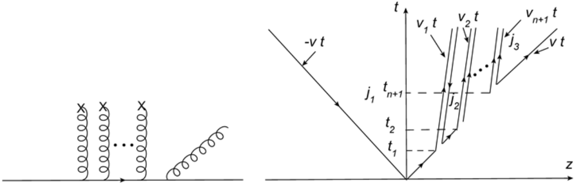
For $\lambda < l_{coh} < L$ - Coherent scattering:



$$\omega \frac{dI}{d\omega dz}\Big|_{LPM} \simeq \frac{\alpha_s}{\pi} N_c \frac{1}{\lambda} \sqrt{\frac{E_{LPM}}{\omega}}.$$

$E_{LPM} = \lambda \mu^2$; $k_\perp \simeq \mu$ - Debye screening mass. Suppression by a factor $\sqrt{\frac{E_{LPM}}{\omega}}$.

In-medium fragmentation of the jet



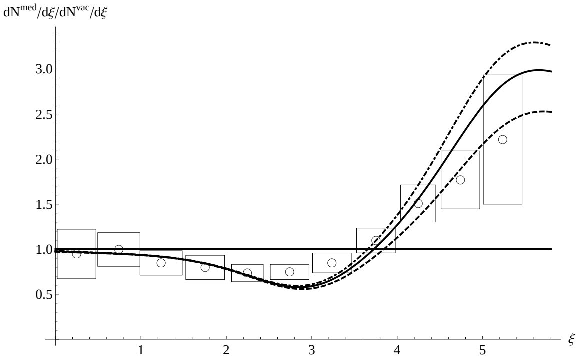
Momentum transfer of order of Debye mass. $v_i \sim 1/\sqrt{2}$. Induced radiation emitted outside medium.
Charge density:

$$\begin{aligned}
j_1^0(x) &= -\delta(z+vt)\theta(-z) + \big\{\delta(z-vt)[\theta(t)-\theta(t-t_1)] \\
&+ \delta[z-vt_1-v_1(t-t_1)]\theta(t-t_1)\big\}\theta(z) \\
j_2^0(x) &= -\delta[z-vt_1-v_1(t-t_1)]\theta(t-t_1) + \delta(z-vt)[\theta(t-t_1)-\theta(t-t_2)] \\
&+ \delta[z-vt_2-v_2(t-t_2)]\theta(t-t_2) \\
j_3^0(x) &= \big[-\delta[z-vt_{n+1}-v_{n+1}(t-t_{n+1})]\theta(t-t_{n+1}) \\
&+ \delta(z-vt)\big]\theta(t-t_{n+1}),
\end{aligned}$$

$$\frac{dN^{med}}{dp} = \frac{m^4}{2E_P}\left(|\tilde{\phi}_{1,\text{ext}}(p)|^2 + \sum|\tilde{\phi}_{2,\text{ext}}(p)|^2 + |\tilde{\phi}_{3,\text{ext}}(p)|^2\right).$$

In-medium fragmentation of the jet (cont'd)

We compute $(dN^{med}/d\xi)/(dN/d\xi)$ in the $1+1$ dimensional theory, as prescribed above, and compare to data ($\xi = \ln \frac{p_{jet}}{p} = \ln \frac{1}{z}$).



The curves correspond to mean free paths of $\lambda = 0.57$, 0.4 and 0.2 fm from top to bottom respectively.

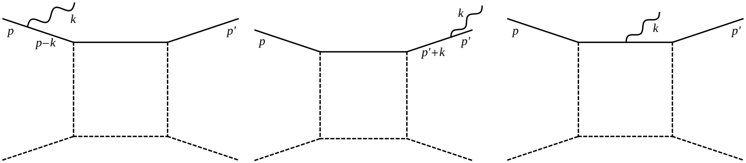
The radiation at large ξ is suppressed for small λ, in analogy with the LPM effect. Suppression for intermediate ξ from partial screening of color charge.

Data taken from the CMS Collaboration. Jet $p_T > 100$ GeV.

Anomalous soft photon production

Low Theorem

Scattering of two spin-zero particles, where one of them is charged



F.E. Low, Phys.Rev. 110 (1958) 974-977

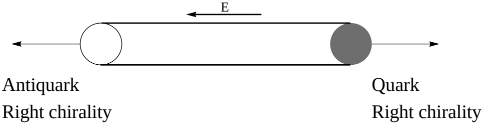
- Total amplitude $M = M^{(1)} + M^{(2)}$;
  - $M^{(1)}$ - diagrams with photons attached to external propagators
  - $M^{(2)}$ - diagrams with photons attached to internal propagators.
- **Low Theorem**: as $k \to 0$, $M^{(1)} \sim 1/k,\ M^{(2)} \sim \text{constant}.$
- As a consequence, soft photons are mainly produced from initial and final charged particles.
- Soft photon yield can be computed from the hadron spectrum

$$\omega \frac{dN^{\gamma}}{d^3k} = \frac{\alpha}{(2\pi)^2} \sum_n \int d^3p_1 \cdots d^3p_n \sum_{i,j} \frac{-Q_iQ_j(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} \frac{dN^{hadr}}{d^3p_1 \cdots d^3p_n}$$

Anomalous soft photon production

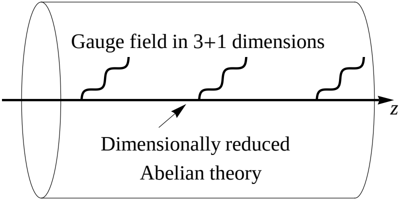
- Contrary to expectations, nearly all experiments observe a significant enhancement of soft photon production (factor $2 \div 5$) - "Anomalous soft photon production."
- For example, DELPHI measurements of soft photons in Z^0 decay to hadrons. Photon spectrum similar to Bremsstrahlung, but by a factor of **four** larger [Eur. Phys. J. C 67, 343 (2010)].
- DELPHI measurements of e^+e^- to $\mu^+\mu^-$ show no deviation from Bremsstrahlung [Eur. Phys. J. C 57, 499 (2008)].
- Theoretical models: [E. V. Shuryak, Phys. Lett. B 231, 175 (1989)], [P. Lichard and L. Van Hove, Phys. Lett. B 245, 605 (1990)], [G. W. Botz, P. Haberl and O. Nachtmann, Z. Phys. C 67, 143 (1995)], [C. -Y. Wong, Phys. Rev. C 81, 064903 (2010)],[Y. Hatta and T. Ueda, Nucl. Phys. B 837, 22 (2010)], etc. None of them explain all the features of the phenomenon.
- Presents challenge to the foundation of the theory. The process needs to be understood in order to account for the background in soft photon production in heavy ion collisions for example.

Enhancement of soft photons



$$\boxed{\Delta Q_5(t) = N_R - N_L = \frac{g}{\pi}\int d^2x\, E^{tot}_{01}(x) = 2[\cos(mt) - 1]}$$

Anomaly induces oscillation of vector charge along jet axis.



$$\boxed{\frac{dN_\gamma}{d^3p} = \left(B_{2/3}\left(\frac{2}{3}\right)^2 + B_{1/3}\left(\frac{1}{3}\right)^2\right)\frac{1}{(2\pi)^3}\frac{1}{2p^0}e^2\frac{4v^2}{(p_0^2 - v^2p_z^2)^2}p_\perp^2\left(1 + \frac{m^2}{p_\perp^2 - m^2}\right)^2}$$

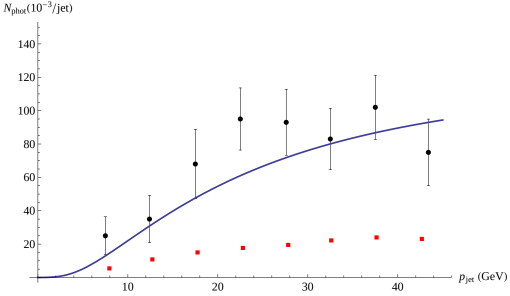
As $p_\perp \to 0$, $\frac{dN_\gamma}{d^3p} \to 0$ - Low theorem recoreved.

Phenomenology of soft photon production

- Quark-antiquark potential in the Schwinger model

$$V(r) = \frac{g\sqrt{\pi}}{2}\left(1 - e^{-\frac{g}{\sqrt{\pi}}r}\right)$$

- String tension $\kappa^2 = \frac{\pi}{2}m^2$, fluctuates [**A. Bialas, Phys. Lett. B 466, 301 (1999)**]:
  $P(\kappa^2) = \sqrt{\frac{2}{<\kappa^2>}}e^{-\frac{\kappa^2}{2<\kappa^2>}}$.
- Finite width: $\frac{1}{p_\perp^2 - m^2} \to \frac{1}{p_\perp^2 - m^2 + i\gamma^2}$.



Data from [[**DELPHI Collaboration], Eur. Phys. J. C 67, 343 (2010)**]

D. E. Kharzeev and FL, Phys.Rev. D87 (2013) 077501, arXiv:1308.2716 [hep-ph].

$$N_{\mathrm{phot}} = \int dm \sqrt{\frac{\pi}{2}} P\left(\frac{\pi}{2}m^2\right) \times \left(\int d^3p \frac{dN}{d^3p}\right)$$

$$p_\perp < 80\,\mathrm{MeV},\ 0.2 < E_\gamma < 1\,\mathrm{GeV}$$

$$\gamma = \sqrt{m_R \Gamma_R} = 0.003\ \mathrm{GeV}.$$

Typical values:
for η, $\gamma = 8\cdot 10^{-4}$ GeV;
for ω, $\gamma = 8\cdot 10^{-2}$ GeV.

Summary and Outlook

- We have used the Schwinger model to study real-time dynamics of the confining string, motivated from the dual superconductor model of the QCD vacuum and dimensional reduction in high energy jets.
- Using this model we addressed:
  - In-medium jet fragmentation and
  - Anomalous soft photon production.

- It would be interesting to try and incorporate this model in the already existing Monte Carlo event generators.
- A systematic derivation of the dimensionally reduced theory would motivate this project even further.