One-loop lepton masses, extended Higgs sectors and renormalization

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Motivation

Model setup

One-loop calculation
- Scalar sector
- Leptonic sector
- Gauge dependencies
- Tadpole contributions
- Heavy Majoranas

Conclusions

Motivation

Open questions concerning:

1. Smallness of ν-masses

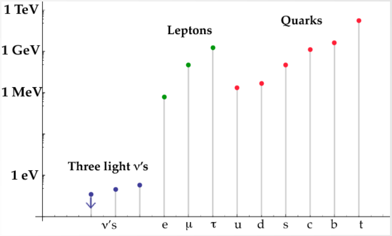
$$\sum_j m_{\nu_j} \lesssim (0.1 - 1.1)\,\mathrm{eV} \text{ (@ 90\% C.L.)}$$

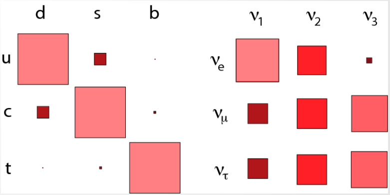
2. Mild hierarchy in ν-mass spectrum:

$$\frac{\Delta m_{21}^2}{|\Delta m_{3l}^2|} \sim 10^{-2}, \quad \frac{\Delta m_{cu}^2}{\Delta m_{tu}^2} \sim 10^{-5}$$

3. Peculiar features of lepton mixing matrix

4. Majorana nature of neutrinos





Source: [Stone, 2013]

⇒ **An abundance of models tries to explain these features**

Motivation: renormalizable mass models

- Instructive example: μ-τ-symmetry provokes **maximal atmospheric mixing** [Grimus, Lavoura, 2003]
- Implemented in **renormalizable 3HDM** with Majorana ν's

$$S^T M_{\nu,\text{light}}\, S = \left(M_{\nu,\text{light}}\right)^*$$

$$S = \begin{array}{c} \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

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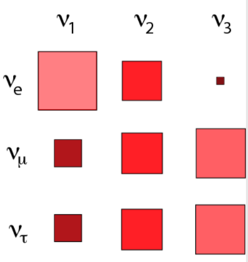

Source: [Stone, 2013]

$$U^T\, M_{\nu,\text{light}}\, U = \text{diag}(m_{\nu,1}, m_{\nu,2}, m_{\nu,3})$$

$$\Rightarrow |U_{\mu i}| = |U_{\tau i}| \quad \forall i \quad \text{or}\ \theta_{23} = 45^\circ,\ \delta = \pm\frac{\pi}{2}.$$

- **Can also have pre-/postdictions of the kind:** $m_\mu / m_\tau \ll 1$

Motivation

How do such predictions behave under radiative corrections?

- *Renormalization of the multi-Higgs-doublet Standard Model and one-loop lepton mass corrections* [Grimus, ML, 2018]

Model setup

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Study a general model, later apply specific parameter choices/symmetries/seesaw mechanism

$$\mathcal{L} = \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{Kin}} + \mathcal{L}_{\mathrm{S}} + \mathcal{L}_{\mathrm{Yuk}} + \mathcal{L}_{\mathrm{Maj}} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}.$$

- $SU(2)_L \times U(1)_Y$ with n_H scalar doublets

$$\mathcal{L}_{\mathrm{S}} = \left(D_\mu \Phi_k\right)^\dagger D^\mu \Phi_k - \mu_{ij}^2 \Phi_i^\dagger \Phi_j - \lambda_{ijkl} \left(\Phi_i^\dagger \Phi_j\right) \left(\Phi_k^\dagger \Phi_l\right)$$

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- Mass generation via Yukawa interaction and Majorana mass term

$$\mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{Maj}} = -\bar{e}_R \Phi_k^\dagger \Gamma_k \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} - \bar{\nu}_R \tilde{\Phi}_k^\dagger \Delta_k \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} - \frac{1}{2} \bar{\nu}_R M_R \nu_R^c + \text{H.c.}$$

$$\Gamma_k \in \mathbb{C}^{(n_L \times n_L)}, \quad \Delta_k \in \mathbb{C}^{(n_R \times n_L)}, \quad k = 1, \ldots, n_H$$

- Gauge fixing with R_ξ-gauge (avoids scalar-vector boson mixing @ tree-level)

Model setup

Charged lepton masses

- Masses are generated via spontaneous symmetry breaking:

$$\Phi_k = \begin{pmatrix} \varphi_k^+ \\ \varphi_k^0 \end{pmatrix} = \begin{pmatrix} \varphi_k^+ \\ \frac{1}{\sqrt{2}} \left(v_k + \varphi_k^{0'} \right) \end{pmatrix}$$

- v_k: solutions to the n_H equations $(\mu_{ij}^2 + \lambda_{ijkl} v_k^* v_l) v_j = 0$ with $v = \sqrt{v_k v_k^*} = 246\,\mathrm{GeV}$
- Charged lepton masses:

$$\mathcal{L}_{\mathrm{mass},\ell} = -\bar{e}_R \underbrace{\frac{v_k^*}{\sqrt{2}} \Gamma_k}_{\equiv M_l} e_L + \mathrm{H.c.}$$

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- M_ℓ: general complex 3×3 matrix. Bi-diagonalisation with $e_R = W_R \gamma_R \ell$ and $e_L = W_L \gamma_L \ell$:

$$\hat{m}_\ell = W_R^\dagger M_\ell W_L = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

Model setup
Neutrino masses

- Neutrino masses: can combine Dirac and Majorana terms

$$\mathcal{L}_{\text{mass},\nu} = -\overline{\nu}_R \underbrace{\frac{v_k}{\sqrt{2}}\Delta_k}_{M_D} \nu_L - \frac{1}{2}\overline{\nu}_R M_R \nu_R^c + \text{H.c.}$$

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$$= \frac{1}{2}\begin{pmatrix}\nu_L^T & (\nu_R^c)^T\end{pmatrix} C^{-1} \underbrace{\begin{pmatrix} 0 & {M_D}^T \\ M_D & M_R \end{pmatrix}}_{M_{D+R}} \begin{pmatrix}\nu_L \\ \nu_R^c\end{pmatrix} + \text{H.c.}$$

$$(\nu_R^c)^T C^{-1} = (C\gamma_0^T \nu_R^*)^T C^{-1} = -\overline{\nu}_R$$

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- M_{D+R} is symmetric $\Rightarrow \mathcal{U}^T M_{D+R}\,\mathcal{U} = \text{diag}(m_{\nu,1}, \ldots, m_{\nu,n_L+n_R})$ with $\mathcal{U} = \begin{pmatrix}U_L \\ U_R^*\end{pmatrix}$ and $\nu_R = U_R \gamma_R\, \chi$, $\nu_L = U_L \gamma_L \chi$
- Can assume typical mass scales $m_D \ll m_R$ for invoking **seesaw mechanism** upon diagonalisation:

$$M_{\nu,\text{light}} = -M_D^T\, M_R^{-1}\, M_D + \mathcal{O}(m_D^2/m_R^2)$$

$$(\nu_R^c)^T C^{-1} = (C\gamma_0^T \nu_R^*)^T C^{-1} = -\overline{\nu}_R$$

One-loop corrections and renormalization

One-loop calculation

Goal: finite one-loop lepton mass corrections

- Potentially far more parameters than process-independent physical observables
  $\Rightarrow \overline{\text{MS}}$-scheme somewhat unavoidable

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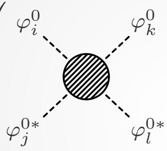
- Potentially far more parameters than process-independent physical observables
  $\Rightarrow$ $\overline{\text{MS}}$-scheme somewhat unavoidable
- **$\overline{\text{MS}}$-Renormalization of scalar sector:** $\{\delta\mu_{ij}^2, \delta\tilde{\lambda}_{ijkl}, \delta v_k\}$
- **$\overline{\text{MS}}$-Renormalization of leptonic sector:** $\{\delta\Delta_k, \delta\Gamma_k, \delta M_R\}$

Renormalization of scalar sector
Quartic coupling

- Determine $\delta\tilde{\lambda}_{ijkl}$ from $\langle\Omega|\, T\varphi_i^0\varphi_j^{0*}\varphi_k^0\varphi_l^{0*}\,|\Omega\rangle$ in **unbroken phase** to simply save some computational effort
- Using dimensional regularisation in $d = 4 - 2\varepsilon$, this is:

$$2i\,\delta\tilde{\lambda}_{ijkl} \equiv \left(\text{[diagram]} \right)_{c_\infty} , \quad c_\infty = \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi)$$



Sufficient for our purposes: $\delta\tilde{\lambda}_{ijkl} = \delta\lambda_{ijkl} + \delta\lambda_{ilkj}$

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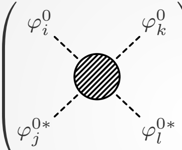


$\Big)_{c_\infty}, \quad c_\infty = \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi)$

- Serves as input for mass counterterm in **broken phase**:

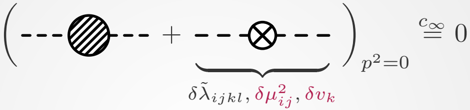
S_b^0

$$S_{b'}^0 = -\frac{i}{2}\left[\delta\mu_{ij}^2 + \delta\tilde{\lambda}_{ijkl}v_k^* v_l\right]\left(V_{ib}^* V_{jb'} + V_{ib'}^* V_{jb}\right) - \frac{i}{4}\delta\tilde{\lambda}_{ijkl}\left[v_i^* v_k^* V_{jb} V_{lb'} + v_j v_l V_{ib}^* V_{kb'}^*\right] + \dots$$

Sufficient for our purposes: $\delta\tilde{\lambda}_{ijkl} = \delta\lambda_{ijkl} + \delta\lambda_{ilkj}$

Renormalization of scalar sector
$\delta\mu^2$ and δv

- Knowing $\delta\tilde{\lambda}_{ijkl}$, find $\delta\mu^2_{ij}$ and δv_k by demanding finite scalar self-energy:



- Need independent δv_k for $\xi_V \neq 0$ [Stoeckinger et al., 2013].
  (Ansatz: $\delta v_k = \alpha_V \xi_V v_k$)

$$\Rightarrow \delta v_k = \frac{c_\infty}{16\pi^2}\left(\frac{g^2\xi_W}{2} + \frac{g^2\xi_Z}{4c_W^2}\right) v_k$$

- Uniquely determines $\delta\mu^2_{ij}$

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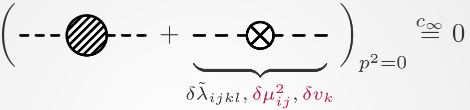

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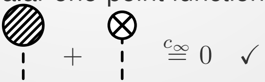
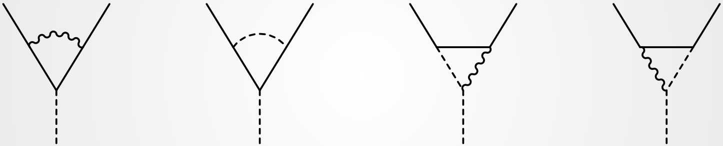
- Uniquely determines $\delta\mu^2_{ij}$
- Check finiteness of scalar one-point function:



Simultaneous finiteness of one- and two-point fct.: $\delta v_k \neq 0$

Renormalization of leptonic sector
Yukawa couplings

- Determine Yukawa counterterms $\delta\Gamma_k$ and $\delta\Delta_k$ from divergencies in vertex corrections, again in **unbroken phase**:

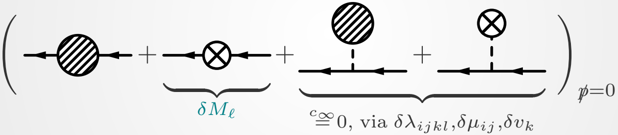


- Find remarkably simple result for neutral leptons

$$\delta\Delta_k = -\frac{c_\infty}{16\pi^2}\left[\left(\frac{g^2\xi_W}{2} + \frac{g^2\xi_Z}{4c_W^2}\right)\Delta_k + \Delta_j\Gamma_k^\dagger\Gamma_j\right],$$

Finiteness of leptonic two-point fct.

- Insert Yukawa- and VEV-counterterms in lepton mass-counterterms
- For charged leptons: $\delta M_\ell = \frac{1}{\sqrt{2}}\left(\delta v_k^* \Gamma_k + v_k^* \delta\Gamma_k\right)$
  $\Rightarrow$ **No freedom left** in the choice of δM_ℓ



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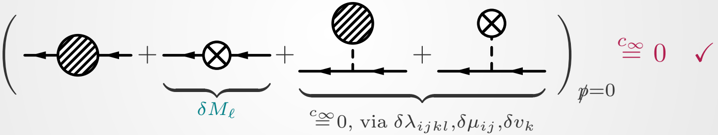

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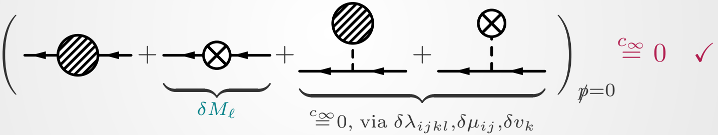
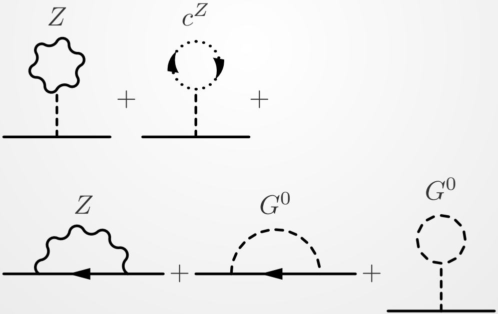


- **δv_k and $\delta\Gamma_k$ suffice for finiteness**
- **Choice of δv_k is unique** (in finite tadpole setup)
- Similarly for neutrinos: $\delta M_D = \frac{1}{\sqrt{2}} \left(\delta v_k \Delta_k + v_k \delta\Delta_k \right)$
- **Note: $\delta M_R = 0$ at one-loop**

Gauge dependencies

Gauge dependencies I

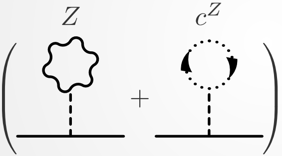
- All relevant correlation functions made finite
- Consistency check: **gauge-parameter independence of one-loop masses**



Gauge dependencies I

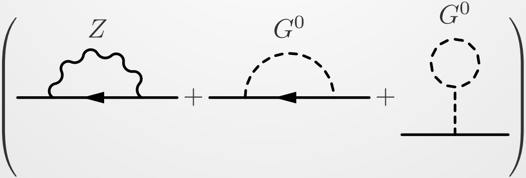
- All relevant correlation functions made finite
- Consistency check: **gauge-parameter independence of one-loop masses**
- Can analytically show this for on-shell lepton self-energies, *i.e.* here: when $\not{p} \to \hat{m}_\nu$ [Weinberg, 1973]

$$\frac{\mathrm{d}}{\mathrm{d}\xi_Z}$$



$$= 0.$$

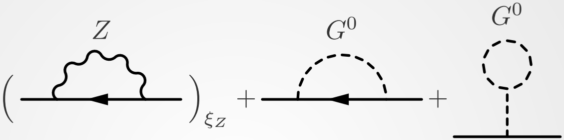
$$\frac{\mathrm{d}}{\mathrm{d}\xi_Z}$$



on-shell $= 0$

Gauge dependencies II

Full ξ_Z-dependence of self energies contained in:



$$\propto \int \frac{\mathrm{d}^d k}{(2\pi)^d} \Big\{ -\frac{1}{2} \left[(\not{p} - \hat{m}_\nu) F_{LR}^2 + F_{RL}^2 (\not{p} - \hat{m}_\nu) \right]$$

$$+ (\not{p} - \hat{m}_\nu) F_{LR} \, S^{(\nu)}(p-k) \, F_{RL} (\not{p} - \hat{m}_\nu)$$

$$+ (\not{p} - \hat{m}_\nu) F_{LR} \, S^{(\nu)}(p-k) \, (F_{RL} \hat{m}_\nu - \hat{m}_\nu F_{LR})$$

$$+ (\hat{m}_\nu F_{LR} - F_{RL} \hat{m}_\nu) \, S^{(\nu)}(p-k) \, F_{RL} (\not{p} - \hat{m}_\nu) \Big\} \, S^{(G^0)}(k, \xi_Z)$$

$$F_{LR} \equiv U_L^\dagger U_L \gamma_L - U_L^T U_L^* \gamma_R$$

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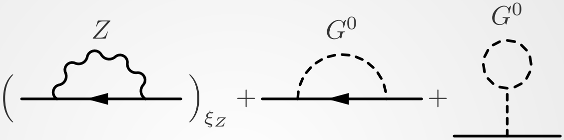


$$\propto \int \frac{\mathrm{d}^d k}{(2\pi)^d} \Big\{ -\frac{1}{2} \left[(\not{p} - \hat{m}_\nu) F_{LR}^2 + F_{RL}^2 (\not{p} - \hat{m}_\nu) \right] \leftarrow \text{Goldstone tadpole}$$
$$+ (\not{p} - \hat{m}_\nu) F_{LR}\, S^{(\nu)}(p-k)\, F_{RL} (\not{p} - \hat{m}_\nu)$$
$$+ (\not{p} - \hat{m}_\nu) F_{LR}\, S^{(\nu)}(p-k)\, (F_{RL} \hat{m}_\nu - \hat{m}_\nu F_{LR})$$
$$+ (\hat{m}_\nu F_{LR} - F_{RL} \hat{m}_\nu)\, S^{(\nu)}(p-k)\, F_{RL} (\not{p} - \hat{m}_\nu) \Big\}\, S^{(G^0)}(k, \xi_Z)$$

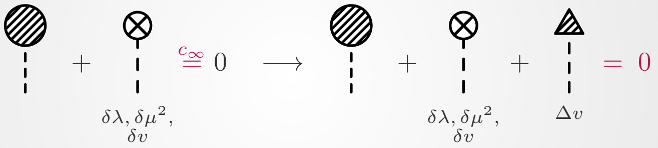
Terms of these types do not contribute to mass corrections, only shifts of the propagator residues $\rightarrow$ field strength renormalization

$\Rightarrow$ **Gauge-parameter independent one-loop masses**

VEV-shifts

Finite tadpole contributions I

- We have seen: need **(finite) tadpole contributions for gauge-parameter independence** of one-loop masses
- Can introduce finite VEV shifts Δv_k to absorb these $\Rightarrow$ **vanishing one-point function**



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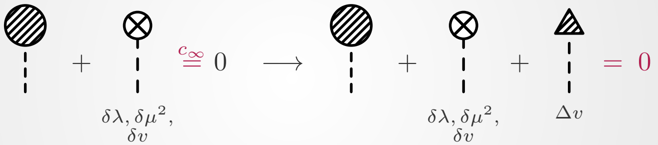
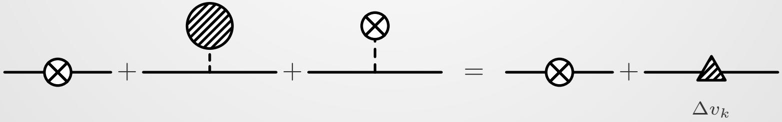


- Equivalence of inserting all tadpole contributions for a given observable versus shifting $v_k \to v_k + \Delta v_k$ in Lagrangian



Finite tadpole contributions II

Charged leptons

- Insertion of finite VEV-shifts yields:

$$= \frac{-i}{\sqrt{2}} \sum_{b=2}^{2n_H} \left[G_b \gamma_L + G_b^\dagger \gamma_R \right] \times \frac{i}{-M_b^2} \times \frac{i}{2} M_b^2 \left(\Delta v_i^* V_{ib} + V_{ib}^* \Delta v_i \right)$$

$$\Downarrow \qquad \boxed{G_b = \left(W_R^\dagger \Gamma_k W_L \right) V_{kb}^*}$$

$$= -\frac{i}{\sqrt{2}} \Big(W_R^\dagger \underbrace{\Delta v_k^* \Gamma_k}_{\Delta M_l} W_L \gamma_L + W_L^\dagger \Delta v_k \Gamma_k^\dagger W_R \gamma_R \Big)$$

- Compare to:

$$\mathcal{L}_{\text{mass},\ell} = -\overline{e_R}\, M_\ell\, e_L + \text{H.c.} = -\frac{1}{\sqrt{2}} \bar{\ell}\, W_R^\dagger v_k^* \Gamma_k W_L \gamma_L\, \ell + \text{H.c.}$$

- **VEV-shifts induce finite mass-shifts**

Final results

- Eventually calculate finite mass shifts (for neutrinos) via

$$\Sigma(p) = \not{p}\left(\Sigma_L^{(A)}(p^2)\gamma_L + \Sigma_R^{(A)}(p^2)\gamma_R\right) + \Sigma_L^{(B)}(p^2)\gamma_L + \Sigma_R^{(B)}(p^2)\gamma_R.$$

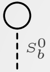
$$\Delta m_i = m_i \left(\Sigma_{\nu L}^{(A)}\right)_{ii}(m_i^2) + \mathrm{Re}\left(\Sigma_{\nu L}^{(B)}\right)_{ii}(m_i^2) \quad (i = 1, \ldots, n_L + n_R)$$

- Having shown gauge-parameter independence, can use specific gauge for simpler calculation
- Presentation of **full analytic results for leptonic self-energies in Feynman gauge (including tadpoles)** [Grimus, ML, 2018]

VEV-shifts of heavy Majoranas

Finite VEV-shifts of heavy Majoranas

- Majorana tadpole contributions:



$$\left(\quad \right)_{\text{fin}} = \Delta t_b^{(\chi)} = -\frac{\sqrt{2}}{16\pi^2}\,\mathrm{Tr}\left[\hat{m}_\nu^3 \left(\mathbb{1} - \ln\frac{\hat{m}_\nu^2}{\mathcal{M}^2}\right)\left(F_b + F_b^\dagger\right)\right]$$

- $\hat{m}_\nu \simeq \begin{pmatrix} \mathcal{O}(m_D^2/m_R) & \mathbf{0} \\ \mathbf{0} & \mathcal{O}(m_R) \end{pmatrix}$, *e.g.* $m_D \sim v$, $m_R \sim 10^{14}\,\mathrm{GeV}$
- Lead to VEV-shifts $\Delta v_l^{(\chi)} = \sum_{b=2}^{2n_H} \frac{\Delta t_b^{(\chi)}}{M_b^2} V_{lb}$

Yukawa couplings: $F_b = \frac{1}{2}\left(U_R^\dagger \Delta_k U_L + U_L^T \Delta_k^T U_R^*\right) V_{kb}$

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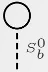

$$\left(\text{tadpole } S_b^0 \right)_{\text{fin}} = \Delta t_b^{(\chi)} = -\frac{\sqrt{2}}{16\pi^2} \operatorname{Tr}\left[\hat{m}_\nu^3 \left(\mathbb{1} - \ln \frac{\hat{m}_\nu^2}{\mathcal{M}^2} \right) \left(F_b + F_b^\dagger \right) \right]$$

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- Lead to VEV-shifts $\Delta v_l^{(\chi)} = \sum_{b=2}^{2n_H} \frac{\Delta t_b^{(\chi)}}{M_b^2} V_{lb}$
- No sufficient suppression: **Contributions very large** for high m_R
- Crude estimate:

$$\Delta v^{(\chi)} \lesssim 10\,\text{GeV} \Rightarrow m_R \lesssim 10^3\,\text{TeV}$$

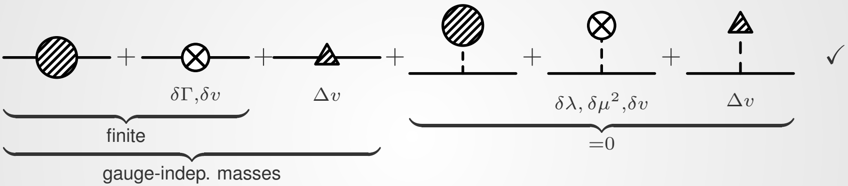
- **Bound for necessity of effective field theory?**

Yukawa couplings: $F_b = \frac{1}{2}\left(U_R^\dagger \Delta_k U_L + U_L^T \Delta_k^T U_R^* \right) V_{kb}$

Finite VEV-shifts of heavy Majoranas

Just a feature of our tadpole scheme?

- VEV-shifts as a matter of **bookkeeping**:
  Finite tadpole contr. in Δv_k:



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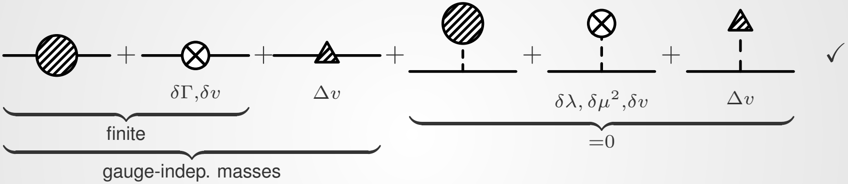
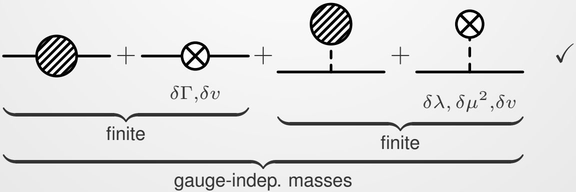


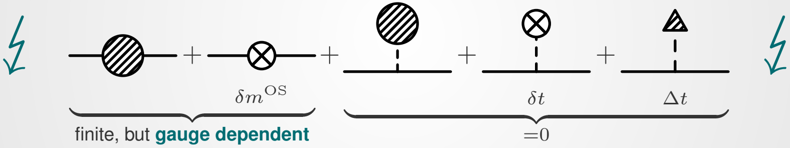
- $\Delta v_k = 0$:



Finite VEV-shifts of heavy Majoranas

Gauge-parameter independence of one-loop masses shows that tadpoles must be taken into account

- If tadpoles are “renormalized away”, gauge-dependence in masses might be introduced, *e.g.* in mass renormalization via



- In General: Potential source for confusion
  Bookkeeping is important!
- **Potentially problematic behavior of Majorana tadpoles in $\overline{\text{MS}}$**

Conclusions

- An **abundance of renormalizable neutrino mass** models available, often with **many new scalars**
- Want to check **perturbative stability of mass and mixing predictions** in generally applicable way $\Rightarrow$ **mHDSM**

Conclusions

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- **VEV counterterm obligatory** when $\xi_{W,Z} \neq 0$ to achieve finite scalar one- & two-point functions
- **Finite tadpole contributions obligatory** for gauge-parameter independent one-loop masses
- Can absorb these contributions in **finite VEV-shifts**
- Potentially **problematic contributions from heavy ν_R in $\overline{\text{MS}}$**
- Implies bound for **necessity of effective theory**
- Generality of mHDSM helpful for finding **simpler definition of gauge-independent renormalized mixing angles?**

Conclusions

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Thank you!

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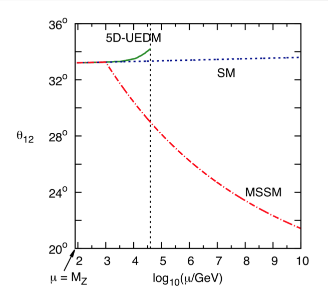
Backup slides

Relevance of radiative corrections?

- Precision era of ν-physics in reach
- Large threshold corrections in BSM models



Ohlsson, Zhou Nat. Com. 5 (2014) 5153

Estimate: Finite VEV-shifts of heavy Majoranas

- y_D : Yukawa coupling of Dirac neutrino mass terms; largest contribution to VEV-shift via lightest scalar

$$\Delta t^{(\chi)} \sim -\frac{\sqrt{2}}{32\pi^2} m_R^3 \, y_D \frac{m_D}{m_R}$$

$$\Rightarrow \Delta v^{(\chi)} \sim \frac{\Delta t^{(\chi)}}{M_H^2} \sim -\frac{\sqrt{2}}{32\pi^2} \frac{m_R^2 m_D}{M_H^2} y_D$$

- Seesaw mechanism: $y_D \sim m_D/v = \sqrt{m_\nu m_R}/v$

$$\Rightarrow |m_R^3| \sim \frac{32\pi^2}{\sqrt{2}} |\Delta v| \frac{v M_H^2}{m_\nu}$$

- $m_\nu = 10^{-10}\,\mathrm{GeV}$, $M_H^2 = 125\,\mathrm{GeV}$, $\Delta v \sim 10\,\mathrm{GeV}$

$$\Rightarrow m_R \sim 4 \times 10^3\,\mathrm{TeV}$$

Mixing angle renormalization
General idea (preliminary)

- Possible definition for one-loop mixing:

$$U_{1-\text{loop}} \equiv \delta U^\dagger U_{\text{tree}} \delta U = U_{\text{tree}} + \delta\theta^\dagger U_{\text{tree}} + U_{\text{tree}} \delta\theta, \quad \delta U = \mathbb{1} + \delta\theta$$

- Unitarity demands anti-hermitian correction, *i.e.* $\delta\theta^\dagger = -\delta\theta$:

$$U^\dagger_{1-\text{loop}} U_{1-\text{loop}} = \mathbb{1} + \delta\theta^\dagger + \delta\theta + U^\dagger_{\text{tree}} \left(\delta\theta^\dagger + \delta\theta\right) U_{\text{tree}} \stackrel{!}{=} \mathbb{1}$$

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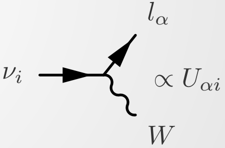
- Mixing physically observable in

$$\mathcal{L}_{cc} = g_B \bar{l}_B W^\mu_B \gamma_\mu U_{\text{PMNS}} \nu_B$$
$$= g_B \bar{l} W^\mu_B \gamma_\mu (Z_l^{1/2})^\dagger U_{\text{PMNS}} Z_\nu^{1/2} \nu$$



- Then with $Z^{1/2}_{l/\nu} = \mathbb{1} + \frac{1}{2}\delta_{l/\nu}$:

$$U_{1-\text{loop}} \equiv U_{\text{PMNS}} + \frac{1}{2}\left[(\delta_l^{AH})^\dagger U_{\text{PMNS}} + U_{\text{PMNS}} \delta_\nu^{\text{AH}}\right]$$

- Looks like a natural choice!

Mixing angle renormalization

Gauge dependence (preliminary)

- Problem: on-shell field strength renormalization is gauge dependent

$$\frac{1}{2}(\delta_L)_{ij} = \frac{-1}{m_i^2 - m_j^2}\left[m_j^2 \Sigma_{ij}^{AL} + m_i m_j \Sigma_{ij}^{AR} + m_j \Sigma_{ij}^{BR} + m_i \Sigma_{ij}^{BL}\right]_{p^2=m_j^2}$$

- Same goes for **anti-hermitian** part (AH):

$$(\delta_L^{AH})_{ij} =$$
$$\frac{-1}{m_i^2 - m_j^2}\Big[m_i^2 \Sigma_{ij}^{AL}(m_i^2) + m_j^2 \Sigma_{ij}^{AL}(m_j^2) + m_i m_j \left(\Sigma_{ij}^{AR}(m_i^2) + \Sigma_{ij}^{AR}(m_j^2)\right)$$
$$+ m_j \left(\Sigma_{ij}^{BR}(m_i^2) + \Sigma_{ij}^{BR}(m_j^2)\right) + m_i \left(\Sigma_{ij}^{BL}(m_i^2) + \Sigma_{ij}^{BL}(m_j^2)\right)\Big]$$

- All ξ-dependent terms should cancel, but at least for finite corrections: **cancellations not obvious**! (Note the different arguments in the self-energies)

Mixing angle renormalization

- Various renormalization methods available in the literature for quark mixing, *e.g.* using symmetric point $p^2 = 0$:

$$V_R^{\mathrm{CKM}} = V_0^{\mathrm{CKM}} + \frac{1}{2}\left((\delta\mathcal{Z}_{u,L}^{AH})^\dagger V_0^{\mathrm{CKM}} + (V_0^{\mathrm{CKM}})^\dagger \delta\mathcal{Z}_{d,L}^{AH}\right)$$

$$(\delta\mathcal{Z}^{AH})_{ij} = \frac{m_i^2 + m_j^2}{m_i^2 - m_j^2}\left(\Sigma_{ij}^L(0) + 2\Sigma_{ij}^S(0)\right)$$

- Alternatively (bluntly speaking): brute force calculation of self-energies, then use only terms that do not depend on ξ [**?**]
- ...the question persists:
  - **What is the canonical method for mixing angle renormalization?**

Mixing angle renormalization

Pinch-technique

Explicitly gauge-independent $\Sigma(p^2)$ for defining renormalized mixing angles: (scalars) Krause et al., JHEP **1609** (2016) 143

$$\overline{\Sigma}(p^2) = \Sigma^{\text{tad}}\Big|_{\xi_V=1}(p^2) + \Sigma^{\text{add}}(p^2),$$

Σ^{tad}: full self-energy w. tadpoles, Σ^{add}: additional explicitly gauge-independent part from toy two-to-two scattering (intermediate scalars H_i):

$$\Gamma^{H_i XX}\frac{i}{p^2 - m_{H_i}^2} i\Sigma^{PT}_{H_i H_j}(p^2)\frac{i}{p^2 - m_{H_j}^2}\Gamma^{H_j YY}.$$

Γ: vertex function, Σ^{PT}: self-energy-like fct. Major insight:

$$\overline{\Sigma}_{H_i H_j}(p^2) = \Sigma^{\text{tad}}_{H_i H_j}(p^2) + \Sigma^{\text{PT}}_{H_i H_j}(p^2) = \Sigma^{\text{tad}}_{H_i H_j}\Big|_{\xi_V=1}(p^2) + \Sigma^{\text{add}}_{H_i H_j}(p^2),$$

$\Rightarrow \overline{\Sigma}$ gauge-independent. Downside: trades gauge dependencies for prescription in the definition of $\overline{\Sigma}$.
p^*-scheme: use *symmetric point* $p^2 \neq 0$ at which self-energies are evaluated: $(p^*)^2 = (M_{H_i}^2 + M_{H_j}^2)/2$.

Scalar mass two-point counterterm

The counterterm pertaining to the scalar self-energy $-i\Pi_{bb'}(p^2)$ is given by

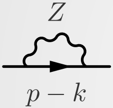
S_b^0  $S_{b'}^0$

$$
\begin{aligned}
S_b^0 \;\text{---}\!\otimes\!\text{---}\; S_{b'}^0 = &-\frac{i}{2}\left[\delta\mu_{ij}^2 + \delta\tilde{\lambda}_{ijkl}v_k^* v_l\right]\left(V_{ib}^* V_{jb'} + V_{ib'}^* V_{jb}\right) \\
&-\frac{i}{4}\delta\tilde{\lambda}_{ijkl}\left[v_i^* v_k^* V_{jb} V_{lb'} + v_j v_l V_{ib}^* V_{kb'}^*\right] \\
&-\frac{i}{2}\tilde{\lambda}_{ijkl}\left(\delta v_k^* v_l + v_k^* \delta v_l\right)\left(V_{ib}^* V_{jb'} + V_{ib'}^* V_{jb}\right) \\
&-\frac{i}{4}\tilde{\lambda}_{ijkl}\left[\left(\delta v_i^* v_k^* + v_i^* \delta v_k^*\right) V_{jb} V_{lb'} + \left(\delta v_j v_l + v_j \delta v_l\right) V_{ib}^* V_{kb'}^*\right]
\end{aligned}
$$

Gauge dependencies

Z-contribution

Using propagator-shift trick yields



$$\supset \int \frac{\mathrm{d}^4 k}{(2\pi)^4} F_{RL} \not{k} S^{(\nu)}(p-k) \not{k} F_{LR} S^{(G^0)}(k, \xi_Z)$$

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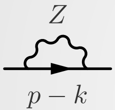

$$\supset \int \frac{\mathrm{d}^4 k}{(2\pi)^4} F_{RL} \not{k} S^{(\nu)}(p-k) \not{k} F_{LR} S^{(G^0)}(k,\xi_Z)$$

$$= \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \Big[-F_{RL} \left(\not{p} - \hat{m}_\nu\right) F_{LR} + F_{RL} \left(\not{p} - \hat{m}_\nu\right) S^{(\nu)}(p-k) \left(\not{p} - \hat{m}_\nu\right) F_{LR} \Big] S^{(G^0)}(k,\xi_Z)$$

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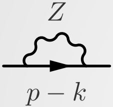

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$$+ F_{RL} \left(\not{p} - \hat{m}_\nu \right) S^{(\nu)}(p-k) \left(\not{p} - \hat{m}_\nu \right) F_{LR} \Big] S^{(G^0)}(k, \xi_Z)$$

$$F_{RL} \equiv U_L^\dagger U_L \gamma_R - U_L^T U_L^* \gamma_L,$$

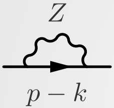
$$F_{LR} \equiv U_L^\dagger U_L \gamma_L - U_L^T U_L^* \gamma_R,$$

$$\begin{pmatrix} U_L & U_R^* \end{pmatrix} \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix} \begin{pmatrix} U_L \\ U_R^* \end{pmatrix} \equiv \hat{m}_\nu \quad \Rightarrow U_L^* \hat{m}_\nu U_L^\dagger = 0$$

Gauge dependencies

Z-contribution

Using propagator-shift trick yields



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$$= \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \Bigg[-\frac{1}{2} \left(\not{p} F_{LR}^2 + F_{RL}^2 \not{p}\right)$$

$$+ \left(\not{p} F_{LR} - F_{RL} \hat{m}_\nu\right) S^{(\nu)}(p-k) \left(F_{RL} \not{p} - \hat{m}_\nu F_{LR}\right) \Bigg] S^{(G^0)}(k, \xi_Z)$$

Vector boson propagators

Various useful ways of writing vector boson propagators:

$$S_{\mu\nu}^{(V)}(k) = \frac{-g_{\mu\nu} + k_\mu k_\nu / k^2}{k^2 - M_V^2} - \frac{k_\mu k_\nu}{k^2} \frac{\xi_V}{k^2 - \xi_V M_V^2}$$

$$= \frac{-g_{\mu\nu} + k_\mu k_\nu / k^2}{k^2 - M_V^2} - \frac{k_\mu k_\nu}{M_V^2} \frac{1}{k^2 - \xi_V M_V^2}$$