Renormalization of mixing angles in extended Higgs sectors

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in collaboration with
A.Denner and J.-N.Lang, JHEP 1811 (2018) 104

see also work with L.Altenkamp, M.Boggia and H.Rzehak,
JHEP 1709 (2017) 134, JHEP 1803 (2018) 110, JHEP 1804 (2018) 062

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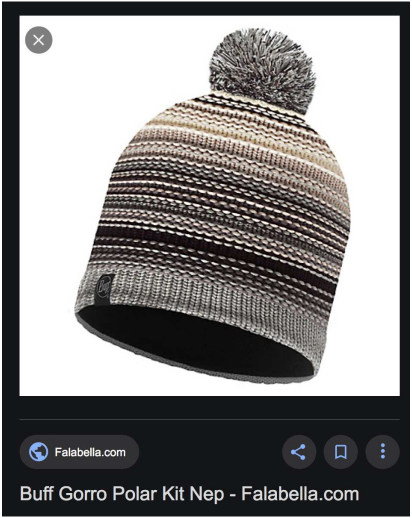

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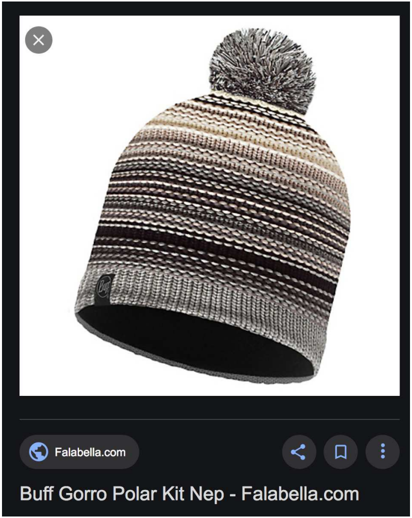

But then ...

... and here we are.

Contents

Introduction

Singlet Extension of the SM (SESM)

Lagrangian: restriction to real, $\mathbb{Z}_2$-symmetric case!

$$\mathcal{L}_{\mathrm{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{1}{2}(\partial\sigma)^2 - V(\Phi,\sigma),$$

$$V = -\mu_2^2 \Phi^\dagger\Phi + \frac{1}{4}\lambda_2 (\Phi^\dagger\Phi)^2 + \lambda_{12}\sigma^2\Phi^\dagger\Phi - \mu_1^2\sigma^2 + \lambda_1\sigma^4$$

Complex scalar SU(2) doublet & real scalar singlet: $v_{1,2} = \text{vevs}$

$$\Phi = \begin{pmatrix} \phi^+ \\ (\eta_2 + i\chi + v_2)/\sqrt{2} \end{pmatrix}, \qquad \sigma = v_1 + \eta_1, \qquad Y_{\mathrm{W}}(\Phi) = 1$$

$\hookrightarrow$ "mass basis" h, H: $\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} H \\ h \end{pmatrix}, \quad R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

Transformation of input parameters:

original set: $\{\lambda_1, \lambda_2, \lambda_{12}, \mu_1^2, \mu_2^2, g_1, g_2\}$

$\downarrow$

mass basis: $\{ \underbrace{M_{\mathrm{H}}, M_{\mathrm{h}}, M_{\mathrm{W}}, M_{\mathrm{Z}}, e}_{\text{renormalized on-shell}}, \underbrace{\lambda_{12}}_{\overline{\mathrm{MS}}}, \alpha \}$

Renormalization:

Bojarski et al. '15
Kanemura et al. '15,'17
Denner et al. '17,'18
Altenkamp et al. '18

Two-Higgs-Doublet Model (THDM)

Lagrangian: restriction to CP-conserving case!

$$\mathcal{L}_{\rm Higgs} = (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2) - V(\Phi_1,\Phi_2),$$
$$D_\mu = \partial_\mu - {\rm i}g_2 I^a_{\rm W} W^a_\mu + \tfrac{\rm i}{2}g_1 Y_{\rm W} B_\mu$$

Higgs potential:

$$\begin{aligned} V = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) \\ & + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ & + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \tfrac{1}{2}\lambda_5\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right] \end{aligned}$$

Two complex scalar SU(2) doublets: $v_{1,2}$ = vevs

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(\eta_1 + i\chi_1 + v_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(\eta_2 + i\chi_2 + v_2) \end{pmatrix}, \quad Y_{\rm W}(\Phi_{1,2}) = 1$$

Transition to the "mass basis":

CP-even neutral fields: $\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} = R(\alpha)\begin{pmatrix}H\\ h\end{pmatrix}$

CP-odd neutral fields: $\begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix} = R(\beta)\begin{pmatrix}G_0\\ A_0\end{pmatrix}, \quad \tan\beta = \frac{v_2}{v_1}$

charged fields: $\begin{pmatrix}\phi_1^\pm\\ \phi_2^\pm\end{pmatrix} = R(\beta)\begin{pmatrix}G^\pm\\ H^\pm\end{pmatrix}$

Higgs potential after diagonalization:

$$V = -t_\mathrm{h} h - t_\mathrm{H} H + \frac{1}{2}M_\mathrm{h}^2 h^2 + \frac{1}{2}M_\mathrm{H}^2 H^2 + \frac{1}{2}M_{\mathrm{A_0}}^2 A_0^2 + M_{\mathrm{H^+}}^2 H^+ H^- + \ldots$$

Transformation of input parameters:

original set: $\{\lambda_1, \ldots, \lambda_5, m_{11}^2, m_{22}^2, m_{12}^2, g_1, g_2\}$

$\downarrow$

mass basis: $\{\underbrace{M_\mathrm{H}, M_\mathrm{h}, M_{\mathrm{A_0}}, M_{\mathrm{H^+}}, M_\mathrm{W}, M_\mathrm{Z}, e}_{\text{renormalized on-shell}}, \underbrace{\lambda_5}_{\overline{\mathrm{MS}}}, \alpha, \beta\}$

Renormalization:
Santos/Barroso '97; Kanemura et al. '04; Lopez-Val/Sola '09; Degrande '14;
Krause et al. '16; Denner et al. '16,'18; Altenkamp '17

Parametrization of extended Higgs sectors

- New free parameters in Higgs extension:
  1 parameter for each new gauge-invariant, renormalizable monomial in $\mathcal{L}$
- Phenomenologically preferred input parameters:
  - Higgs-boson masses M_{H_i}
    $\hookrightarrow$ experimental accessibility, clear theoretical meaning
    Clean definition via "on-shell" (OS) renormalization:
    $M^2_{\mathrm{H}_i}$ = location of propagator pole
  - Higgs mixing angles (at least for CP-even scalars)
    $\hookrightarrow$ rescale Higgs couplings to known particles
    $\hookrightarrow$ sensitivity via couplings measurements

    But: Issues in renormalization (=field-theoretical foundation)
    $\hookrightarrow$ discussed in this talk in some detail
- Typically further free parameters remain
  (e.g. quartic scalar self-couplings)

Extended Higgs sectors
and renormalization issues

Renormalization of mixing angles – preliminaries

Distinguish 2 cases:

- Mixing angle $\alpha \neq$ free parameter of theory:
  renormalization of α is only bookkeeping, like field renormalization
  $\hookrightarrow$ no physical effect!
- Mixing angle $\alpha =$ free parameter of theory:
  renormalization of α defines dependence of S-matrix on α
  $\hookrightarrow$ discussed in the following

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Different renormalization schemes I and II

- input parameters c_i have different physical meaning in I, II:
  $c_{i,II} = c_{i,I} + \Delta c_i$ with $\Delta c_i \neq 0$ for a given scenario
  $\hookrightarrow$ parameter conversion in scheme change necessary

- predicted observables F are parametrized differently in I, II:
  $F = F_I(c_{i,I}) = F_{II}(c_{i,II}) = F_{II}(c_{i,I} + \Delta c_i)$

  But: $F_I^{\mathrm{(N)LO}}(c_{i,I}) \neq F_{II}^{\mathrm{(N)LO}}(c_{i,II})$ in finite orders
  $\hookrightarrow$ higher orders should reduce ren. scheme dependence

Multiplicative renormalization of the $(\mathrm{H},\mathrm{h}) \equiv (\mathrm{H}_1,\mathrm{H}_2)$ system:

Renormalization transformation ("B" → "bare" parameters)

$$\alpha_\mathrm{B} = \alpha + \delta\alpha,$$

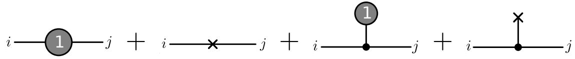
$$\begin{pmatrix} H_{1,\mathrm{B}} \\ H_{2,\mathrm{B}} \end{pmatrix} = (Z^H)^{1/2} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}, \quad (Z^H)^{1/2} = \begin{pmatrix} 1+\frac{1}{2}\delta Z^H_{11} & \frac{1}{2}\delta Z^H_{12} \\ \frac{1}{2}\delta Z^H_{21} & 1+\frac{1}{2}\delta Z^H_{22} \end{pmatrix}$$

Renormalization conditions (fix ren. constants $\delta\alpha, \delta Z_{ij}$)

- δZ^H_{ij} fixed by OS field renormalization (=external self-energy corrs.)

$$i \ne j: \quad \delta Z^H_{ij} = \frac{2\Sigma_{ij}(M^2_{\mathrm{H}_j})}{M^2_{\mathrm{H}_i} - M^2_{\mathrm{H}_j}} = \text{ singular for } M_{\mathrm{H}_1} \to M_{\mathrm{H}_2} \;!$$

Σ_{ij} =



($\times$ = tadpole counterterms)

- $\delta\alpha$ delivers 2 kinds of contributions:
  - from field rotation via $R(\alpha+\delta\alpha)$: $\quad -\delta\alpha + \frac{1}{2}\delta Z^H_{12}, \quad \delta\alpha + \frac{1}{2}\delta Z^H_{21}$
    $\hookrightarrow$ $\delta\alpha$ can cancel singularity in δZ^H_{ij}
  - from parameter relations: $\quad (M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})\delta\alpha = \text{non-singular}.$

Issues in previous proposals to fix $\delta\alpha$:

- Procedures based on $\Sigma_{ij}(p^2)$ at some momentum transfer p^2
  - $+$ process independence
  - $-$ gauge dependence
  - $-$ physical meaning and generalizability of ad hoc procedures unclear

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- $\overline{\mathrm{MS}}$ prescription: $(D = 4 - 2\epsilon)$

$$\delta\alpha_{\overline{\mathrm{MS}}} = \frac{\Sigma_{12}\big|_{\text{UD-div}}}{M_{\mathrm{H}_1}^2 - M_{\mathrm{H}_2}^2} \equiv \beta(\alpha)\left(\frac{1}{\epsilon} + \ln(4\pi) - \gamma_{\mathrm{E}}\right)$$

$\Rightarrow$ running mixing angle $\alpha(\mu_{\mathrm{r}})$, where $\dfrac{\partial\alpha(\mu_{\mathrm{r}})}{\partial \ln \mu_{\mathrm{r}}^2} = \beta(\alpha)$

  - \+ process independence, simplicity
  - \+ scale dependence as diagnostic tool to check perturbative stability
  - − singularities in degeneracy limits

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- Process-specific OS conditions: e.g. $\Gamma^{\mathrm{h}\to XY} \equiv \Gamma^{\mathrm{h}\to XY}_{\mathrm{LO}}$
  - $+$ gauge independence
  - $-$ process dependence
  - $-$ "contamination" of $\delta\alpha$ by all types of different corrections
    - $\hookrightarrow$ perturbative instabilities & "dead corners"

Desirable properties for the renormalized mixing angles:

Freitas, Stöckinger '02; Denner, S.D., Lang '18

- gauge independence
  $\hookrightarrow$ S-matrix = gauge-independent function of input parameters
- symmetry wrt. mixing degrees of freedom
- process independence
- perturbative stability
  $\hookrightarrow$ higher-order corrections should not get artificially large
- smoothness for degenerate masses or extreme mixing angles
  $\hookrightarrow$ no singularities like $1/(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})$ or $1/\sin\alpha$, $1/\cos\alpha$, etc.

Renormalization schemes for the SESM and THDM

(i) $\overline{\mathrm{MS}}$ schemes

$\overline{\text{MS}}$ schemes: simple application (just include UV divergences in δZ...)

But: issues with tadpole renormalization

- On-shell conditions: no physical effect of tadpoles (just bookkeeping)
- $\overline{\text{MS}}$ conditions: parameters related to masses depend on tadpole ren.!

Two commonly used tadpole treatments:

1. Vanishing renormalized tadpoles t_S: $t_{S,0} = t_S + \delta t_S = 0 + \delta t_S$ e.g. Denner '93

- (tadpole loops Γ^S) $+ \delta t_S = 0$ $\Rightarrow$ explicit tadpoles can be ignored
- (implicit) tadpole contributions δt_S in counterterms (gauge dependence!)
- drawback: $t_{S,0} = \delta t_S$ enters relations between bare input parameters
  $\hookrightarrow$ gauge-dependent terms $\propto \delta t_S$ enter relations
  between renormalized parameters and predicted observables

2. Vanishing bare tadpoles $t_{S,0}$: $t_{S,0} = 0$ Fleischer/Jegerlehner '80; Actis et al. '06

- explicit tadpole loops Γ^S have to included everywhere,
  technical variant: remove Γ^S from 2-pt. fcts. by shift $v_S \to v_S + \Delta v_s$
- advantage: no δt_S terms in relations between bare parameters
  $\hookrightarrow$ gauge-independent counterterms and relations between
  renormalized parameters and observables

Renormalization schemes for the SESM and THDM

(ii) OS schemes

OS renormalization schemes for the SESM and THDM Denner, S.D., Lang '18

Idea: Renormalization condition on ratio of S-matrix elements

$$\frac{\mathcal{M}^{H_1\to XY}}{\mathcal{M}^{H_2\to XY}} \stackrel{!}{=} \frac{\mathcal{M}_0^{H_1\to XY}}{\mathcal{M}_0^{H_2\to XY}} = \rho(\alpha) = \text{function of } \alpha \text{ only}$$

- $X, Y \stackrel{!}{=}$ neutral, otherwise problem with IR divergences
- vertex corrections δ^{H_iXY} to $\delta\alpha$ might be avoidable for clever choice of X, Y

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SESM: extension by fermionic singlet ψ with Yukawa coupling $y_\psi \to 0$

$$\mathcal{L}_\psi = \mathrm{i}\bar{\psi}\partial\!\!\!/\psi - y_\psi\bar{\psi}\psi\underbrace{(v_1 + H_1 c_\alpha - H_2 s_\alpha)}_{=\sigma} = \text{ gauge invariant}$$

- ratio $\rho(\alpha) = -c_\alpha/s_\alpha$ for $XY = \bar{\psi}\psi$
- vertex corrections $\delta^{H_i\bar{\psi}\psi} \to 0$ for $y_\psi \to 0$
- $\delta\alpha = \frac{1}{2}(\delta Z^H_{11} - \delta Z^H_{22})c_\alpha s_\alpha + \frac{1}{2}(\delta Z^H_{12}c_\alpha^2 - \delta Z^H_{21}s_\alpha^2)$

  $=$ gauge independent, symmetric in $\mathrm{H}_1/\mathrm{H}_2$, perturbatively stable

OS renormalization schemes for the SESM and THDM Denner, S.D., Lang '18

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THDM: refinements required (no gauge-inv. coupling to fermion singlet)
$\hookrightarrow$ use right-handed neutrinos, $\delta\beta$, vertex corrections contribute, ...

OS renormalization schemes for the THDM Denner, S.D., Lang '18

Problem:

no Higgs singlet $\rightarrow$ no gauge-invariant $h\bar{\psi}\psi$ operator with singlet ψ

Solution:

Add two right-handed singlet neutrinos $\nu_{1/2,\mathrm{R}}$ of THDM types 1/2

$$\mathcal{L}_{\nu_\mathrm{R}} = \mathrm{i}\bar{\nu}_{1\mathrm{R}}\partial\!\!\!/\nu_{1\mathrm{R}} + \mathrm{i}\bar{\nu}_{2\mathrm{R}}\partial\!\!\!/\nu_{2\mathrm{R}} - \left[y_{\nu_1}\bar{L}_{1\mathrm{L}}\tilde{\Phi}_1\nu_{1\mathrm{R}} + y_{\nu_2}\bar{L}_{2\mathrm{L}}\tilde{\Phi}_2\nu_{2\mathrm{R}} + \text{h.c.}\right]$$

Renormalization conditions for α, β from appropriate ratios

$$\frac{\mathcal{M}^{H_1\to\nu_1\bar{\nu}_1}}{\mathcal{M}^{H_2\to\nu_1\bar{\nu}_1}} \stackrel{!}{=} \frac{\mathcal{M}_0^{H_1\to\nu_1\bar{\nu}_1}}{\mathcal{M}_0^{H_2\to\nu_1\bar{\nu}_1}} = -\frac{c_\alpha}{s_\alpha}, \qquad \frac{\mathcal{M}^{A_0\to\nu_1\bar{\nu}_1}}{\mathcal{M}^{H_1\to\nu_1\bar{\nu}_1}} \stackrel{!}{=} \frac{\mathcal{M}_0^{A_0\to\nu_1\bar{\nu}_1}}{\mathcal{M}_0^{H_1\to\nu_1\bar{\nu}_1}} \propto \frac{s_\beta}{c_\alpha}$$

deliver gauge-independent renormalization constants: ("OS1" scheme)

$$\delta\alpha = (\delta_{H_1\nu_1\bar{\nu}_1} - \delta_{H_2\nu_1\bar{\nu}_1})c_\alpha s_\alpha + \tfrac{1}{2}(\delta Z^H_{11} - \delta Z^H_{22})c_\alpha s_\alpha + \tfrac{1}{2}(\delta Z^H_{12}c_\alpha^2 - \delta Z^H_{21}s_\alpha^2),$$

$$\delta\beta = \frac{s_\beta}{c_\beta}\left[\delta_{H_1\nu_1\bar{\nu}_1} - \delta_{A_0\nu_1\bar{\nu}_1} - \tfrac{1}{2}(\delta Z_{A_0A_0} - \delta Z^H_{11}) - \tfrac{1}{2}\frac{s_\alpha}{c_\alpha}\delta Z^H_{21}\right] + \tfrac{1}{2}\delta Z_{G_0A_0} - \frac{s_\beta}{c_\beta}\frac{s_\alpha}{c_\alpha}\delta\alpha$$

Comments:

- vertex corrections $\delta_{S\nu_i\bar{\nu}_i}$ unavoidable in spite of $y_{\nu_i} \to 0$
- but: singular factors $1/c_\alpha, 1/s_\beta$ in $\delta\beta$ can be avoided

"OS12" renormalization schemes for the THDM Denner, S.D., Lang '18

Modification:

replace matrix elements $\mathcal{M}^{S\to\nu_j\bar{\nu}_j}$ by appropriate formfactors $F^{S\to\nu_j\bar{\nu}_j}$

$$\mathcal{M}^{H_i\to\nu_j\bar{\nu}_j} = [\bar{u}_\nu v_\nu]_{H_i} F^{H_i\to\nu_j\bar{\nu}_j}, \quad \mathcal{M}^{A_0\to\nu_j\bar{\nu}_j} = [\bar{u}_\nu \mathrm{i}\gamma_5 v_\nu]_{A_0} F^{A_0\to\nu_j\bar{\nu}_j}$$

Renormalization condition for β alone:

$$0 \stackrel{!}{=} \frac{F^{A_0\to\nu_1\bar{\nu}_1} c_\beta}{c_\alpha F^{H_1\to\nu_1\bar{\nu}_1} - s_\alpha F^{H_2\to\nu_1\bar{\nu}_1}} + \frac{F^{A_0\to\nu_2\bar{\nu}_2} s_\beta}{s_\alpha F^{H_1\to\nu_2\bar{\nu}_2} + c_\alpha F^{H_2\to\nu_2\bar{\nu}_2}}$$

Renormalization constant $\delta\beta$:

$$\delta\beta = \tfrac{1}{2} c_\beta s_\beta \left[(c_\alpha^2 - s_\alpha^2)(\delta Z_{11}^H - \delta Z_{22}^H) - 2c_\alpha s_\alpha (\delta Z_{12}^H + \delta Z_{21}^H) \right] + \tfrac{1}{2}\delta Z_{G_0 A_0}$$
$$+ c_\beta s_\beta \left(\delta_{A_0\nu_2\bar{\nu}_2} + c_\alpha^2 \delta_{H_1\nu_1\bar{\nu}_1} + s_\alpha^2 \delta_{H_2\nu_1\bar{\nu}_1} - \delta_{A_0\nu_1\bar{\nu}_1} - s_\alpha^2 \delta_{H_1\nu_2\bar{\nu}_2} - c_\alpha^2 \delta_{H_2\nu_2\bar{\nu}_2} \right)$$

Comments:

- $\delta\alpha$ fixed similar as above
- both $\delta\alpha, \delta\beta$ gauge invariant and perturbatively stable without "dead corners"

Renormalization schemes
for the SESM and THDM

(iii) symmetry-inspired schemes

Mixing of physical states and "rigid invariance"

Idea: UV divergences can be removed via renormalization in unbroken phase of theory ‘t Hooft ’71; Lee, Zinn-Justin ’72–’74

$\hookrightarrow$ field renormalization matrix $(Z^H)^{1/2}$ can be taken diagonal in "η basis":

$$(Z^H)^{1/2}\big|_{\rm UV} = R^{\rm T}(\alpha+\delta\alpha)\begin{pmatrix} 1+\frac{1}{2}\delta Z_1^\eta & 0 \\ 0 & 1+\frac{1}{2}\delta Z_2^\eta \end{pmatrix} R(\alpha)\Big|_{\rm UV}$$

$\Rightarrow$ Relations among UV divergences in δZ^H_{ij} and $\delta\alpha$:

$$\begin{aligned}
\delta Z^H_{11}\big|_{\rm UV} &= c_\alpha^2\delta Z_1^\eta\big|_{\rm UV} + s_\alpha^2\delta Z_2^\eta\big|_{\rm UV},\\
\delta Z^H_{22}\big|_{\rm UV} &= s_\alpha^2\delta Z_1^\eta\big|_{\rm UV} + c_\alpha^2\delta Z_2^\eta\big|_{\rm UV},\\
\delta Z^H_{12}\big|_{\rm UV} + \delta Z^H_{21}\big|_{\rm UV} &= 2c_\alpha s_\alpha(\delta Z_2^\eta - \delta Z_1^\eta)\big|_{\rm UV},\\
\delta Z^H_{12}\big|_{\rm UV} - \delta Z^H_{21}\big|_{\rm UV} &= 4\delta\alpha\big|_{\rm UV}
\end{aligned}$$

Kanemura et al. ’03
Krause et al. ’16
Denner et al. ’18

$\Rightarrow$ $\delta\alpha$ can be defined via symmetry relation

$$\delta\alpha = \frac{1}{4}\left(\delta Z^H_{12} - \delta Z^H_{21}\right) = \frac{\Sigma^H_{12}(M^2_{\rm H_2}) + \Sigma^H_{12}(M^2_{\rm H_1})}{2(M^2_{\rm H_1} - M^2_{\rm H_2})}$$

Note: $\frac{1}{2}\delta Z^H_{12} - \delta\alpha = \delta\alpha + \frac{1}{2}\delta Z^H_{21} = \frac{1}{4}\left(\delta Z^H_{12} + \delta Z^H_{21}\right) =$ regular for $M_{\rm H_1} \to M_{\rm H_2}$

Mixing of physical and unphysical states and background-field invariance

Problem: Gauge-fixing terms break rigid invariance.
$\hookrightarrow$ modification of method necessary for mixing with Goldstone fields

Solution: quantization via Background-Field Method (BFM) Abbott '81, ...

BFM – basic features and EW higher orders: Denner, S.D., Weiglein '94

- fields split into "quantum" and "background" parts: $\phi \to \phi + \hat{\phi}$

  ϕ: gauge fixed, appear in loops in diagrams

  $\hat{\phi}$: sources of gauge-invariant effective action, on trees in diagrams
- vertex functions obey "classical" (ghost-free) Ward identities
  $\hookrightarrow$ many desirable properties of vertex functions
- Ward identities can keep their forms after renormalization
  $\hookrightarrow$ simple relations between renormalization constants,
  e.g. electric charge ren. constant $Z_e = Z_{\hat{\gamma}\hat{\gamma}}^{-1/2}$ as in QED

  $\Rightarrow$ use analogous relations to fix $\delta\alpha$, $\delta\beta$, ... ($\delta\alpha$ as from rigid invariance)

Application to the SESM and THDM Denner, S.D., Lang '18

Relations involving $\delta\beta$ in the THDM:

$$\delta Z_1^{\hat\eta} = -2\delta Z_e - \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\frac{\delta c_\mathrm{W}^2}{c_\mathrm{W}^2} + \frac{\delta M_\mathrm{W}^2}{M_\mathrm{W}^2} + 2\frac{\delta c_\beta}{c_\beta} + \text{tadpoles},$$

$$\delta Z_2^{\hat\eta} = -2\delta Z_e - \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\frac{\delta c_\mathrm{W}^2}{c_\mathrm{W}^2} + \frac{\delta M_\mathrm{W}^2}{M_\mathrm{W}^2} + 2\frac{\delta s_\beta}{s_\beta} + \text{tadpoles}$$

$\Rightarrow$ with above relations for $\delta Z_i^{\hat\eta}$:

$$\delta\beta = \tfrac{1}{2}c_\beta s_\beta\left[(s_\alpha^2 - c_\alpha^2)(\delta Z_{11}^{\hat H} - \delta Z_{22}^{\hat H}) + 2c_\alpha s_\alpha(\delta Z_{12}^{\hat H} + \delta Z_{21}^{\hat H})\right] + \text{tadpoles}$$

(similar results obtained by Krause et al. '16)

Comments on BFM schemes (BFMS):

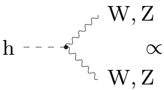
- $\delta\alpha$, $\delta\beta$ depend on choice of symmetry relations and on gauge
  But: S-matrix depends on α, β in a gauge-independent way
- process independence
- absence of singularities for mass degeneracy or $s_\alpha, s_\beta, \dots \to 0$

NLO corrections
to h/H$\to$WW/ZZ$\to$4fermions

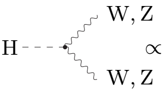
Light versus heavy Higgs decays $\mathrm{h/H} \to \mathrm{WW/ZZ} \to 4f$

$\hookrightarrow$ SESM and THDM new in Monte Carlo program PROPHECY4F

- different resonance patterns:
  - $M_\mathrm{h} = 125\,\mathrm{GeV}$: at least one $\mathrm{W/Z}$ off-shell
    $\mathrm{h} \to \mathrm{WW^*/ZZ^*} \to 4f, \quad \Gamma^{\mathrm{h}\to 4f} \sim 1\,\mathrm{MeV}$
  - $M_\mathrm{H} > 2M_\mathrm{Z}$: on-shell decays $\mathrm{H} \to \mathrm{WW/ZZ}$ possible
    $\mathrm{H} \to \mathrm{WW/ZZ} \to 4f, \quad \Gamma^{\mathrm{H}\to 4f} \sim 100\,\mathrm{MeV}$
- LO prediction suppressed by small mixing factors:



$\propto \cos\gamma \gtrsim 0.9(0.95)$



$\propto \sin\gamma \lesssim 0.4(0.3)$

LHC result: $\quad \mu = \left.\frac{\Gamma_\mathrm{exp}}{\Gamma_\mathrm{SM}}\right|_{\mathrm{Higgs}\to\mathrm{WW/ZZ}} = 1 \pm 20\%(10\%) \sim \cos^2\gamma$

$\Rightarrow$ Potentially large corrections to $\mathrm{H} \to \mathrm{WW/ZZ}$

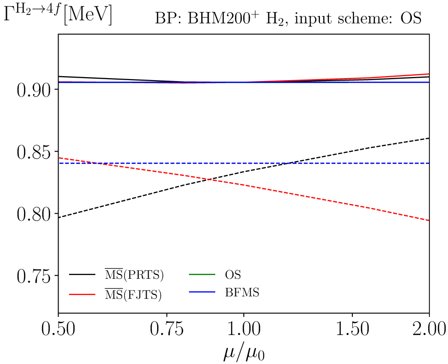
Perturbatively stable renormalization schemes particularly important!

Numerical results for the SESM

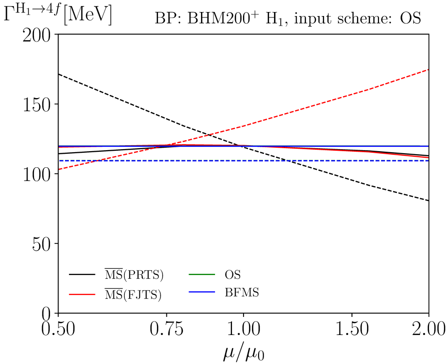
Total decay widths for $\mathrm{h}, \mathrm{H} \to 4f$ at NLO

Altenkamp et al. '17, Denner et al. '18

$\mathrm{h} \to 4f$:



$\mathrm{H} \to 4f$:



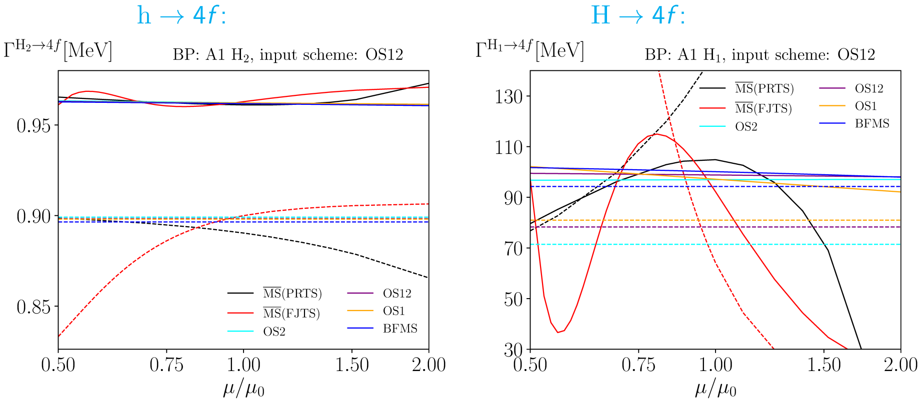
Transition from LO to NLO:

- $\overline{\mathrm{MS}}$ schemes: drastic reduction of ren. scale dependence
- comparison of schemes: drastic reduction of ren. scheme dependence, i.e. good agreement of all scheme after conversion of input
- overall uncertainty of NLO prediction $\lesssim 0.5\%$ as in SM

Numerical results for the THDM

Total decay widths for $\mathrm{h/H} \to 4f$ at NLO

Altenkamp et al. '17, Denner et al. '18



Transition from LO to NLO:

- $\overline{\mathrm{MS}}$ schemes: useful results for $\mathrm{h} \to 4f$ in "moderate scenarios"
  But: perturbative instability in extreme scenarios and for $\mathrm{H} \to 4f$
- OS & BFM schemes: perfect agreement after conversion of input
- NLO uncertainty estimate should include ren. scheme dependence (including well-behaved schemes)

Conclusions

Extended Higgs sectors

- simple models reflect features of more comprehensive BSM theories
- renormalizablity allows for precise predictions
  $\hookrightarrow$ SESM and THDM studied as examples

Renormalization of extended Higgs sectors

- masses $\to$ e.g. on-shell renormalization
- self-couplings $\to$ e.g. $\overline{\mathrm{MS}}$ prescription
- issues with mixing angles
  - gauge (in)dependence
  - symmetry wrt. mixing degrees of freedom
  - process (in)dependence
  - perturbative stability
  - smoothness for degenerate masses or extreme mixing angles
- new renormalization schemes suggested for SESM & THDM based on
  - on-shell conditions
  - symmetry arguments ("rigid" or background-field gauge invariance)

$\Rightarrow$ many desirable properties & good performance in practice

New schemes implemented in PROPHECY4F for $\mathrm{h}/\mathrm{H} \to 4f$ and 2HDECAY (Krause et al. '18) for $1 \to 2$-particle Higgs decays

Phenomenological analysis of $\mathrm{h/H} \to \mathrm{WW/ZZ} \to 4f$ at NLO

- detailed comparison of renormalization schemes in SESM & THDM
  - consistent scheme conversion essential
  - renormalization scheme dependence discussed at (N)LO
- $\mathrm{h} \to 4f$: Prophecy4f — SM-like behaviour, small BSM effects, no distortions of distributions by BSM effects
- $\mathrm{H} \to 4f$: Prophecy4f — large impact of corrections due to suppressed H couplings $\hookrightarrow$ well-behaved renormalization schemes essential
- SESM:
  - generally very robust NLO results
- THDM:
  - no sensitivity of $\mathrm{h} \to 4f$ to the type of THDM
  - $\overline{\mathrm{MS}}$ mixing angles problematic in delicate scenarios (large heavy Higgs masses, degeneracy limit $M_\mathrm{H} \to M_\mathrm{h}$, etc.)
  - prefect performance of new OS & symmetry-inspired schemes

More results & outlook

- similar results for Higgs production via VBF & VH in Hawk
- renormalization schemes generalizable to other SM extensions

Backup slides

THDM Yukawa couplings:

Avoid FCNC at tree level!

$\hookrightarrow$ Couple each fermion flavour only to one Φ_n ($\mathbb{Z}_2$ symmetry)

$$\mathcal{L}_{\mathrm{Yukawa}} = -\bar{L}'^{\mathrm{L}} Y^l l'^{\mathrm{R}} \Phi_{n_1} - \bar{Q}'^{\mathrm{L}} Y^u u'^{\mathrm{R}} \tilde{\Phi}_{n_2} - \bar{Q}'^{\mathrm{L}} Y^d d'^{\mathrm{R}} \Phi_{n_3} + h.c.$$

THDM type	u_i	d_i	e_i	$\mathbb{Z}_2$ symmetry
Type I	Φ_2	Φ_2	Φ_2	$\Phi_1 \to -\Phi_1$
Type II	Φ_2	Φ_1	Φ_1	$(\Phi_1, d_i, e_i) \to -(\Phi_1, d_i, e_i)$
Lepton-specific	Φ_2	Φ_2	Φ_1	$(\Phi_1, e_i) \to -(\Phi_1, e_i)$
Flipped	Φ_2	Φ_1	Φ_2	$(\Phi_1, d_i) \to -(\Phi_1, d_i)$

Yukawa couplings modified by THDM factors $\xi^f_{\mathrm{H,h,A_0}}$:

	Type I	Type II	Lepton-specific	Flipped
ξ^l_{H}	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos\alpha/\cos\beta$	$\sin\alpha/\sin\beta$
ξ^u_{H}	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$
ξ^d_{H}	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$
ξ^l_{h}	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$
ξ^u_{h}	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$
ξ^d_{h}	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$
$\xi^l_{\mathrm{A_0}}$	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	$\cot\beta$
$\xi^u_{\mathrm{A_0}}$	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
$\xi^d_{\mathrm{A_0}}$	$\cot\beta$	$-\tan\beta$	$\cot\beta$	$-\tan\beta$

Different $\overline{\mathrm{MS}}$ schemes for the THDM: Altenkamp, S.D., Rzehak '17

- $\overline{\mathrm{MS}}(\alpha)$: see also by Krause et al. '16; Denner et al. '16
  - input: β, λ_5, α
  - tadpole treatment a): $t_S = 0$
  - gauge dependent: results tied to 't Hooft–Feynman gauge
- $\mathrm{FJ}(\alpha)$: see also by Krause et al. '16; Denner et al. '16
  - input: β, λ_5, α
  - FJ tadpole treatment b): $t_{S,0} = 0$
  - gauge independent

Different $\overline{\text{MS}}$ schemes for the THDM: Altenkamp, S.D., Rzehak '17

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- $\text{FJ}(\alpha)$: see also by Krause et al. '16; Denner et al. '16
  - input: β, λ_5, α
  - FJ tadpole treatment b): $t_{S,0} = 0$
  - gauge independent
- $\overline{\text{MS}}(\lambda_3)$:
  - as $\overline{\text{MS}}(\alpha)$, but α replaced by coupling λ_3 as input
  - gauge independent only in R_ξ gauges at NLO
- $\text{FJ}(\lambda_3)$:
  - as FJ(α), but α replaced by coupling λ_3 as input
  - gauge independent

$\hookrightarrow$ Study renormalization scheme and scale dependence in predictions

Running of $\overline{\mathrm{MS}}$ parameters: (numerical solution of ren. group eqs.)

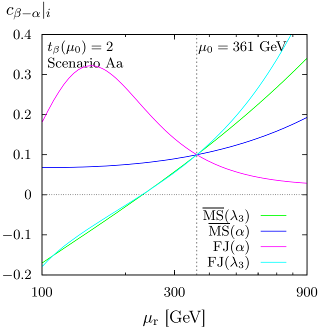
Example: $\cos(\beta-\alpha) \equiv c_{\beta-\alpha}$ in a THDM low-mass scenario of Type I

Scenario A: $M_\mathrm{h} = 125\,\mathrm{GeV}$, $c_{\beta-\alpha} = +0.1$ (Aa=A1),
$c_{\beta-\alpha} = -0.1$ (Ab),
$c_{\beta-\alpha} = +0.2$ (A2),

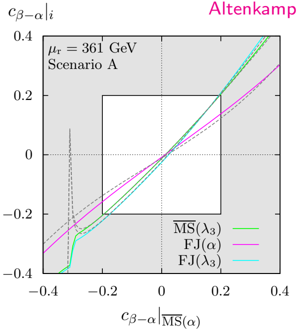
$M_\mathrm{H} = 300\,\mathrm{GeV}$, $M_{\mathrm{A}_0} = M_{\mathrm{H}^+} = 460\,\mathrm{GeV}$, $\lambda_5 = -1.9$, $\tan\beta = 2$

default scale: $\mu_0 = \frac{1}{5}(M_\mathrm{h} + M_\mathrm{H} + M_{\mathrm{A}_0} + 2M_{\mathrm{H}^+}) = 361\,\mathrm{GeV}$

Altenkamp et al. '17



Strong dependence of running on renormalization scheme

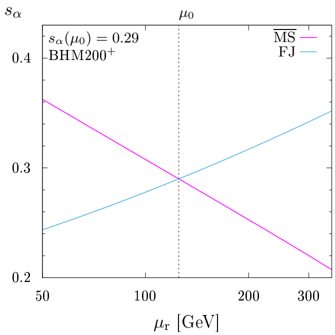


Sizeable conversion effects!

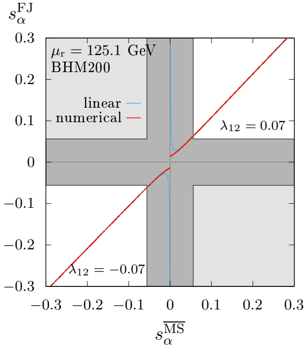
$\overline{\rm MS}$ schemes for the SESM: Altenkamp, Boggia, S.D. '18

Example: $\sin\alpha \equiv s_\alpha$ in a SESM low-mass scenario

Scenario BHM200$^\pm$:
$M_{\rm h} = 125.1\,\rm GeV$, $M_{\rm H} = 200\,\rm GeV$,
$s_\alpha = \pm 0.29$, $\lambda_{12} = \pm 0.07$, default scale: $\mu_0 = M_{\rm h}$



strong dependence of running on ren. scheme

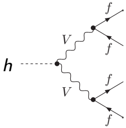


moderate conversion effects

(shaded areas non-perturbative or theoretically impossible)

Survey of Feynman diagrams for NLO corrections to $\mathrm{h}\to\mathrm{WW}/\mathrm{ZZ}\to 4f$

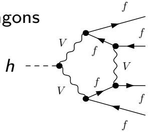
Lowest order:



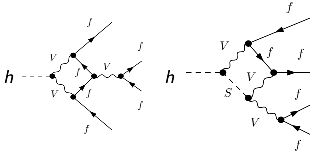
$$= \mathcal{M}_{\mathrm{SM,LO}} \times \begin{cases} \cos\alpha, & \text{SESM} \\ \sin(\beta-\alpha), & \text{THDM} \end{cases}$$

Typical one-loop diagrams: # diagrams = $\mathcal{O}(200-400)$

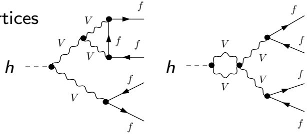
pentagons



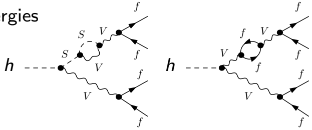
boxes



vertices



self-energies



+ counterterms

+ tree graphs with real gluon or photons

(analogously for $\mathrm{H}\to\mathrm{WW}/\mathrm{ZZ}\to 4f$)

NLO corrections to $h \to 4f$ in the SESM & THDM implemented in ...

Prophecy4f is hosted by Hepforge, IPPP Durham

Prophecy4f

A Monte Carlo generator for a Proper description of the Higgs decay into 4 fermions

- Home
- Downloads
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- Release History
- Contact

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Prophecy4f is a Monte Carlo integrator for Higgs decays $H \to WW/ZZ \to 4$ fermions

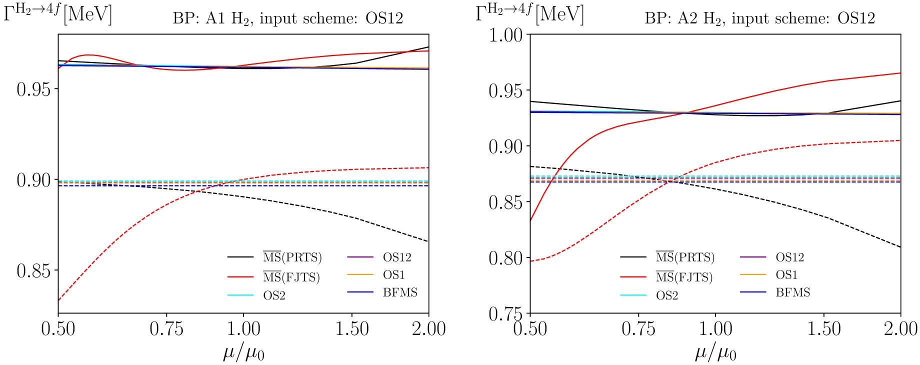
It includes:

- all four-fermion final states
- NLO QCD and electroweak corrections
- all interferences at LO and NLO
- effects beyond NLO from heavy-Higgs effects
- alternatively an Improved Born Approximation (IBA) with leading effects of the corrections
- production of unweighted events for leptonic final states
- optional inclusion of a 4th fermion generation (w/ or w/o leading two-loop improvements)

New PROPHECY4F version available on request (on hepforge soon)

Total decay widths for $\mathrm{h} \to 4f$ at NLO

Altenkamp et al. '17, Denner et al. '18

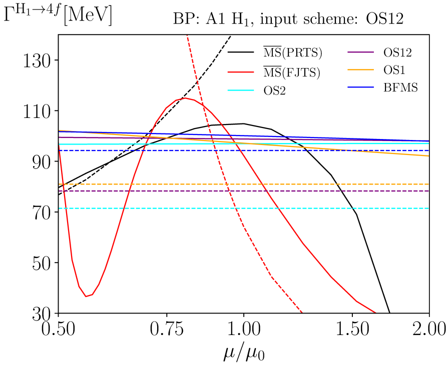


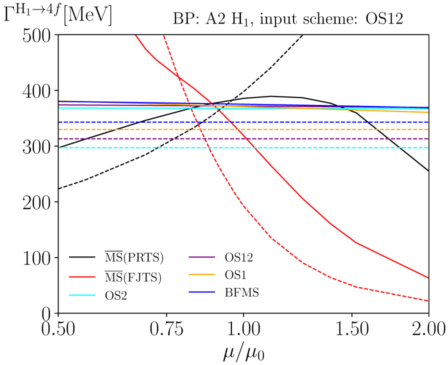
Transition from LO to NLO:

- $\overline{\mathrm{MS}}$ schemes: sizeable ren. scale dependence remains at NLO
  (Note: behaviour of schemes deteriorates in more extreme scenarios)
- OS & BFM schemes: perfect agreement after conversion of input
- NLO uncertainty estimate should include ren. scheme dependence
  (including well-behaved schemes)

Total decay widths for $\mathrm{H} \to 4f$ at NLO

Denner et al. '18



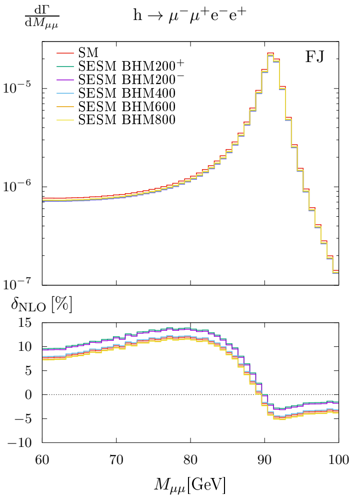


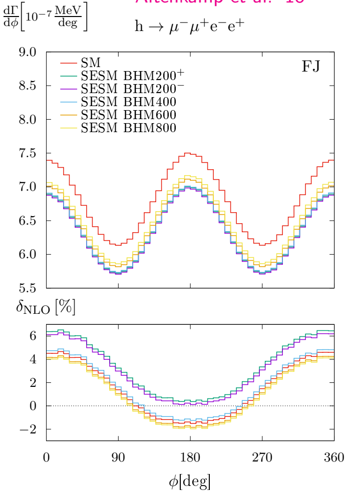
Transition from LO to NLO:

- $\overline{\mathrm{MS}}$ schemes: no reduction of ren. scale dependence at NLO
  $\hookrightarrow$ schemes useless in such cases
- OS & BFM schemes: very good agreement after conversion of input
- NLO uncertainty estimate should include ren. scheme dependence (including well-behaved schemes)

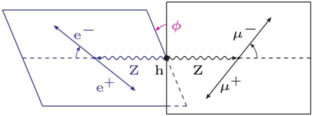
NLO corrections to leptonic distributions [$\overline{\mathrm{MS}}$(FJTS) scheme]

Altenkamp et al. '18



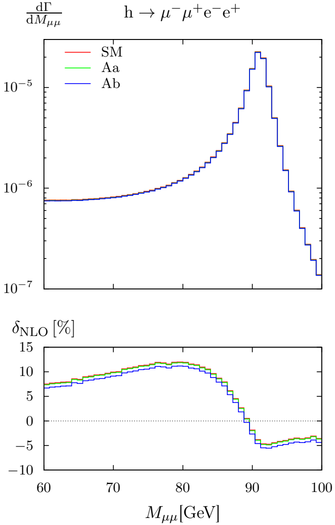


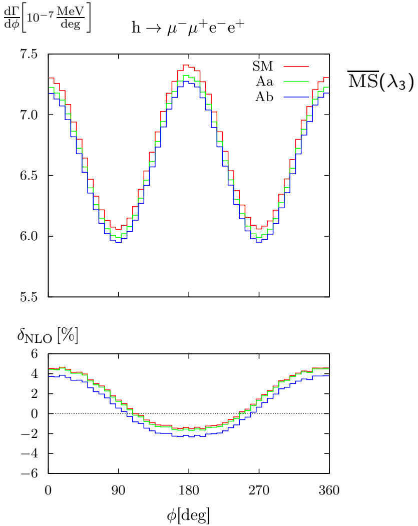
correction $\delta_{\mathrm{SESM}} \approx \delta_{\mathrm{SM}} + \mathrm{const.}$
mainly due to external hH mixing



NLO corrections to leptonic distributions in scenario A

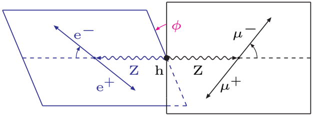
Altenkamp et al. '17





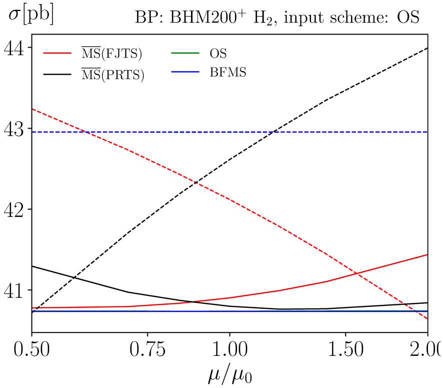
correction $\delta_{THDM} \approx \delta_{SM} + \text{const.}$

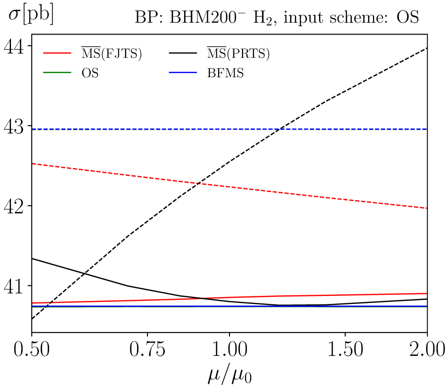
mainly due to external hH mixing



$\mathrm{pp} \to \mathrm{h}\mu^+\nu_\mu + X$ at NLO in the SESM

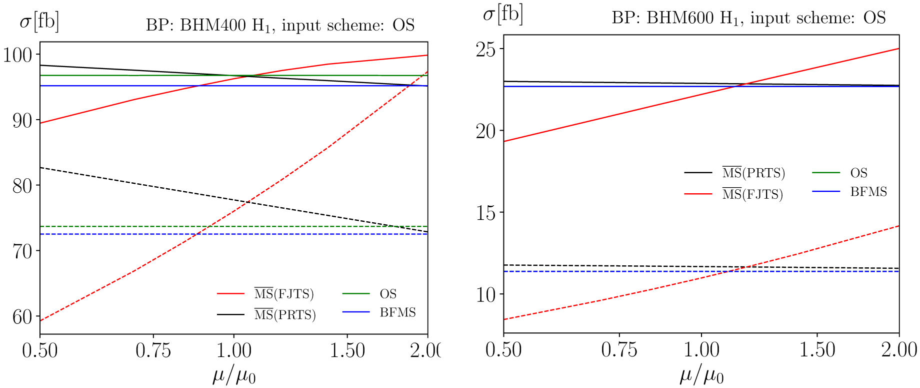
Denner et al. '18





$\mathrm{pp} \to \mathrm{H}\mu^+\nu_\mu + X$ at NLO in the SESM

Denner et al. '18

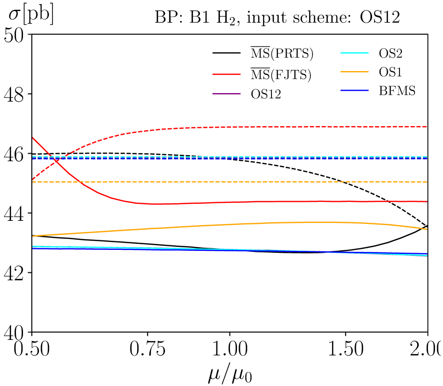


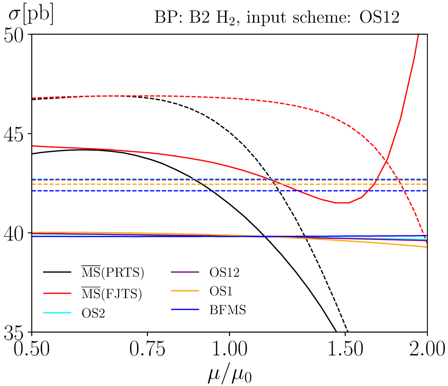
$M_\mathrm{H} = 400\,\mathrm{GeV}, s_\alpha = 0.26, \lambda_{12} = 0.17$

$M_\mathrm{H} = 600\,\mathrm{GeV}, s_\alpha = 0.22, \lambda_{12} = 0.23$

$\mathrm{pp} \to \mathrm{h}\mu^+\nu_\mu + X$ at NLO in the THDM

Denner et al. '18



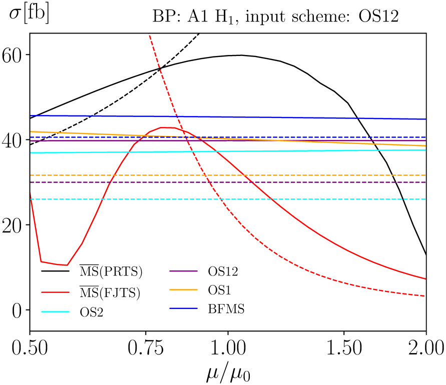


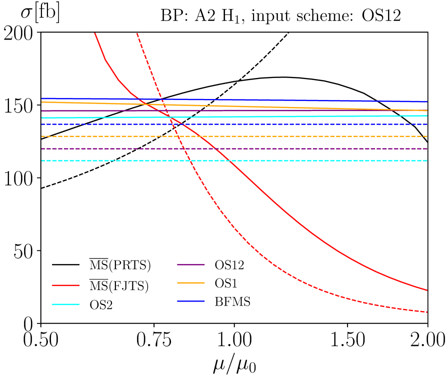
Scenarios B1,B2: $M_\mathrm{h} = 125\,\mathrm{GeV}, \quad \mu_0 = \frac{1}{5}(M_\mathrm{h} + M_\mathrm{H} + M_{\mathrm{A}_0} + 2M_{\mathrm{H}^+})$

B1:	$c_{\beta-\alpha} = 0.15,$	$M_\mathrm{H} = 600\,\mathrm{GeV},$	$M_{\mathrm{A}_0} = M_{\mathrm{H}^+} = 690\,\mathrm{GeV},$	$\lambda_5 = -1.9,$	$\tan\beta = 4.5$
B2:	$0.30,$	$200\,\mathrm{GeV},$	$420\,\mathrm{GeV},$	$-2.5746,$	3

$pp \to H\mu^+\nu_\mu + X$ at NLO in the THDM

Denner et al. '18





VBF h production at NLO in the THDM

Denner et al. '18