The Goldstone boson catastrophe arising in higher-order corrections to Higgs boson masses, and its possible solutions

[J. Braathen and MDG, 1609.06977] [J. Braathen, MDG and F. Staub, 1706.05372]
[MDG and S. Paßehr, to appear]

Mark D. Goodsell

LPTHE
LABORATOIRE DE PHYSIQUE
THEORIQUE ET HAUTES ENERGIES

cnrs

SORBONNE UNIVERSITÉ

PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR
ANR

Overview

- GBC
- TLDR

Classic BSM perspective on the Higgs mass

For many years the standard example has been the MSSM for $\sim$ TeV-scale SUSY:

- Quartic predicted to be determined entirely by gauge couplings at tree level – in large M_H limit have

$$\lambda = \frac{1}{8}(g_Y^2 + g_2^2)\cos^2 2\beta = \frac{M_Z^2}{2v^2}\cos^2 2\beta$$

- Hence $\rightarrow m_h(\text{tree}) \leqslant M_Z$
- $\delta m_h^2(\text{loops}) \geqslant (125\text{GeV})^2 - (M_Z)^2 \geqslant (86\text{GeV})^2 \gtrsim m_h^2(\text{tree})$
- Can have $\delta m_h(\text{two loops}) \lesssim$ 10 GeV $\rightarrow \delta m_h^2(\text{two loops}) \sim 15\% m_h^2$!
- While at three-loop order, have $\delta m_h \sim$ few hundred MeV, $\rightarrow \delta m_h^2(\text{three loops}) \lesssim 1\% m_h^2$

This has prompted much work on precision calculations of the Higgs mass in BSM theories.

Equivalently we need two-loop threshold corrections in the EFT approach, and this can be extracted from the same calculations by matching pole masses.

But there is a technical barrier for any theory other than the gaugeless limit of the MSSM: the Goldstone Boson Catastrophe.

This includes the Standard Model where it was studied by [Martin, '14], [Elias-Miro, Espinosa, Konstandin, '14]!

- Consider the Abelian Goldstone Model of one complex scalar $\Phi = \frac{1}{\sqrt{2}}(v + h + iG)$ and tree-level potential

$$V = \mu^2 |\Phi|^2 + \lambda |\Phi|^4.$$

- This is a nice prototype for the Standard Model in Landau gauge – but a subtle difference is that the Goldstone boson is physical!
- At tree level, the tadpole equation gives $\mu^2 + \lambda v^2 = 0$, and the masses are $m_G^2 = \mu^2 + \lambda v^2$, $M_h^2 = \mu^2 + 3\lambda v^2$.
- Expand the tree-level potential:

$$\begin{aligned} V^{(0)} &\supset hv(\mu^2 + \lambda v^2) + \frac{1}{2}(\mu^2 + \lambda v^2) G^2 + \frac{1}{2}(\mu^2 + 3\lambda v^2) h^2 + ... \\ &\equiv hv m_G^2 + \frac{1}{2} m_G^2 G^2 + \frac{1}{2}(m_G^2 + m_h^2) h^2 + ... \end{aligned}$$

- But we use $m_G^2 \equiv \mu^2 + \lambda v^2$ to calculate loops, and once we include loop corrections we have

$$0 = \mu^2 + \lambda v^2 + \frac{1}{v} \frac{\partial \Delta V}{\partial h}\bigg|$$

Treatment of tadpoles

The "Martin" approach:

- Fix vacuum expectation values and adjust masses order by order, cf $\mu^2 = -\lambda v^2 - \frac{1}{v}\frac{\partial \Delta V}{\partial h}$ in SM:

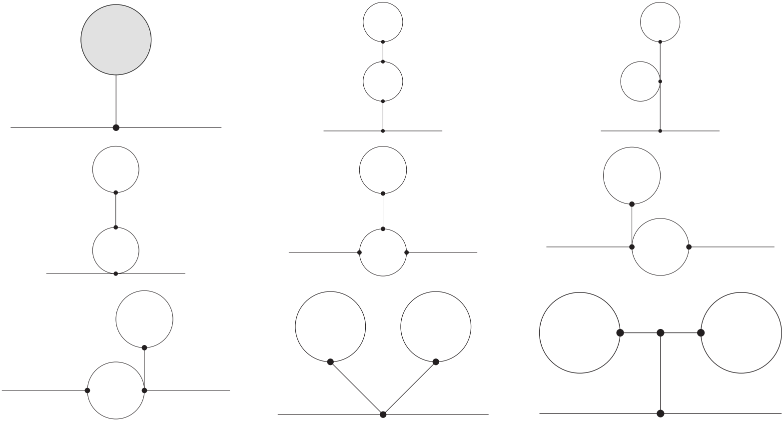
$$m_G^2 = \mathcal{O}(1-\text{loop})$$

  may be negative!
- Gauge invariance of result is not manifest
- Avoids all "internal" diagrams, in principle just need to calculate the genuine two-loop tadpoles ...
- ... except that, as we saw, to solve the GBC/respect perturbation series, we need some extra equivalent contributions.

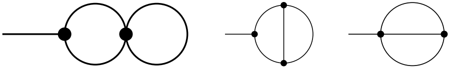
Alternative approaches that are not often used in BSM: Jaergerlehner-Fleischer (see Rui Santos' talk yesterday) $\leftrightarrow$ vev renormalisation; and $\overline{\text{MR}}$ Kniehl et al:

- Work with tree-level expectation values and masses: $m_G^2 = 0$
- Include all internal (but still 1PI) diagrams)
- Now we would need to include reducible diagrams in all processes, e.g. Z self-energy ...

Tadpoles



The blob represents the three genuine two-loop tadpole topologies:



Formulation of the GBC

Recall that

$$m_G^2 = \pm\mathcal{O}(1 - \text{loop}) \qquad \text{or} \qquad m_G^2 = 0$$

At one loop, this is benign enough:

- For tadpoles proportional to hGG coupling

$$T \sim \lambda v \int \frac{d^d q}{q^2 - m_G^2} \propto m_G^2 (\overline{\log}\, m_G^2 - 1)$$

- For masses, the self-energy diagrams give

$$\Pi \sim \lambda^2 v^2 \int \frac{d^d q}{(q^2 - m_G^2)((q+p)^2 - m_G^2} \propto (\overline{\log}\, p^2 - 2)$$

- So we see that we need to include momentum at one loop for this model (or the Standard Model in Landau gauge) – more later.

Beyond one loop

At two loops we find that the tadpole equations give (with $A(x) \equiv x(\log x/Q^2 - 1)$)

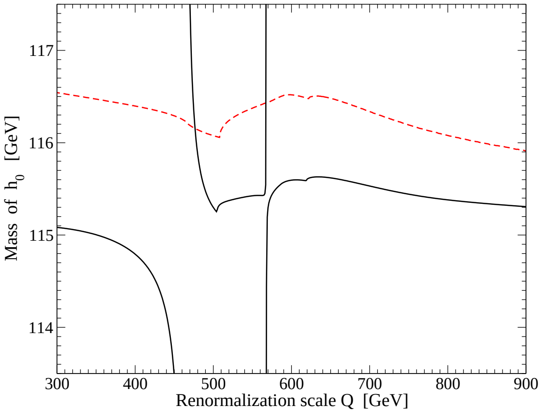
$$0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\left[3A(m_h^2) + A(m_G^2)\right]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\log \frac{m_G^2}{Q^2}}{(16^2)^2}\left[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{M_h^2} A(M_h^2)\right] + \overbrace{\ldots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}}$$

The problem then extends to two-loop self energies, and becomes even worse for three-loop tadpoles etc.

GB Catastrophe in the MSSM

The problem was identified early on when trying to use the effective potential approach on the full MSSM potential – From S. Martin [hep-ph/0211366]:



Solid line: including EW effects, dashed line without

This shows both the GB catastrophe near $Q = 568$ GeV and the 'Higgs boson catastrophe' near 463 GeV.

So what happened after 2002?

- Martin's calculation was in any case not publically available, nor were there closed-form expressions.
- Instead, until recently almost all spectrum generators for the MSSM (`SPheno`, `SoftSUSY`, `FeynHiggs`) used routines from P. Slavich for $\alpha_s \alpha_t$ and (Yukawa^4) corrections – performed in the gaugeless limit at two loops.
- But: in the MSSM the quartic coupling is given at tree-level by the gauge couplings:

$$\mathcal{L} \supset -\frac{g_Y^2 + g_2^2}{8} (|H_u|^2 - |H_d|^2)^2 \underset{g_Y, g_2 \to 0}{\longrightarrow} 0.$$

- This means that the Goldstone boson does not couple to the Higgs, so the dangerous terms are absent!
- For a long time the problem remained neglected: (full) electroweak corrections were never computed beyond the SM

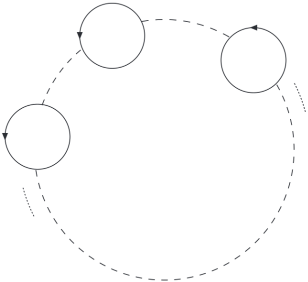
Resummation

A solution for the Standard Model was proposed in [Martin, ’14], [Elias-Miro, Espinosa, Konstandin, ’14]:

The daisy diagram contributes the most singular term at any fixed loop order; it has the most soft Goldstone propagators – each term looks like

$$\int d^4q \frac{(\Pi_{GG}(q^2))^n}{(q^2 - m_G^2)^n} \sim (\Pi_{GG}(0))^n \frac{\partial^n f(m_G^2)}{\partial(m_G^2)^n}$$

$$f(m_G^2) = -\frac{i}{2} C \int d^d q \log(-q^2 + m_G^2)$$



- $f(x) \equiv \frac{1}{4} x^2 (\overline{\log}\, x - \frac{3}{2})$.
- But if we sum it to all orders, then we will just find $f(m_G^2 + \Pi_{GG}(0))$

Both papers agree that we should use instead use the resummed potential

$$\hat{V}_{\text{eff}} \equiv V_{\text{eff}} + \frac{1}{16\pi^2}\left[f(m_G^2 + \Delta) - \sum_{n=0}^{l-1} \frac{\Delta^n}{n!} \left(\frac{\partial}{\partial m_G^2}\right)^n f(m_G^2)\right].$$

Generalising

If we want to apply this to general theories, however, we have two problems:

1. Identifying the Goldstone boson(s) among the scalars: in general the fields can mix!
2. Taking derivatives of the potential as a function of masses and couplings generally means taking derivatives of mixing matrices.

[Martin, Kumar '16] applied this to find the tadpoles in the MSSM with CP conservation, where they could use 2×2 matrices and do all the derivatives explicitly.

We can do better by taking all of the derivatives <u>implicitly</u>.

We can do better still by adopting a different solution.

On-shell scheme

[Braathen, MDG '16]

We saw that we can cure the IR divergences by resumming the Goldstone boson propagators, so that the effective mass in the loop functions became $m_G^2 + \Delta = 0$. But we can do this more directly by just putting the Goldstone boson on shell:

$$(m_G^2)^{\text{run.}} \equiv (m_G^2)^{\text{OS}} - \Pi_{GG}((m_G^2)^{\text{OS}})$$

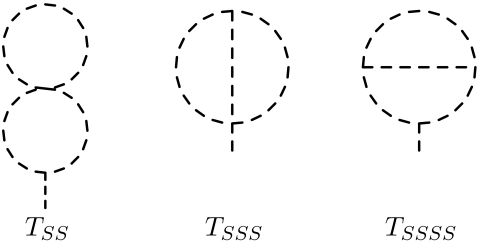
We can do this directly in the tadpole equations – and also the self-energies! So then there should be no need to take derivatives of couplings ... exactly what we want. For example, applying the above shift to the one loop tadpole gives a two-loop correction:

$$\frac{\partial V}{\partial v} \supset \frac{\lambda v}{16\pi^2} A(m_G^2) = \frac{\lambda v}{16\pi^2}\left[\underbrace{A((m_G^2)^{\text{OS}}}_{\to 0} - \underbrace{\Pi_{GG}((m_G^2)^{\text{OS}}) \log \frac{(m_G^2)^{\text{OS}}}{Q^2}}_{\text{cancels divergent part}} + \underbrace{...}_{3-\text{loop}}\right]$$

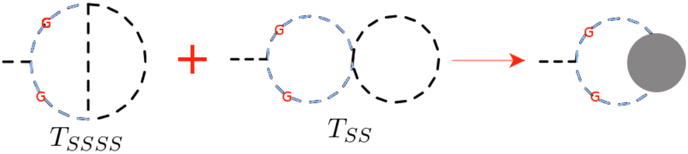
We also see that $\Pi_{GG}((m_G^2)^{\text{OS}}) = \Pi_g(0)$ (at least at this loop order) automatically!

Illustration

To see why this works, let us look at the scalar-only case. There are three classes of tadpole diagrams:

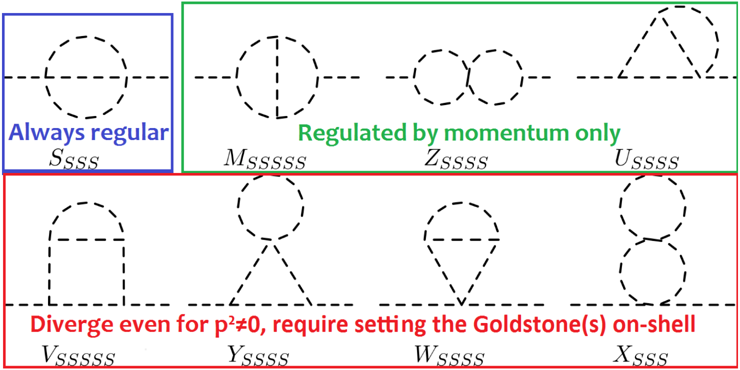


We find that the divergences only come from the T_{SS} and T_{SSSS} topologies, and they correspond to a Goldstone self-energy as a subdiagram and exactly cancel out against the on-shell shift:



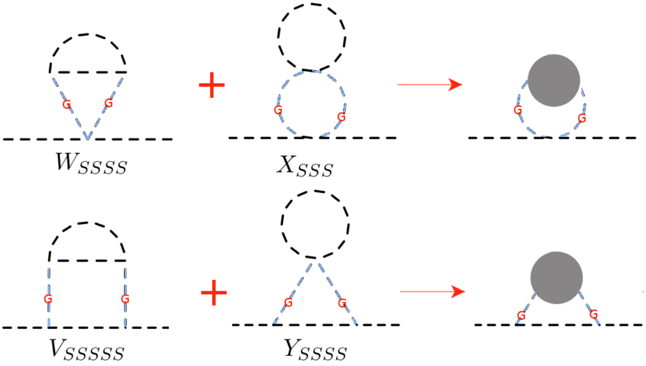
Mass diagrams

We also find that we can apply our on-shell scheme to the cancellation of divergences in self-energies! This seemed hopeless in the former approaches ... We can divide the topologies into three categories:



Mass diagram divergences

Again we find that the divergences in m_G^2 arise from Goldstone boson propagator subdiagrams:



... and once more the one-loop shifts from our on-shell scheme exactly cancel the divergences (but leave a finite momentum-dependent piece).

Generalised effective potential limit

Since we see that there are classes of diagrams that are divergent when the $p^2 \equiv s \neq 0$ and the Goldstone bosons are on-shell, the obvious response is that we cannot avoid using momentum dependence – but this is computationally expensive. Instead, we can expand the self-energies as:

$$\Pi^{(2)}_{ij}(s) = \frac{\overline{\log}(-s)}{s}\Pi^{(2)}_{-1\,l,ij} + \frac{1}{s}\Pi^{(2)}_{-1,ij} + \Pi^{(2)}_{l^2,ij}\,\overline{\log}^2(-s) + \Pi^{(2)}_{l,ij}\,\overline{\log}(-s) + \Pi^{(2)}_{0,ij} + \sum_{k=1}^{\infty}\Pi^{(2)}_{k,ij}\frac{s^k}{k!}$$

If we discard all terms $\mathcal{O}(s)$ and higher, we have a generalised effective potential approximation! We can find closed forms for the singular terms, e.g.

$$U(0,0,0,u) = (\overline{\log}\,u - 1)\,\overline{\log}(-s) - \frac{\pi^2}{6} + \frac{5}{2} - 2\,\overline{\log}\,u - \frac{1}{2}\overline{\log}^2\,u + \mathcal{O}(s).$$

This turns out to be a very good approximation, e.g. even in the Standard Model:

	SARAH/SPheno		SMH (code by Martin & Robertson)	
ξ	1	0.01	0	
2ℓ momentum dependence	partial $s = m_h^2\|^{\text{tree}}$	partial $s = m_h^2\|^{\text{tree}}$	full iterative	none $s = 0$
$m_h^{2\ell}$ (GeV)	125.083	125.134	125.176	125.121

... alternatively it would be good to have the full momentum dependence!

"Consistent Tadpoles"

[Braathen, MDG, Staub '17]

Another way to solve the problem: expand the masses perturbatively $\rightarrow$

$$\mu^2 = -\lambda v^2 - \frac{1}{v}\frac{\partial \Delta V(\mu)}{\partial v}$$
$$= (\mu^2)^{\text{tree}} - \frac{1}{v}\frac{\Delta V((\mu^2)^{\text{tree}})}{\partial v} + \frac{1}{v^2}\left[\left(\frac{\partial^2 \Delta V}{\partial v \partial \mu^2}\right)\left(\frac{\partial \Delta V}{\partial v}\right)\right]_{\mu^2=(\mu^2)^{\text{tree}}}$$

In our example only m_G and m_h depend on μ, and it only enters the loop functions through those masses, so writing $T \equiv \frac{\partial \Delta V}{\partial v} \equiv T^{(1)} + T^{(2)} + \ldots$ and $\mu^2 = (\mu^2)^{\text{tree}} + (\mu^2)^{(1)} + (\mu^2)^{(2)}$ we have

$$(\mu^2)^{(1)} = -\frac{1}{v}T^{(1)}\Big|_{\mu^2=-\lambda v^2}$$
$$(\mu^2)^{(2)} = \underbrace{-\frac{1}{v}T^{(2)}\Big|_{\mu^2=-\lambda v^2} + \frac{1}{v}T^{(1)}\left[\frac{\partial T^{(1)}}{\partial m_G^2}\right.}_{\text{IR divergences cancel}} + \left.\underbrace{\frac{\partial T^{(1)}}{\partial m_h^2}}_{\text{IR safe}}\right]_{\mu^2=-\lambda v^2}$$

This is termed "consistently solving the tadpole equations." But it solves the problem in the same way as our on-shell approach!

Key difference is we get $(1-\text{loop})^2$ finite shifts to tadpoles and self-energies for all terms that depend on μ^2 (or equivalent parameter):

$$\Delta T^{(2)} = -\frac{1}{v} T^{(1)} \left[\frac{\partial T^{(1)}}{\partial m_G^2} + \frac{\partial T^{(1)}}{\partial m_h^2} \right]$$

and also for self-energies:

$$\Delta \Pi^{(2)} \supset -\frac{1}{v} T^{(1)} \left[\frac{\partial \Pi^{(1)}}{\partial m_G^2} + \frac{\partial \Pi^{(1)}}{\partial m_h^2} \right]$$

New: this is equivalent to including a counterterm for the μ^2 parameter modulo an important subtlety in Landau gauge/gaugeless limit.

It is similar – but not identical – to including tadpole/vev counterterms (Jaegerlehner-Fleischer), or working at tree-level and including tadpole diagrams (`mr`/Kniehl et al).

All three new approaches solve the GBC in the same way.

Two different catastrophes

Using “Martin-style” tadpole conditions the potential has at best a phase/singularities:

- We must work with non-zero but small Goldstone boson masses (in Landau gauge).
- We also get a “Higgs boson catastrophe”:

$$2\lambda v^2 + \frac{1}{v}\frac{\partial \Delta V}{\partial h} < 0 \rightarrow (m_h^2)^{\text{tree}} < 0!$$

  This can even appear in other gauge choices!!
- Allows us/forces us to use the generic results of [Martin, 2003] which did not allow for infra-red divergences.

If we work with “consistent tadpole equations”:

- Exactly massless Goldstones, need to properly regularise the infra-red divergences.
- Work with small m_G^2 and expand $\rightarrow$ can use existing results in gaugeless limit
- **New work: Use dimensional regularisation to handle IR divergences!** Need to recompute the results for these cases to extract the $\frac{1}{\epsilon_{IR}} + \text{finite}$ pieces. This is the approach that we will take in future!

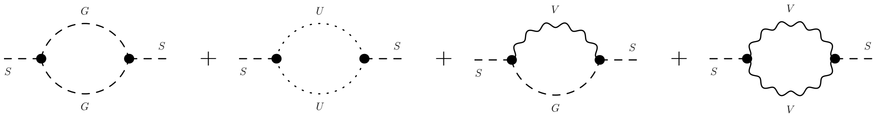
Gaugeless vs gauged diagrams

The second place it rears its head is in diagrams regulated by momenta.

- If we work in the gaugeless limit, or a theory with genuine Goldstone bosons, we have genuine IR singularities, recall:

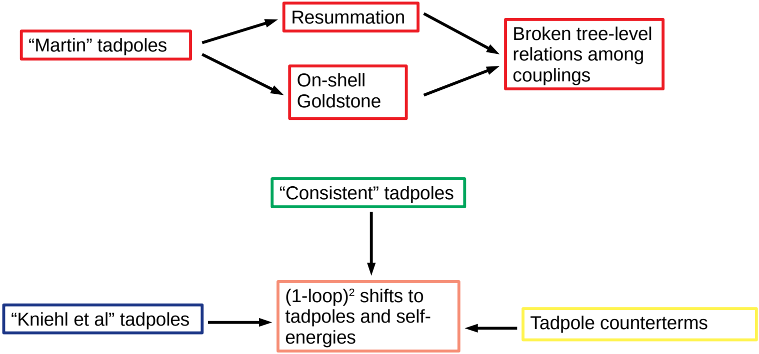
$$\Pi \sim \lambda^2 v^2 \int \frac{d^d q}{(q^2 - m_G^2)((q+p)^2 - m_G^2)} \propto (\overline{\log}\, p^2 - 2)$$

- $\longrightarrow$ use “Generalised effective potential” or full momentum dependence
- If we add gauge couplings, these should cancel out:



- $\longrightarrow$ can use dimensional regularisation to take care of $1/\epsilon_{IR}$ poles, or momentum dependence.

Summary

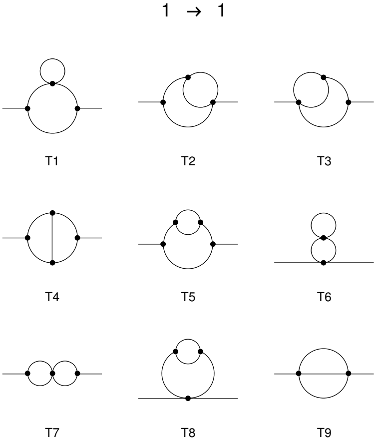


“Martin” tadpoles
Resummation
On-shell Goldstone
Broken tree-level relations among couplings
“Consistent” tadpoles
“Kniehl et al” tadpoles
(1-loop)2 shifts to tadpoles and self-energies
Tadpole counterterms

Calculating the full generic two-loop result

To go away from the gaugeless limit we need the full self-energies/tadpoles.

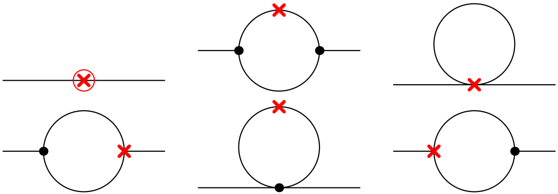
There are 9 irreducible self-energy topologies:



Topologies 2 and 3 are equal for real scalar self-energies.

Renormalising the result

- The typical approach to renormalisation in explicit models is to compute unrenormalised diagrams, counterterms, and insertions separately: we add topologies



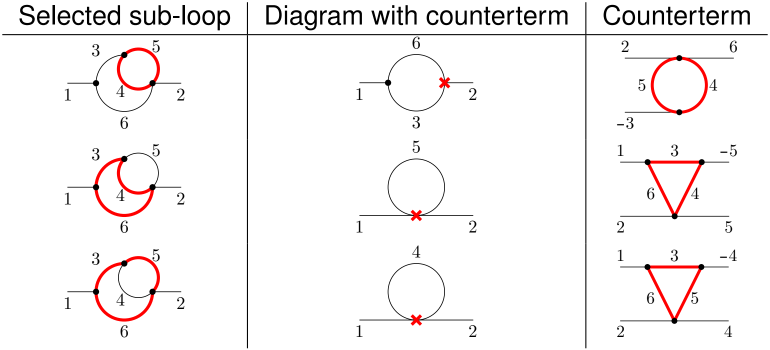
- This would mean giving the result only in terms of unrenormalised loop integrals, or the ϵ^0 pieces, and also just giving the result of these diagrams: the user would have to compute the counterterms for each model.
- This is simple, but inefficient: there are also many cancellations between these diagrams and the genuine two-loop integrals, in particular terms of the form

$$\frac{1}{\epsilon}\int d^d q \frac{1}{q^2 - m_1^2} \frac{1}{(q-p)^2 - m_2^2}$$

- Indeed it is known that the $\mathcal{O}(\epsilon)$ pieces of the subdivergences cancel out in the loop integrals when we use the basis of functions available in `TSIL`.

BPHZ method

Instead we use the BPHZ method of renormalising, where we subtract off the subdivergences:



The forest formula ensures that this is equivalent.

Classes level

Next we need to populate the topologies with fields and evaluate them. For this we need generic vertices; a generic QFT looks like

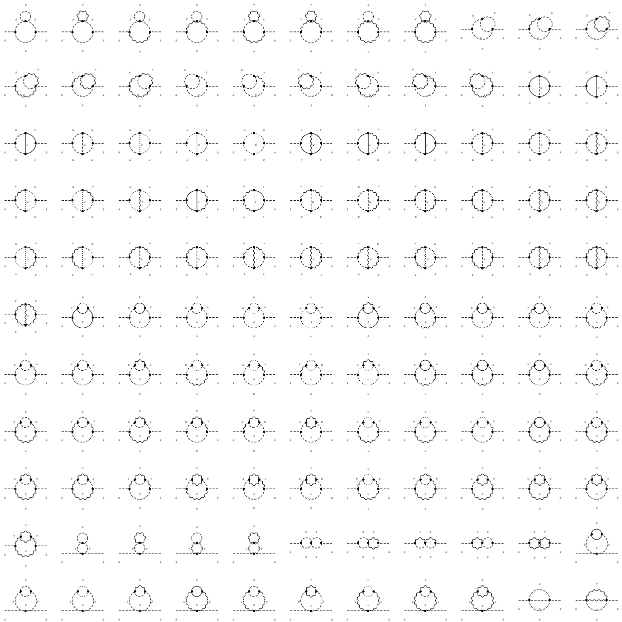
$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_{SF} + \mathcal{L}_{SV} + \mathcal{L}_{FV} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Sghost}} \,.$$

where

$$\begin{aligned}
\mathcal{L}_S &\equiv -\frac{1}{6}\, a_{ijk}\, \Phi_i\, \Phi_j\, \Phi_k - \frac{1}{24}\, \lambda_{ijkl}\, \Phi_i\, \Phi_j\, \Phi_k\, \Phi_l \,, \\
\mathcal{L}_{SF} &\equiv -\frac{1}{2}\, y^{IJk}\, \psi_I\, \psi_J\, \Phi_k - \frac{1}{2}\, y_{IJk}\, \overline{\psi}^I\, \overline{\psi}^J\, \Phi_k \,, \\
\mathcal{L}_{FV} &\equiv g^{aJ}_I\, A^a_\mu\, \overline{\psi}^I\, \overline{\sigma}^\mu\, \psi_J \,, \\
\mathcal{L}_{SV} &\equiv \frac{1}{2}\, g^{abi}\, A^a_\mu\, A^{\mu b}\, \Phi_i + \frac{1}{4}\, g^{abij}\, A^a_\mu\, A^{\mu b}\, \Phi_i\, \Phi_j + g^{aij}\, A^a_\mu\, \Phi_i\, \partial^\mu \Phi_j \,, \\
\mathcal{L}_{\text{gauge}} &\equiv g^{abc}\, A^a_\mu\, A^b_\nu\, \partial^\mu A^{\nu c} - \frac{1}{4}\, g^{abe}\, g^{cde}\, A^{\mu a}\, A^{\nu b}\, A^c_\mu\, A^d_\nu + g^{abc}\, A^a_\mu\, \omega^b\, \partial^\mu \overline{\omega}^c \,, \\
\mathcal{L}_{\text{Sghost}} &\equiv \xi\, \hat{g}^{abi}\, \Phi_i\, \overline{\omega}^a\, \omega^b \,.
\end{aligned}$$

This is in terms of real scalars and vectors, Weyl fermions, and ghosts.
Actually for the computer algebra we shall use four-component Majorana fermions.

Brute force



Evaluating

- Generate the amplitudes with `FeynArts`
- Evaluate the two-loop amplitudes in general gauge using `TwoCalc`. Gives results in terms of scalar integrals and tensors

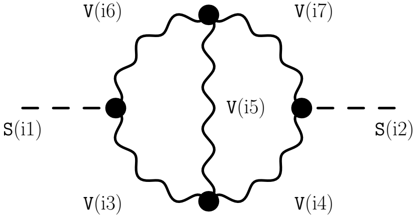
$$Y^{j_1 \cdots j_o}_{i_1 \cdots i_n} = \int \frac{d^d q_1 \, d^d q_2}{\left[\imath \, \pi^2 \, (2 \, \pi \, \mu)^{d-4}\right]^2} \frac{k^2_{j_1} \cdots k^2_{j_o}}{\left(k^2_{i_1} - m^2_{i_1}\right) \cdots \left(k^2_{i_n} - m^2_{i_n}\right)}$$

- We perform the BPHZ renormalisation in MS$'$ and DR$'$ schemes using our own code to calculate counterterms and match them into the insertions, and `FormCalc, OneCalc` to evaluate the insertion diagrams.
- We keep the result in an unexpanded form because the integral reduction differs depending on whether there are IR singularities/special values of the momenta/degenerate masses
- We have derived all the necessary integral reductions, either from in `TwoCalc`, by `TARCER`, or in many cases by hand, to reduce to a basis of integrals that can be evaluated in `TSIL`.

Gauge choice

We shall give the results explicitly in Feynman gauge:

- Not because of the GBC (this still exists for theories with genuine Goldstone bosons!)
- Nor reducing number of diagrams (Landau gauge has fewer classes)
- Because the expressions are much shorter: in particular

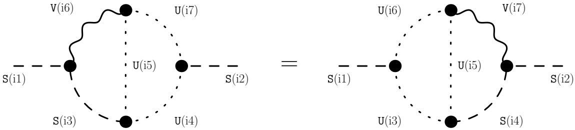


Gauge	Integrals
Feynman	6
Landau	896
R_ξ	924

However, we have the expressions for Landau/general gauge, which we will be able to use later to demonstrate gauge independence.

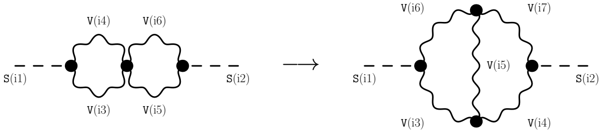
Simplifying

- Initially we have 121 self-energy (and 25 tadpole) diagrams.
- We can trivially reduce the number of self-energies to 92 from relating topologies 2 and 3 and also exchanges not identified by `FeynArts`:



- We can further reduce this to 58 classes! First we do this by exchanging quartic vector couplings for products of triple vector couplings:

$$i\frac{\partial^4 \mathcal{L}}{\partial A^{\mu_a}\partial A^{\mu_b}\partial A^{\mu_c}\partial A^{\mu_d}} = -2ig^{abe}g^{cde}\eta^{\mu_a\mu_b}\eta^{\mu_c\mu_d} - 2ig^{ace}g^{bde}\eta^{\mu_a\mu_c}\eta^{\mu_b\mu_d} - 2ig^{ade}g^{cbe}\eta^{\mu_a\mu_d}\eta^{\mu_c\mu_d}$$



Ghost busting

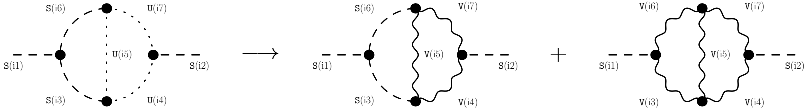
The ghost-ghost-vector couplings are trivially given by g^{abc}, but the scalar-ghost-ghost couplings are more subtle:

$$\hat{g}^{abi} = \frac{1}{2}\, g^{abi} - \frac{1}{2}\, g^{abc}\,(F_D)_i^c$$

where

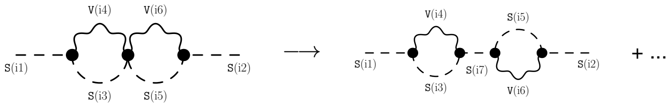
$$(F_D)_j^a = \begin{cases} 0\,, & a > N_G \\ 0\,, & j > N_G \\ m_a\,\delta_{aj}\,, & a, j \leqslant N_G \end{cases}$$

i.e. it depends on whether the scalar is a Goldstone or not!
But with some care we can bust all of the ghosts, e.g.

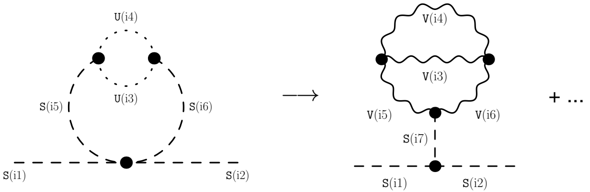


Special cases

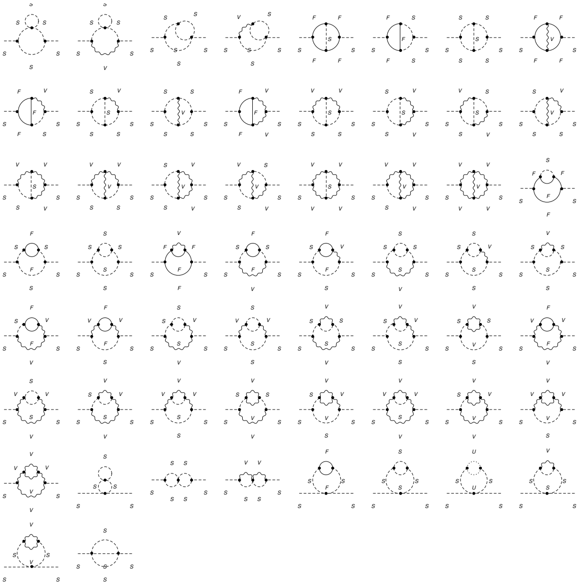
We find two amusing special cases which can either be left unreduced or lead to a non-1PI-irreducible topology:



or one with an “internal” propagator:



Remaining classes



Conclusions

Now have a complete generic two-loop calculation for scalar self energies, available as a package `TLDR` at

`http://tldr.hepforge.org`

But this is just the start:

- Needs to be implemented in code(s), in particular `SARAH`, and models, in particular the THDM and (N)MSSM.
- Will be useful for charged/coloured scalars, not just the Higgs/heavy neutral scalars.
- Can eliminate all Goldstone bosons from the calculation, sum with reducible diagrams to get an explicitly gauge-invariant result
- Same technology can easily produce vector and fermion self-energies.
- Can apply these to fixed-order or EFT calculation (through pole-matching).
- Essential part of calculation of Higgs decays, ...
- Longer term: also four-fermi interaction at zero momentum for muon decays ...