Stefan Liebler
ITP KIT

Revisiting the hMSSM

Karlsruhe – October 2019

Karlsruhe Institute of Technology

HELMHOLTZ | GEMEINSCHAFT

Outline

1 Setting the stage
2 Low-energy effective 2HDM
3 The improved hMSSM
4 Conclusions

The MSSM Higgs sector contains two doublets $H_d = (\phi_d^{0*}, -\phi_d^-)$ and $H_u = (\phi_u^+, \phi_u^0)$, which enter the tree-level potential as follows

$$\begin{aligned} V_{\rm MSSM}^{\rm LO} =& (m_{H_d}^2 + \mu^2)|H_d|^2 + (m_{H_u}^2 + \mu^2)|H_u|^2 - B\mu\epsilon_{ij}(H_d^i H_u^j + \text{h.c.}) \\ &+ \frac{g^2 + g'^2}{8}(|H_d|^2 - |H_u|^2)^2 + \frac{g^2}{2}|H_d^* H_u|^2 \,. \end{aligned}$$

After EWSB the neutral components

$$\phi_d^0 = \frac{1}{\sqrt{2}}(v_d + \sigma_d + i\xi_d) \qquad \text{and} \qquad \phi_u^0 = \frac{1}{\sqrt{2}}(v_u + \sigma_u + i\xi_u)$$

acquire VEVs and in the CP-odd sector we are left with a pseudoscalar with mass ${M_A}^2 = 2B\mu/\sin(2\beta)$ with $t_\beta := \tan\beta = v_u/v_d$. The mass matrix of the CP-even sector turns into

$$\mathcal{M}_{\text{tree}}^2 = \begin{pmatrix} \mathcal{M}_{dd}^2 & \mathcal{M}_{du}^2 \\ \mathcal{M}_{du}^2 & \mathcal{M}_{uu}^2 \end{pmatrix} = \begin{pmatrix} M_A^2 s_\beta^2 + M_Z^2 c_\beta^2 & -(M_A^2 + M_Z^2)s_\beta c_\beta \\ -(M_A^2 + M_Z^2)s_\beta c_\beta & M_A^2 c_\beta^2 + M_Z^2 s_\beta^2 \end{pmatrix} .$$

Diagonalization yields $\tan(2\alpha) = \tan(2\beta)\frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$ and $M_h^2 \leq M_Z^2 \cos^2(2\beta)$.
$\rightarrow$ Higher-order corrections are needed.

Higher-order corrections lift the light CP-even Higgs mass to $M_h = 125\,$GeV:

$$\mathcal{M}^2_{\text{loop}} = \begin{pmatrix} M_A^2 s_\beta^2 + M_Z^2 c_\beta^2 & -(M_A^2 + M_Z^2) s_\beta c_\beta \\ -(M_A^2 + M_Z^2) s_\beta c_\beta & M_A^2 c_\beta^2 + M_Z^2 s_\beta^2 \end{pmatrix} + \begin{pmatrix} \Delta\mathcal{M}^2_{dd} & \Delta\mathcal{M}^2_{du} \\ \Delta\mathcal{M}^2_{du} & \Delta\mathcal{M}^2_{uu} \end{pmatrix} .$$

Idea of the hMSSM:

[Djouadi Maiani Moreau Polosa Quevillon Riquer 1307.5205, 1304.1787, 1305.2172, 1502.05653]

If the dominant correction is $\Delta\mathcal{M}^2_{uu}$ and all other corrections are small, one can invert the relation and obtain $\Delta\mathcal{M}^2_{uu}$ as a function of the eigenvalue M_h:

$$\epsilon := \Delta\mathcal{M}^2_{uu} = \frac{M_h^2(M_A^2 + M_Z^2 - M_h^2) - M_A^2 M_Z^2 c_{2\beta}^2}{M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 - M_h^2} .$$

Then all other masses and mixing angles are fixed to

$$M_H^2 = M_A^2 + M_Z^2 - M_h^2 + \epsilon, \qquad M_{H^\pm} = M_A^2 + M_W^2 ,$$

$$\tan(2\alpha) = \tan(2\beta) \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2 + \epsilon / \cos 2\beta} .$$

Aim 1 of this talk: Understand the underlying assumptions of this approach.

Currently used Higgs self-couplings in the hMSSM approach:

$$\lambda_{hhh} = 3\frac{M_Z^2}{v}c_{2\alpha}s_{\alpha+\beta} + \frac{3c_\alpha^3}{vs_\beta}\epsilon, \qquad \lambda_{Hhh} = \frac{M_Z^2}{v}\left(2s_{2\alpha}s_{\alpha+\beta} - c_{2\alpha}c_{\alpha+\beta}\right) + \frac{3s_\alpha c_\alpha^2}{vs_\beta}\epsilon$$

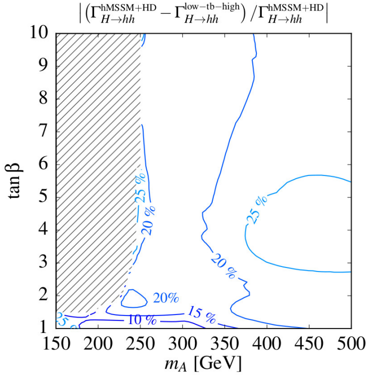
Though:

[LHCHXSWG-2015-002] revealed differences in the decay $H \to hh$ between a full MSSM calculation and the hMSSM approach.

However, this comparison is slightly misleading:

✗ "low-tb-high" corresponds to a calculation with FeynHiggs at full one-loop including the resummation of logs.

✗ hMSSM+HDecay instead is a tree-level calculation employing the above loop-corrected coupling.



Aim 2 of this talk:
Get a better understanding
of the Higgs-self couplings in the hMSSM.

Outline

1 Setting the stage

2 Low-energy effective 2HDM

3 The improved hMSSM

4 Conclusions

Go back to square one!
The MSSM Higgs sector and thus also the hMSSM is nothing else than an effective low-energy 2HDM Higgs sector, i.e. we have two Higgs doublets H_1 and H_2, that enter the tree-level potential as follows

$$V_{\rm 2HDM}^{\rm LO} = m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1^\dagger H_2 + \text{h.c.}) + \frac{\lambda_1}{2}|H_1|^4 + \frac{\lambda_2}{2}|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1^\dagger H_2|^2\,.$$

Supersymmetry fixes all couplings

$$\lambda_1 = \lambda_2 = \frac{g^2+g'^2}{4}, \qquad \lambda_3 = \frac{g^2-g'^2}{4}, \qquad \lambda_4 = -\frac{g^2}{4}$$

and we identify $m_3^2 = B\mu$, which relates to M_A. Corrections to this low-energy 2HDM from heavy (s)particles, that are integrated out, can be calculated in the effective potential approach (EPA), where the Lagrangian takes the form
[Beneke Ruiz-Femenia Spinrath 0810.3768]

$$\mathcal{L}_{\rm 2HDM}^{\text{eff}} = \sum_{i,j\in\{1,2\}} Z_{ij}^{\text{eff}}(D_\mu H_i)^\dagger(D_\mu H_j) - V_{\rm 2HDM}^{\text{eff}}\,.$$

Correction to the MSSM Higgs masses in the EPA (Diagrammatic with $p^2 = 0$):
$\mathcal{O}(\alpha_t)$: [Okada Yamaguchi Yanagida '91, Haber Hempfling '91, Ellis Ridolfi Zwirner '91]
$\mathcal{O}(\alpha_b)$, EW: [Brignole '92, Chankowski Pokorski Rosiek '93, Dabelstein '94, Pierce et al. '96]
Correction to the MSSM Higgs self-couplings in the EPA:
$\mathcal{O}(\alpha_t, \alpha_b)$: [Barger et al. '92, Hollik Penaranda '01, Dobado et al. '02]
For our purpose: [SL Mühlleitner Spira Stadelmaier 1810.10979]
We consider $\mathcal{O}(\alpha_t)$ corrections in the gaugeless limit:

$$V_{\text{2HDM}}^{\text{eff}} = V_{\text{2HDM}}^{\text{LO}} + V^{\text{NLO}}(t) + V^{\text{NLO}}(\tilde{t}) + \mathcal{O}(\alpha_t^2)$$

The individual contributions from the top quark and stop sector (using field-dependent masses) are given by ($C_\epsilon = \Gamma(1+\epsilon)(4\pi)^\epsilon$):

$$V^{\text{NLO}}(t) = \frac{3}{(4\pi)^2} C_\epsilon \left\{ \overline{m}_t^4 \left[\frac{1}{\epsilon} + \frac{3}{2} - \log \frac{\overline{m}_t^2}{Q^2} \right] \right\}$$

$$V^{\text{NLO}}(\tilde{t}) = -\frac{3}{(4\pi)^2} \frac{1}{2} C_\epsilon \left\{ \overline{m}_{\tilde{t}_1}^4 \left[\frac{1}{\epsilon} + \frac{3}{2} - \log \frac{\overline{m}_{\tilde{t}_1}^2}{Q^2} \right] + \overline{m}_{\tilde{t}_2}^4 \left[\frac{1}{\epsilon} + \frac{3}{2} - \log \frac{\overline{m}_{\tilde{t}_2}^2}{Q^2} \right] \right\} .$$

For the Higgs masses this implies $\Delta\mathcal{M}^2_{ij} = \Delta\mathcal{M}^2_{ij}(t) + \Delta\mathcal{M}^2_{ij}(\tilde{t})$.
The (potentially) dominant corrections to $\mathcal{M}^2_{uu}$ are

$$\Delta\mathcal{M}^2_{uu}(t) = \frac{12}{(4\pi)^2 v^2 s^2_\beta} m_t^4 \left[2\Delta_\varepsilon + 2\log\left(\frac{Q^2}{m_t^2}\right)\right],$$

$$\Delta\mathcal{M}^2_{uu}(\tilde{t}) = \frac{12}{(4\pi)^2 v^2 s^2_\beta} m_t^4 \left[-2\Delta_\varepsilon + A_t^2 C_t^2 g_t + 2A_t C_t \log\left(\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}}\right) + 2\log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{Q^2}\right)\right]$$

using

$$C_t = \frac{X_t}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}, \quad g_t = 2 - \frac{m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \log\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}}, \quad X_t = A_t - \mu/t_\beta \text{ : stop mixing}.$$

Summing the two contributions yields a UV finite correction

$$\Delta\mathcal{M}^2_{uu} = \frac{12}{(4\pi)^2 v^2 s^2_\beta} m_t^4 \left[A_t^2 C_t^2 g_t + 2A_t C_t \log\left(\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}}\right) + 2\log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)\right].$$

For $M_S = M_{\tilde{t}_L} = M_{\tilde{t}_R}$ this can be expanded in large M_S

$$\Delta\mathcal{M}^2_{uu} = \frac{3G_F}{\sqrt{2}\pi^2 s^2_\beta} m_t^4 \left[\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t A_t}{M_S^2}\left(1 - \frac{X_t A_t}{12 M_S^2}\right) + \dots\right].$$

The other two elements receive corrections from the stop sector, which yield

$$\Delta\mathcal{M}^2_{dd}(\tilde{t}) = \frac{12}{(4\pi)^2 v^2 s_\beta^2} m_t^4 C_t^2 \mu^2 g_t \,,$$

$$\Delta\mathcal{M}^2_{du}(\tilde{t}) = -\frac{12}{(4\pi)^2 v^2 s_\beta^2} m_t^4 C_t \mu \left[A_t C_t g_t + \log\left(\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}}\right)\right] .$$

For $M_S = M_{\tilde{t}_L} = M_{\tilde{t}_R}$ we can rotate into mass eigenstates and get

$$\Delta M_h^2 = \frac{3G_F}{\sqrt{2}\pi^2} m_t^4 \left[\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12 M_S^2}\right) + \ldots\right] .$$

Expression from the previous slide:

$$\Delta\mathcal{M}^2_{uu} = \frac{3G_F}{\sqrt{2}\pi^2 s_\beta^2} m_t^4 \left[\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t A_t}{M_S^2}\left(1 - \frac{X_t A_t}{12 M_S^2}\right) + \ldots\right] .$$

Remember that $X_t = A_t - \mu/t_\beta$.

We are now able to formulate the assumptions of the hMSSM approach:
For vanishing μ/M_S the elements $\Delta\mathcal{M}^2_{dd}(\tilde{t})$ and $\Delta\mathcal{M}^2_{du}(\tilde{t})$ vanish.
It then yields $X_t/M_S \approx A_t/M_S$ and thus

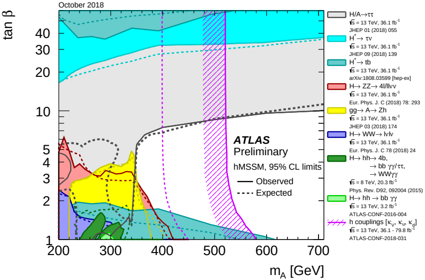
$$\epsilon := \Delta\mathcal{M}^2_{uu} = \Delta M_h^2/s_\beta^2\,.$$

We can thus formulate the assumptions of the hMSSM approach:

✓ Low values of $\tan\beta$ as one neglects (s)bottom contributions!

✓ Low value of μ/M_S, such that $\Delta\mathcal{M}^2_{dd}$ and $\Delta\mathcal{M}^2_{du}$ are subdominant.

$\leftrightarrow$ Electroweakinos that are lighter than squark spectrum!



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We calculated the corrections to the Higgs self-couplings in the EPA approach:

$$\Delta\lambda_{uuu}(t) = \frac{72}{(4\pi)^2 v^3 s_\beta^3} m_t^4 \left[\Delta_\varepsilon - \frac{2}{3} + \log\left(\frac{Q^2}{m_t^2}\right)\right],$$

$$\Delta\lambda_{uuu}(\tilde{t}) = \frac{72}{(4\pi)^2 v^3 s_\beta^3} m_t^4 \left[-\Delta_\varepsilon + \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{Q^2}\right)\right] + \ldots$$

Adding all other combinations $\Delta\lambda_{ddd}$, $\Delta\lambda_{ddu}$, $\Delta\lambda_{duu}$, rotating into mass eigenstates and expanding in inverse powers of $M_S = M_{\tilde{t}_L} = M_{\tilde{t}_R}$ yields

$$\Delta\lambda_{Hhh} = \frac{72 s_\alpha c_\alpha^2}{(4\pi)^2 v^3 s_\beta^3} m_t^4 \left[\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{A_t^2}{M_S^2}\left(1 - \frac{A_t^2}{12 M_S^2}\right) - \frac{2}{3} + \ldots\right].$$

We can identify ϵ defined in the mass corrections and thus rewrite $\Delta\lambda_{Hhh}$ as

$$\Delta\lambda_{Hhh} = \frac{3 s_\alpha c_\alpha^2}{v s_\beta}\left[\epsilon + \frac{24}{(4\pi)^2 v^2 s_\beta^2} m_t^4 \left(-\frac{2}{3} + \frac{2m_t^2}{3M_S^2} - \frac{m_t^2 A_t^2}{M_S^4} + \frac{m_t^2 A_t^4}{3M_S^6} - \frac{m_t^2 A_t^6}{30 M_S^8}\right)\right].$$

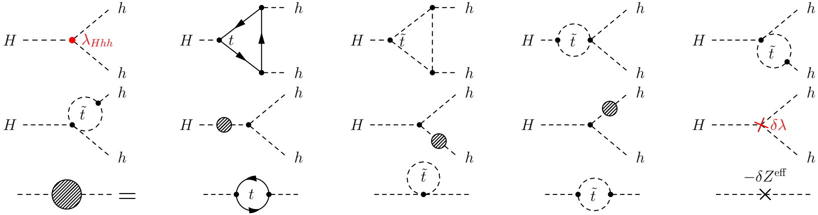
Definition of the "improved hMSSM" [SL Mühlleitner Spira Stadelmaier 1810.10979]:
For the Higgs self-couplings use the modified correction

$$\bar{\epsilon} = \epsilon - \frac{24}{(4\pi)^2 v^2 s_\beta^2} m_t^4 \frac{2}{3}\,,$$

which includes a missing contribution from the top-quark! Obtain ϵ according to the hMSSM approach from M_h, M_A and $\tan\beta$, define $\bar{\epsilon}$, that enters all Higgs self-couplings, e.g.

$$\lambda_{hhh} = 3\frac{M_Z^2}{v} c_{2\alpha} s_{\alpha+\beta} + \frac{3c_\alpha^3}{v s_\beta}\bar{\epsilon}\,, \qquad \lambda_{Hhh} = \frac{M_Z^2}{v}(2 s_{2\alpha} s_{\alpha+\beta} - c_{2\alpha} c_{\alpha+\beta}) + \frac{3 s_\alpha c_\alpha^2}{v s_\beta}\bar{\epsilon}\,.$$

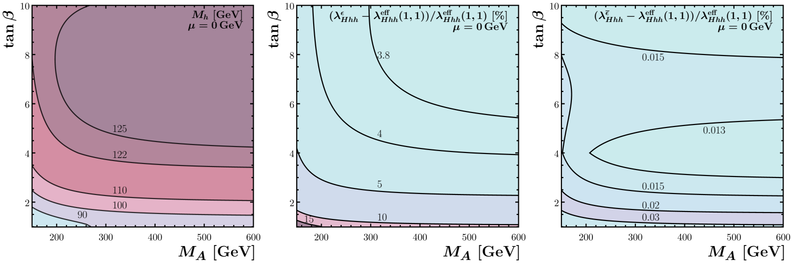
We calculate $\Gamma(H \to hh)$ augmented by momentum-dependent corrections:



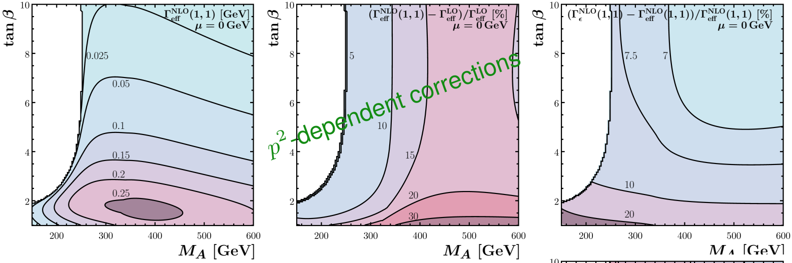
Numerical results for one scenario:

$$M_S = 1.5\,\text{TeV}\,,\, \mu = 0\,\text{GeV}\,,\, X_t = \begin{cases} 2950\,\text{GeV} & \tan\beta \leq 4 \\ (2950 - \frac{400}{3}(\tan\beta - 4))\,\text{GeV} & \tan\beta > 4 \end{cases}.$$

For $\mu = 0\,\text{GeV}$ the hMSSM approach reproduces exactly m_H and α.
We show the light Higgs mass m_h and λ^{ϵ}_{Hhh} and $\lambda^{\bar{\epsilon}}_{Hhh}$ in comparison to the exact effective value of $\lambda^{\text{eff}}_{Hhh}(t = 1, \tilde{t} = 1)$ including top and stop contributions (at zero momentum) at one-loop:



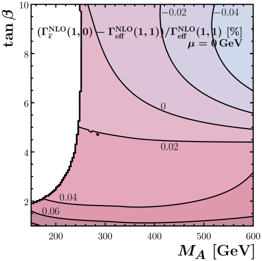
Our numerical results for the partial width $\Gamma(H \to hh)$:
see also [Brignole Zwirner '92, Williams Weiglein '07, Chalons Djouadi Quevillon '17]



Comments:

✓ Using the "improved hMSSM", i.e. $\bar{\epsilon}$ in λ_{Hhh}, reproduces $\Gamma(H \to hh)$ with exact top and stop corrections accurately, for $\mu = 0\,$GeV.

✓ p^2-dependent one-loop corrections are of relevance! For the hMSSM p^2-dependent corrections from the top-quark are sufficient!



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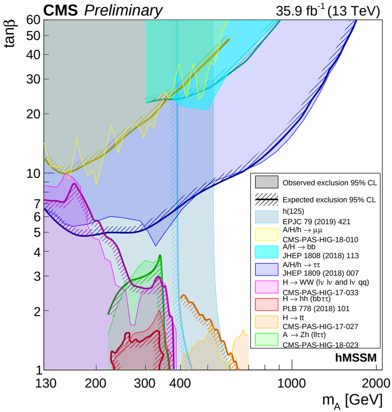
The hMSSM is a nice approach to the MSSM Higgs sector that uses M_h as input. We discussed the hMSSM as low-energy effective 2HDM, where both top and stops are integrated out:

✓ Remember the assumptions of the hMSSM, i.e. low $\tan\beta$ and low μ/M_S.
✓ Use corrected $\bar{\epsilon}$ for the Higgs self-couplings.
✓ Augment $\Gamma(H \to hh)$ with momentum-dependent corrections from top-quark.



To be done:
✗ Understand differences for $\mu > 0\,$GeV.
✗ Identify the scheme of m_t in $\bar{\epsilon}$.
✗ Go beyond the gaugeless limit.

Don't forget that the hMSSM is only a good approximation.
"Proper" MSSM scenarios offer other aspects, e.g. light electroweakinos.
[Bahl SL Stefaniak 1901.05933]

Details: $\mathcal{A}^{\text{NLO}}(t,\tilde{t}) = \mathcal{A}^{\text{virt}}(t,\tilde{t}) + \mathcal{A}^{\text{ext}}(t,\tilde{t}) + \mathcal{A}^{\text{ext,eff}}(t,\tilde{t}) + \mathcal{A}^{\delta\lambda}(t,\tilde{t})\,,$

$$\delta Z_H = \Sigma'_{HH}(M_H^2)\,, \quad \delta Z_h = \Sigma'_{hh}(M_h^2)\,, \quad \delta Z_{Hh}(p^2) = \frac{\Sigma_{Hh}(p^2)}{M_H^2 - M_h^2}$$

$$\delta Z_H^{\text{eff}} = \Sigma'_{HH}(0)\,, \quad \delta Z_h^{\text{eff}} = \Sigma'_{hh}(0)\,, \quad \delta Z_{Hh}^{\text{eff}}(p^2) = \frac{p^2\Sigma'_{Hh}(0)}{M_H^2 - M_h^2}$$

$$\mathcal{A}^{\text{ext}}(t,\tilde{t}) = \lambda_{Hhh}(\tfrac{1}{2}\delta Z_H + \delta Z_h) + \lambda_{hhh}\delta Z_{Hh}(M_H^2) - 2\lambda_{HHh}\delta Z_{Hh}(M_h^2)$$

$$\mathcal{A}^{\text{ext,eff}}(t,\tilde{t}) = \lambda_{Hhh}(-\tfrac{1}{2}\delta Z_H^{\text{eff}} - \delta Z_h^{\text{eff}}) - \lambda_{hhh}\delta Z_{Hh}^{\text{eff}}(M_H^2) + 2\lambda_{HHh}\delta Z_{Hh}^{\text{eff}}(M_h^2)$$

$$\delta\alpha = -\frac{s_{4\alpha}}{4}\left(\frac{\Delta\mathcal{M}_{du}^2}{\mathcal{M}_{du}^2} - \frac{\Delta\mathcal{M}_{uu}^2 - \Delta\mathcal{M}_{dd}^2}{\mathcal{M}_{uu}^2 - \mathcal{M}_{dd}^2}\right)$$

$$\mathcal{A}^{\delta\lambda}(t,\tilde{t}) = \frac{\partial\lambda_{Hhh}}{\partial\alpha}\delta\alpha + \mathcal{A}^{\text{eff}} = \lambda_{hhh}\delta\alpha - 2\lambda_{HHh}\delta\alpha + \mathcal{A}^{\text{eff}}$$

$$\mathcal{A}^{\text{eff}}(t,\tilde{t}) = -\Delta\lambda_{Hhh}(t,\tilde{t}) = -\left.\mathcal{A}^{\text{virt}}(t,\tilde{t})\right|_{q_i^2=0}$$