Higgs Boson Pair Production via Gluon Fusion: NLO QCD Corrections & the Top Mass Scheme

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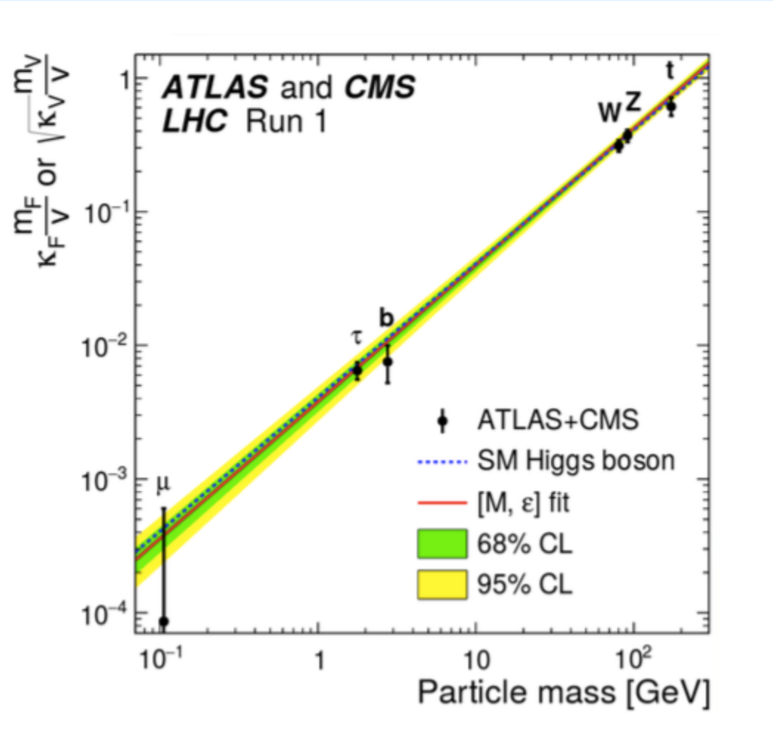
Outline

- Motivation
- Previous work
- NLO cross section
  - Virtual corrections
  - Real corrections
- Numerical analysis
  - Differential cross section
  - Total hadronic cross section
  - Factorisation/Renormalisation scale dependence
  - Top mass uncertainty
- Conclusions

Motivation

- Detection of a Higgs boson with a mass ~ 125 GeV
- Higgs mass, coupling strengths, spin and CP already determined
- *S*elf-coupling strength still unknown



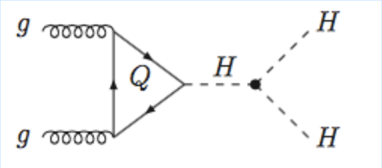
$$V(\phi) = \frac{\lambda}{2}\left\{|\phi|^2 - \frac{v^2}{2}\right\}^2$$

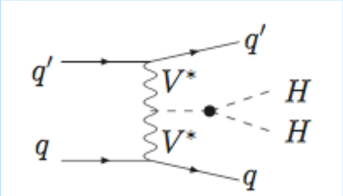
$$\lambda_{h^3} = 3\frac{m_h^2}{v}$$

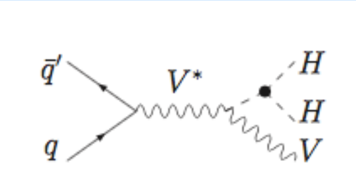
$$\lambda_{h^4} = 3\frac{m_h^2}{v^2}$$

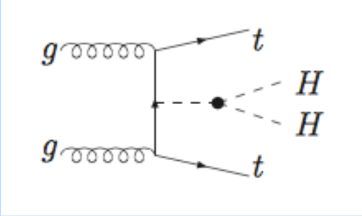
Higgs boson pair production

Production channels

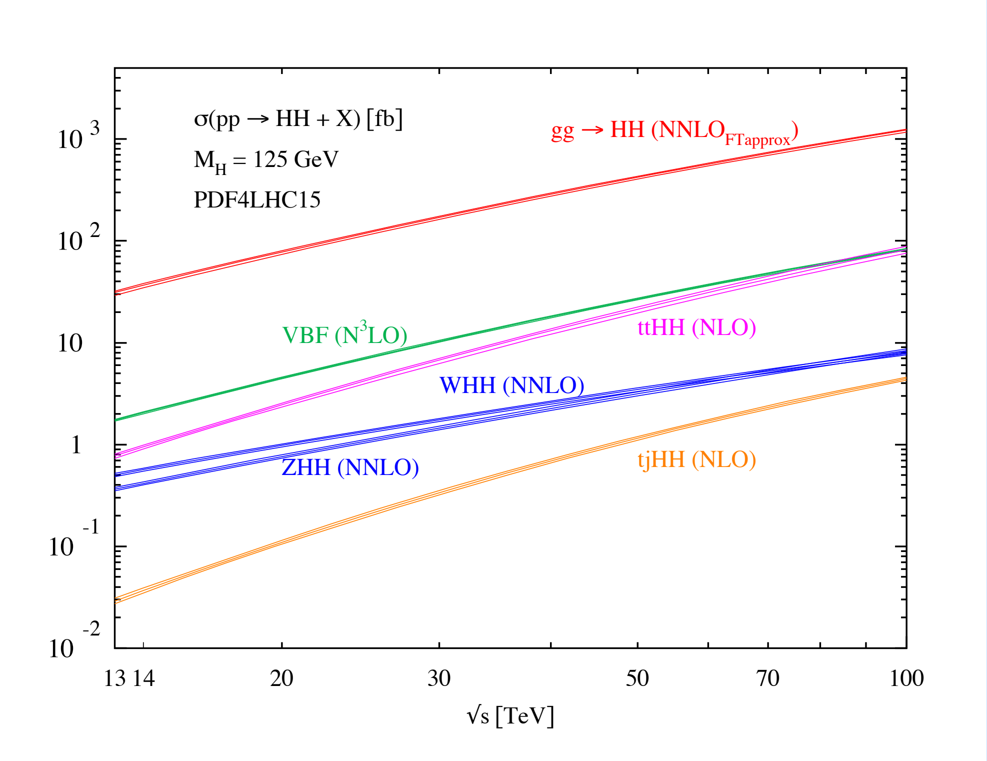








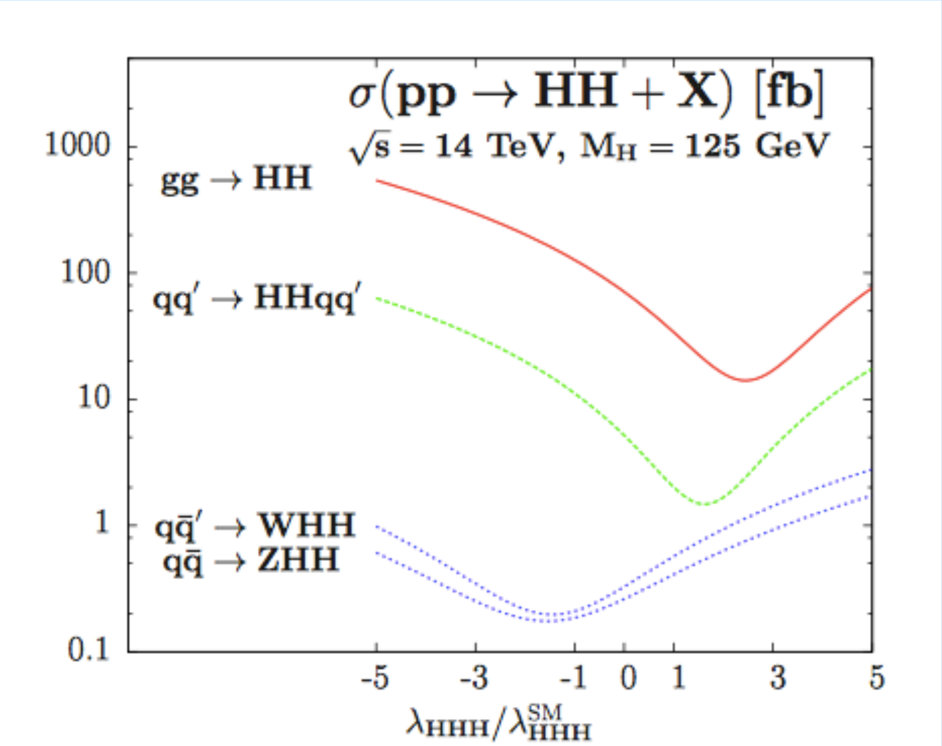
Cross sections



HH White Paper

Motivation

Uncertainties:



$$gg \to HH : \frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$

Baglio, Djouadi, Gröber,
Mühlleitner, Quevillon, Spira

Current Status

- Virtual & real (N)NLO QCD corrections in large top mass limit (HTL): ~100%

  Dawson,Dittmaier,Spira
  de Florian,Mazzitelli
  Grigo,Melnikov,Steinhauser

- Large top mass expansion: ~ ±10%

  Grigo, Hoff, Melnikov, Steinhauser

- NLO mass effects of the real NLO correction alone ~ -10 %

  Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

- NLO QCD corrections including the full top mass dependence: - 15 % NLO mass effects

  Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke

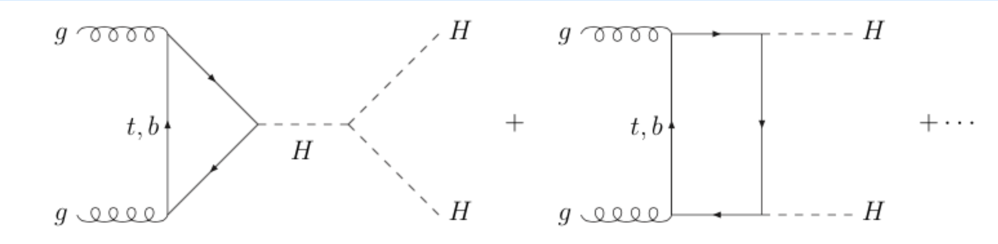
  Baglio,Campanario,SG,Mühlleitner, Spira,Streicher

- New expansion/extrapolation methods:
  - $1/m_t^2$ expansion & conformal mapping & Padé approximants — Gröber,Maier,Rauh
  - p_T^2 expansion — Bonciani,Degrassi,Giardino,Gröber
  - high-energy — Davies,Mishima,Steinhauser,Wellmann

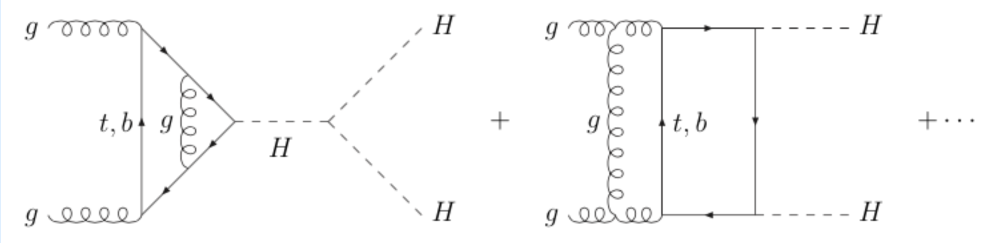
NLO Corrections

$$\sigma_{\rm NLO}(pp \to HH + X) = \sigma_{\rm LO} + \Delta\sigma_{\rm virt} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

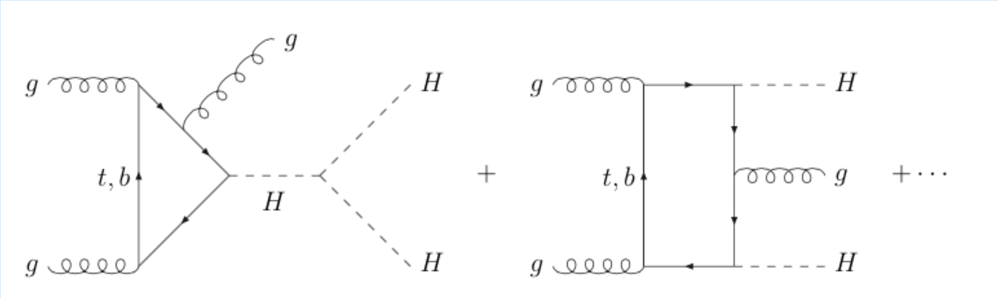
σ_{LO}:



$\Delta\sigma_{virt}$:



$\Delta\sigma_{ij}$:



NLO Corrections

$$\sigma_{\text{NLO}}(pp \to HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

$$\begin{aligned}
\sigma_{\text{LO}} &= \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \\
\Delta\sigma_{\text{virt}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \, C \\
\Delta\sigma_{gg} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -zP_{gg}(z) \log \frac{M^2}{\tau s} \right. \\
&\qquad \left. + d_{gg}(z) + 6[1 + z^4 + (1-z)^4] \left(\frac{\log(1-z)}{1-z} \right)_+ \right\} \\
\Delta\sigma_{gq} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s (1-z)^2} \right. \\
&\qquad \left. + d_{gq}(z) \right\} \\
\Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \, d_{q\bar{q}}(z)
\end{aligned}$$

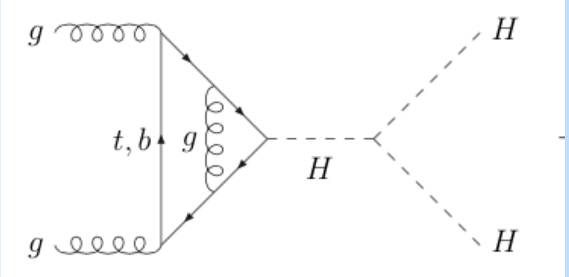
$$C \to \pi^2 + \frac{11}{2} + C_{\triangle\triangle}, \qquad d_{gg} \to -\frac{11}{2}(1-z)^3, \qquad d_{gq} \to \frac{2}{3}z^2 - (1-z)^2, \qquad d_{q\bar{q}} \to \frac{32}{27}(1-z)^3$$

Virtual Corrections

47 two-loop box diagrams + 8 triangular diagrams + 2 one-particle reducible diagrams

Triangular Diagrams

⟵ single Higgs case

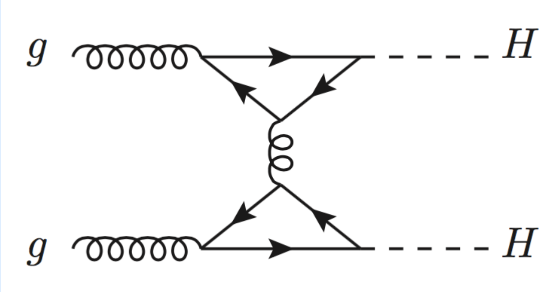


One-particle reducible diagrams

⟶ analytical results for $C_{\triangle\triangle}$

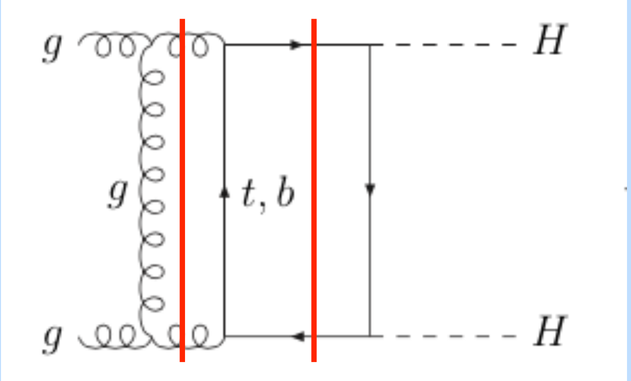
$(H \to Z\gamma)$

see e.g. Degrassi, Giardino, Gröber



Virtual Corrections

Box diagrams



- Matrix element form factors for individual diagrams using Feynman rules (by hand $\rightarrow$ *Reduce, Mathematica, Form*)

- Feynman parametrisation $\rightarrow$ additional 6-dimensional integrals

- Ultraviolet divergences: endpoint subtractions of the 6-dimensional Feynman integral

$$\int_0^1 dx \, \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx \, \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx \, \frac{f(x)-f(1)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \, \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

Virtual Corrections

- Infrared and collinear divergences: proper subtraction of the integrand

denominator: $N = ar^2 + br + c$ $\quad a = \mathcal{O}(\rho)$ $\quad \rho_s = \frac{\hat{s}}{m_Q^2}$

$N_0 = br + c$ $\quad b = 1 + \mathcal{O}(\rho)$

$c = -\rho_s x(1-x)(1-s)t$

$$\int_0^1 \mathrm{d}\vec{x}\mathrm{d}r \frac{rH(\vec{x},r)}{N^{3+2\epsilon}} = \int_0^1 \mathrm{d}\vec{x}\mathrm{d}r \Big\{ \Big(\frac{rH(\vec{x},r)}{N^{3+2\epsilon}} - \frac{rH(\vec{x},0)}{N_0^{3+2\epsilon}} \Big) + \frac{rH(\vec{x},0)}{N_0^{3+2\epsilon}} \Big\}$$

Taylor expansion in ϵ $\qquad$ analytical r-integration

- Integration by parts due to numerical instabilities at the thresholds

$m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \to m_t^2(1 - i\bar{\epsilon})$ with $\bar{\epsilon} \ll 1$

$$\int_0^1 \mathrm{dx} \frac{\mathrm{f(x)}}{\mathrm{(a+bx)^3}} = \frac{\mathrm{f(0)}}{\mathrm{2a^2b}} - \frac{\mathrm{f(1)}}{\mathrm{2b(a+b)^2}} + \int_0^1 \mathrm{dx} \frac{\mathrm{f'(x)}}{\mathrm{2b(a+bx)^2}}$$

Virtual Corrections

Differential cross section

$$Q^2 \frac{d\Delta\sigma_{virt}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{virt}(Q^2)\Big|_{\tau=\frac{Q^2}{s}}$$

$\frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau} = \text{gluon luminosity}$

$\hat{\sigma}_{virt} = \text{virtual part of the partonic cross section}$

$(Q^2 = m_{HH}^2)$

⟶ 7 dimensional integrals (6 Feynman and one phase space integration)

⟶ use Vegas for numerical integration (P. Lepage)

Virtual Corrections

Differential cross section

$$Q^2 \frac{d\Delta\sigma_{virt}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{virt}(Q^2)\Big|_{\tau=\frac{Q^2}{s}}$$

$$\frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau} = \text{gluon luminosity}$$

$$\hat{\sigma}_{virt} = \text{virtual part of the partonic cross section}$$

$$(Q^2 = m^2_{HH})$$

⟶ 7 dimensional integrals (6 Feynman and one phase space integration)

⟶ use Vegas for numerical integration (P. Lepage)

⟶ numerical instabilities

Virtual Corrections

Numerical Instabilities

1. due to phase-space integration over Mandelstam variable *t*

   ⟶ cut-off at $t = 10^{-8}$ for individual diagrams (total sum finite)

   ⟶ logarithmic substitution with $y = \log \frac{t - t_-}{m_t^2}$

2. due to the small imaginary parts $\bar{\epsilon}$ of the top mass above the thresholds, where
   $m_t^2 \to m_t^2(1 - i\bar{\epsilon})$
   need value in narrow width approximation where $\bar{\epsilon} \to 0$
   calculate partonic cross section for different $\bar{\epsilon}$ ⟶ Richardson extrapolation

Virtual Corrections

Renormalisation

α_s and m_t need to be renormalised

→ α_s in $\overline{MS}$ with $N_F = 5$

→ m_t on shell (→ central prediction)

$$\delta\sigma = \delta\alpha_s \frac{\delta\sigma_{LO}}{\delta\alpha_s} + \delta m_t \frac{\delta\sigma_{LO}}{\delta m_t}$$

Subtraction of the heavy-top limit → virtual mass effects only (infrared finite)

$$\Delta C_{mass} = C^0 - C^0_{HTL}$$

Adding back the results in the heavy-top limit (HPAIR)

$$C = \underbrace{C_{HTL}}_{\text{HPAIR}} + \Delta C_{mass}$$

Real Corrections

Processes: $gg, q\bar{q} \to HHg;\ gq \to HHq$

- Full matrix elements generated with FeynArts and FormCalc
- Matrix elements in the heavy-top limit rescaled locally by massive LO matrix elements ⟶ subtracted ⟶ free of divergences

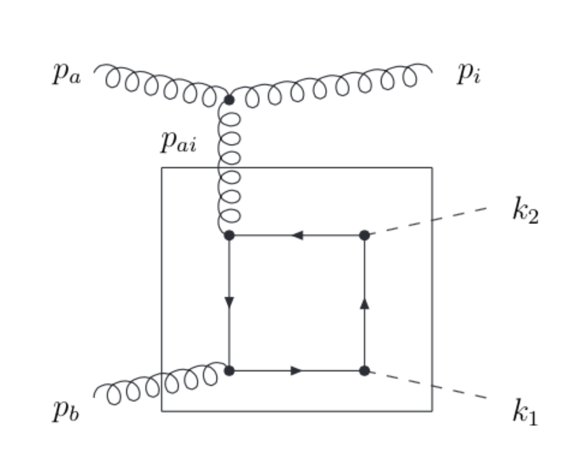
$$\frac{|\mathcal{M}^R(p_i,k_j)|^2}{|\mathcal{M}_{LO}(\tilde{p}_i,\tilde{k}_j)|^2} - \frac{|\mathcal{M}^R_{HTL}(p_i,k_j)|^2}{|\mathcal{M}_{LO}(\tilde{p}_i,\tilde{k}_j)|^2} = \frac{\Delta^{RM}(p_i,k_j)}{|\mathcal{M}_{LO}(\tilde{p}_i,\tilde{k}_j)|^2}$$

$\mathcal{M}^R(p_i,k_j)$ = real NLO matrix element

$\mathcal{M}^R_{HTL}(p_i,k_j)$ = real NLO matrix element in the HTL

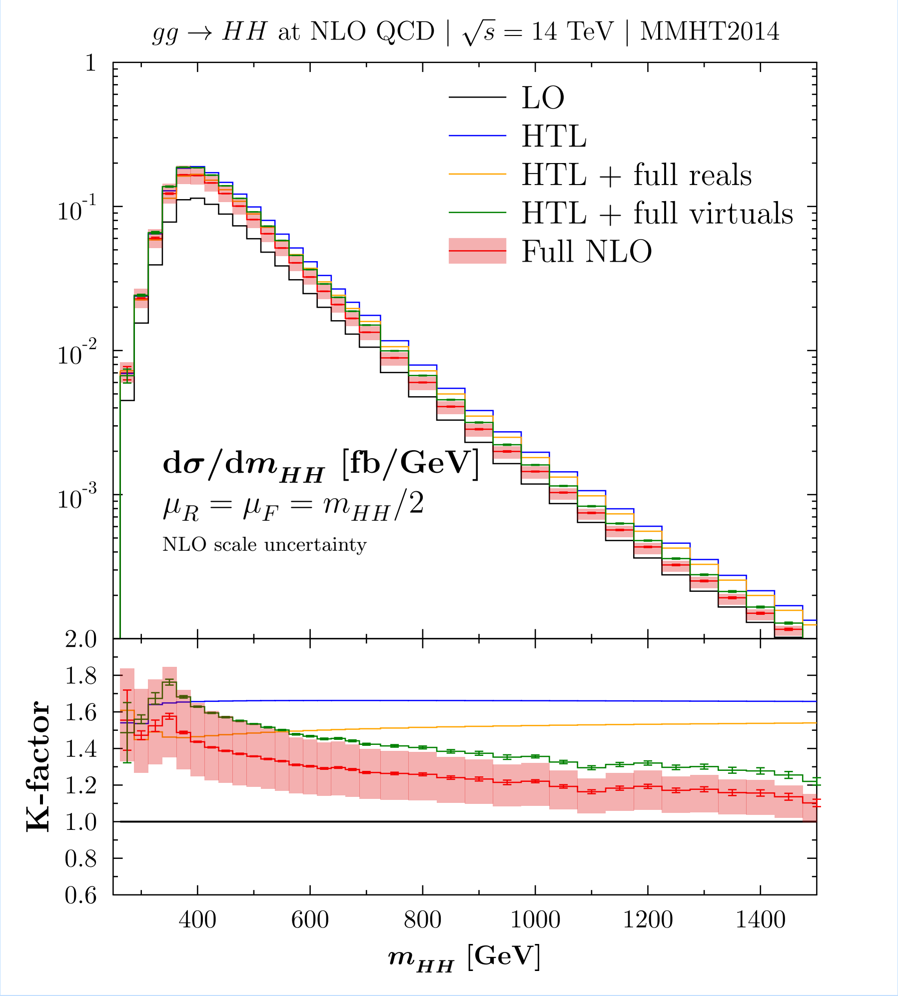
$\mathcal{M}_{LO}(\tilde{p}_i,\tilde{k}_j)$ = LO matrix element

$\Delta^{RM}(p_i,k_j)$ = remaining finite part of the NLO matrix element



- Adding back the results in the heavy-top limit (HPAIR)

Differential cross section

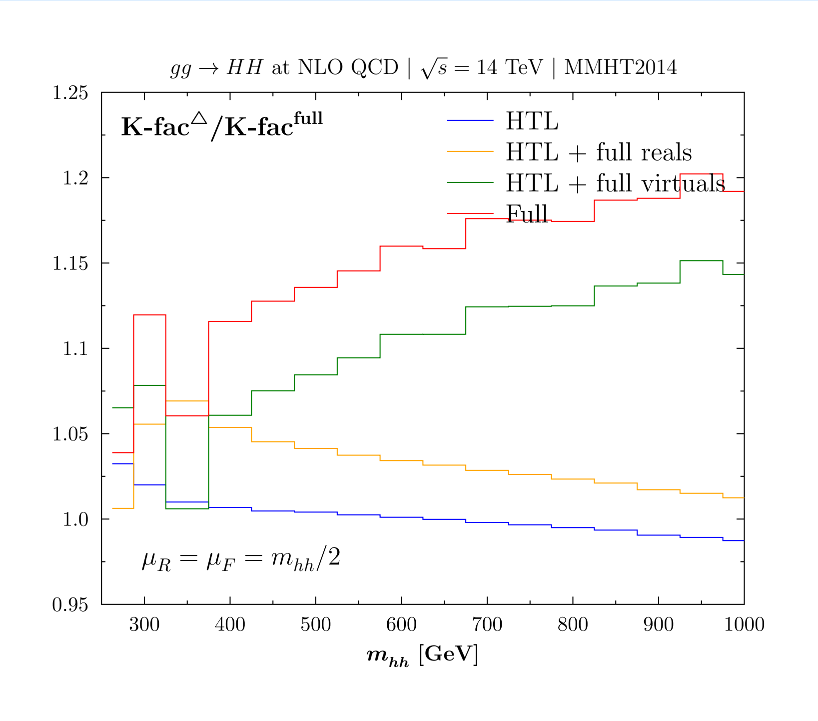


K-factor

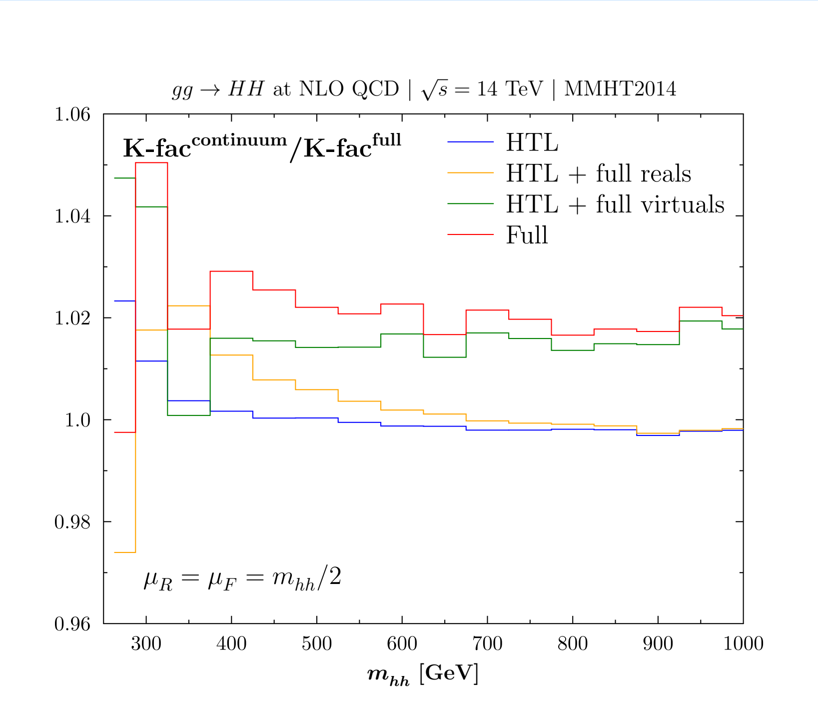
$$K = \frac{\sigma_{\mathrm{NLO}}}{\sigma_{\mathrm{LO}}}$$

Differential cross section

Triangular contributions



Box contributions



Total hadronic cross section

Range 275-300 GeV:

- extension of Boole's rule (*h* step size of 5 GeV)

$$\int_{x_0}^{x_5} f(x)\mathrm{d}x \approx \frac{5h}{288}[19f(x_0) + 75f(x_1) + 50f(x_2) + 50f(x_3) + 75f(x_4) + 19f(x_5)]$$

error: $-\frac{275}{12096}h^7 f(x_6) \qquad (x_0 < x_6 < x_5)$

Range 300-1500 GeV:

- Richardson extrapolation for differential cross sections with bin sizes of 50, 100, 200 and 400 GeV
- Trapezoidal rule

$$\int_a^b f(x)\mathrm{d}x \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

error: $-\frac{nh^3}{12}f''(\zeta) \qquad (a < \zeta < b)$

Total hadronic cross section

Energy	m_t = 173 GeV	m_t = 172.5 GeV
13 TeV	$27.80(9)^{+13.8\%}_{-12.8\%}$ fb	$27.73(7)^{+13.8\%}_{-12.8\%}$ fb
14 TeV	$32.91(10)^{+13.6\%}_{-12.6\%}$ fb	$32.78(7)^{+13.5\%}_{-12.5\%}$ fb
27 TeV	$127.7(2)^{+11.5\%}_{-10.4\%}$ fb	$127.0(2)^{+11.7\%}_{-10.7\%}$ fb
100 TeV	$1149(2)^{+10.8\%}_{-10.0\%}$ fb	$1140(2)^{+10.7\%}_{-10.0\%}$ fb

HH White Paper (to appear soon)

Scale dependence

Factorisation / renormalisation scale dependence

varying both scales by a factor of two around central value of $\mu_F = \mu_R = m_{hh}/2$

Differential cross section:

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=300\,\mathrm{GeV}} = 0.02978(8)^{+15.3\%}_{-13.0\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=400\,\mathrm{GeV}} = 0.1609(4)^{+14.4\%}_{-12.8\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=600\,\mathrm{GeV}} = 0.03204(9)^{+10.9\%}_{-11.5\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=1200\,\mathrm{GeV}} = 0.000435(4)^{+7.1\%}_{-10.6\%}\ \mathrm{fb/GeV}$$

Total cross section:

$$\sigma(gg\to HH) = 32.78(7)^{+13.5\%}_{-12.5\%} \quad (\mathrm{PDF4LHC15})$$

$$(\sqrt{s} = 14\,\mathrm{TeV})$$

Baglio,Campanario,G,Mühlleitner,Spira,Streicher: 1811.05692

Top mass uncertainty

Uncertainty due to m_t: differential cross section

→ uncertainty related to the scheme and scale choice of the top mass

→ calculated the total NLO results for the differential cross section for the $\overline{MS}$ top mass at different scale choices

→ $\overline{MS}$ top mass scale in the range [*Q*/4,*Q*], m_t

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=300\,\mathrm{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=400\,\mathrm{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=600\,\mathrm{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=1200\,\mathrm{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \mathrm{fb/GeV}$$

$(\sqrt{s} = 14\,\mathrm{TeV})$

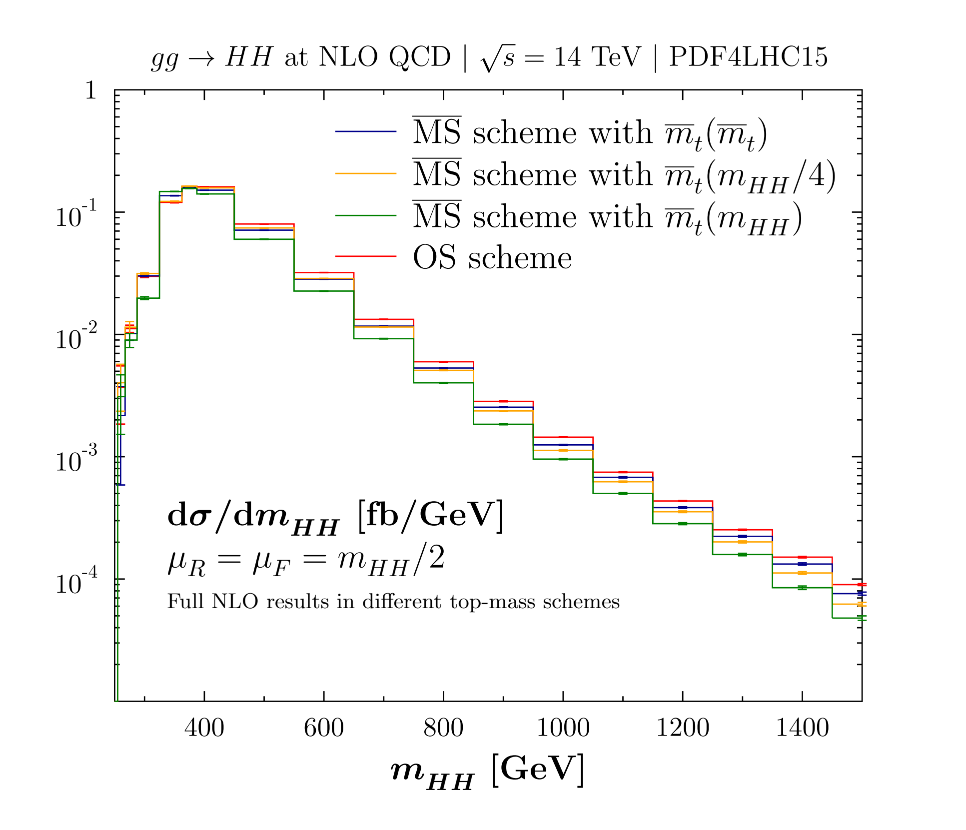
Top mass uncertainty

Uncertainty due to m_t: total hadronic cross section

Take for individual Q values the maximum / minimum differential cross section and integrate

$$\sigma(gg \to HH) = 32.78(7)^{+4.0\%}_{-17\%}$$

with PDF4LHC15



NNLO and beyond

Low Q^2 region:

- HTL known up to N^3LO, $\mathcal{O}\left(\frac{1}{m_t^{2n}}\right)$ up to NNLO – SV

Threshold:

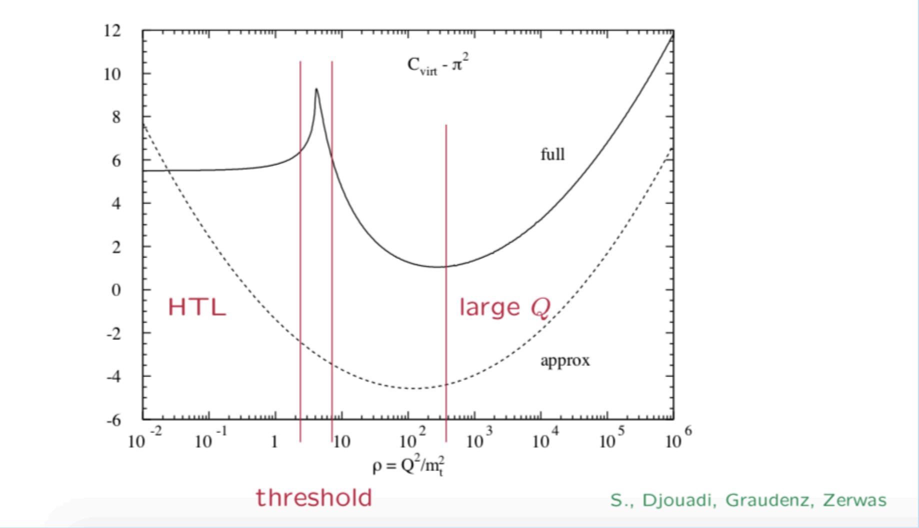
- non relativistic Greens functions ($\longleftarrow$ triangles, boxes?)

Large Q^2 region:

- triangles: resummation of leading and subleading (abelian,non-abelian?) logs exists
- boxes: unknown?

Intermediate region above threshold:

- unclear



Conclusions

- Calculation of two-loop integral with three free parameter ratios
- NLO top mass effects of ~ -15% (inclusive), up to ~ -30 % (exclusive) compared to HTL result
- Factorisation / renormalisation scale dependence: ~ 15% uncertainties
- Top mass scheme and scale uncertainties: $\lesssim$ 30% (differential)

  total cross section at 14 TeV: $\sigma(gg \to HH) = 32.78(7)^{+4.0\%}_{-17\%}$

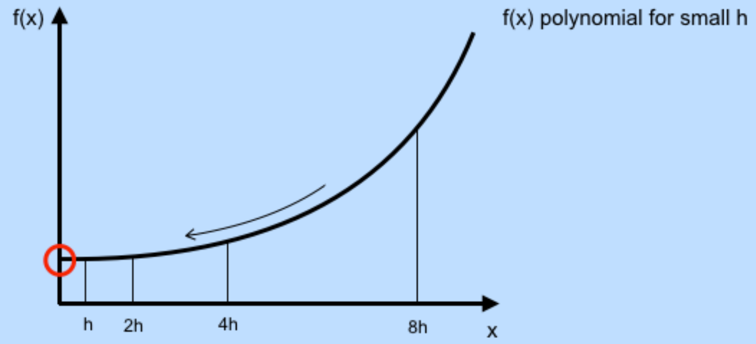
BACK-UP

Richardson extrapolation

→ sequence acceleration method to obtain a better convergence behaviour

Approximation polynomial

$$M_{i+1}(h) = \frac{t^{k_i} M_i(\frac{h}{t}) - M_i(h)}{t^{k_i} - 1}$$



h and *h/t* the two step sizes and k_i the truncation error

$$M_2[f(h), f(2h)] = 2f(h) - f(2h) \qquad = f(0) + \mathcal{O}(h^2)$$

$$M_4[f(h), f(2h), f(4h)] = (8f(h) - 6f(2h) + f(4h))/3 \qquad = f(0) + \mathcal{O}(h^3)$$

$$M_8[f(h), f(2h), f(4h), f(8h)] = (64f(h) - 56f(2h) + 14f(4h) - f(8h))/21 \qquad = f(0) + \mathcal{O}(h^4)$$

In our case $h = \bar{\epsilon}$ and $\bar{\epsilon}_n = 0.05 \times 2^n \quad n = 0 \ldots 9$

Theoretical error from Richardson extrapolation estimated by the difference of the fifth and the

Uncertainty due to m_t for single Higgs

→ $\overline{MS}$ top mass in the range [Q/4,Q]

$$\sigma(gg \to H)\big|_{m_H=125\,\mathrm{GeV}} = 42.17^{+0.4\%}_{-0.5\%}\,\mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=300\,\mathrm{GeV}} = 9.85^{+7.5\%}_{-0.3\%}\,\mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=400\,\mathrm{GeV}} = 9.43^{+0.1\%}_{-0.9\%}\,\mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=600\,\mathrm{GeV}} = 1.97^{+0.0\%}_{-15.9\%}\,\mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=900\,\mathrm{GeV}} = 0.230^{+0.0\%}_{-22.3\%}\,\mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=1200\,\mathrm{GeV}} = 0.0402^{+0.0\%}_{-26.0\%}\,\mathrm{pb}$$

$$C \to \frac{C_A - C_F}{12}\left[\log\frac{Q^2}{m_t^2} - i\pi\right]^2 - C_F\left[\log\frac{Q^2}{m_t^2} - i\pi\right] + 3C_F\log\frac{\mu_t^2}{m_t^2} + \mathcal{O}(1)$$

resummation for large Q

- Abelian logs (C_F): $H \to \gamma\gamma$

Kotsky, Yakovlev, PLB 418 (1998) 335 (LL)
Akhoury, Wang, Yakovlev, PRD 64 (2001) 113008 (NLL)

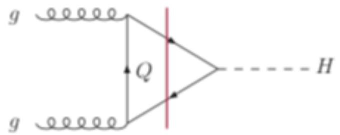
- non-Abelian logs (C_A): LL related to IR singularities $\to$ exponentiate

Liu, Penin, PRL 119 (2017) no.26, 262001; JHEP 1811 (2018) 158

- non-Abelian NLL?
- remainder (NNLL)?
- boxes? (more scales)

stolen from M.Spira (Les Houches '19)

- threshold: $\mathcal{P}$-wave QCD potential $\rightarrow$ Coulomb singularities



- matrix element $\propto \beta^2$, phase space $\propto \beta$
  imaginary part $\propto \beta^3$ @ LO

- Coulomb singularity at each order in imaginary part:

$$C_{Coul} = \frac{Z}{1 - e^{-Z}} = 1 + \frac{Z}{2} + \cdots \text{ with } Z = C_F \frac{\pi \alpha_s}{\beta}$$

$\Rightarrow$ step in imaginary part @ N^{3}LO
$\Rightarrow$ log. sing. in real part @ N^{3}LO ($\leftarrow$ dispersion integral)

- solution: non-relativistic Green-function in threshold range
  [real part renormalized, finite top width]

Melnikov, S., Yakovlev, ZPC 64 (1994) 401

- remainder?

stolen from M.Spira (Les Houches '19)