NLO Higgs EFT
(and the k-Framework)

Raquel Gomez-Ambrosio

Durham University

NLO Higgs EFT

~~(and the k-Framework)~~

Raquel Gomez-Ambrosio

Durham University

May 15, 2015

!!

NLO Higgs Effective Field Theory and κ-framework*

Margherita Ghezzi †, **Raquel Gomez-Ambrosio** ‡, **Giampiero Passarino** § **and Sandro Uccirati** ¶

Dipartimento di Fisica Teorica, Università di Torino, Italy
INFN, Sezione di Torino, Italy

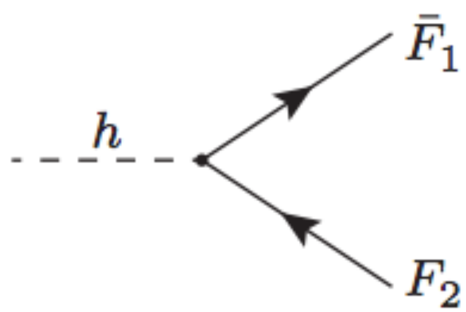
A consistent framework for studying Standard Model deviations is developed. It assumes that New Physics becomes relevant at some scale beyond the present experimental reach and uses the Effective Field Theory approach by adding higher-dimensional operators to the Standard Model Lagrangian and by computing relevant processes at the next-to-leading order, extending the original κ-framework.

Outline

- **Introduction**
- **NLO EFT : Renormalisation**
    - 1-point functions
    - 2-point functions
    - 3-point functions
    - Finite renormalisation
- **Literature/State of the art**
- **Other examples: Singlet Extension and THDM**

The Kappa Framework

- The k-framework has some limitations:
  - Mixed linear and quadratic terms (interpretation…)
  - It doesn't accommodate "new kinematics"
  - It's inconsistent beyond LO



$$= ie\,\kappa_{ffS}\left(C_L\frac{1-\gamma_5}{2} + C_R\frac{1+\gamma_5}{2}\right)$$

$$\mathcal{M}|_{\rm UV} = \frac{\alpha}{4\pi}\frac{\mathcal{M}_0}{4s^2 M_W^2}\frac{2}{4-D}(1-\kappa_{ffS}^2)\left(\sum_l m_l^2 + 3\sum_q m_q^2\right)$$

From arXiv: 1711.09875

The SMEFT ~~Framework~~

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_4 + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_i^n}{\Lambda^{n-4}} \mathscr{O}_i^{(d=n)}$$

- Some assumptions
- Renormalizability ?

Definition *The Higgs couplings can be extracted from Green's functions in well-defined kinematic limits, e.g. residue of the poles after extracting the parts which are* $1P$ *reducible. These are well-defined QFT objects, that we can probe both in production and in decays; from this perspective,* VH *production or vector-boson-fusion are on equal footing with* gg *fusion and Higgs decays. Therefore, the first step requires computing these residues which is the main result of this paper.*

NLO-EFT #0 : Tadpoles

$$\boxed{\overline{\beta}_{\mathrm{h}} = i\, g^2\, M_{\mathrm{W}}^2\, \left(\overline{\beta}_{\mathrm{h}}^{(4)} + g_6\, \overline{\beta}_{\mathrm{h}}^{(6)}\right)\,,}$$

$$\boxed{\langle 0\,|\overline{\mathrm{H}}|\,0\,\rangle = 0}$$

$$\overline{\beta}_{\mathrm{h}}^{(n)} = \underbrace{\beta_{-1}^{(n)}\,\Delta_{\mathrm{UV}}\left(M_{\mathrm{W}}^2\right)}_{\text{Pole}} + \underbrace{\beta_0^{(n)}}_{\text{Finite}} + \underbrace{\beta_{\mathrm{fin}}^{(n)}}_{\text{Logs}}\,. \qquad \Delta_{\mathrm{UV}}(x) = \frac{2}{\varepsilon} - \gamma_{\mathrm{E}} - \ln\pi - \ln\frac{x}{\mu^2}$$

More details in the backup....

NLO-EFT #1 : Gauge Fixing

$$\mathscr{L}_{\mathrm{gf}} = -\mathscr{C}^{+}\mathscr{C}^{-} - \frac{1}{2}\mathscr{C}_{\mathrm{Z}}^{2} - \frac{1}{2}\mathscr{C}_{\mathrm{A}}^{2},$$

$$\mathscr{C}^{\pm} = -\xi_{\mathrm{W}}\,\partial_\mu\,\mathrm{W}_\mu^{\pm} + \xi_{\pm}M\phi^{\pm}, \quad \mathscr{C}_{\mathrm{Z}} = -\xi_{\mathrm{Z}}\,\partial_\mu\,\mathrm{Z}_\mu + \xi_0\frac{M}{\mathrm{c}_\theta}\phi^0, \quad \mathscr{C}_{\mathrm{A}} = \xi_{\mathrm{A}}\,\partial_\mu\,\mathrm{A}_\mu\,.$$

$$\xi_i = 1 + \Delta R_i$$

Feynman rules for the physical fields remain unchanged, shifts go into FP Lagrangian

No dim-6 ghost operators ? !

NLO-EFT #2: two point functions

Wave functions and counterterms

$$\Phi = \mathrm{Z}_\Phi \, \Phi_{\mathrm{ren}}\,, \qquad p = \mathrm{Z}_p \, p_{\mathrm{ren}} \qquad \mathrm{Z}_i = 1 + \frac{g^2}{16\pi^2}\left(\mathrm{dZ}_i^{(4)} + g_6\,\mathrm{dZ}_i^{(6)}\right)$$

We construct self-energies, Dyson resum them and require that all propagators are UV finite. In a second step we construct 3 -point (or higher) functions, check their dim-4-finiteness and remove the remaining dim-6 UV divergences by mixing the Wilson coefficients Wi:

$$\mathrm{W}_i = \sum_j Z^{\mathrm{W}}_{ij}\,\mathrm{W}^{\mathrm{ren}}_j \qquad Z^{\mathrm{W}}_{ij} = \delta_{ij} + \frac{g^2}{16\pi^2}\,\mathrm{dZ}^{\mathrm{W}}_{ij}\,\Delta_{\mathrm{UV}}$$

Renormalized Wilson coefficients are scale dependent and the logarithm of the scale can be resummed in terms of the LO coefficients of the anomalous dimension matrix

NLO-EFT #2: two point functions

Wave functions and counterterms

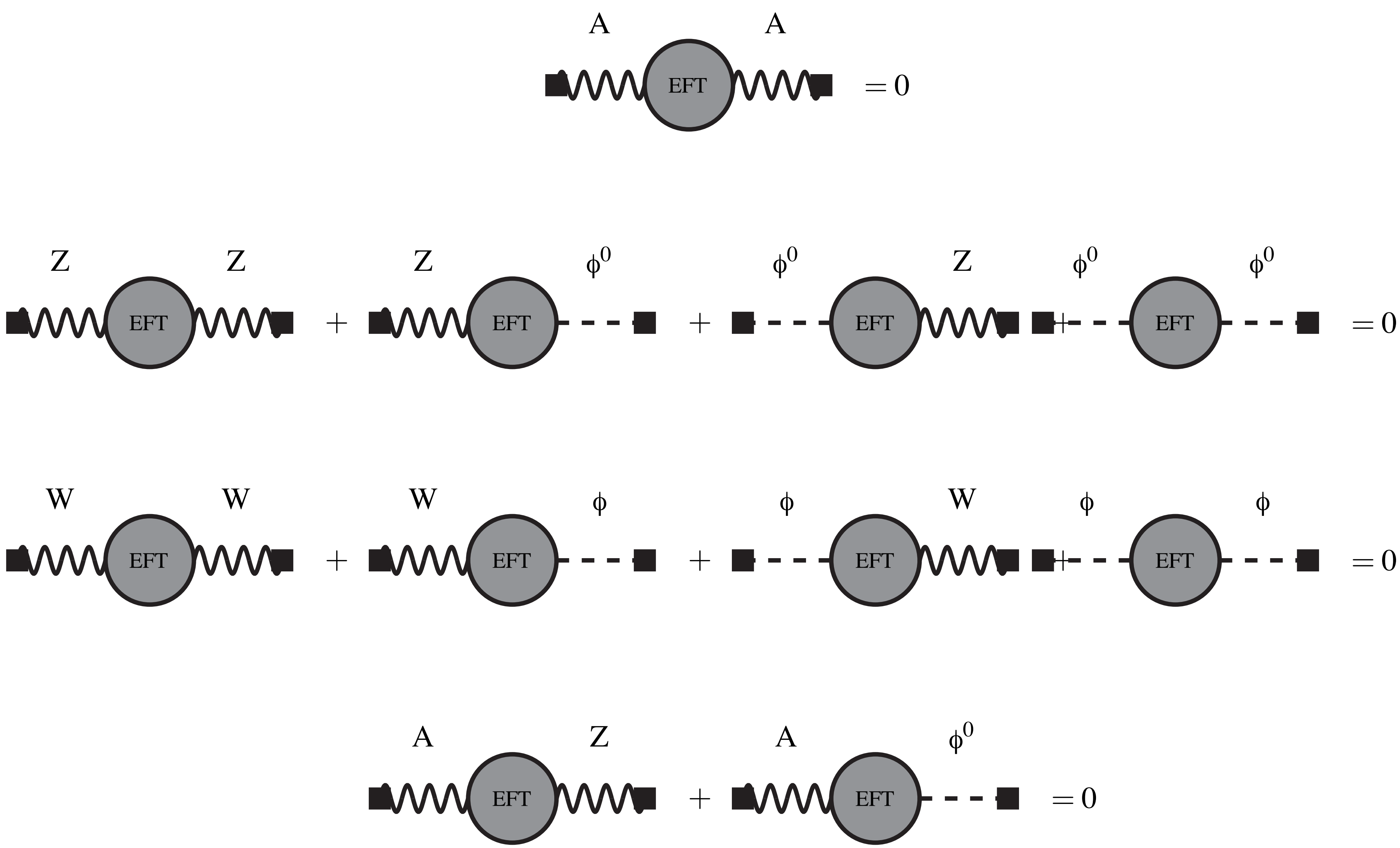
$$\Phi = Z_\Phi \, \Phi_{\rm ren} \,, \qquad p = Z_p \, p_{\rm ren} \qquad Z_i = 1 + \frac{g^2}{16\pi^2}\left(dZ_i^{(4)} + g_6 \, dZ_i^{(6)}\right)$$

Discussion point:

$$\boxed{W_i = \sum_j Z_{ij}^{W} \, W_j^{\rm ren}}$$

After we use different operators to remove residual dim-6 divergences, the interpretation of single operator effects becomes ill-defined

NLO-EFT #3 : WST Identities



NLO-EFT #4 : Dyson Propagators

$$\hat{\Delta}^{\mathrm{VV}}_{\mu\nu} = -\frac{\delta_{\mu\nu}}{s - M^2_{\mathrm{V}} + \frac{g^2}{16\pi^2}\,\mathrm{D}^{\mathrm{VV}}} + \frac{g^2}{16\pi^2}\,\frac{\mathrm{P}^{\mathrm{VV}}\,p_\mu p_\nu}{\left(s - M^2_{\mathrm{V}} + \frac{g^2}{16\pi^2}\,\mathrm{D}^{\mathrm{VV}}\right)\left(s - M^2_{\mathrm{V}} + \frac{g^2}{16\pi^2}\,\mathrm{D}^{\mathrm{VV}} - \frac{g^2}{16\pi^2}\,\mathrm{P}^{\mathrm{VV}}\,s\right)}\,.$$

F. ex. Higgs:

$$g^{-2}\,\hat{\Delta}^{-1}_{\mathrm{HH}}(s) = -g^{-2}\,\mathrm{Z}_{\mathrm{H}}\left(s - M^2_{\mathrm{H}}\right) - \frac{1}{16\pi^2}\,\Sigma_{\mathrm{HH}}$$

F. ex. fermions:

$$\mathrm{G}^{-1}_{\mathrm{f}}(p) = \overline{\mathrm{Z}}_{\mathrm{f}}\left(i\,\not{p} + m_{\mathrm{f}}\right)\mathrm{Z}_{\mathrm{f}} - \mathrm{S}_{\mathrm{f}}\,,$$

$$\mathrm{Z}_{\mathrm{f}} = \mathrm{Z}_{\mathrm{R\,f}}\,\gamma^- + \mathrm{Z}_{\mathrm{L\,f}}\,\gamma^+\,, \qquad \overline{\mathrm{Z}}_{\mathrm{f}} = \mathrm{Z}_{\mathrm{L\,f}}\,\gamma^+ + \mathrm{Z}_{\mathrm{R\,f}}\,\gamma^- \qquad \gamma^{\pm} = \frac{1}{2}\left(1 \pm \gamma^5\right)\,,$$

$$\mathrm{Z}_{\mathrm{I\,f}} = 1 - \frac{1}{2}\,\frac{g^2}{16\pi^2}\left[\mathrm{dZ}^{(4)}_{\mathrm{I\,f}} + g_6\,\mathrm{dZ}^{(6)}_{\mathrm{I\,f}}\,\Delta_{\mathrm{UV}}\right]\,, \qquad m_{\mathrm{f}} = M_{\mathrm{f}}\left(1 + \frac{g^2}{16\pi^2}\,\mathrm{dZ}_{m_{\mathrm{f}}}\,\Delta_{\mathrm{UV}}\right)$$

NLO-EFT #5 : Finite Renormalization

$$\mathrm{M}^2 = \mathrm{Z_M}\,\mathrm{M}^2_{\mathrm{ren}} \qquad \mathrm{Z_M} = 1 + \frac{g^2}{16\,\pi^2}\left(\mathrm{dZ}^{(4)}_{\mathrm{M}} + g_6\,\mathrm{dZ}^{(6)}_{\mathrm{M}}\right)\Delta_{\mathrm{UV}}$$

$$\mathrm{p} = \mathrm{Z_p}\,\mathrm{p}_{\mathrm{ren}} \qquad \mathrm{Z_p} = 1 + \frac{g^2}{16\,\pi^2}\left(\mathrm{dZ}^{(4)}_{\mathrm{p}} + g_6\,\mathrm{dZ}^{(6)}_{\mathrm{p}}\right)\Delta_{\mathrm{UV}}\,.$$

We discuss 3 schemes for the physical parameters

For the EFT coefficients MS-bar scheme is the only option

NLO-EFT #5 : Finite Renormalization

We discuss 3 schemes for the physical parameters

- On-shell renormalization:

$$m_0^2 = M_{\mathrm{OS}}^2 \left[1 + \frac{g^2}{16\,\pi^2}\,\mathrm{Re}\,\Sigma_{\mathrm{VV}\,;\,\mathrm{fin}}\left(M_{\mathrm{OS}}^2\right)\right] \Longrightarrow m_0^2 = s_{\mathrm{V}} \left[1 + \frac{g^2}{16\,\pi^2}\,\Sigma_{\mathrm{VV}\,;\,\mathrm{fin}}\left(M_{\mathrm{OS}}^2\right)\right]$$

$$\boxed{M_{\mathrm{V\,ren}} = M_{\mathrm{V}\,;\mathrm{OS}} + \frac{g_{\mathrm{ren}}^2}{16\,\pi^2}\left(\mathrm{d}\mathscr{Z}_{M_{\mathrm{V}}}^{(4)} + g_6\,\mathrm{d}\mathscr{Z}_{M_{\mathrm{V}}}^{(6)}\right)}$$

$$M_{\mathrm{ren}}^2 = M_{\mathrm{W}\,;\mathrm{OS}}^2 \left[1 + \frac{g_{\mathrm{ren}}^2}{16\,\pi^2}\left(\mathrm{d}\mathscr{Z}_{M_{\mathrm{W}}}^{(4)} + g_6\,\mathrm{d}\mathscr{Z}_{M_{\mathrm{W}}}^{(6)}\right]\right),$$

$$M_{\mathrm{H\,ren}}^2 = M_{\mathrm{H}\,;\mathrm{OS}}^2 \left[1 + \frac{g_{\mathrm{ren}}^2}{16\,\pi^2}\left(\mathrm{d}\mathscr{Z}_{M_{\mathrm{H}}}^{(4)} + g_6\,\mathrm{d}\mathscr{Z}_{M_{\mathrm{H}}}^{(6)}\right)\right],$$

$$\mathrm{c}_{\theta}^{\mathrm{ren}} = \mathrm{c}_{\mathrm{W}} \left[1 + \frac{g_{\mathrm{ren}}^2}{16\,\pi^2}\left(\mathrm{d}\mathscr{Z}_{\mathrm{c}_{\theta}}^{(4)} + g_6\,\mathrm{d}\mathscr{Z}_{\mathrm{c}_{\theta}}^{(6)}\right)\right],$$

NLO-EFT #5 : Finite Renormalization

We discuss 3 schemes for the physical parameters

- GFermi renormalization:

$$g_{\mathrm{ren}} = g_{\mathrm{exp}} + \frac{g^2_{\mathrm{exp}}}{16\,\pi^2}\left(\mathrm{d}\mathscr{Z}^{(4)}_g + g_6\,\mathrm{d}\mathscr{Z}^{(6)}_g\right)$$

$$\frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{g^2}{8\,M^2}\left\{1 + \frac{g^2}{16\,\pi^2}\left[\delta_{\mathrm{G}} + \frac{1}{M^2}\,\Sigma_{\mathrm{WW}}(0)\right]\right\},$$

$$\boxed{g^2_{\mathrm{ren}} = 4\,\sqrt{2}\,G_{\mathrm{F}}\,M^2_{\mathrm{W\,;OS}}\left\{1 + \frac{G_{\mathrm{F}} M^2_{\mathrm{W\,;OS}}}{2\,\sqrt{2}\,\pi^2}\left[\delta_{\mathrm{G}} + \frac{1}{M^2}\,\Sigma_{\mathrm{WW\,;\,fin}}(0)\right]\right\}}$$

NLO-EFT #5 : Finite Renormalization

We discuss 3 schemes for the physical parameters

- Alpha renormalization:

$$g^2\,\mathrm{s}_\theta^2 = 4\,\pi\,\alpha\left[1 - \frac{\alpha}{4\,\pi}\,\frac{\Pi_{\mathrm{AA}}(0)}{\mathrm{s}_\theta^2}\right];$$

$$g^2_{\mathrm{ren}} = g^2_{\mathrm{A}}\left[1 + \frac{\alpha}{4\,\pi}\,\mathrm{d}\mathscr{Z}_g\right], \quad \mathrm{c}_\theta^{\mathrm{ren}} = \hat{\mathrm{c}}_\theta\left[1 + \frac{\alpha}{4\,\pi}\,\mathrm{d}\mathscr{Z}_{\mathrm{c}_\theta}\right], \quad M_{\mathrm{ren}} = M_{\mathrm{Z\,;OS}}\,\hat{\mathrm{c}}_\theta^2\left[1 + \frac{\alpha}{8\,\pi}\,\mathrm{d}\mathscr{Z}_{M_{\mathrm{W}}}\right]$$

$$g^2_{\mathrm{A}} = \frac{4\,\pi\,\alpha}{\hat{\mathrm{s}}_\theta^2} \qquad \hat{\mathrm{s}}_\theta^2 = \frac{1}{2}\left[1 - \sqrt{1 - 4\,\frac{\pi\,\alpha}{\sqrt{2}\,G_{\mathrm{F}}\,M^2_{\mathrm{Z\,;OS}}}}\right]$$

Useful for S,T,U params. and more intuitive SMEFT shifts

NLO-EFT #6 : Three point functions

Example: H to AA

The amplitude for the process $H(P) \to A_\mu(p_1) A_\nu(p_2)$ is:

$$A^{\mu\nu}_{HAA} = \mathscr{T}_{HAA} \frac{p_2^\mu p_1^\nu - p_1 \cdot p_2 \delta^{\mu\nu}}{M_H^2}$$

$$\mathscr{T}_{HAA} = i \frac{g^3}{16\pi^2} \left(\mathscr{T}^{(4)}_{HAA} + g_6 \underbrace{\mathscr{T}^{(6),b}_{HAA}}_{\text{UVdivergent}} \right) + i g g_6 \underbrace{\mathscr{T}^{(6),a}_{HAA}}_{\text{UVfinite}},$$

$$\mathscr{T}^{ren}_{HAA} = i \frac{g^3_{ren}}{16\pi^2} \left(\mathscr{T}^{(4)}_{HAA} + g_6 \mathscr{T}^{(6),b}_{HAA;fin} + \mathscr{T}^{(6),R}_{HAA} \ln \frac{\mu_R^2}{M^2} \right) + i g_{ren} g_6 \mathscr{T}^{(6),a}_{HAA}$$

More details in the backup….

NLO-EFT #6 : Three point functions

Example: H to bb

$$\Delta^{\rm LO}(\mu) \equiv \frac{\overline{\Gamma}_\ell^{(4,0)}(\mu) + \overline{\Gamma}_\ell^{(6,0)}(\mu)}{\overline{\Gamma}_\ell^{(4,0)}(m_H)} \qquad \Delta^{\rm NLO}(\mu) \equiv \Delta^{\rm LO}(\mu) + \frac{\overline{\Gamma}_\ell^{(4,1)}(\mu) + \overline{\Gamma}_\ell^{(6,1)}(\mu)}{\overline{\Gamma}_\ell^{(4,0)}(m_H)} \,.$$

$$\boxed{\Delta^{\rm LO}(m_H) = 1 + \frac{(\overline{v}^{(\ell)})^2}{\Lambda_{\rm NP}^2}\left[3.74\tilde{C}_{HWB} + 2.00\tilde{C}_{H\Box} - 1.41\frac{\overline{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{bH} + 1.24\tilde{C}_{HD}\right]}$$

Pecjak et al 1904.06358

NLO-EFT #6 : Three point functions
Example: H to bb

$$
\begin{aligned}
\Delta^{\rm NLO}(m_H) = 1.13 + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\rm NP}^2}\Bigg\{ & 4.16\tilde{C}_{HWB} + 2.40\tilde{C}_{H\Box} - 1.73\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{bH} + 1.33\tilde{C}_{HD} \\
& + 2.75\tilde{C}_{HG} - 0.12\tilde{C}_{Hq}^{(3)} + \Bigg(- 7.9\tilde{C}_{Ht} + 5.8\tilde{C}_{Hq}^{(1)} + 3.1\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{qtqb}^{(1)} - 3.1\tilde{C}_{tH} + 2.7\tilde{C}_{HW} \\
& + 2.4\tilde{C}_{H} - 1.9\frac{\bar{v}^{(\ell)}}{\overline{e}^{(\ell)}\,\overline{m}_b^{(\ell)}}\tilde{C}_{bW} - 1.3\tilde{C}_{qb}^{(8)} - 1.3\frac{\tilde{C}_{tW}}{\overline{e}^{(\ell)}} - 1.0\tilde{C}_{qb}^{(1)}\Bigg) \times 10^{-2} \\
& + \Bigg(- 9\left[\frac{\tilde{C}_{tB}}{\overline{e}^{(\ell)}} + \left(\tilde{C}_{Hq}^{(3)}\right)_{22} + \left(\tilde{C}_{Hq}^{(3)}\right)_{11} - \tilde{C}_{HB} + \tilde{C}_{Hu} + \tilde{C}_{Hc}\right] - 8\frac{\bar{v}^{(\ell)}}{g_s\overline{m}_b^{(\ell)}}\tilde{C}_{bG} - 7\tilde{C}_{W} \\
& + 6\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{qtqb}^{(8)} + 4\Bigg[\tilde{C}_{Hl}^{(1)} + \left(\tilde{C}_{Hl}^{(1)} - \tilde{C}_{Hq}^{(1)}\right)_{22} + \left(\tilde{C}_{Hl}^{(1)} - \tilde{C}_{Hq}^{(1)}\right)_{11} + \tilde{C}_{H\tau} + \tilde{C}_{H\mu} + \tilde{C}_{He} \\
& + \tilde{C}_{Hs} + \tilde{C}_{Hd} - \frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{Htb}\Bigg] - 3\left[\tilde{C}_{Hl}^{(3)} + \left(\tilde{C}_{Hl}^{(3)}\right)_{22} + \left(\tilde{C}_{Hl}^{(3)}\right)_{11}\right] + 2\tilde{C}_{Hb}\Bigg) \times 10^{-3} \\
& - 4 \times 10^{-5}\frac{\bar{v}^{(\ell)}}{\overline{e}^{(\ell)}\,\overline{m}_b^{(\ell)}}\tilde{C}_{bB}\Bigg\}\,.
\end{aligned}
$$

Pecjak et al 1904.06358

Other examples in the literature

- Charge renormalisation — This paper and Pecjak ‘19
- Z to ff — Dawson ‘18
- Z & W poles — Dawson, Giardino ‘19
- H to Z ff — Buchalla ‘13
- H to VV — This paper and Dawson ‘18
- And others…. — But not that many!

FR-PHENO-2016-004

Low energy behaviour of standard model extensions

Michele Boggia[a,1], Raquel Gomez-Ambrosio[b,1], Giampiero Passarino[b,1]

[a] *Albert-Ludwigs-Universität Freiburg, Physikalisches Institut*
D-79104, Freiburg, Germany

[b] *Dipartimento di Fisica Teorica, Università di Torino, Italy*
INFN, Sezione di Torino, Italy

Study of SESM and THDM
in SMEFT (mass eigenbasis)

Abstract

The integration of heavy scalar fields is discussed in a class of BSM models, containing more that one representation for scalars and with mixing. The interplay between integrating out heavy scalars and the Standard Model decoupling limit is examined. In general, the latter cannot be obtained in terms of only one large scale and can only be achieved by imposing further assumptions on the couplings. Systematic low-energy expansions are derived in the more general, non-decoupling scenario, including mixed tree-loop and mixed heavy-light generated operators. The number of local operators is larger than the one usually reported in the literature.

Keywords: Perturbative calculations, Extensions of electroweak Higgs sector, Effective Field Theory
PACS: 12.38.Bx, 12.60.Fr, 14.80.Ec, 12.60.-i

Raquel

NLO-EFT #0 : Tadpoles

$$\boxed{\overline{\beta}_{\mathrm{h}} = i\, g^2 M_{\mathrm{W}}^2 \left(\overline{\beta}_{\mathrm{h}}^{(4)} + g_6\, \overline{\beta}_{\mathrm{h}}^{(6)} \right) ,} \qquad \overline{\beta}_{\mathrm{h}}^{(n)} = \beta_{-1}^{(n)}\, \Delta_{\mathrm{UV}} \left(M_{\mathrm{W}}^2 \right) + \beta_0^{(n)} + \beta_{\mathrm{fin}}^{(n)} .$$

$$\beta_{-1}^{(4)} = - \sum_{\mathrm{gen}} \left(x_{\mathrm{l}}^2 + 3\, x_{\mathrm{d}}^2 + 3\, x_{\mathrm{u}}^2 \right) + \frac{1}{8} \left(12 + 2\, x_{\mathrm{H}} + 3\, x_{\mathrm{H}}^2 \right) + \frac{1}{8}\, \frac{6 + \mathrm{c}_\theta^2\, x_{\mathrm{H}}}{\mathrm{c}_\theta^4}$$

$$\beta_0^{(4)} = - \frac{1}{2}\, \frac{1 + 2\, \mathrm{c}_\theta^4}{\mathrm{c}_\theta^4}$$

$$\beta_{\mathrm{fin}}^{(4)} = - \frac{1}{4}\, a_0^{\mathrm{fin}} \left(M_{\mathrm{W}} \right) \left(6 + x_{\mathrm{H}} \right) - \frac{3}{8}\, a_0^{\mathrm{fin}} \left(M_{\mathrm{H}} \right) x_{\mathrm{H}}^2 - \frac{1}{8}\, \frac{6 + \mathrm{c}_\theta^2\, x_{\mathrm{H}}}{\mathrm{c}_\theta^4}\, a_0^{\mathrm{fin}} \left(M_{\mathrm{Z}} \right)$$
$$+ \sum_{\mathrm{gen}} \left[3\, a_0^{\mathrm{fin}} \left(M_{\mathrm{u}} \right) x_{\mathrm{u}}^2 + 3\, a_0^{\mathrm{fin}} \left(M_{\mathrm{d}} \right) x_{\mathrm{d}}^2 + a_0^{\mathrm{fin}} \left(M_{\mathrm{l}} \right) x_{\mathrm{l}}^2 \right]$$

Backup

NLO-EFT #0 : Tadpoles

$$
\begin{aligned}
\beta^{(6)}_{-1} = {}& \frac{1}{8}\,\frac{12+x_{\mathrm{H}}}{c^2_\theta}\,a_{\phi\,\mathrm{W}} + \frac{1}{8}\,(36+2x_{\mathrm{H}}+3x^2_{\mathrm{H}})\,a_{\phi\,\mathrm{W}} - \frac{1}{4}\sum_{\mathrm{gen}}\Big[3\,(4\,a_{\mathrm{u}\,\phi}+4\,a_{\phi\,\Box}+4\,a_{\phi\,\mathrm{W}}-a_{\phi\,\mathrm{D}})\,x^2_{\mathrm{u}} \\
& - 3\,(4\,a_{\mathrm{d}\,\phi}-4\,a_{\phi\,\Box}-4\,a_{\phi\,\mathrm{W}}+a_{\phi\,\mathrm{D}})\,x^2_{\mathrm{d}} + (4\,a_{\phi\,\Box}+4\,a_{\phi\,\mathrm{W}}-a_{\phi\,\mathrm{D}}-4\,a_{\mathrm{L}\,\phi})\,x^2_{\mathrm{l}}\Big] \\
& + \frac{3}{16}\,(4\,a_{\phi\,\Box}+4\,a_{\phi\,\mathrm{W}}+a_{\phi\,\mathrm{D}}+8\,s^2_\theta\,a_{\phi\,\mathrm{B}}-8\,c_\theta\,s_\theta\,a_{\phi\,\mathrm{WB}})\,\frac{1}{c^4_\theta} + \frac{1}{32}\,(4\,a_{\phi\,\Box}-a_{\phi\,\mathrm{D}})\,(12-2x_{\mathrm{H}}+7x^2_{\mathrm{H}}) \\
& - \frac{1}{32}\,(4\,a_{\phi\,\Box}+a_{\phi\,\mathrm{D}}+96\,c^2_\theta\,a_\phi)\,\frac{x_{\mathrm{H}}}{c^2_\theta}
\end{aligned}
$$

$$
\beta^{(6)}_{0} = -\frac{1}{8}\Big[8\,s^2_\theta\,a_{\phi\,\mathrm{B}}+4\,(1+2c^4_\theta)\,a_{\phi\,\Box}+4\,(1+2c^2_\theta+6c^4_\theta)\,a_{\phi\,\mathrm{W}}+(1-2c^4_\theta)\,a_{\phi\,\mathrm{D}}-8\,c_\theta\,s_\theta\,a_{\phi\,\mathrm{WB}}\Big]\,\frac{1}{c^4_\theta}
$$

$$
\begin{aligned}
\beta^{(6)}_{\mathrm{fin}} = {}& 3\,a^{\mathrm{fin}}_0\left(M_{\mathrm{H}}\right)\,a_\phi\,x_{\mathrm{H}} - \frac{1}{4}\,a^{\mathrm{fin}}_0\left(M_{\mathrm{W}}\right)\,(18+x_{\mathrm{H}})\,a_{\phi\,\mathrm{W}} - \frac{1}{8}\,a^{\mathrm{fin}}_0\left(M_{\mathrm{Z}}\right)\,\frac{12+x_{\mathrm{H}}}{c^2_\theta}\,a_{\phi\,\mathrm{W}} \\
& + \frac{1}{4}\sum_{\mathrm{gen}}\Big[3\,(4\,a_{\mathrm{u}\,\phi}+4\,a_{\phi\,\Box}+4\,a_{\phi\,\mathrm{W}}-a_{\phi\,\mathrm{D}})\,a^{\mathrm{fin}}_0\left(M_{\mathrm{u}}\right)\,x^2_{\mathrm{u}} - 3\,(4\,a_{\mathrm{d}\,\phi}-4\,a_{\phi\,\Box}-4\,a_{\phi\,\mathrm{W}}+a_{\phi\,\mathrm{D}})\,a^{\mathrm{fin}}_0\left(M_{\mathrm{d}}\right)\,x^2_{\mathrm{d}} \\
& + (4\,a_{\phi\,\Box}+4\,a_{\phi\,\mathrm{W}}-a_{\phi\,\mathrm{D}}-4\,a_{\mathrm{L}\,\phi})\,a^{\mathrm{fin}}_0\left(M_{\mathrm{l}}\right)\,x^2_{\mathrm{l}}\Big] \\
& - \frac{3}{16}\,(4\,a_{\phi\,\Box}+4\,a_{\phi\,\mathrm{W}}+a_{\phi\,\mathrm{D}}+8\,s^2_\theta\,a_{\phi\,\mathrm{B}}-8\,c_\theta\,s_\theta\,a_{\phi\,\mathrm{WB}})\,\frac{1}{c^4_\theta}\,a^{\mathrm{fin}}_0\left(M_{\mathrm{Z}}\right) - \frac{1}{16}\,(4\,a_{\phi\,\Box}-a_{\phi\,\mathrm{D}})\,a^{\mathrm{fin}}_0\left(M_{\mathrm{W}}\right)\,(6-x_{\mathrm{H}}) \\
& + \frac{1}{32}\,(4\,a_{\phi\,\Box}+a_{\phi\,\mathrm{D}})\,a^{\mathrm{fin}}_0\left(M_{\mathrm{Z}}\right)\,\frac{x_{\mathrm{H}}}{c^2_\theta} - \frac{1}{32}\,(28\,a_{\phi\,\Box}+12\,a_{\phi\,\mathrm{W}}-7\,a_{\phi\,\mathrm{D}})\,a^{\mathrm{fin}}_0\left(M_{\mathrm{H}}\right)\,x^2_{\mathrm{H}}
\end{aligned}
$$

Backup

Example: H to AA

$$\mathrm{A}^{\mu\nu}_{\mathrm{HAA}} = \mathcal{T}_{\mathrm{HAA}}\, T^{\mu\nu}\,, \quad M^2_{\mathrm{H}}\, T^{\mu\nu} = p^{\mu}_2\, p^{\nu}_1 - p_1 \cdot p_2\, \delta^{\mu\nu}$$

Define subamplitudes

$$\frac{3}{8}\,\frac{M_{\mathrm{W}}}{M^2_{\mathrm{t}}}\,\mathcal{T}^{\mathrm{t}}_{\mathrm{HAA\,;LO}} = 2 + \left(M^2_{\mathrm{H}} - 4\,M^2_{\mathrm{t}}\right)\,\mathrm{C}_0\left(-M^2_{\mathrm{H}}\,,0\,,0\,;M_{\mathrm{t}}\,,M_{\mathrm{t}}\,,M_{\mathrm{t}}\right)\,,$$

$$\frac{9}{2}\,\frac{M_{\mathrm{W}}}{M^2_{\mathrm{b}}}\,\mathcal{T}^{\mathrm{b}}_{\mathrm{HAA\,;LO}} = 2 + \left(M^2_{\mathrm{H}} - 4\,M^2_{\mathrm{b}}\right)\,\mathrm{C}_0\left(-M^2_{\mathrm{H}}\,,0\,,0\,;M_{\mathrm{b}}\,,M_{\mathrm{b}}\,,M_{\mathrm{b}}\right)\,,$$

$$\frac{1}{M_{\mathrm{W}}}\,\mathcal{T}^{\mathrm{W}}_{\mathrm{HAA\,;LO}} = -6 - 6\,\left(M^2_{\mathrm{H}} - 2\,M^2_{\mathrm{W}}\right)\,\mathrm{C}_0\left(-M^2_{\mathrm{H}}\,,0\,,0\,;M_{\mathrm{W}}\,,M_{\mathrm{W}}\,,M_{\mathrm{W}}\right)$$

$$\boxed{\mathcal{T}_{\mathrm{HAA}} = i\,\frac{g^3}{16\,\pi^2}\,\left(\mathcal{T}^{(4)}_{\mathrm{HAA}} + g_6\,\mathcal{T}^{(6),b}_{\mathrm{HAA}}\right) + i\,g\,g_6\,\mathcal{T}^{(6),a}_{\mathrm{HAA}}}$$

Backup

Example: H to AA

Dim 6 part

$$\mathscr{T}_{\mathrm{HAA}} = i\,\frac{g^3}{16\,\pi^2}\,\Big(\mathscr{T}^{(4)}_{\mathrm{HAA}} + g_6\,\mathscr{T}^{(6),b}_{\mathrm{HAA}}\Big) + i\,g\,g_6\,\mathscr{T}^{(6),a}_{\mathrm{HAA}}$$

$$\mathscr{T}^{(6),a}_{\mathrm{HAA}} = 2\,\frac{M^2_{\mathrm{H}}}{M}\,\left(\mathrm{s}^2_\theta\,a_{\phi\,\mathrm{W}} + \mathrm{c}^2_\theta\,a_{\phi\,\mathrm{B}} + \mathrm{s}_\theta\,\mathrm{c}_\theta\,a_{\phi\,\mathrm{WB}}\right)$$

$$\mathscr{T}^{\mathrm{ren}}_{\mathrm{HAA}} = \mathscr{T}_{\mathrm{HAA}}\left[1 + \frac{g^2}{16\,\pi^2}\left(\mathrm{dZ_A} + \frac{1}{2}\,\mathrm{dZ_H} + 3\,\mathrm{dZ}_g\right)\Delta_{\mathrm{UV}}\right],$$

$$\mathrm{c}_\theta = \mathrm{c}^{\mathrm{ren}}_\theta\left(1 + \frac{g^2}{16\,\pi^2}\,\mathrm{dZ}_{\mathrm{c}_\theta}\,\Delta_{\mathrm{UV}}\right), \qquad g = g_{\mathrm{ren}}\left(1 + \frac{g^2_{\mathrm{ren}}}{16\,\pi^2}\,\mathrm{dZ}_g\,\Delta_{\mathrm{UV}}\right)$$

$$\mathscr{T}^{\mathrm{ren}}_{\mathrm{HAA}} = i\,\frac{g^3_{\mathrm{ren}}}{16\,\pi^2}\left(\mathscr{T}^{(4)}_{\mathrm{HAA}} + g_6\,\mathscr{T}^{(6),b}_{\mathrm{HAA}\,;\,\mathrm{fin}}\right) + i\,g_{\mathrm{ren}}\,g_6\,\mathscr{T}^{(6),a}_{\mathrm{HAA}\,;\,\mathrm{ren}} + i\,\frac{g^3_{\mathrm{ren}}}{16\,\pi^2}\,g_6\,\mathscr{T}^{(6)}_{\mathrm{HAA}\,;\,div},$$

$$\mathscr{T}^{(6)}_{\mathrm{HAA}\,;\,div} = \mathscr{T}^{(6),b}_{\mathrm{HAA}\,;\,\mathrm{div}}\,\Delta_{\mathrm{UV}}\left(M^2_{\mathrm{W}}\right) + \frac{M^2_{\mathrm{H\,ren}}}{M_{\mathrm{ren}}}\left\{\left[\mathrm{dZ}^{(4)}_{\mathrm{H}} - \mathrm{dZ}^{(4)}_{M_{\mathrm{W}}} + 2\,\mathrm{dZ}^{(4)}_{\mathrm{A}} - 2\,\mathrm{dZ}^{(4)}_{g}\right] a_{\mathrm{AA}} - 2\,\frac{\mathrm{c}^{\mathrm{ren}}_\theta}{s^{\mathrm{ren}}_\theta}\,\mathrm{dZ}^{(4)}_{\mathrm{c}_\theta}\,a_{\mathrm{AZ}}\right\}\Delta_{\mathrm{UV}}$$

$$\mathscr{T}^{(6),a}_{\mathrm{HAA}\,;\,\mathrm{ren}} = 2\,\frac{M^2_{\mathrm{H\,ren}}}{M_{\mathrm{ren}}}\,a_{\mathrm{AA}},$$

Backup