
KIT-NEP ‘19

# Higher-order Higgs masses and mixing matrix

Sebastian Paßehr

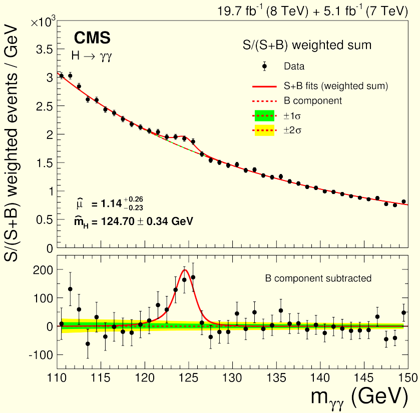
Institute for Theoretical Particle Physics and Cosmology,
RWTH Aachen University

KITNEP workshop
Karlsruher Institut für Technologie
Karlsruhe, Germany
7th of October 2019

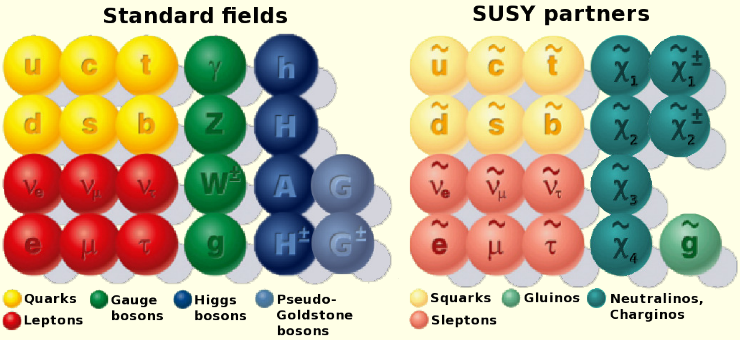
# Contents

Higgs-like particle discovered: [ATLAS, arXiv:1207.7214 [hep-ex]], [CMS, arXiv:1207.7235 [hep-ex]],

e. g. signal in $H \to \gamma\gamma$, [CMS, arXiv:1407.0558 [hep-ex]]




- very good agreement with SM Higgs boson
- but: SM has many deficiencies
- test models beyond the Standard Model, e. g. Supersymmetry
- experimental value:
  $125.09 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})$ GeV
  [ATLAS, CMS, arXiv:1503.07589]
  $125.18 \pm 0.16$ GeV
  [Particle Data Group, Physical Review D. 98 (3): 030001]




two complex $SU(2)$-Higgs doublets:

$$\mathcal{H}_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}\left(\phi_1^0 - i\,\chi_1^0\right) \\ -\phi_1^- \end{pmatrix}, \qquad \mathcal{H}_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}\left(\phi_2^0 + i\,\chi_2^0\right) \end{pmatrix},$$

positive real vacuum expectation values $v_1$, $v_2$

# Higgs potential of the MSSM

non-kinetic part of the Lagrangian involving only Higgs fields:

$$V_H^{\text{MSSM}} = V_{\text{Higgs}}^{\text{MSSM}} + V_{\text{breaking}}^{\text{MSSM}} \,,$$

$$V_{\text{Higgs}}^{\text{MSSM}} = \frac{1}{8}\left(g_Y^2 + g_{\text{w}}^2\right)\left(|\mathcal{H}_2|^2 - |\mathcal{H}_1|^2\right)^2 + \frac{1}{2}\,g_{\text{w}}^2\,|\mathcal{H}_1^\dagger \mathcal{H}_2|^2 + |\mu|^2\left(|\mathcal{H}_1|^2 + |\mathcal{H}_2|^2\right) \,,$$

$$V_{\text{breaking}}^{\text{MSSM}} = \tilde{m}_1^2\,|\mathcal{H}_1|^2 + \tilde{m}_2^2\,|\mathcal{H}_2|^2 + \left(\mu\, b_\mu\, \mathcal{H}_1 \cdot \mathcal{H}_2 + \text{h.c.}\right) \,,$$

minimization of potential relates bilinear and quartic terms $\Rightarrow$ mass prediction

$$m_h^2 = \frac{1}{2}\left(m_A^2 + m_Z^2\right)\left(1 - \sqrt{1 - 4\,\frac{m_A^2\, m_Z^2}{\left(m_A^2 + m_Z^2\right)^2}\,\cos^2 2\beta}\right) \leq m_Z^2\,\cos^2 2\beta$$

large loop corrections in order to match experimental value,
on-shell renormalization not possible for **all** Higgs bosons
(for approximately on-shell $h$ see talk by S. Liebler)

# Contents

# Mass determination at higher orders

Higgs masses at $k$ loop order given by poles of propagator matrix

$$\mathbf{\Delta}^{(k)}_{\text{Higgs}}\left(p^2\right) = i\left[p^2\mathbf{1} - \mathbf{M}^{(k)}_{\text{Higgs}}\left(p^2\right)\right]^{-1},$$

matrix of renormalized two-point vertex functions:

$$\widehat{\mathbf{\Gamma}}^{(k)}_{\text{Higgs}}\left(p^2\right) = -\left[\mathbf{\Delta}^{(k)}_{\text{Higgs}}\left(p^2\right)\right]^{-1},$$

masses determined by

$$\det\left[\widehat{\mathbf{\Gamma}}^{(k)}_{\text{Higgs}}\left(p^2\right)\right]_{p^2 = x_i^2} = 0\,, \quad M_i^2 = \mathfrak{Re}\left[x_i^2\right]\,,$$

various treatments of det[...] on the market,
some violate gauge invariance and/or perturbative order
(also see talk by H. Bahl)

- lowest order: $\mathbf{M}^{(k)}_{\text{Higgs}}(p^2)\Big|_{k\,=\,0} = \mathbf{M}^{(0)}_{\text{Higgs}}$ , diagonal,
- higher order: $\mathbf{M}^{(k)}_{\text{Higgs}}(p^2)\Big|_{k\,\geq\,1} = \mathbf{M}^{(0)}_{\text{Higgs}} - \sum_{j\,=\,1}^{k} \widehat{\boldsymbol{\Sigma}}^{(j)}_{\text{Higgs}}(p^2)$ ,

  shift by renormalized self-energies (here for the MSSM)

$$\widehat{\boldsymbol{\Sigma}}^{(j)}_{\text{Higgs}}(p^2) = \begin{pmatrix} \hat{\Sigma}^{(j)}_{h}(p^2) & \hat{\Sigma}^{(j)}_{hH}(p^2) & \hat{\Sigma}^{(j)}_{hA}(p^2) & \hat{\Sigma}^{(j)}_{hG}(p^2) & \hat{\Sigma}^{(j)}_{hZ}(p^2) \\ \hat{\Sigma}^{(j)}_{hH}(p^2) & \hat{\Sigma}^{(j)}_{HH}(p^2) & \hat{\Sigma}^{(j)}_{HA}(p^2) & \hat{\Sigma}^{(j)}_{HG}(p^2) & \hat{\Sigma}^{(j)}_{HZ}(p^2) \\ \hat{\Sigma}^{(j)}_{hA}(p^2) & \hat{\Sigma}^{(j)}_{HA}(p^2) & \hat{\Sigma}^{(j)}_{AA}(p^2) & \hat{\Sigma}^{(j)}_{AG}(p^2) & \hat{\Sigma}^{(j)}_{AZ}(p^2) \\ \hat{\Sigma}^{(j)}_{hG}(p^2) & \hat{\Sigma}^{(j)}_{HG}(p^2) & \hat{\Sigma}^{(j)}_{AG}(p^2) & \hat{\Sigma}^{(j)}_{GG}(p^2) & \hat{\Sigma}^{(j)}_{ZG}(p^2) \\ \hat{\Sigma}^{(j)}_{hZ}(p^2) & \hat{\Sigma}^{(j)}_{HZ}(p^2) & \hat{\Sigma}^{(j)}_{AZ}(p^2) & \hat{\Sigma}^{(j)}_{GZ}(p^2) & \hat{\Sigma}^{(j)}_{ZZ}(p^2) \end{pmatrix},$$

- minimization at higher order: necessary condition $\frac{\partial V^{(k)}_{H}}{\partial \phi} = 0$ ,

  typically: substitute parameters of mass dimension 2 in potential by

  tadpole contributions (linear terms in $\phi$) $\to T^{(j)}_{h}, T^{(j)}_{H}, T^{(j)}_{A}$ ,

  cause for Goldstone-Boson Catastrophe (see talk by M. Goodsell) ,

  (for vacuum stability in SUSY see talks by W. G. Hollik and E. Bagnaschi) .

charged current at one-loop order in $R_\xi$ gauge:

$$\Sigma_{ij}^{(1)}(p^2) \supset \frac{g_{h_i WW}\, g_{h_j WW}}{4\, M_W^4} \left[m_i^2\, m_j^2 - p^4\right] B_0\left(p^2, \xi\, M_W^2, \xi\, M_W^2\right)$$
$$+ \frac{2\, \Re\left[g_{h_i H^+ W}\, g^*_{h_j H^+ W}\right]}{M_W^2} \left[m_{H^\pm}\left(2\, p^2 - m_i^2 - m_j^2\right) + m_i^2\, m_j^2 - p^4\right] B_0\left(p^2, m_{H^\pm}^2, \xi\, M_W^2\right)$$
$$+ \ldots A_0\left(\xi\, M_W^2\right),$$

gauge-invariant, strictly perturbative poles at two-loop order:

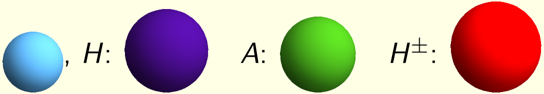
$$x_i^2 = m_i^2 \qquad \text{(0L)}$$
$$+ \Sigma_{ii}^{(1)}(m_i^2) \qquad \text{(1L)}$$
$$+ \Sigma_{ii}^{(2)}(m_i^2) + \Sigma_{ii}^{(1)}(m_i^2)\, \Sigma_{ii}^{(1)\,\prime}(m_i^2) + \sum_{j \neq i} \frac{1}{m_i^2 - m_j^2}\, \Sigma_{ij}^{(1)}(m_i^2)\, \Sigma_{ji}^{(1)}(m_i^2)\,, \qquad \text{(2L)}$$

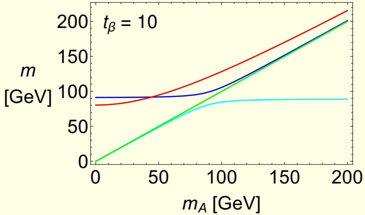
so far, no full two-loop result for BSM available (wait for tomorrow) .

# Higgs particles in the MSSM

- lowest order mass eigenstates:

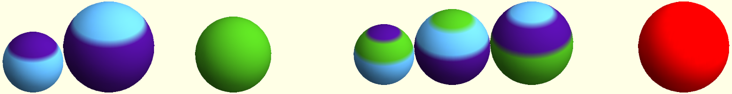
| *CP* even | *CP* odd | charged |
|---|---|---|
| $h$: , $H$: | $A$: | $H^{\pm}$: |

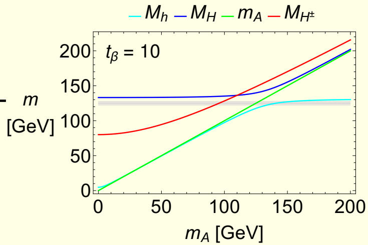





- higher orders: off-diagonal entries in $\hat{\mathbf{\Sigma}}^{(j)}_{\text{Higgs}}(p^2)$ induce mixing,

| real MSSM | | complex MSSM | both |
|---|---|---|---|
| *CP* even | *CP* odd | *CP* mixed | charged |





# Contents

five massive, physical Higgs bosons $h$, $H$, $A$, $H^{\pm}$

three massless, unphysical Goldstone bosons $G$, $G^{\pm}$
(only acquire masses by gauge-fixing)

$$\begin{pmatrix} h \\ H \end{pmatrix} = \mathbf{D}_{\alpha} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}, \quad \begin{pmatrix} A \\ G \end{pmatrix} = \mathbf{D}_{\beta_n} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \quad \begin{pmatrix} H^{\pm} \\ G^{\pm} \end{pmatrix} = \mathbf{D}_{\beta_c} \begin{pmatrix} \phi_1^{\pm} \\ \phi_2^{\pm} \end{pmatrix}$$

with rotation matrix

$$\mathbf{D}_x = \begin{pmatrix} -\sin(x) & \cos(x) \\ \cos(x) & \sin(x) \end{pmatrix},$$

applying minimization conditions (i. e. vanishing tadpoles):

$$\beta_n = \beta\,, \qquad \beta_c = \beta\,, \qquad \tan(2\,\alpha) = \frac{m_A^2 + M_Z^2}{m_A^2 - M_Z^2} \tan(2\,\beta)\,.$$

two point vertex function at one-loop order

$$\widehat{\mathbf{\Gamma}}^{(1)}_{hHA}\left(p^2\right) = i\left[p^2\mathbf{1} - \mathbf{M}^{(1)}_{hHA}\left(p^2\right)\right]$$

with mass matrix at one-loop order

$$\mathbf{M}^{(1)}_{hHA}\left(p^2\right) = \mathbf{M}^{(0)}_{hHA} - \widehat{\mathbf{\Sigma}}^{(1)}_{hHA}\left(p^2\right)$$

$\Rightarrow$ physical mass eigenstates at the tree level: $h,\ H,\ A$ with masses $m_h,\ m_H,\ m_A$
physical mass eigenstates at one-loop order: $h_1,\ h_2,\ h_3$ with masses $m_{h_1},\ m_{h_2},\ m_{h_3}$

$$\text{since } \mathbf{M}^{(1)}_{hHA}\left(m^2_{h_i}\right) \nsim \mathbf{M}^{(1)}_{hHA}\left(m^2_{h_j}\right),\ m_{h_i} \neq m_{h_j}$$

$$\nexists \text{ orthogonal } \mathbf{U} \;\middle|\; \left(h_1,\ h_2,\ h_3\right) = \mathbf{U}\left(h,\ H,\ A\right)$$

i. e. no orthogonal $\mathbf{U}$ diagonalizes $\widehat{\mathbf{\Gamma}}^{(1)}_{hHA}(p^2)$ for all $p^2$

however, for each $p^2$ we can find orthogonal $\mathbf{U}^{p^2}$ which diagonalizes $\widehat{\mathbf{\Gamma}}^{(1)}_{hHA}(p^2)$

particularly: determine $\mathbf{U}^{m^2_{h_1}}$, $\mathbf{U}^{m^2_{h_2}}$, $\mathbf{U}^{m^2_{h_3}}$

construct matrix

$$\mathbf{Z}^{\text{mix}} = \left(1,\ 0,\ 0\right) \mathbf{U}^{m^2_{h_1}} + \left(0,\ 1,\ 0\right) \mathbf{U}^{m^2_{h_2}} + \left(0,\ 0,\ 1\right) \mathbf{U}^{m^2_{h_3}}$$

indeed:

$$\left[\mathbf{M}^{(1)}_{hHA}\left(m^2_{h_i} - i\,\Gamma_{h_i}\, m_{h_i}\right)\right]_{kl} \mathbf{Z}^{\text{mix}}_{il} = \left(m^2_{h_i} - i\,\Gamma_{h_i}\, m_{h_i}\right) \mathbf{Z}^{\text{mix}}_{ik}$$

# Z matrix, normalization

kinetic term of effective Lagrangian for loop-corrected Higgs bosons:

$$\mathbf{Z}_{il}^{\text{mix}} \left[ \left. \frac{d(\widehat{\boldsymbol{\Delta}}_{hHA}^{(1)})^{-1}}{dp^2} \right|_{p^2 = m_{h_i}^2 - i\, \Gamma_{h_i}\, m_{h_i}} \right]_{kl} \mathbf{Z}_{ik}^{\text{mix}} = \mathbf{Z}_{il}^{\text{mix}} \left[ \mathbf{1} + \left. \frac{d\widehat{\boldsymbol{\Sigma}}_{hHA}^{(1)}}{dp^2} \right|_{p^2 = m_{h_i}^2 - i\, \Gamma_{h_i}\, m_{h_i}} \right]_{kl} \mathbf{Z}_{ik}^{\text{mix}} = 1$$

$$\Rightarrow \quad \mathbf{Z}_i^{\text{fac}} = \left\{ \mathbf{Z}_{il}^{\text{mix}} \left[ \mathbf{1} + \left. \frac{d\widehat{\boldsymbol{\Sigma}}_{hHA}^{(1)}}{dp^2} \right|_{p^2 = m_{h_i}^2 - i\, \Gamma_{h_i}\, m_{h_i}} \right]_{kl} \mathbf{Z}_{ik}^{\text{mix}} \right\}^{-1}$$

we derive the **Z** matrix

$$\mathbf{Z}_{il} = \sqrt{\mathbf{Z}_i^{\text{fac}}}\, \mathbf{Z}_{il}^{\text{mix}} ,$$

popular approximations:

- diagonalize $\widehat{\boldsymbol{\Gamma}}_{ij}(p^2 = 0)$ by orthogonal matrix $\mathbf{U}^0$ ,
- diagonalize $\widehat{\boldsymbol{\Gamma}}_{ij}\left(p^2 = \frac{1}{2}\left(m_i^2 + m_j^2\right)\right)$ by orthogonal matrix $\mathbf{U}^m$ .

example: $H \to t\bar{t}$ in decoupling limit, $M_H \sim m_{H^\pm} \gg M_{\text{EW}}$,
consider only top-, bottom Yukawa couplings $y_t, y_b$,
derive full one-loop amplitude $\mathcal{A}^{1\text{L}}_{H_{\text{phys}}tt}$ from strict loop expansion of

$$\mathcal{A}^{\text{vert}}_{H_{\text{phys}}tt} = \mathbf{Z}_{Hh_i}\,\mathcal{A}^{\text{vert}}_{h_i tt} \to 1 \cdot \mathcal{A}^{1\text{L}}_{Htt} + \left(1 - \frac{1}{2}\,\frac{\mathrm{d}\hat{\Sigma}^{(1)}_{HH}}{\mathrm{d}p^2}\bigg|_{p^2 \sim m^2_{H^\pm}}\right)\mathcal{A}^{\text{tree}}_{Htt} - \frac{\hat{\Sigma}^{(1)}_{Hh}}{p^2 - m_h^2}\bigg|_{p^2 \sim m^2_{H^\pm}}\,\mathcal{A}^{\text{tree}}_{htt}\,,$$

we find

$$\frac{\mathcal{A}^{1\text{L}}_{Htt}}{\mathcal{A}^{\text{tree}}_{Htt}} = -\frac{1}{16\,\pi^2}\,y_b^2\,c_\beta^2\,\log\frac{m^2_{H^\pm}}{M^2_{\text{EW}}}\,,$$

$$-\frac{1}{2}\,\frac{\mathrm{d}\hat{\Sigma}_{HH}}{\mathrm{d}p^2}\bigg|_{p^2 \sim m^2_{H^\pm}} = \frac{3}{32\,\pi^2}\left[y_t^2\,c_\beta^2 + y_b^2\,s_\beta^2\right]\log\frac{m^2_{H^\pm}}{M^2_{\text{EW}}}\,,$$

$$-\frac{\hat{\Sigma}_{Hh}}{p^2 - m_h^2}\bigg|_{p^2 \sim m^2_{H^\pm}} = -\frac{3}{16\,\pi^2}\left[y_t^2 - y_b^2\right]s_\beta\,c_\beta\,\log\frac{m^2_{H^\pm}}{M^2_{\text{EW}}}\,,$$

each term contributes large logarithms,

cannot be captured by $\mathbf{U}^0$ or $\mathbf{U}^m$,

same result for:
$A \to t\bar{t}$, $H^+ \to t\bar{b}$ .

# Summary

- Higgs-mass prediction in SUSY models
- no on-shell renormalization of all states possible
- loop corrections required to compete with measurement
- loop-induced mixing by off-diagonal self-energies
- non-unitary mixing matrix at higher order
- strict loop expansion necessary to get correct logarithms: contributions from genuine vertex and mixing
- popular approximations fail