

EFTs with Two Higgs Doublets

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Outline

- 1 Standard Model Effective Field Theory
- 2 2HDM Effective Field Theory: the dim-six basis
- 3 Operator counting: Hilbert series
- 4 2HDM-EFT: physical basis
- 5 Phenomenology
- 6 Summary

Standard Model Effective Field Theory

In searches for new physics we can distinguish among:

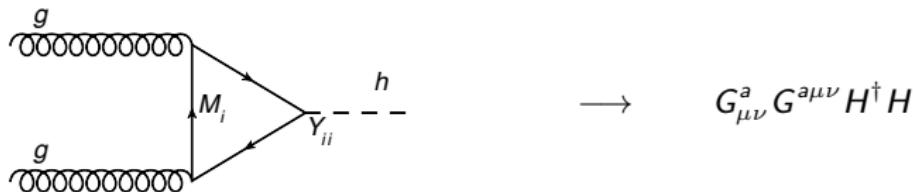
- **Direct searches**
Searches for new resonances.
- **Top-down approach: BSM models (model-dependent)**
Unknowns: model parameters.
- **Bottom-up approach: EFT ("model-independent")**
Unknowns: Wilson coefficients

Assumptions:

- The dynamical degrees of freedom at the EW scale are those of the SM
- New Physics appears at some high scale $\Lambda \gg v$ (decoupling)
- Absence of mixing of new heavy scalars with the SM Higgs doublet
- $SU(2)_L \times U(1)_Y$ is **linearly realized** at high energies

Effective Field Theories

Local operators parametrize the effects of the exchange of new heavy particles:



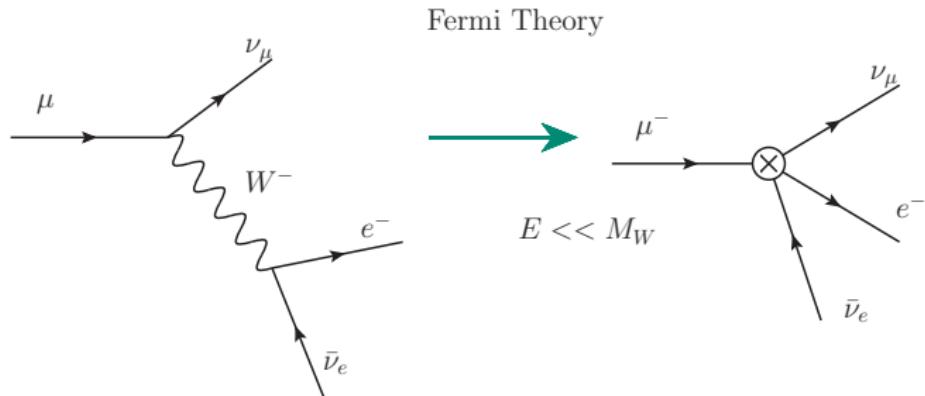
Integrate out the heavy fields and obtain the effective operator.

SM example: the limit of infinite top mass

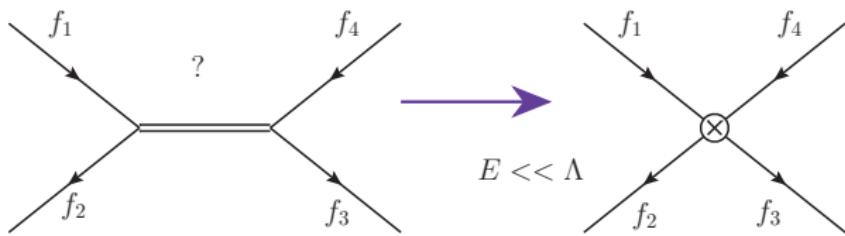
$$\Delta \mathcal{L}_{gg h} = \frac{g_S^2}{48\pi^2} G_{\mu\nu}^a G^{a\mu\nu} \frac{h}{v}$$

The coefficient is determined by matching the full theory with the effective theory.

Effective Field Theories



SMEFT



SMEFT Lagrangian

Higgs doublet - EW symmetry is linearly realized

$$\mathcal{L}_{HEFT} = \mathcal{L}_{SM} + \sum_{n>4} \sum_i \frac{c_i}{\Lambda^{n-4}} O_i^{D=n}$$

$$\mathcal{L}_{HEFT} = \mathcal{L}_{SM} + \frac{1}{\Lambda} \mathcal{L}^{D=5} + \frac{1}{\Lambda^2} \mathcal{L}^{D=6} + \frac{1}{\Lambda^3} \mathcal{L}^{D=7} + \frac{1}{\Lambda^4} \mathcal{L}^{D=8} + \dots$$

- $\mathcal{L}^{D=5}$ and $\mathcal{L}^{D=7}$: lepton number violating
- $\mathcal{L}^{D=8}$ and higher: parametrically subleading
- $\mathcal{L}^{D=6}$: leading effect

$$\mathcal{L}_{HEFT} = \mathcal{L}_{SM} + \frac{1}{\Lambda^2} \mathcal{L}^{D=6}$$

C. N. Leung, S. T. Love and S. Rao, Z. Phys. C 31 (1986) 433
 Buchmüller and Wyler, NPB 268 (1986) 621
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SMEFT

GIMR/Warsaw basis

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{c\varphi}$	$(\varphi^\dagger \varphi) (\bar{l}_r e_r \varphi)$
$Q_{\bar{G}}$	$f^{ABC} \tilde{G}^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi \square}$	$(\varphi^\dagger \varphi) \square (\varphi^\dagger \varphi)$	$Q_{w\varphi}$	$(\varphi^\dagger \varphi) (\bar{q}_p u_r \tilde{e})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^*$ $(\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi) (\bar{q}_p d_r \varphi)$
$Q_{\bar{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^I G^{A\mu\nu}$	Q_{cW}	$(\bar{l}_r \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi) (\bar{l}_r \gamma^\mu l_r)$
$Q_{\varphi \bar{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^I G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_r \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu^I \varphi) (\bar{l}_r \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_\mu^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi) (\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi \bar{W}}$	$\varphi^\dagger \varphi \tilde{W}_\mu^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi) (\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu^I \varphi) (\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \bar{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi) (\bar{u}_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \varphi W_\mu^I W^{I\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi) (\bar{d}_p \gamma^\mu d_r)$
$Q_{\varphi \bar{W}B}$	$\varphi^\dagger \varphi W_\mu^I W^{I\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi) (\bar{u}_p \gamma^\mu d_r)$

- 15 bosonic operators
- 19 single-fermionic-current operators

15+19+25=59 independent operators (for 1 fermion generation)

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_r \gamma_\mu l_r) (\bar{l}_s \gamma^\mu l_s)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r) (\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r) (\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r) (\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r) (\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_s) (\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r) (\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r) (\bar{q}_s \gamma^\mu q_t)$	Q_{cu}	$(\bar{e}_p \gamma_\mu e_r) (\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r) (\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{cd}	$(\bar{e}_p \gamma_\mu e_r) (\bar{d}_s \gamma^\mu d_t)$	$Q_{qs}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{u}_s \gamma^\mu u_t)$
		Q_{ud}	$(\bar{u}_p \gamma_\mu u_r) (\bar{d}_s \gamma^\mu d_t)$	$Q_{qs}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{u}_s \gamma^\mu T^A d_t)$
		$Q_{qd}^{(1)}$	$(\bar{u}_p \gamma_\mu T^A u_r) (\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{d}_s \gamma^\mu d_t)$
		$Q_{qd}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r) (\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{d}_s \gamma^\mu d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating			
Q_{ledq}	$(\bar{l}_r e_r) (d_s q_t^I)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_s^{\alpha I})^T C u_r^\beta] [(q_s^{\alpha J})^T C l_t^k]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_j^I u_r) \varepsilon_{jk} (q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(q_p^{\alpha I})^T C q_r^{\beta k}] [(u_s^*)^T C e_t]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^I T^A u_r) \varepsilon_{jk} (q_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk \text{ min}} [(q_p^{\alpha I})^T C q_r^{\beta k}] [(q_s^{\alpha m})^T C l_t^n]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_r^I e_r) \varepsilon_{jk} (q_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma} (r^I \varepsilon)_{jk} (r^I \varepsilon)_{mn} [(q_p^{\alpha I})^T C q_r^{\beta k}] [(q_s^{\alpha m})^T C l_t^n]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_r^I \sigma_{\mu\nu} e_r) \varepsilon_{jk} (q_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma} [(d_p^{\alpha I})^T C u_r^\beta] [(u_s^*)^T C e_t]$		

- 25 four-fermion operators (assuming barionic number conservation)

SMEFT - From 1 to 3 fermion generations

- Add **flavour indices** to all operators
- From 59 to 2499 operators!
- Assume some **flavour structure** to avoid severe constraints from FCNC

Class	N_{op}	n_g	CP-even		CP-odd	
			1	3	n_g	1
1	4	2	2	2	2	2
2	1	1	1	1	0	0
3	2	2	2	2	0	0
4	8	4	4	4	4	4
5	3	$3n_g^2$	3	27	$3n_g^2$	3
6	8	$8n_g^2$	8	72	$8n_g^2$	8
7	8	$\frac{1}{2}n_g(9n_g + 7)$	8	51	$\frac{1}{2}n_g(9n_g - 7)$	1
8 : $(\bar{L}L)(\bar{L}L)$	5	$\frac{1}{4}n_g^2(7n_g^2 + 13)$	5	171	$\frac{7}{4}n_g^2(n_g - 1)(n_g + 1)$	0
8 : $(\bar{R}R)(\bar{R}R)$	7	$\frac{1}{8}n_g(21n_g^3 + 2n_g^2 + 31n_g + 2)$	7	255	$\frac{1}{8}n_g(21n_g + 2)(n_g - 1)(n_g + 1)$	0
8 : $(\bar{L}L)(\bar{R}R)$	8	$4n_g^2(n_g^2 + 1)$	8	360	$4n_g^2(n_g - 1)(n_g + 1)$	0
8 : $(\bar{L}R)(\bar{R}L)$	1	n_g^4	1	81	n_g^4	1
8 : $(\bar{L}R)(\bar{L}R)$	4	$4n_g^4$	4	324	$4n_g^4$	4
8 : All	25	$\frac{1}{8}n_g(107n_g^3 + 2n_g^2 + 89n_g + 2)$	25	1191	$\frac{1}{8}n_g(107n_g^3 + 2n_g^2 - 67n_g - 2)$	5
Total	59	$\frac{1}{8}(107n_g^4 + 2n_g^3 + 213n_g^2 + 30n_g + 72)$	53	1350	$\frac{1}{8}(107n_g^4 + 2n_g^3 + 57n_g^2 - 30n_g + 48)$	23
						1149

$$1 = F^3$$

$$2 = H^6$$

$$3 = H^4 D^2$$

$$4 = F^2 H^2$$

$$5 = \phi^2 H^3$$

$$6 = \psi^2 FH$$

$$7 = \psi^2 H^2 D$$

Alonso, Jenkins, Manohar and Trott, JHEP 1404 (2014) 159

The EFT of the 2HDM

Suppose that a new particle is discovered around the EW scale.

We still do not know the UV complete theory, and we would like to extend the EFT approach to include the new field.

Is it a viable option? 2 possibilities:

- there is a gap between the new d.o.f. and the other NP resonances → YES
- the new d.o.f. is heavy and we suppose that it is at the same scale of the rest of the NP particles → NO

In the following:

- We suppose the existence of a second Higgs doublet, not so far from the EW scale, and of a gap w.r.t. the other NP resonances;
- we parametrize our ignorance of the UV-complete theory building an EFT with the dynamical degrees of freedom of the 2HDM.

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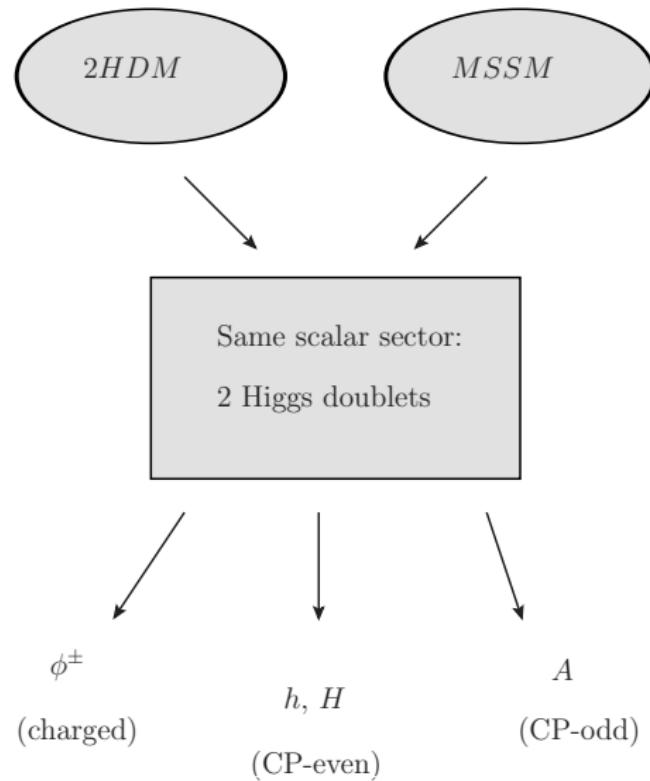
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The Two Higgs Doublet Model



Motivations:

- MSSM
- Axion models
- Models explaining baryon asymmetry

Vector boson and fermion content of the 2HDM:
the same as the SM.

The Two Higgs Doublet Model

$$\begin{aligned}\mathcal{L}_{2HDM}^{(4)} = & -\frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + (D_\mu \varphi_1)^\dagger (D^\mu \varphi_1) + (D_\mu \varphi_2)^\dagger (D^\mu \varphi_2) \\ & - \mathcal{V}(\varphi_1, \varphi_2) + i (\bar{q} \not{D} I + \bar{q} \not{D} q + \bar{u} \not{D} u + \bar{d} \not{D} d) + \mathcal{L}_Y,\end{aligned}$$

$$\begin{aligned}\mathcal{V}(\varphi_1, \varphi_2) = & m_{11}^2 \varphi_1^\dagger \varphi_1 + m_{22}^2 \varphi_2^\dagger \varphi_2 - m_{12}^2 (\varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1) + \frac{\lambda_1}{2} (\varphi_1^\dagger \varphi_1)^2 + \frac{\lambda_2}{2} (\varphi_2^\dagger \varphi_2)^2 \\ & + \lambda_3 \varphi_1^\dagger \varphi_1 \varphi_2^\dagger \varphi_2 + \lambda_4 \varphi_1^\dagger \varphi_2 \varphi_2^\dagger \varphi_1 + \frac{\lambda_5}{2} \left[(\varphi_1^\dagger \varphi_2)^2 + (\varphi_2^\dagger \varphi_1)^2 \right] \\ & + \lambda_6 (\varphi_1^\dagger \varphi_1) (\varphi_1^\dagger \varphi_2) + \lambda_7 (\varphi_2^\dagger \varphi_2) (\varphi_1^\dagger \varphi_2)\end{aligned}$$

$$\begin{aligned}m_{11}^2, m_{22}^2, \lambda_{1,2,3,4} \text{ real} \\ m_{12}^2, \lambda_{5,6,7} \text{ complex}\end{aligned}$$

The Two Higgs Doublet Model

FCNC can be avoided imposing an appropriate Z_2 symmetry:

$$Z_2 : \quad \phi_1 \rightarrow -\phi_1 \quad \text{or} \quad \phi_2 \rightarrow -\phi_2$$

We keep the m_{12} -term, that softly breaks the symmetry.

$$V(\varphi_1, \varphi_2) = m_{11}^2 \varphi_1^\dagger \varphi_1 + m_{22}^2 \varphi_2^\dagger \varphi_2 - m_{12}^2 (\varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1) + \frac{\lambda_1}{2} (\varphi_1^\dagger \varphi_1)^2 + \frac{\lambda_2}{2} (\varphi_2^\dagger \varphi_2)^2$$

$$+ \lambda_3 \varphi_1^\dagger \varphi_1 \varphi_2^\dagger \varphi_2 + \lambda_4 \varphi_1^\dagger \varphi_2 \varphi_2^\dagger \varphi_1 + \frac{\lambda_5}{2} \left[(\varphi_1^\dagger \varphi_2)^2 + (\varphi_2^\dagger \varphi_1)^2 \right]$$

$$+ \lambda_6 (\varphi_1^\dagger \varphi_1) (\varphi_1^\dagger \varphi_2) + \lambda_7 (\varphi_2^\dagger \varphi_2) (\varphi_1^\dagger \varphi_2)$$

$$m_{11}^2, m_{22}^2, \lambda_{1,2,3,4} \text{ real}$$

$$m_{12}^2, \lambda_{5,6,7} \text{ complex}$$

Rotation to the physical basis (2HDM)

Lagrangian for the **mass terms** of the CP-odd (η_a), CP-even (ρ_a) and charged (ϕ_a^+) Higgses:

$$\mathcal{L}_{M_H}^{(4)} = \frac{1}{2} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}^T \mathbf{m}_\eta^2 \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \phi_1^- \\ \phi_2^- \end{pmatrix}^T \mathbf{m}_{\phi^\pm}^2 \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}^T \mathbf{m}_\rho^2 \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

with

$$\begin{aligned} \mathbf{m}_\eta^2 &= (\nu_1 \nu_2 \lambda_5 - m_{12}^2) \begin{pmatrix} -\frac{\nu_2}{\nu_1} & 1 \\ 1 & -\frac{\nu_1}{\nu_2} \end{pmatrix} \\ \mathbf{m}_\rho^2 &= \begin{pmatrix} \lambda_1 \nu_1^2 + m_{12}^2 \frac{\nu_2}{\nu_1} & \nu_1 \nu_2 (\lambda_3 + \lambda_4 + \lambda_5) - m_{12}^2 \\ \nu_1 \nu_2 (\lambda_3 + \lambda_4 + \lambda_5) - m_{12}^2 & \lambda_2 \nu_2^2 + m_{12}^2 \frac{\nu_1}{\nu_2} \end{pmatrix} \\ \mathbf{m}_{\phi^\pm}^2 &= \left[\frac{\nu_1 \nu_2}{2} (\lambda_4 + \lambda_5) - m_{12}^2 \right] \begin{pmatrix} -\frac{\nu_2}{\nu_1} & 1 \\ 1 & -\frac{\nu_1}{\nu_2} \end{pmatrix} \end{aligned}$$

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Rotation to the physical basis:

	eigenvalues	physical states	Goldstone bosons	rotation angle
$\mathbf{m}_{\phi^\pm}^2$	$0; m_\pm^2$	H^\pm (charged)	G^\pm	$\beta \equiv \arctan \frac{v_2}{v_1}$
\mathbf{m}_η^2	$0; m_A^2$	A (CP-odd)	G_0	$\beta \equiv \arctan \frac{v_2}{v_1}$
\mathbf{m}_ρ^2	$m_h^2; m_H^2$	h, H (CP-even)	—	α

2HDM-EFT

Assumptions for the effective theory of the 2HDM:

$$\mathcal{L}_{2HDM} = \mathcal{L}_{2HDM}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

- Its gauge group contains the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ as a subgroup.
- It contains two Higgs doublets as dynamical degrees of freedom, either as fundamental or composite fields.
- At low energies it reproduces the 2HDM, barring the existence of weakly coupled *light* particles, like axions or sterile neutrinos.

Dim-5 the generalization of the [Weinberg operator](#):

$$Q_{\nu\nu}^{11} = (\tilde{\varphi}_1^\dagger I_p)^T C (\tilde{\varphi}_1^\dagger I_r), \quad Q_{\nu\nu}^{22} = (\tilde{\varphi}_2^\dagger I_p)^T C (\tilde{\varphi}_2^\dagger I_r)$$

Crivellin, MG, Procura, JHEP 1609 (2016) 160

2HDM-EFT

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$$\mathcal{L}_{2HDM} = \mathcal{L}_{2HDM}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

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Crivellin, MG, Procura, JHEP 1609 (2016) 160

2HDM-EFT operators

φ^6
$Q_\varphi^{111} = (\varphi_1^\dagger \varphi_1)^3$
$Q_\varphi^{112} = (\varphi_1^\dagger \varphi_1)^2 (\varphi_2^\dagger \varphi_2)$
$Q_\varphi^{122} = (\varphi_1^\dagger \varphi_1) (\varphi_2^\dagger \varphi_2)^2$
$Q_\varphi^{222} = (\varphi_2^\dagger \varphi_2)^3$
$Q_\varphi^{(1221)1} = (\varphi_1^\dagger \varphi_2) (\varphi_2^\dagger \varphi_1) (\varphi_1^\dagger \varphi_1)$
$Q_\varphi^{(1221)2} = (\varphi_1^\dagger \varphi_2) (\varphi_2^\dagger \varphi_1) (\varphi_2^\dagger \varphi_2)$
$Q_\varphi^{(1212)1} = (\varphi_1^\dagger \varphi_2)^2 (\varphi_1^\dagger \varphi_1) + h.c.$
$Q_\varphi^{(1212)2} = (\varphi_1^\dagger \varphi_2)^2 (\varphi_2^\dagger \varphi_2) + h.c.$

- Higgs doublets only
- They modify the Higgs potential

Crivellin, MG, Procura, JHEP 1609 (2016) 160

2HDM-EFT operators

\square	$\varphi^4 D^2$	φD
$Q_{\square}^{1(1)} = (\varphi_1^\dagger \varphi_1) \square (\varphi_1^\dagger \varphi_1)$	$Q_{\varphi D}^{(1)11(1)} = [(D_\mu \varphi_1)^\dagger \varphi_1] [\varphi_1^\dagger (D^\mu \varphi_1)]$	$Q_{\varphi D}^{(1)21(2)} = [(D_\mu \varphi_1)^\dagger \varphi_2] [\varphi_1^\dagger (D^\mu \varphi_2)] + h.c.$
$Q_{\square}^{2(2)} = (\varphi_2^\dagger \varphi_2) \square (\varphi_2^\dagger \varphi_2)$	$Q_{\varphi D}^{(2)22(2)} = [(D_\mu \varphi_2)^\dagger \varphi_2] [\varphi_2^\dagger (D^\mu \varphi_2)]$	$Q_{\varphi D}^{(1)12(2)} = [(D_\mu \varphi_1)^\dagger \varphi_1] [\varphi_2^\dagger (D^\mu \varphi_2)] + h.c.$
$Q_{\square}^{1(2)} = (\varphi_1^\dagger \varphi_1) \square (\varphi_2^\dagger \varphi_2)$	$Q_{\varphi D}^{(1)22(1)} = [(D_\mu \varphi_1)^\dagger \varphi_2] [\varphi_2^\dagger (D^\mu \varphi_1)]$	$Q_{\varphi D}^{12(12)} = [\varphi_1^\dagger \varphi_2] [(D_\mu \varphi_1)^\dagger (D^\mu \varphi_2)] + h.c.$
	$Q_{\varphi D}^{(2)11(2)} = [(D_\mu \varphi_2)^\dagger \varphi_1] [\varphi_1^\dagger (D^\mu \varphi_2)]$	$Q_{\varphi D}^{12(21)} = [\varphi_1^\dagger \varphi_2] [(D_\mu \varphi_2)^\dagger (D^\mu \varphi_1)] + h.c.$

- Four Higgs doublets and two derivatives
- They modify the kinetic terms of the Higgs fields, the Higgs-gauge boson interactions and the W and Z masses

Crivellin, MG, Procura, JHEP 1609 (2016) 160

2HDM-EFT operators

$\varphi^2 X^2$	
GG, WW, BB	WB
$Q_{\varphi X}^{11} = (\varphi_1^\dagger \varphi_1) X_{\mu\nu} X^{\mu\nu}$	$Q_{\varphi WB}^{11} = (\varphi_1^\dagger \tau^I \varphi_1) W_{\mu\nu}^I B^{\mu\nu}$
$Q_{\varphi X}^{22} = (\varphi_2^\dagger \varphi_2) X_{\mu\nu} X^{\mu\nu}$	$Q_{\varphi WB}^{22} = (\varphi_2^\dagger \tau^I \varphi_2) W_{\mu\nu}^I B^{\mu\nu}$

$Q_{\varphi \tilde{X}}^{11} = (\varphi_1^\dagger \varphi_1) \tilde{X}_{\mu\nu} X^{\mu\nu}$	$Q_{\varphi \tilde{W}B}^{11} = (\varphi_1^\dagger \tau^I \varphi_1) \tilde{W}_{\mu\nu}^I B^{\mu\nu}$
$Q_{\varphi \tilde{X}}^{22} = (\varphi_2^\dagger \varphi_2) \tilde{X}_{\mu\nu} X^{\mu\nu}$	$Q_{\varphi \tilde{W}B}^{22} = (\varphi_2^\dagger \tau^I \varphi_2) \tilde{W}_{\mu\nu}^I B^{\mu\nu}$

$$X = G^A, W^I \text{ or } B$$

- Operators with two Higgs doublets and two field strength tensors
- They modify the Higgs-gauge boson interactions

Crivellin, MG, Procura, JHEP 1609 (2016) 160

2HDM-EFT operators

$\Psi^2 \varphi^2 D$	
(1)	(3)
$Q_{\varphi ud}^1 = i(\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{u}_p \gamma^\mu d_r)$	
$Q_{\varphi ud}^2 = i(\tilde{\varphi}_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{u}_p \gamma^\mu d_r)$	
$Q_{\varphi l}^{(1)1} = (\varphi_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{l}_p \gamma^\mu l_r)$	$Q_{\varphi l}^{(3)1} = (\varphi_1^\dagger i\overleftrightarrow{D}_\mu^I \varphi_1)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi l}^{(1)2} = (\varphi_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{l}_p \gamma^\mu l_r)$	$Q_{\varphi l}^{(3)2} = (\varphi_2^\dagger i\overleftrightarrow{D}_\mu^I \varphi_2)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi e}^1 = (\varphi_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{e}_p \gamma^\mu e_r)$	
$Q_{\varphi e}^2 = (\varphi_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{e}_p \gamma^\mu e_r)$	
$Q_{\varphi q}^{(1)1} = (\varphi_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{q}_p \gamma^\mu q_r)$	$Q_{\varphi q}^{(3)1} = (\varphi_1^\dagger i\overleftrightarrow{D}_\mu^I \varphi_1)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi q}^{(1)2} = (\varphi_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{q}_p \gamma^\mu q_r)$	$Q_{\varphi q}^{(3)2} = (\varphi_2^\dagger i\overleftrightarrow{D}_\mu^I \varphi_2)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi u}^1 = (\varphi_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{u}_p \gamma^\mu u_r)$	
$Q_{\varphi u}^2 = (\varphi_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{u}_p \gamma^\mu u_r)$	
$Q_{\varphi d}^1 = (\varphi_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{d}_p \gamma^\mu d_r)$	
$Q_{\varphi d}^2 = (\varphi_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{d}_p \gamma^\mu d_r)$	

- Operators containing two fermions, two Higgs doublets and a covariant derivative
- They contribute to the fermion-Z and fermion-W couplings after EWSB

Crivellin, MG, Procura,
JHEP 1609 (2016) 160

2HDM-EFT operators

$\Psi^2 \varphi X$		
G	W	B
$Q_{dG}^1 = (\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi_1 G_{\mu\nu}^A$	$Q_{dW}^1 = (\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi_1 W_{\mu\nu}^I$	$Q_{dB}^1 = (\bar{q}_p \sigma^{\mu\nu} d_r) \varphi_1 B_{\mu\nu}$
$Q_{dG}^2 = (\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi_2 G_{\mu\nu}^A$	$Q_{dW}^2 = (\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi_2 W_{\mu\nu}^I$	$Q_{dB}^2 = (\bar{q}_p \sigma^{\mu\nu} d_r) \varphi_2 B_{\mu\nu}$
$Q_{uG}^1 = (\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi}_1 G_{\mu\nu}^A$	$Q_{uW}^1 = (\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi}_1 W_{\mu\nu}^I$	$Q_{uB}^1 = (\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi}_1 B_{\mu\nu}$
$Q_{uG}^2 = (\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi}_2 G_{\mu\nu}^A$	$Q_{uW}^2 = (\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi}_2 W_{\mu\nu}^I$	$Q_{uB}^2 = (\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi}_2 B_{\mu\nu}$
	$Q_{eW}^1 = (\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi_1 W_{\mu\nu}^I$	$Q_{eB}^1 = (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi_1 B_{\mu\nu}$
	$Q_{eW}^2 = (\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi_2 W_{\mu\nu}^I$	$Q_{eB}^2 = (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi_2 B_{\mu\nu}$

$$\sigma^{\mu\nu} = i [\gamma^\mu, \gamma^\nu]/2$$

- Operators containing two fermion fields, one Higgs doublet and a field strength tensor
- They give rise to dipole interactions after EWSB

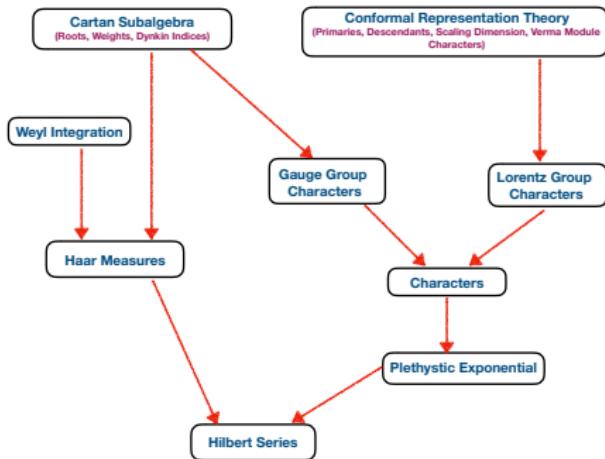
2HDM-EFT operators

$\Psi^2 \varphi^3$		
e	d	u
$Q_{e\varphi}^{111} = (\bar{l}_p e_r \varphi_1)(\varphi_1^\dagger \varphi_1)$	$Q_{d\varphi}^{111} = (\bar{q}_p d_r \varphi_1)(\varphi_1^\dagger \varphi_1)$	$Q_{u\varphi}^{111} = (\bar{q}_p u_r \tilde{\varphi}_1)(\varphi_1^\dagger \varphi_1)$
$Q_{e\varphi}^{122} = (\bar{l}_p e_r \varphi_1)(\varphi_2^\dagger \varphi_2)$	$Q_{d\varphi}^{122} = (\bar{q}_p d_r \varphi_1)(\varphi_2^\dagger \varphi_2)$	$Q_{u\varphi}^{122} = (\bar{q}_p u_r \tilde{\varphi}_1)(\varphi_2^\dagger \varphi_2)$
$Q_{e\varphi}^{222} = (\bar{l}_p e_r \varphi_2)(\varphi_2^\dagger \varphi_2)$	$Q_{d\varphi}^{222} = (\bar{q}_p d_r \varphi_2)(\varphi_2^\dagger \varphi_2)$	$Q_{u\varphi}^{222} = (\bar{q}_p u_r \tilde{\varphi}_2)(\varphi_2^\dagger \varphi_2)$
$Q_{e\varphi}^{211} = (\bar{l}_p e_r \varphi_2)(\varphi_1^\dagger \varphi_1)$	$Q_{d\varphi}^{211} = (\bar{q}_p d_r \varphi_2)(\varphi_1^\dagger \varphi_1)$	$Q_{u\varphi}^{211} = (\bar{q}_p u_r \tilde{\varphi}_2)(\varphi_1^\dagger \varphi_1)$

- Operators with two fermion fields and three Higgs doublets
- They modify the relation between fermion masses and Higgs-fermion couplings

Crivellin, MG, Procura, JHEP 1609 (2016) 160

Counting: how many operators?



Hilbert Series

$$\mathcal{H}[\Phi] = \prod_{j=1}^n \int_{\mathcal{G}_j} d\mu_j \text{PE}[\Phi, R]$$

(partition function of the operator basis)

- Equations of Motion
(ideals in a commutative ring)
- Integration by Parts
(total derivatives are the descendants of the conformal group)

Anisha et al., JHEP 1909 (2019) 035

Counting: how many operators?

Conformal Group

- Free theories are conformal
- Mass terms and interactions are perturbations of the free theory
- EFT is a perturbation around the free theory

Generating representations of the Conformal Group

- Fundamental fields Φ_a
- and the infinite tower of derivatives (symmetrized, acting on Φ_a)
- with EOM removed \rightarrow e.g. $D^2\phi$, $D\psi$, $D_\mu X^{\mu\nu}$
- and only primaries retained (remove descendant operators \rightarrow IBP)

Counting: how many operators?

Local operators are given by the tensor product of the generating representations of the conformal group

All possible tensor products are generated by the plethistic exponential:

$$\text{PE} [\phi_R \chi_R (x_1, \dots, x_r)] = \exp \left[\sum_{n=1}^{\infty} \frac{1}{n} (\pm 1)^{n+1} \phi_R^n \chi_R (x_1, \dots, x_r) \right]$$

- ± 1 accounts for bosons/fermions
- χ_R are the characters of the generating representations R

$$\chi_{\phi_a} = \chi_{\phi_a, SO(d+2, \mathbb{C})} \chi_{\phi_a, \text{gauge}}$$

Counting: how many operators?

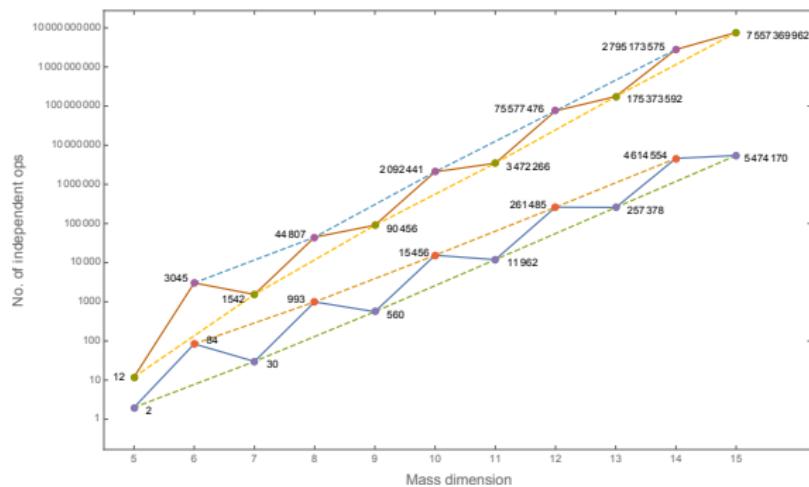
Refs on Hilbert Series applied to EFT

- Feng, Hanany and He, JHEP 0703 (2007) 090, arXiv:hep-th/0701063 **gauge invariants**
- Hanany, Jenkins, Manohar and Torri, JHEP 1103 (2011) 096, arXiv:1010.3161 **flavour invariants**
- Lehman, Phys.Rev. D90 (2014) no.12, 12502, arXiv:1410.4193 **dim-7**
- Henning, Lu, Melia and Murayama, Commun.Math.Phys. 347 (2016) no.2, 363-388, arXiv:1507.07240
- Lehman and Martin, JHEP 1602 (2016) 081, arXiv:1510.00372 **dim-8**
- Wells and Zhang, JHEP 1601 (2016) 123, arXiv:1510.08462
- Henning, Lu, Melia and Murayama, JHEP 1708 (2017) 016, arXiv:1512.03433 **dim-8+**
- Henning, Lu, Melia and Murayama, JHEP 1710 (2017) 199, arXiv:1706.08520
- Barzinji, Trott and Vasudevan, Phys.Rev. D98 (2018) no.11, 116005, arXiv:1806.06354
- Anisha, Das Bakshi, Chakrabortty and Prakash, JHEP 1909 (2019) 035, arXiv:1905.11047
2HDM-EFT, MLRSM-EFT
- ...

Counting: how many operators?

SMEFT

$$\mathcal{H}(\mathcal{D}, \{\phi_a\}) = H(\mathcal{D}, Q, Q^\dagger, L, L^\dagger, H, H^\dagger, u, u^\dagger, d, d^\dagger, e, e^\dagger, B_L, B_R, W_L, W_R, G_L, G_R).$$



Hermitian conjugates are counted separately!

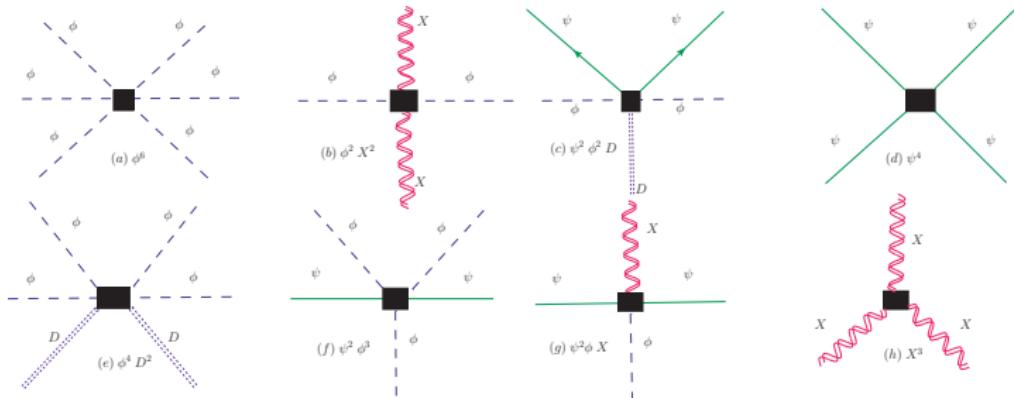
$d = 6, n_f = 1$:

$\mathcal{H}_6 = 84 = 76 + 8$ (baryon number violating)

Usual counting in the Warsaw basis:

$\mathcal{H}_6 = 63 = 59 + 4$

Counting of the 2HDM-EFT operators



(a) ϕ^6 [6 + 7×2 = 20]

(c) $\psi^2 \phi^2 D$ [14 + 10×2 = 34]

(e) $\phi^4 D^2$ [8 + 6×2 = 20]

(g) $\psi^2 \phi X$ [16×2 = 32]

(b) $\phi^2 X^2$ [32]

(d) ψ^4 [20 + 5×2 = 30] + [4×2 = 8] (BNV)

(f) $\psi^2 \phi^3$ [24×2 = 48]

(h) X^3 [4]

TOTAL: 220 (+ 8 BNV) operators!

Counting of $\varphi^4 D^2$ operators

$\varphi^4 D^2$	
\square	φD
$Q_{\square}^{1(1)} = (\varphi_1^\dagger \varphi_1) \square (\varphi_1^\dagger \varphi_1)$	$Q_{\varphi D}^{(1)11(1)} = [(D_\mu \varphi_1)^\dagger \varphi_1] [\varphi_1^\dagger (D^\mu \varphi_1)]$
$Q_{\square}^{2(2)} = (\varphi_2^\dagger \varphi_2) \square (\varphi_2^\dagger \varphi_2)$	$Q_{\varphi D}^{(2)22(2)} = [(D_\mu \varphi_2)^\dagger \varphi_2] [\varphi_2^\dagger (D^\mu \varphi_2)]$
$Q_{\square}^{1(2)} = (\varphi_1^\dagger \varphi_1) \square (\varphi_2^\dagger \varphi_2)$	$Q_{\varphi D}^{(1)22(1)} = [(D_\mu \varphi_1)^\dagger \varphi_2] [\varphi_2^\dagger (D^\mu \varphi_1)]$
	$Q_{\varphi D}^{(2)11(2)} = [(D_\mu \varphi_2)^\dagger \varphi_1] [\varphi_1^\dagger (D^\mu \varphi_2)]$
	$Q_{\varphi D}^{(1)21(2)} = [(D_\mu \varphi_1)^\dagger \varphi_2] [\varphi_1^\dagger (D^\mu \varphi_2)] + h.c.$
	$Q_{\varphi D}^{(1)12(2)} = [(D_\mu \varphi_1)^\dagger \varphi_1] [\varphi_2^\dagger (D^\mu \varphi_2)] + h.c.$
	$Q_{\varphi D}^{12(12)} = [\varphi_1^\dagger \varphi_2] [(D_\mu \varphi_1)^\dagger (D^\mu \varphi_2)] + h.c.$
	$Q_{\varphi D}^{12(21)} = [\varphi_1^\dagger \varphi_2] [(D_\mu \varphi_2)^\dagger (D^\mu \varphi_1)] + h.c.$

$$\varphi^\dagger \overset{\leftrightarrow}{D}_\mu \varphi \equiv \varphi^\dagger \left(D_\mu - \overset{\leftrightarrow}{D}_\mu \right) \varphi \text{ and } \varphi^\dagger \left(D_\mu + \overset{\leftrightarrow}{D}_\mu \right) \varphi = \partial_\mu (\varphi^\dagger \varphi) :$$

$$\partial_\mu (\varphi_1^\dagger \varphi_1) \partial_\mu (\varphi_1^\dagger \varphi_1) \quad \left(\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_1 \right) \left(\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_1 \right)$$

$$\partial_\mu (\varphi_2^\dagger \varphi_2) \partial_\mu (\varphi_2^\dagger \varphi_2) \quad \left(\varphi_2^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2 \right) \left(\varphi_2^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2 \right)$$

$$\partial_\mu (\varphi_1^\dagger \varphi_1) \partial_\mu (\varphi_2^\dagger \varphi_2) \quad \left(\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_1 \right) \left(\varphi_2^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2 \right)$$

$$\partial_\mu (\varphi_1^\dagger \varphi_2) \partial_\mu (\varphi_2^\dagger \varphi_1) \quad \left(\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2 \right) \left(\varphi_2^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_1 \right)$$

$$\partial_\mu (\varphi_1^\dagger \varphi_2) \partial_\mu (\varphi_1^\dagger \varphi_2) + h.c. \quad \left(\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2 \right) \left(\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2 \right) + h.c.$$

Hilbert Series:

12 (+ 8 Z_2 -violating) operators

- Crivellin, MG, Procura, JHEP 1609 (2016) 160
- Karmakar and Rakshit, JHEP 1710 (2017) 048
- Anisha, Das Bakshi, Chakrabortty and Prakash, JHEP 1909 (2019) 035

2HDM-EFT: kinetic terms

$$\begin{aligned} \mathcal{L}_{H_{\text{kin}}}^{(4)+(6)} = & \frac{1}{2} \begin{pmatrix} \partial_\mu \rho_1 \\ \partial_\mu \rho_2 \end{pmatrix}^T \begin{pmatrix} 1 + \frac{2\Delta_{\square}^{11}}{\Lambda^2} + \frac{\Delta_{\varphi D}^{11}}{2\Lambda^2} & \frac{\Delta_{\square}^{12}}{\Lambda^2} + \frac{\Delta_{\varphi D}^{12}}{2\Lambda^2} \\ \frac{\Delta_{\square}^{12}}{\Lambda^2} + \frac{\Delta_{\varphi D}^{12}}{2\Lambda^2} & 1 + \frac{2\Delta_{\square}^{22}}{\Lambda^2} + \frac{\Delta_{\varphi D}^{22}}{2\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu \rho_1 \\ \partial_\mu \rho_2 \end{pmatrix} \\ & + \frac{1}{2} \begin{pmatrix} \partial_\mu \eta_1 \\ \partial_\mu \eta_2 \end{pmatrix}^T \begin{pmatrix} 1 + \frac{\Delta_{\varphi D}^{11}}{2\Lambda^2} & \frac{\Delta_{\varphi D}^{12}}{2\Lambda^2} \\ \frac{\Delta_{\varphi D}^{12}}{2\Lambda^2} & 1 + \frac{\Delta_{\varphi D}^{22}}{2\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu \eta_1 \\ \partial_\mu \eta_2 \end{pmatrix} \\ & + \begin{pmatrix} \partial_\mu \phi_1^+ \\ \partial_\mu \phi_2^+ \end{pmatrix}^\dagger \begin{pmatrix} 1 & \frac{\Delta_{\varphi D}^+}{2\Lambda^2} \\ \frac{\Delta_{\varphi D}^+}{2\Lambda^2} & 1 \end{pmatrix} \begin{pmatrix} \partial_\mu \phi_1^+ \\ \partial_\mu \phi_2^+ \end{pmatrix} \end{aligned}$$

Example: $\mathcal{O}_{\phi\square} = \partial_\mu(\phi^\dagger \phi) \partial^\mu(\phi^\dagger \phi)$

$$\frac{c_{\phi\square}}{v^2} \mathcal{O}_{\phi\square} = c_{\phi\square} \partial_\mu h \partial^\mu h + \dots$$

$$\Delta \mathcal{L}_h = \frac{1}{2} (1 + 2c_{\phi\square}) \partial_\mu h \partial^\mu h + \dots \quad \Rightarrow \quad \bar{h} = (1 + 2c_{\phi\square})^{\frac{1}{2}} h$$

2HDM-EFT: kinetic terms

$$\begin{aligned}
L_{H_{\text{kin}}}^{(4)+(6)} = & \frac{1}{2} \begin{pmatrix} \partial_\mu \rho_1 \\ \partial_\mu \rho_2 \end{pmatrix}^T \begin{pmatrix} 1 + \frac{2\Delta_{\square}^{11}}{\Lambda^2} + \frac{\Delta_{\varphi D}^{11}}{2\Lambda^2} & \frac{\Delta_{\square}^{12}}{\Lambda^2} + \frac{\Delta_{\varphi D}^{12}}{2\Lambda^2} \\ \frac{\Delta_{\square}^{12}}{\Lambda^2} + \frac{\Delta_{\varphi D}^{12}}{2\Lambda^2} & 1 + \frac{2\Delta_{\square}^{22}}{\Lambda^2} + \frac{\Delta_{\varphi D}^{22}}{2\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu \rho_1 \\ \partial_\mu \rho_2 \end{pmatrix} \\
& + \frac{1}{2} \begin{pmatrix} \partial_\mu \eta_1 \\ \partial_\mu \eta_2 \end{pmatrix}^T \begin{pmatrix} 1 + \frac{\Delta_{\varphi D}^{11}}{2\Lambda^2} & \frac{\Delta_{\varphi D}^{12}}{2\Lambda^2} \\ \frac{\Delta_{\varphi D}^{12}}{2\Lambda^2} & 1 + \frac{\Delta_{\varphi D}^{22}}{2\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu \eta_1 \\ \partial_\mu \eta_2 \end{pmatrix} \\
& + \begin{pmatrix} \partial_\mu \phi_1^+ \\ \partial_\mu \phi_2^+ \end{pmatrix}^\dagger \begin{pmatrix} 1 & \frac{\Delta_{\varphi D}^+}{2\Lambda^2} \\ \frac{\Delta_{\varphi D}^+}{2\Lambda^2} & 1 \end{pmatrix} \begin{pmatrix} \partial_\mu \phi_1^+ \\ \partial_\mu \phi_2^+ \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\rho_1 &\rightarrow \rho_1 \left(1 - \frac{\Delta_{\varphi D}^{11} + 4\Delta_{\square}^{11}}{4\Lambda^2} \right) - \left(\frac{\Delta_{\varphi D}^{12} + 4\Delta_{\square}^{12}}{4\Lambda^2} \right) \rho_2 & \phi_1^+ &\rightarrow \phi_1^+ - \frac{\Delta_{\varphi D}^+}{4\Lambda^2} \phi_2^+ \\
\rho_2 &\rightarrow \rho_2 \left(1 - \frac{\Delta_{\varphi D}^{22} + 4\Delta_{\square}^{22}}{4\Lambda^2} \right) - \frac{\Delta_{\varphi D}^{12} + 4\Delta_{\square}^{12}}{4\Lambda^2} \rho_1 & \phi_2^+ &\rightarrow \phi_2^+ - \frac{\Delta_{\varphi D}^+}{4\Lambda^2} \phi_1^+ \\
\eta_1 &\rightarrow \eta_1 \left(1 - \frac{\Delta_{\varphi D}^{11}}{4\Lambda^2} \right) - \frac{\Delta_{\varphi D}^{12}}{4\Lambda^2} \eta_2 & \eta_2 &\rightarrow \eta_2 \left(1 - \frac{\Delta_{\varphi D}^{22}}{4\Lambda^2} \right) - \frac{\Delta_{\varphi D}^{12}}{4\Lambda^2} \eta_1
\end{aligned}$$

2HDM-EFT: mass terms

$$\begin{aligned} L_{M_H}^{(4)+(6)} = & \frac{1}{2} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}^T (m_\eta^2 + \Delta m_\eta^2) \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \\ & + \begin{pmatrix} \phi_1^- \\ \phi_2^- \end{pmatrix}^T (m_{\phi^\pm}^2 + \Delta m_{\phi^\pm}^2) \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} \\ & + \frac{1}{2} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}^T (m_\rho^2 + \Delta m_\rho^2) \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} \end{aligned}$$

$$\Delta m_\eta^2 = \Delta m_{\varphi D\eta}^2 + \Delta m_{\varphi^6 \eta}^2$$

$$\Delta m_\rho^2 = \Delta m_{\varphi D\rho}^2 + \Delta m_{\varphi^6 \rho}^2$$

$$\Delta m_{\phi^\pm}^2 = \Delta m_{\varphi D\phi^\pm}^2 + \Delta m_{\varphi^6 \phi^\pm}^2$$

2HDM:

$$m_{\phi^\pm}^2 \text{ and } m_\eta^2$$



$$\beta \equiv \arctan \frac{v_2}{v_1}$$

$$m_\rho^2$$



$$\alpha$$

2HDM-EFT:

$$m_{\phi^\pm}^2 + \Delta m_{\phi^\pm}^2 \text{ and}$$

$$m_\rho^2 + \Delta m_\rho^2$$



$$\beta_\phi^\pm, \beta_\eta \neq \beta$$

$$m_\rho^2 + \Delta m_\rho^2$$



$$\alpha' \neq \alpha$$

Yukawa sector

The couplings of the Higgs doublets with fermions in the 2HDM

$$\mathcal{L}_Y = -Y_1^e \bar{l} \varphi_1 e - Y_2^e \bar{l} \varphi_2 e - Y_1^d \bar{q} \varphi_1 d - Y_2^d \bar{q} \varphi_2 d - Y_1^u \bar{q} \tilde{\varphi}_1 u - Y_2^u \bar{q} \tilde{\varphi}_2 u + h.c.$$

(Require $Y_1^f = 0$ or $Y_2^f = 0$ to avoid FCNC)

Paschos-Glashow-Weinberg theorem:

If all right-handed fermions with the same quantum numbers couple to the same Higgs multiplet, then FCNC are absent.

model	u_R	d_R	e_R
Type I	φ_2	φ_2	φ_2
Type II	φ_2	φ_1	φ_1
Lepton – specific	φ_2	φ_2	φ_1
Flipped	φ_2	φ_1	φ_2

Yukawa sector

The couplings of the Higgs doublets with fermions in the 2HDM-EFT

$$\mathcal{L}_Y + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i \quad \mathcal{O}_{ijk} \sim (\bar{f}_L f_R \phi_i) (\phi_j^\dagger \phi_k) \quad i, j, k = 1, 2$$

After the EW symmetry breaking:

- new contributions to the fermion masses

$$m^f = \frac{v_1 Y_1^f}{\sqrt{2}} + \frac{v_2 Y_2^f}{\sqrt{2}} + \frac{1}{2\sqrt{2}\Lambda^2} (v_1^3 C_{f\varphi}^{111} + v_1 v_2^2 C_{f\varphi}^{122} + v_2^3 C_{f\varphi}^{222} + v_1^2 v_2 C_{f\varphi}^{211})$$

- Modifications to the Higgs-fermion-fermion and Higgs-Higgs-fermion-fermion couplings

Crivellin, MG, Procura, JHEP 1609 (2016) 160

Experimental bounds

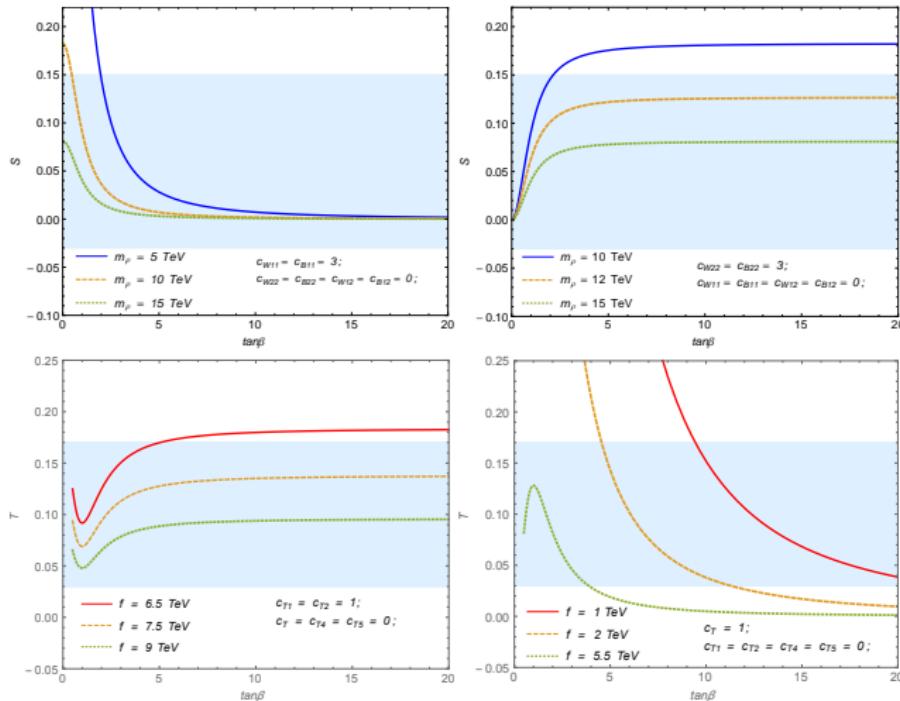
- Bosonic operators: $\varphi^4 D^2$, φ^6 , $\varphi^2 D^2 X$, $\varphi^2 X^2$
[Karmakar and Rakshit, JHEP 1710 \(2017\) 048](#)
- Fermionic operators: $\varphi^2 \psi^2 D$
constraints from [EWPT](#) (Z , W fermionic decays)

Constraints come from:

- anomalous TGCs ($\varphi^2 D^2 X$)
- oblique parameters ($\varphi^2 D^2 X$, $\varphi^4 D^2$)
- Higgs decays
- Associated Higgs production ($\varphi^2 \psi^2 D$)
- Higgs pair production

Bounds on linear [combinations](#) of Wilson coefficients!

Experimental bounds



1 σ bands:

- $S = [-0.03, 0.15]$
- $T = [0.03, 0.17]$
- $U = 0$

Karmakar and Rakshit,
JHEP 1710 (2017) 048

Summary

- The SMEFT approach can be extended to include new light degrees of freedom.
- In this talk, the effective Lagrangian for the dynamical degrees of freedom of the 2HDM has been presented.
- Counting with the method of the Hilbert series, the 2HDM-EFT basis counts 220 (+8 baryon-violating) operators (without assuming Z_2 symmetry).
- The dim-6 operators affect the rotations to the physical basis and modify $\tan \beta$. The CP-even and charged Higgs matrices are not diagonalized by the same angle anymore.
- Experimental measurements constrain linear combination of Wilson coefficients.

BACKUP

2HDM-EFT, general case

$\varphi^4 D^2$		
$O_{H1} = (\partial_\mu \varphi_1 ^2)^2$	$O_{T1} = (\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_1)^2$	$O_{(1)21(2)} = (\varphi_1^\dagger D_\mu \varphi_2)(D^\mu \varphi_1^\dagger \varphi_2)$
$O_{H2} = (\partial_\mu \varphi_2 ^2)^2$	$O_{T2} = (\varphi_2^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2)^2$	$O_{(1)12(2)} = (\varphi_1^\dagger D_\mu \varphi_1)(D^\mu \varphi_2^\dagger \varphi_2)$
$O_{H1H2} = \partial_\mu \varphi_1 ^2 \partial^\mu \varphi_2 ^2$	$O_{T3} = (\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2)^2 + h.c.$	$O_{(1)22(1)} = (\varphi_1^\dagger D_\mu \varphi_2)(D^\mu \varphi_1^\dagger \varphi_1)$
$O_{H12} = (\partial_\mu (\varphi_1^\dagger \varphi_2 + h.c.))^2$	$O_{T4} = (\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2)(\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_1) + h.c.$	$O_{(2)11(2)} = (\varphi_2^\dagger D_\mu \varphi_1)(D^\mu \varphi_1^\dagger \varphi_2)$
$O_{H1H12} = \partial_\mu \varphi_1 ^2 \partial^\mu (\varphi_1^\dagger \varphi_2 + h.c.)$	$O_{T5} = (\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2)(\varphi_2^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2) + h.c.$	
$O_{H2H12} = \partial_\mu \varphi_2 ^2 \partial^\mu (\varphi_1^\dagger \varphi_2 + h.c.)$		

$(D \varphi)(D \varphi)X$	$(\varphi D \varphi)(D X)$
$O_{\varphi B11} = ig'(D_\mu \varphi_1^\dagger D_\nu \varphi_1)B^{\mu\nu}$	$O_{B11} = \frac{ig'}{2}(\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_1)D_\nu B^{\mu\nu}$
$O_{\varphi B22} = ig'(D_\mu \varphi_2^\dagger D_\nu \varphi_2)B^{\mu\nu}$	$O_{B22} = \frac{ig'}{2}(\varphi_2^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2)D_\nu B^{\mu\nu}$
$O_{\varphi B12} = ig'(D_\mu \varphi_1^\dagger D_\nu \varphi_2)B^{\mu\nu} + h.c.$	$O_{B12} = \frac{ig'}{2}(\varphi_1^\dagger \overset{\leftrightarrow}{D}_\mu \varphi_2)D_\nu B^{\mu\nu} + h.c.$
$O_{\varphi W11} = ig(D_\mu \varphi_1^\dagger \vec{\sigma} D_\nu \varphi_1)\vec{W}^{\mu\nu}$	$O_{W11} = \frac{ig}{2}(\varphi_1^\dagger \vec{\sigma} \overset{\leftrightarrow}{D}_\mu \varphi_1)D_\nu \vec{W}^{\mu\nu}$
$O_{\varphi W22} = ig(D_\mu \varphi_2^\dagger \vec{\sigma} D_\nu \varphi_2)\vec{W}^{\mu\nu}$	$O_{W22} = \frac{ig}{2}(\varphi_2^\dagger \vec{\sigma} \overset{\leftrightarrow}{D}_\mu \varphi_2)D_\nu \vec{W}^{\mu\nu}$
$O_{\varphi W12} = ig(D_\mu \varphi_1^\dagger \vec{\sigma} D_\nu \varphi_2)\vec{W}^{\mu\nu} + h.c.$	$O_{W12} = \frac{ig}{2}(\varphi_1^\dagger \vec{\sigma} \overset{\leftrightarrow}{D}_\mu \varphi_2)D_\nu \vec{W}^{\mu\nu} + h.c.$

$\varphi^2 X^2$	
$O_{BB11} = g'^2(\varphi_1^\dagger \varphi_1)B_{\mu\nu}B^{\mu\nu}$	$O_{GG11} = g_s^2(\varphi_1^\dagger \varphi_1)G_{\mu\nu}^a G^{a\mu\nu}$
$O_{BB22} = g'^2(\varphi_2^\dagger \varphi_2)B_{\mu\nu}B^{\mu\nu}$	$O_{GG22} = g_s^2(\varphi_2^\dagger \varphi_2)G_{\mu\nu}^a G^{a\mu\nu}$
$O_{BB12} = g'^2(\varphi_1^\dagger \varphi_2 + h.c.)B_{\mu\nu}B^{\mu\nu}$	$O_{GG12} = g_s^2(\varphi_1^\dagger \varphi_2 + h.c.)G_{\mu\nu}^a G^{a\mu\nu}$

2HDM-EFT, general case

$\varphi^3 \psi^2$		
$O_{e\varphi}^{111} = (\bar{t} e \varphi_1) \varphi_1^\dagger \varphi_1$ $O_{e\varphi}^{122} = (\bar{t} e \varphi_1) \varphi_2^\dagger \varphi_2$ $O_{e\varphi}^{112} = (\bar{t} e \varphi_1)(\varphi_1^\dagger \varphi_2 + h.c.)$ $O_{e\varphi}^{211} = (\bar{t} e \varphi_2) \varphi_1^\dagger \varphi_1$ $O_{e\varphi}^{222} = (\bar{t} e \varphi_2) \varphi_2^\dagger \varphi_2$ $O_{e\varphi}^{212} = (\bar{t} e \varphi_2)(\varphi_1^\dagger \varphi_2 + h.c.)$	$O_{d\varphi}^{111} = (\bar{q} d \varphi_1) \varphi_1^\dagger \varphi_1$ $O_{d\varphi}^{122} = (\bar{q} d \varphi_1) \varphi_2^\dagger \varphi_2$ $O_{d\varphi}^{112} = (\bar{q} d \varphi_1)(\varphi_1^\dagger \varphi_2 + h.c.)$ $O_{d\varphi}^{211} = (\bar{q} d \varphi_2) \varphi_1^\dagger \varphi_1$ $O_{d\varphi}^{222} = (\bar{q} d \varphi_2) \varphi_2^\dagger \varphi_2$ $O_{d\varphi}^{212} = (\bar{q} d \varphi_2)(\varphi_1^\dagger \varphi_2 + h.c.)$	$O_{u\varphi}^{111} = (\bar{q} u \tilde{\varphi}_1) \varphi_1^\dagger \varphi_1$ $O_{u\varphi}^{122} = (\bar{q} u \tilde{\varphi}_1) \varphi_2^\dagger \varphi_2$ $O_{u\varphi}^{112} = (\bar{q} u \tilde{\varphi}_1)(\varphi_1^\dagger \varphi_2 + h.c.)$ $O_{u\varphi}^{211} = (\bar{q} u \tilde{\varphi}_2) \varphi_1^\dagger \varphi_1$ $O_{u\varphi}^{222} = (\bar{q} u \tilde{\varphi}_2) \varphi_2^\dagger \varphi_2$ $O_{u\varphi}^{212} = (\bar{q} u \tilde{\varphi}_2)(\varphi_1^\dagger \varphi_2 + h.c.)$

$\varphi^2 \psi^2 D$		
$O_{\varphi ud}^{11} = i(\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{u} \gamma^\mu d)$ $O_{\varphi ud}^{22} = i(\tilde{\varphi}_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{u} \gamma^\mu d)$ $O_{\varphi ud}^{12} = i(\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{u} \gamma^\mu d) + h.c.$ $O_{\varphi e}^{11} = (\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{e} \gamma^\mu e)$ $O_{\varphi e}^{22} = (\tilde{\varphi}_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{e} \gamma^\mu e)$ $O_{\varphi e}^{12} = (\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{e} \gamma^\mu e) + h.c.$	$O_{\varphi u}^{11} = (\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{u} \gamma^\mu u)$ $O_{\varphi u}^{22} = (\tilde{\varphi}_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{u} \gamma^\mu u)$ $O_{\varphi u}^{12} = (\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{u} \gamma^\mu u) + h.c.$ $O_{\varphi d}^{11} = (\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{d} \gamma^\mu d)$ $O_{\varphi d}^{22} = (\tilde{\varphi}_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{d} \gamma^\mu d)$ $O_{\varphi d}^{12} = (\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{d} \gamma^\mu d) + h.c.$	$O_{\varphi q}^{11(1)} = (\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_1)(\bar{q} \gamma^\mu q)$ $O_{\varphi q}^{22(1)} = (\tilde{\varphi}_2^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{q} \gamma^\mu q)$ $O_{\varphi q}^{12(1)} = (\tilde{\varphi}_1^\dagger i\overleftrightarrow{D}_\mu \varphi_2)(\bar{q} \gamma^\mu q) + h.c.$ $O_{\varphi q}^{11(3)} = (\tilde{\varphi}_1^\dagger i\tau^I i\overleftrightarrow{D}_\mu \varphi_1)(\bar{q} \tau^I \gamma^\mu q)$ $O_{\varphi q}^{22(3)} = (\tilde{\varphi}_2^\dagger i\tau^I i\overleftrightarrow{D}_\mu \varphi_2)(\bar{q} \tau^I \gamma^\mu q)$ $O_{\varphi q}^{12(3)} = (\tilde{\varphi}_1^\dagger i\tau^I i\overleftrightarrow{D}_\mu \varphi_2)(\bar{q} \tau^I \gamma^\mu q) + h.c.$

$\varphi \psi^2 X$		
$O_{uG}^1 = (\bar{q} \sigma_{\mu\nu} t^a u) \tilde{\varphi}_1 G^{a\mu\nu}$ $O_{uG}^2 = (\bar{q} \sigma_{\mu\nu} t^a u) \tilde{\varphi}_2 G^{a\mu\nu}$ $O_{dG}^1 = (\bar{q} \sigma_{\mu\nu} t^a d) \varphi_1 G^{a\mu\nu}$ $O_{dG}^2 = (\bar{q} \sigma_{\mu\nu} t^a d) \varphi_2 G^{a\mu\nu}$	$O_{uW}^1 = (\bar{q} \sigma_{\mu\nu} \sigma^i u) \tilde{\varphi}_1 W^{i\mu\nu}$ $O_{uW}^2 = (\bar{q} \sigma_{\mu\nu} \sigma^i u) \tilde{\varphi}_2 W^{i\mu\nu}$ $O_{dW}^1 = (\bar{q} \sigma_{\mu\nu} \sigma^i d) \varphi_1 W^{i\mu\nu}$ $O_{dW}^2 = (\bar{q} \sigma_{\mu\nu} \sigma^i d) \varphi_2 W^{i\mu\nu}$ $O_{eW}^1 = (\bar{l} \sigma_{\mu\nu} \sigma^i e) \varphi_1 W^{i\mu\nu}$ $O_{eW}^2 = (\bar{l} \sigma_{\mu\nu} \sigma^i e) \varphi_2 W^{i\mu\nu}$	$O_{uB}^1 = (\bar{q} \sigma_{\mu\nu} u) \tilde{\varphi}_1 B^{\mu\nu}$ $O_{uB}^2 = (\bar{q} \sigma_{\mu\nu} u) \tilde{\varphi}_2 B^{\mu\nu}$ $O_{dB}^1 = (\bar{q} \sigma_{\mu\nu} d) \varphi_1 B^{\mu\nu}$ $O_{dB}^2 = (\bar{q} \sigma_{\mu\nu} d) \varphi_2 B^{\mu\nu}$ $O_{eB}^1 = (\bar{l} \sigma_{\mu\nu} e) \varphi_1 B^{\mu\nu}$ $O_{eB}^2 = (\bar{l} \sigma_{\mu\nu} e) \varphi_2 B^{\mu\nu}$